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Review Article

Statistical and mathematical modeling of temperature dynamics in automotive brake components: A systematic review

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ABSTRACT

Maximizing the effectiveness of braking mechanisms is critical for enhancing car safety and advancing technology. This study investigates temperature fluctuations in brake liners and brake discs during deceleration using statistical and mathematical approaches. Key factors considered include the physical parameters of engagement materials, initial frictional force, and braking duration. The research highlights the importance of controlling heat transfer in rotor brakes, particularly regarding speed and load changes. Understanding the thermodynamic dynamics within brake elements is essential for improving vehicle safety. This comprehensive examination provides insights into the methodologies for predicting temperature changes in automotive braking components and evaluates the precision of these predictions. The findings offer valuable perspectives on thermal regulation in vehicle brake systems, paving the way for future improvements in brake design and performance.

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INTRODUCTION

The design and development of an automotive disc brake require a thorough knowledge of braking principles, different systems, operational aspects of the components, and mechanical and thermal constraints. Based on these aspects, an efficient disc brake can be designed, considering varying operating conditions, thermal stresses, and metallurgical limitations. In the recent trend of advanced disc brake technology, advancements are achieved through various analytical and experimental findings. In this direction, a thorough simulated analysis can provide in-depth knowledge about the operational behavior and performance of these brakes. To gain insight into the parameters combined mechanical and thermal performance analysis models have been formulated, developed, and analyzed in this work. The mechanical and thermal performance of the

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brake disc was tested under various design and operating conditions, including different vehicle speeds, pedal efforts, coefficients of friction, deceleration rates, and uphill and downhill decelerations.

Several competitors' automobiles have particular requirements for the design of brake discs and accompanying features to ensure optimal circulation. These requirements consider area of surface, drag value, weight, cost, and the overall brake mechanism. Nevertheless, the demands made upon a brake pad expand with the vehicle's capacity, acceleration, and velocity. The most popular type automotive pedal is the rotor brake, which generates friction among a brake cushion as well as a disc's edge when applied. This mechanical brake is located near the piston and the tire rim.

This was previously considered a niche issue due to the relative design robustness of the task and received limited attention. On the other hand, recent advances in automotive development and the electrification of braking systems require a deeper understanding of brake thermal properties. This information will undoubtedly enhance temperature forecasts in various driving conditions, which have traditionally been primarily studied for moving vehicles. Considering rotor brake engagement temperature affect both the radial coefficient of resistance and pad deteriorate [1].

Heat dissipation has been the subject of substantial examination and research, with problems related to effective brake cooling affecting all types of brakes. Under typical vehicle operating conditions, convection serves as the primary heat transfer mechanism, dissipating the most heat into the surrounding air. At high temperatures, radiation significantly contributes to heat dissipation, while conduction is the least explored mechanism of heat transmission. In this study, we theoretically investigated the contributions of all three modes using computational fluid dynamics.

It is practical to compute the speed characteristic for a given racing course in order to evaluate and optimize brake disc development. Research focusing on frictional heat associated with braking force is more effective than measuring the vehicle's kinetic energy and the kinetic energy of each component. Furthermore, the findings were further investigated to determine the most practical solution for rapidly measuring the temperature at the point where two objects meet, such as a disc brake and its padding, which interact during a uniform deceleration brake application. Figure 1 depicts a conventional the automotive industry brake system, which includes brakes with discs at the entrance and brakes with drums design at the outer edge of the car.

A brake with discs creates resistance by pushing two sets of pads across a disc. This action causes a shaft, including a car axle, gradually slow down and/or stop rotating. Kinetic energy is turned into wasted energy, which must be dispersed. Hydraulic disc brakes are the most prevalent form of control for transportation automobiles. It is made up of the disc, master cylinder, and caliper on each side of wheel disc, as illustrated in Figure 2.

