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Research Article

Reserve estimation using paid and incurred claims information

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ABSTRACT

Insurance companies need to estimate accurately the possible future claims payments and thus allocate sufficient reserves to avoid financial difficulties. Reserve estimates are usually based on historical data from various sources of information. In classical reserve estimation methods, the reserve estimate is based on either paid claims information only or incurred claims information. Since all claims are eventually settled, in theory, the ultimate claim estimates obtained using paid or incurred claims data are expected to become equal. In practice, however, the ultimate estimates obtained using these two sources of information generally differ. Therefore, methods have been developed that use both sources of information to obtain the same or similar estimates. Munich Chain Ladder, Extended Complementary Loss Ratio and Paid-Incurred Chain methods are among the widely used ones. In the Turkish insurance sector, reserves are estimated using the Chain Ladder method and the Munich method. This study aims to investigate the alternative method for estimating reserves for Turkish Highways Motor Vehicles Compulsory Liability Insurance. Reserves are estimated using all these methods. Based on mean squared error, it is concluded that the Paid Incurred Chain method can be an alternative.

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INTRODUCTION

The essential purpose of insurance companies is to cover the claims and costs of losses and to make profits with the premiums they receive for the risks they cover. Claim reserve expresses the insurance companies' claims that have occurred during the activity period but have not yet been paid and show their obligations to the insureds. If the claim reserve is not determined correctly, companies may face failure to fulfill their obligations to their insureds, loss of capital, and even bankruptcy.

Reserve estimates are usually based on historical data from different sources of information. In most methods, a reserve estimate is obtained based on a single source of information, namely either paid or incurred claims amount. However, better estimates can be obtained as the information about the claim increases, so reserve estimation methods using both information have been developed.

Reserve estimation using both paid and incurred claims data was first discussed by Halliwell [1]. Quarg-Mack [2] developed the Munich Chain Ladder method (MCL) to reduce the difference between reserves obtained using

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paid and incurred claim data with the Chain Ladder (CL) method. In this method, the CL development factor is rearranged by using the paid/incurred claim ratio. However, this method still yields two different reserve estimates. In subsequent studies, it was tried to reach a single reserve estimate by using both pieces of information together. Halliwell [3] made a single reserve calculation using the linear regression model based on the paid and incurred claims information. Dahms [4] extended the Complementary Loss Ratio method by taking the outstanding claims reserve at the end of the previous development year as a measure of risk and using both sources of information together. Posthuma et al. [5] created the claim reserve model using both pieces of information together, assuming that the claims paid and incurred follow a multivariate normal distribution. Wüthrich and Merz [6] developed the Paid-Incurred Chain (PIC) method using Hertig's [7] lognormal chain ladder for paid claims, and Gogol's [8] Bayesian approach for incurred claims by assuming that the ultimate paid and incurred claims are equal. The tail development factors in the Paid-Incurred Chain method were developed by Merz and Wüthrich [9], and the modeling of dependency structure was analyzed by Happ and Wüthrich [10] and Peters et al. [11]. The claims development results were examined by Happ and Wüthrich [12].

Pigeon et al. [13] developed the Individual Paid-Incurred Chain claim reserving method by modifying the Paid-Incurred Chain method for individual claim data. Antonio and Plat [14] estimated the reserve by considering the incurred claims data since they obtained the claim severity using the initial outstanding claims information. Heberle [15] developed an alternative method based on paid and incurred claim data using the Kalman-filter theory. Dupin et al. [16] developed a semi-parametric method considering both paid and incurred claims.

In addition to these studies in which reserve is estimated using paid and incurred claims data, there are also studies in which reserve is estimated using paid claims and claims numbers. The main ones among these studies are Verrall et al. [17] and Martinez et al. [18].