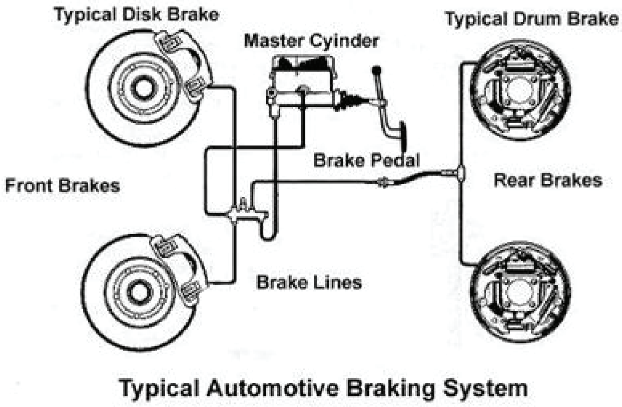

Figure 1. Typical automotive brake system [Oduro (2012) [2], reproduced with acknowledgment].

a) Caliper assembly:- It is made up of an accelerate pad, cylinders, caliper structure, brake rotors, and bleeder anchor, as illustrated in Figure 3.

b) Knuckle for brake rotor support: - It is the stationary rigid component used to support the brake rotor assembly as shown in Figure 4.

c) Brake pad: - It is made of metal components with integrated brake lining. Generally, every brake caliper has two pads for stopping, one on both sides of the braking rotor, as illustrated in Figure 5. Pads for brakes have heat-absorbing qualities because of the substantial quantity of heat created by contact between their surfaces with the rotor. Temperature and resistance cause the pads to wear down gradually, necessitating brake pad maintenance services.

d) Wheel hub: -It is a revolving component that connects to the disc spinning. A bolted attachment is made across the vehicle's tire with the rotating component.

e) Brake rotor: -The rotating part is a spherical disc attached each spinning wheel. The rotating part is designed to promote the conversion of motion energy into thermal energy.Research technique covers the processes taken for carrying accomplish the precise duties of the full study job as outlined in the analysis strategy of Figure 6.

In Figure 6 A Statistical and Research Examination of the Dispersion of Heat Utilizing Computational Fluid Dynamics (CFD), Statistical Computational Simulation, and Verification of Research and Computational Fluid Dynamics Outcomes.

This examination involves a comprehensive analysis of heat dispersion within brake systems, employing advanced methodologies such as Computational Fluid Dynamics (CFD) and statistical computational simulations. The research methodology begins with the use of CFD to model and predict the behavior of heat transfer within the brake components, taking into account factors such as fluid flow, temperature distribution, and thermal conductivity.

Following the CFD analysis, statistical computational simulations are conducted to further understand the