The outline of this paper is as follows. Section 2 involves information about the claims development triangle. Reserve estimation methods based on paid and incurred claim data are given in Section 3, and the mean squared errors of these methods are given in Section 4. Reserve estimations are obtained using the paid and outstanding claims data between the years 2010-2016 for the Highways Motor Vehicles Compulsory Liability Insurance taken from the Republic of Türkiye Ministry of Treasury and Finance. The results are presented in Section 5. The findings are summarized in Section 6.

Claims Development Triangle

Claims data are classified based on the accident year, and payment delays since property insurance often delay reporting and closing claims. That is, claims data are summarized using the table in Figure 1, which shows the change in claims over time, called the claims development triangle.

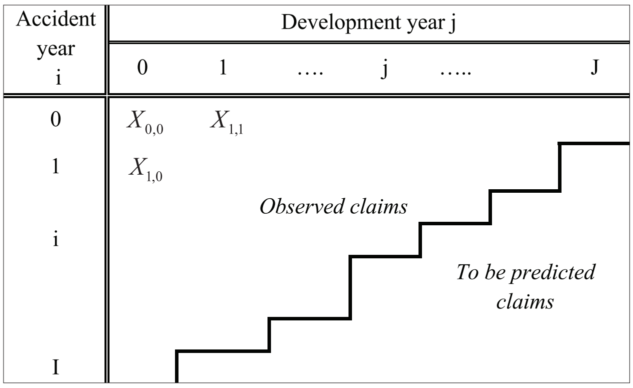


Figure 1: Claims Development Triangle

i is called the accident year, which gives the accident time as $0 \le i \le I$, and *j* is called the development year, which gives the delay in claims payment as $0 \le j \le J$, where *I* is the ultimate accident year, and *J* is the ultimate development year.

The claims development triangle can consist of paid or incurred claims. Claims payments are claims that include only payments made without regard to possible future payments for claims incurred and are entirely objective[19]. Let $X_{i,j}^{P}$ be incremental claims payment. This, $C_{i,j}^{P} = \sum_{i=0}^{j} X_{i,j}^{P}$ demonstrates the cumulative claim payments.

As incurred claims include outstanding claims in addition to paid claims, they are equal to or greater than paid claims and therefore are subjective [19]. Let $X_{i,j}^{I}$ be incremental claims incurred. This, $C_{i,j}^{I} = \sum_{i=0}^{j} X_{i,j}^{I}$ demonstrates the cumulative claims incurred.

Information on all development years for claims payment, incurred claims, and both combined are given as follows respectively:

$$D_J^P = \sigma\left\{C_{i,j}^P; i+j \le J\right\},\ D_J^I = \sigma\left\{C_{i,j}^I; i+j \le J\right\},$$
$$D_J = \sigma\left\{C_{i,j}^P, C_{i,j}^I; i+j \le J\right\}.$$

Information up to any development year *j* for claims payment, incurred claims, and both combined are given as follows respectively:

$$B_j^P = \sigma\left\{C_{i,l}^P; l \le j\right\},\ B_j^I = \sigma\left\{C_{i,l}^I; l \le j\right\},$$
$$B_j = \sigma\left\{C_{i,l}^P, C_{i,l}^I; l \le j\right\}.$$

Reserve Estimation Methods Based on Paid and Incurred Claims

Reserve estimates are based on historical data from different information sources. In most traditional reserve estimation methods, reserve estimates are obtained using either paid or incurred claims. Theoretically, the ultimate claims estimates using either paid or incurred claims are expected to be the same since it is expected that all claims incurred at the end of the development year for each accident year will be paid. In practice, however, the reserve estimates are generally different because the ultimate claims estimates obtained from these two sources of information are quite different. The goal of reserve estimation models that use these two sources of information together is to reduce or eliminate this difference.

Mack's Chain Ladder Method

Mack [20] determined the variance of the claims reserve and hence the confidence interval by expressing the CL method as stochastic without any distribution assumption. The basic assumption in this method is that the development factor f_j from the j^{th} development year to the $j+1^{\text{th}}$ development year is the same for all accident years. The expected value and variance of cumulative paid and incurred claims are as follows:

$$E(C^P_{i,J} \mid D^P_J) = C^P_{i,J-i} \prod_{j=J-i}^{J-1} f^P_j \quad \text{and} \quad E(C^I_{i,J} \mid D^I_J) = C^I_{i,J-i} \prod_{j=J-i}^{J-1} f^I_j, \tag{1}$$

$$Var(C^P_{i,J} \mid D^P_J) = C^P_{i,J-i} \prod_{j=J-i}^{J-1} (\sigma^P_j)^2 \quad \text{and} \quad Var(C^I_{i,J} \mid D^I_J) = C^I_{i,J-i} \prod_{j=J-i}^{J-1} (\sigma^I_j)^2 \tag{2}$$

The parameters in equation (1) are given by:

$$\hat{f}^P_j = \frac{\sum_{i=0}^{J-j-1} C^P_{i,j+1}}{\sum_{i=0}^{J-j-1} C^P_{i,j}} \quad \text{and} \quad \hat{f}^I_j = \frac{\sum_{i=0}^{J-j-1} C^I_{i,j+1}}{\sum_{i=0}^{J-j-1} C^I_{i,j}} \quad 0 \le j \le J-1. \tag{3}$$

The parameters σ^2_j in equation (2) are given by[20]:

$$\begin{aligned} (\hat{\sigma}^P_j)^2 &= \frac{1}{J-j-1} \sum_{i=0}^{J-j-1} C^P_{i,j} \left(\frac{C^P_{i,j+1}}{C^P_{i,j}} - \hat{f}^P_j\right)^2 \\ (\hat{\sigma}^I_j)^2 &= \frac{1}{J-j-1} \sum_{i=0}^{J-j-1} C^I_{i,j} \left(\frac{C^I_{i,j+1}}{C^I_{i,j}} - \hat{f}^I_j\right)^2 \end{aligned} \;,\; 0 \le j \le J-2, \\ (\hat{\sigma}_J)^2 = \min(\sigma^4_{J-2}/\sigma^2_{J-3}, \sigma^2_{J-3}, \sigma^2_{J-2}), \quad J-1. \tag{4}$$

Munich Chain Ladder Method

The Munich Chain Ladder (MCL) is a method of estimating reserves based on both paid and incurred claims. The MCL method has the same basic structure as Mack's distribution-free CL model, but unlike the CL method, it considers the dependence between paid and incurred claims [21].

Paid/incurred ratio showing the relationship between the paid and incurred claims for the i^{th} accident and j^{th} development year is obtained as follows [2].

$$Q_{i,j} = \frac{C^P_{i,j}}{C^I_{i,j}}, \tag{5}$$

The expected value of cumulative paid and incurred claims are as follows:

$$E(C^P_{i,J} \mid D_J) = C^P_{i,J-i} \prod_{j=J-i}^{J-1} \left(f^P_{i,j}\right)^{MZM}, \quad \text{and} \quad E(C^I_{i,J} \mid D_J) = C^I_{i,J-i} \prod_{j=J-i}^{J-1} \left(f^I_{i,j}\right)^{MZM}. \tag{6}$$

Let λ^P and λ^I be the correlation factors showing the relationship between the claims development triangles consisting of paid and incurred claims. Then the development factors in equation (6) are:

$$\left(f^P_{i,j}\right)^{MZM} = f^P_j + \underbrace{\lambda^P \frac{\sigma\left(f^P_{i,j} \mid B^P_j\right)}{\sigma\left(Q^{-1}_{i,j} \mid B^P_j\right)}\left(Q^{-1}_{i,j} - q^{-1}_j\right)}_{\text{correction term}},$$

and

$$\left(f^I_{i,j}\right)^{MZM} = f^I_j + \underbrace{\lambda^I \frac{\sigma\left(f^I_{i,j} \mid B^I_j\right)}{\sigma\left(Q_{i,j} \mid B^I_j\right)}\left(Q_{i,j} - q_j\right)}_{\text{correction term}}.$$

Here, it is seen that the claims development factor in the MCL method is obtained by adding the correction terms to the claims development factors in Mack's CL method given in equation (3).