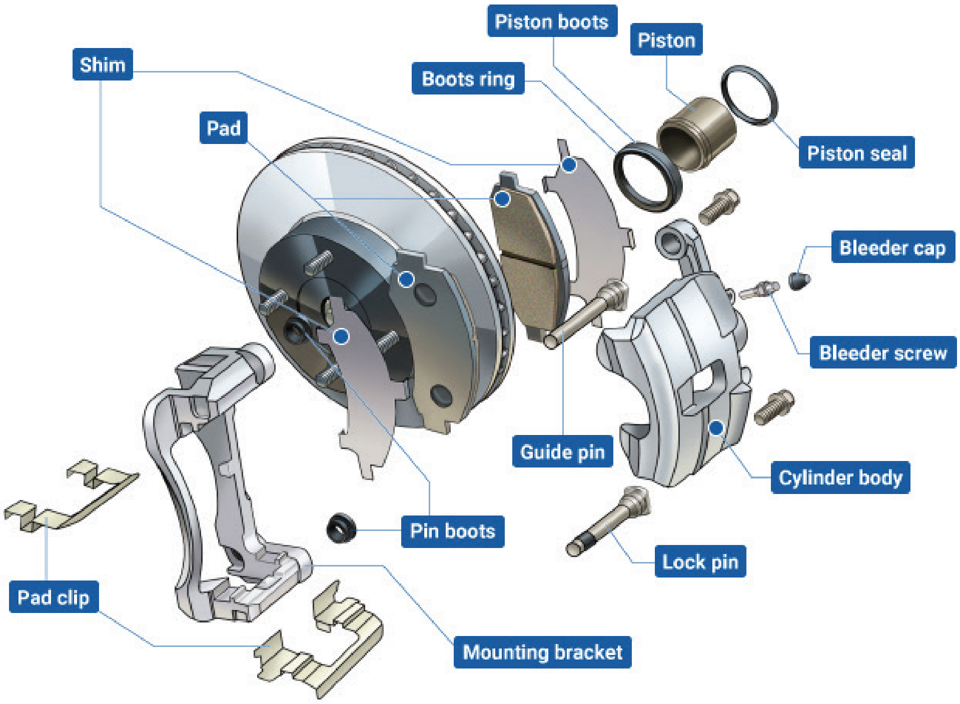

Figure 2. Features of rotor brake mechanisms [GridOto.com (2021) [3], screenshot used for educational purposes; copyright remains with the original publisher].

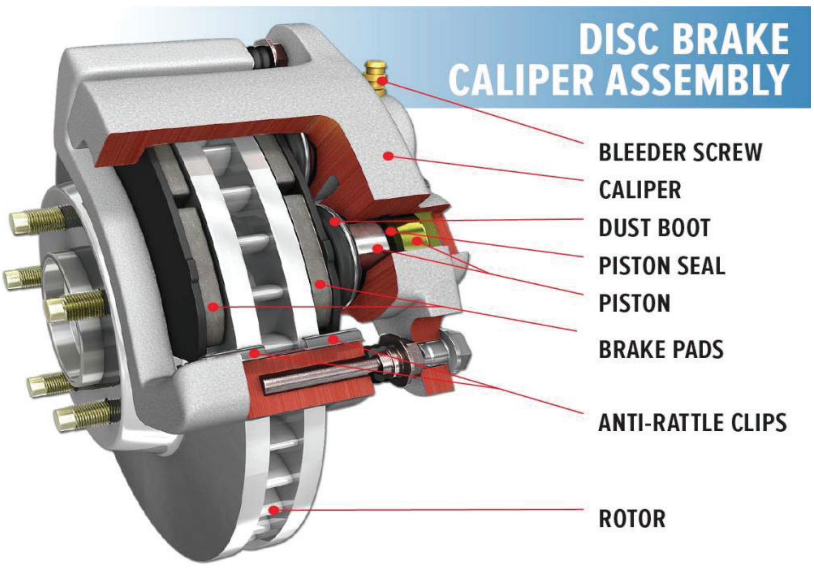

Figure 3. Wheel brake caliper component [GridOto.com (2021) [3], image used for illustrative purposes; all rights belong to the publisher].

variability and reliability of the heat dispersion models. These simulations help identify patterns, correlations, and potential anomalies in the data, providing a deeper insight into the thermal behavior of the brake systems under various operating conditions.

The final phase of the examination involves a rigorous verification process, where the outcomes from the CFD and statistical simulations are compared against empirical research data. This step is crucial for validating the accuracy and credibility of the computational models, ensuring that the

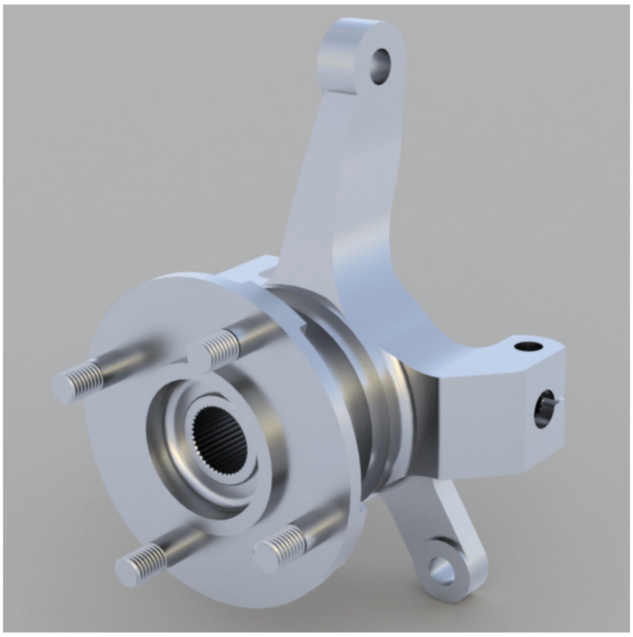

Figure 4. Knuckle for brake rotor support [From GrabCAD (2021) [4], 3D model by user from the GrabCAD community; used under Creative Commons license or as per site usage terms].