Extended Complementary Loss Ratio Method

Extended Complementary Loss Ratio (ECLR) developed by Dahms [4] differs from the regression model of

Mack's CL method. It consists of a regression model that uses case reserves (outstanding) instead of cumulative claims.

Let $R_{i,j} = R_{i,j+1} + X^I_{i,j} - X^P_{i,j}$ be the random variable representing the case reserves (outstanding) at the end of the i^{th} accident and j^{th} development year. Only one reserve estimate is made for each accident year, assuming that case reserves are equal to zero in an ultimate year, i.e. $R_{i,J} = 0$. The expected value of the incremental paid and incurred claims are

$$E(X^P_{i,j+1} \mid B_j) = \alpha_j R_{i,j} \quad \text{and} \quad E(X^I_{i,j+1} \mid B_j) = \beta_j R_{i,j},$$

$$\text{where} \quad \hat{\alpha}_j = \frac{\sum_{i=0}^{J-j-1} X^P_{i,j+1}}{\sum_{i=0}^{J-j-1} R_{i,j}} \quad \text{and} \quad \hat{\beta}_j = \frac{\sum_{i=0}^{J-j-1} X^I_{i,j+1}}{\sum_{i=0}^{J-j-1} R_{i,j}}. \tag{7}$$

Assuming that case reserves are a measure of risk for incremental claims paid and incurred, the expected value of case reserves is recursively calculated as:

$$E(R_{i,j+1} \mid B_j) = (1 - \alpha_j + \beta_j) R_{i,j} = f_j R_{i,j}. \tag{8}$$

Paid-Incurred Chain Method

In this method, where reserve estimation is performed using both paid and incurred claims, Hertig's [7] log-normal CL claims reserve model is used for paid claims, and Gogol's [8] Bayesian claim reserve model is used for incurred claims. The basic assumption of the method is that the ultimate paid and incurred claims are equal to each other, i.e. $C^P_{i,J} = C^I_{i,J}$ [6]. This leads to only one reserve estimate. Another assumption is that the development factors obtained from the paid and incurred claims triangles are log-normally distributed:

$$\xi_{i,j} = \log\left(\frac{C^P_{i,j}}{C^P_{i,j-1}}\right) \sim N(\Phi_j, \sigma_j^2),$$

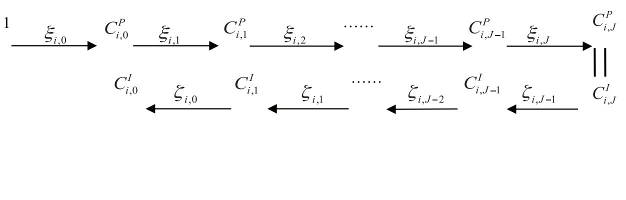
$$i \in \{0,1,\ldots,J\} \text{ and } j \in \{0,1,\ldots,J\},$$

$$\zeta_{i,j} = -\log\left(\frac{C^I_{i,j-1}}{C^I_{i,j}}\right) \sim N(\Psi_l, \tau_l^2),$$

$$k \in \{0,1,\ldots,J\} \text{ and } l \in \{0,1,\ldots,J-1\}.$$

Here, $\xi_{0,0},\ldots,\xi_{J,J},\zeta_{0,0},\ldots,\zeta_{J,J-1}$ have a multivariate normal distribution.