Figure 5. Brake pads in disc brakes [From FrontechChina.com (2021) [5], image used for educational and non-commercial purposes; copyright remains with the original owner]

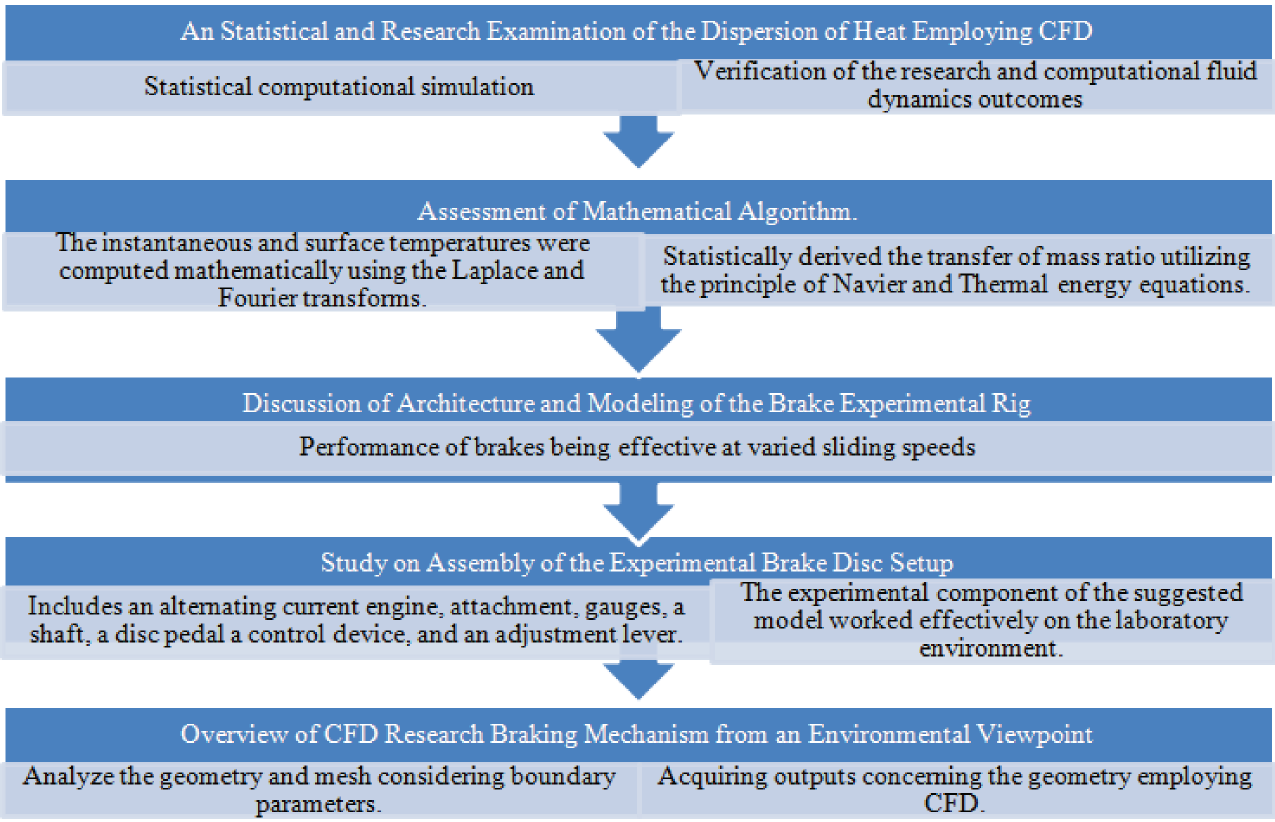

Figure 6. Research methodology adopted in present study.

predictions align with real-world observations. By integrating these approaches, the study aims to enhance the understanding of heat dispersion in brake systems, contributing to the development of more efficient and reliable brake designs.