The basic working structure of the model is as follows:



Distribution of $\log C^P_{i,J}$, given $\{B_j, \Theta\}$

$$\log C^P_{i,J} \mid_{\{B_j,\Theta\}} \sim N\Big(\mu_J + (1-\beta_j)(\log C^P_{i,j} - \eta_j) + \beta_j(\log C^I_{i,j} - \mu_j), (1-\beta_j)(v_J^2 - w_j^2)\Big),$$

and the expected value of cumulative paid claims is

$$E\left[C^P_{i,J} \mid B_j, \Theta\right] = \exp\Big\{\mu_J + (1-\beta_j)(\log C^P_{i,j} - \eta_j) + \beta_j(\log C^I_{i,j} - \mu_j) + (1-\beta_j)(v_j^2 - w_j^2)/2\Big\} \tag{9}$$

where the parameters are

$$\mu_j = \sum_{m=0}^{J} \Phi_m - \sum_{n=j}^{J-1} \Psi_n, \quad \eta_j = \sum_{m=0}^{j} \Phi_m.$$

$$v_j^2 = \sum_{m=0}^{J} \sigma_m^2 + \sum_{n=j}^{J-1} \tau_n^2, \quad \omega_j = \sum_{m=0}^{j} \sigma_m^2.$$

and the credibility weight is $\beta_j = \dfrac{v_J^2 - \omega_j^2}{v_j^2 - \omega_j^2} > 0$.

Assuming that the variance parameters σ_j^2 and τ_j^2 in the parameter vector $\Theta = (\Phi_0,\ldots,\Phi_J,\Psi_0,\ldots,\Psi_{J-1},\ldots,\sigma_0,\ldots\sigma_J,\tau_0,\ldots,\tau_{J-1})$ in equation (9) are known, the mean parameters can be obtained using the Bayesian approach. The assumption that the variance parameters are known means that the posterior distributions can be calculated analytically.

The expected value of the cumulative paid claims is determined as

$$E\left[C^P_{i,J} \mid D_J\right] = \left(C^P_{i,J-i}\right)^{1-\beta_{J-i}} \left(C^I_{i,J-i}\right)^{1-\beta_{J-i}} \exp\left\{(1-\beta_{J-i}) \sum_{l=J-i+1}^{J} \phi_l^{post} + \beta_{J-i} \sum_{l=J-i}^{J-1} \psi_l^{post}\right\} \times \exp\left\{(1-\beta_{J-i}) \frac{v_J^2 - \omega_{J-i}^2}{2} + \frac{\left(s_i^{post}\right)^2}{2}\right\}, \tag{10}$$

assuming that the a priori distributions of the mean parameters are

$$\Phi_m \sim N\left(\phi_m, s_m^2\right), \quad m \in \{0,\ldots,J\} \tag{11}$$

$$\Psi \sim N\left(\psi_n, t_n^2\right), \quad n \in \{0,\ldots,J-1\} \tag{12}$$

The interested reader can consult Merz and Wüthrich [6] for information on the derivation of ϕ_l^{post}, ψ_l^{post} and s_i^{post} in equation (10).

Reserve Estimation Uncertainty

Mean squared error (MSE), a widely used risk measure in actuarial science, is used in selecting the method that best fits the data, in other words, that gives the best reserve estimate among the models used. The mean squared error of the reserve estimate for the methods mentioned in section 3 is given here.

Mean Squared Error for Mack's Chain Ladders Method

Mean squared errors of the overall reserve estimates calculated based on cumulative paid claims and cumulative incurred claims are as follows:

$$mse\left(\hat{R}^P\right)=\sum_{i=1}^{J} mse\left(\hat{R}_i^P\right)+C_{i,J}^P\left(\sum_{t=i+1}^{J}\hat{C}_{t,J}^P\right)\sum_{k=J-i}^{J-1}\frac{2\left(\hat{\sigma}_k^P\right)^2\Big/\left(\hat{f}_k^P\right)^2}{\sum_{n=1}^{J-k-1}C_{n,k}^P},$$

$$mse\left(\hat{R}^I\right)=\sum_{i=1}^{J} mse\left(\hat{R}_i^I\right)+C_{i,J}^I\left(\sum_{t=i+1}^{J}\hat{C}_{t,J}^I\right)\sum_{k=J-i}^{J-1}\frac{2\left(\hat{\sigma}_k^I\right)^2\Big/\left(\hat{f}_k^I\right)^2}{\sum_{n=1}^{J-k-1}C_{n,k}^I}.$$

Mean Squared Error for Extended Complementary Loss Ratio

Mean squared error of the overall reserve estimation is as follows:

$$mse(\hat{R})=\sum_{i=1}^{I} mse(\hat{R}_i)+2\sum_{1\le i_1<i_2\le I}\ \sum_{k_1,k_2=I-i+1}^{J}\hat{X}_{i_1,k_1}^P\hat{X}_{i_2,k_2}^P\left[\sum_{l=J-i_1+1}^{k_1\wedge k_2-1}\frac{\hat{a}_{k_1,k_2,l}}{\sum_{j=0}^{J-k-1}R_{j,l}}\right],$$

$$\text{where}\quad \hat{a}_{k_1,k_2,l}=\begin{cases}\dfrac{\hat{\sigma}_l^2}{\hat{\alpha}_l^2}, & k_1=k_2=l+1,\\[2mm] \dfrac{\hat{\gamma}_l-\hat{\sigma}_l^2}{\hat{\alpha}_l\hat{f}_l}, & k_1>k_2=l+1\quad or\quad k_2>k_1=l+1,\\[2mm] \dfrac{\hat{\sigma}_l^2-2\hat{\gamma}_l+\hat{\tau}_l^2}{\hat{f}_l^2}, & k_1\ge k_2>l+1\quad or\quad k_2\ge k_1>l+1.\end{cases}$$

and $\hat{\sigma}_k^2, \hat{\tau}_k^2, \hat{\gamma}_k^2$ are respectively ,

$$\hat{\sigma}_k^2=\sum_{i=0}^{J-k+1}R_{i,k}\left(\frac{X_{i,k+1}^P}{R_{i,k}}-\hat{\alpha}_k\right)^2,\qquad 1\le k<n-1,$$

$$\hat{\tau}_k^2=\sum_{i=0}^{J-k+1}R_{i,k}\left(\frac{X_{i,k+1}^I}{R_{i,k}}-\hat{\beta}_k\right)^2,\qquad 1\le k<n-1,$$

$$\hat{\gamma}_k^2=\sum_{i=0}^{J-k+1}R_{i,k}\left(\frac{X_{i,k+1}^P}{R_{i,k}}-\hat{\alpha}_k\right)\left(\frac{X_{i,k+1}^I}{R_{i,k}}-\hat{\beta}_k\right),\quad 0\le k<J-1.$$

Mean Squared Error for Paid-Incured Chain Claims Reserving Method

Mean squared error of the overall reserve estimation is as follows:

$$mse(R\mid D_J)=\sum_{1\le k_1,k_2\le J}^{J}\Bigg(\exp\Big(\left(1-\beta_{J-k_1}\right)\left(\nu_J^2-\omega_{J-k_1}^2\right)1_{\{k_1=k_2\}}+\left(e_{k_1}\right)'\Sigma\left(D_J\right)e_{k_2}\Big)-1\Bigg)E\left[P_{k_1,J}\mid D_J\right]E\left[P_{k_2,J}\mid D_J\right]$$

APPLICATION

In this section, the reserve is estimated for compulsory motor insurance in Turkey with the methods in Section 3. Data consists of paid and outstanding claims obtained from the Ministry of Treasury and Finance of the Republic of Turkey for the years 2010-2016. Outstanding claims are claims that have been reported to the insurer but are still in the settlement phase. Incurred claims are determined by adding paid claims to outstanding claims. The data are presented in Appendix. The similarities and differences between the methods used are summarized in Table 1.

Table 2 involves the reserve estimates obtained using Mack's Distribution-Free Chain Ladder, Münich Chain Ladder, Extended Complementary Loss Ratio and Paid-Incurred Chain methods for the paid and incurred claims development triangles. These estimates were obtained with the help of the MATLAB r2013 program.

Table 1. Comparison of methods

Methods	Information used	Distribution assumption	Reserve estimate	MSE	Claims Development Result
Mack's Chain Ladder	Claims paid or incurred	No	Two different	Available	Available
Münich Chain Ladder	Both claims paid and incurred	No	Two different	Not available	Not available
Extended Complementary Loss Ratio	Both claims paid and incurred	No	Only one	Available	Available
Paid-Incured Chain	Both claims paid and incurred	Yes	Only one	Available	Available

Table 2. Claims reserve estimated values

	Methods					
	Mack's Chain Ladder		Münich Chain Ladder		Extended Complementary Loss Ratio	Paid-Incured Chain
	Information used					
Accident years	Paid	Incurred	Paid	Incurred	Both claims paid and incurred	Both claims paid and incurred
0	-	-	-	-	-	-
1	484.441.022	459.944.890	482.864.462	461.674.782	450.344.108	480.781.869
2	707.454.726	925.051.812	918.483.827	918.661.368	732.500.897	709.497.069
3	909.221.716	1.167.898.222	1.025.124.407	984.254.655	1.167.657.635	926.847.199
4	1.343.982.937	2.129.785.947	1.886.015.350	1.865.139.755	1.893.557.672	1.427.428.700
5	2.109.999.716	3.676.524.682	3.267.023.011	3.494.651.313	3.767.517.974	2.407.336.360
6	3.522.194.320	4.487.882.738	3.954.792.981	4.225.006.565	5.933.270.184	3.766.690.833
Total Reserve Estimate	**9.077.294.437**	**12.847.088.291**	**11.534.304.038**	**11.949.388.438**	**13.944.848.470**	**9.718.582.029**

From Table 2, it is seen that when Mack's CL method is applied separately to the paid and incurred claims development triangles, the total reserve estimates are quite different from each other. As the claims incurred in the ultimate development year are expected to be fully paid, the ultimate reserve estimates must be equal. However, it can be seen that the reserve estimate with this method does not meet this expectation. When the MCL method developed to reduce this difference between the estimates is used, it is clear that this difference decreases considerably. Therefore, it can be said that reserve estimates obtained with the MCL method are more realistic but still do not meet the expectation. The estimates obtained with MCL are among estimates obtained with Mack's CL for paid and incurred claims.

A single reserve estimate was obtained with the ECLR method, which uses both sources of information together. Obtaining a single reserve estimate is consistent with the expectation. However, it can be seen that the reserve estimation with the ECLR method is higher than those calculated by other methods due to the fact that estimates are obtained using outstanding claims.

In order to compare reserve estimation using the Paid-Incurred Chain method with the results of other methods, non-informative priors were used. That is, in equations (11) and (12) it is assumed that $s_m^2 \to \infty$ and $t_m^2 \to \infty$. As with the ECLR method, the PIC method used both sources of information together and obtained a single reserve estimate. It can be seen that the reserve estimates calculated with this method are close to the estimates using only the paid claims data in Mack's CL method and lower than the estimates obtained by other methods.

The estimates obtained for the incurred claims data were higher than the estimates obtained for the paid claims data with the MCL and Mack's CL methods. The reason for this is that incurred claims include outstanding claims as well as paid ones. One can consider that it is better to use incurred claim data involving more information, as future conditions may differ from current conditions. However, this may be undesirable for insurance companies, as using only the incurred claims data may cause more reserves to be set aside. Therefore, the estimations obtained by using both sources of information together may be more reasonable for the company.

In order to decide which method gives a better reserve estimation, the mean squared errors of the reserve estimates are calculated, and the square roots of the mean squared errors are given in Table 3.

Table 3. The Square Root of the Mean Squared Error (RMSE)

	Mack's Chain Ladder		Münich Chain Ladder		Extended Complementary Loss Ratio	Paid-Incured Chain
	Paid	Incurred	Paid	Incurred		
RMSE	445.889.728	489.678.795	Not available	Not available	1.559.501.412	369.177.239

Since there are no formulas in the literature for the MSE of the reserve estimate using the MCL method, nothing can be said about the estimation uncertainty. Therefore, it was not possible to comment on the suitability of this method compared to other methods for the data. The RMSE of the reserve estimation calculated with the ECLR method is higher than those with all other methods. The reserve estimate calculated by this method was also higher than that calculated by the other methods. It is seen in Table 3 that the lowest RMSE is obtained with the PIC method. Therefore, it can be said that this method is more suitable for the available data compared to other methods.

CONCLUSION

In non-life insurance, it takes a while for the claim to be resolved, as there is often a delay between reporting and closing the claim. This delay requires the insurer to set aside a reserve for possible claims payments that have not yet been resolved. Being able to determine the claim reserve correctly is very important for insurance companies to protect their financial structures. There are two different approaches used to estimate the claim reserve: deterministic and stochastic. In both approaches, it is aimed to obtain the best estimate of the reserve. But the stochastic approach, unlike the deterministic approach, provides information about the variability and distribution of the claim reserve.

Reserve estimates are usually based on historical data from different sources of information. While traditional reserve estimation methods use a single source of information, estimation methods that use claim information from different sources together have emerged in recent years. The objective of this study is to compare the performance of different reserve methods using both paid and incurred information. For this purpose, the reserves are estimated using the compulsory traffic insurance data in Turkey with the methods of Mack's CL, MCL, ECLR, and PIC. It is founded that MCL reduces the gap between the CL estimations. But it still gives two different reserve estimates. A single reserve estimate is obtained from the methods ECLR and PIC. The reserve estimate obtained by the PIC method is almost similar to the estimates obtained by the CL and MCL methods for paid claims. The estimation uncertainties are compared using the square root of the conditional MSE. Based on RMSE, it is concluded that the PIC method is the most appropriate one for the Turkish Highways Motor Vehicles Compulsory Liability Insurance data used.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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Appendix

Table 1. Cumulative Paid Claims Development Triangle

Accident year	Development year						
	2010	2011	2012	2013	2014	2015	2016
2010	995.798.171	1.438.259.466	1.559.650.271	1.662.755.354	1.786.917.968	1.897.388.105	1.998.562.668
2011	1.189.641.874	1.736.757.582	1.891.777.500	2.064.977.043	2.206.087.115	2.330.766.107	
2012	1.353.354.645	1.931.272.233	2.169.789.044	2.368.695.049	2.535.892.270		
2013	1.373.141.787	2.073.507.051	2.353.701.930	2.459.316.689			
2014	1.645.091.685	2.475.031.357	2.862.810.883				
2015	2.043.090.824	3.248.651.746					
2016	2.388.117.281						

Table 2. Cumulative Incurred Claims Development Triangle

Accident year	Development year						
	2010	2011	2012	2013	2014	2015	2016
2010	1.271.513.059	1.590.836.041	1.755.971.819	1.915.034.788	2.046.574.503	2.165.046.051	2.291.753.130
2011	1.505.498.005	1.961.676.492	2.212.416.307	2.380.275.450	2.530.016.443	2.636.417.398	
2012	1.721.677.567	2.299.078.019	2.572.917.292	2.782.450.708	2.931.756.416		
2013	1.813.312.452	2.536.373.441	2.896.405.488	2.969.761.591			
2014	2.170.842.459	3.154.454.913	3.556.440.946				
2015	2.913.586.967	4.254.558.888					
2016	3.483.927.015						