The evaluation and application of mathematical algorithms to solve complex heat transfer problems in brake systems. The algorithms are designed to accurately compute the instantaneous and surface temperatures of brake components under varying operational conditions. By leveraging advanced mathematical techniques, these algorithms can model the transient heat conduction, convection, and radiation processes that occur during braking. Laplace and Fourier transform are used to handle the time-dependent aspects of heat transfer, enabling the solution of transient heat conduction problems by converting time-domain functions. This aids in solving problems related to the distribution of temperatures over the surface of brake components, especially when dealing with periodic or oscillatory heat sources. By combining these transforms, the study can accurately compute the temperature profiles at any given instant and location on the brake components, providing detailed insights into their thermal behaviour. The next phase involves the statistical derivation of the mass transfer ratio, which is crucial for understanding the distribution and movement of thermal energy within the brake system. Navier-Stokes equations of fluid dynamics describe the motion of fluid substances, incorporating factors like velocity, pressure, density, and viscosity. Thermal energy equations govern the conservation and transfer of thermal energy within the system. They account for the conduction, convection, and radiation processes that contribute to the overall thermal balance. Through the combined use of mathematical algorithms, Laplace and Fourier transforms, and statistical methods based on the Navier-Stokes and thermal energy equations, this comprehensive assessment provides a detailed understanding of the heat transfer and thermal management in brake systems. The results not only enhance the theoretical foundation but also contribute to the practical optimization of brake designs for improved performance and reliability.

The architecture and modelling of the brake experimental rig are crucial components in the study of brake system performance. This section delves into the detailed design, construction, and operational principles of the experimental setup used to test and evaluate brake systems. The brake experimental rig, combined with the analysis of brake performance at varied sliding speeds, provides comprehensive insights into the effectiveness and reliability of brake systems. By understanding how different factors influence brake performance, the study aims to optimize brake design and operation, ensuring safety and efficiency in real-world applications. The study examines how different sliding speeds affect the rate of thermal degradation and identifies the maximum safe operating speeds for the brake system. The rig tests the brake system's ability to generate sufficient braking force and achieve the desired deceleration at different sliding speeds. This helps in evaluating the overall stopping power and responsiveness of the brakes. High sliding speeds can induce vibrations and noise in the brake system, affecting ride comfort and safety. The experimental setup measures these factors to ensure that the brakes perform smoothly and quietly across a range of speeds.

The study on the assembly of the experimental brake disc setup, along with the successful operation of the experimental component in a laboratory environment, provides a comprehensive understanding of brake system performance. The detailed assembly process ensures the accuracy of the setup, while the experimental tests validate theoretical models and offer insights for optimizing brake design. This integrated approach contributes to the development of more efficient and reliable brake systems for real-world applications.

The CFD research on braking mechanisms from an environmental viewpoint provides a comprehensive understanding of the thermal, fluid, and particulate dynamics of brake systems. By meticulously analyzing the geometry and mesh, considering boundary parameters, and acquiring detailed outputs, the study offers valuable insights for designing more efficient, environmentally friendly, and reliable braking systems. This integrated approach not only enhances the performance of brake components but also contributes to sustainable automotive engineering practices.

Literature Review

A brake heat transfer problem is highly transient, with no steady state due to the involvement of conduction, convection, and radiation. During a brief period of complete braking, the disc experiences a substantial heat flux, influenced by its complex geometry, including features like drilling, slotting, and venting. Solving the energy equation under these conditions is a complex task. Analytical investigation, primarily centered on analyzing data [6], is one method for tackling this issue. Uniform heat flow parameters, on the other hand, often overlook the two principal thermal dissipation methods, convection as well as radiation, which are handled in the majority of modern Finite Element models [7].

Furthermore, it is possible to obtain a velocity characteristic for a particular race track, which permits the analysis of temperature patterns and stress levels over time. Under a single, consistent deceleration brake purpose, interactions between elements, including the discs and friction pads, are visible [8]. The primary function of a brake system serves to slow or stop an automobile inside an adequate span and longitude. The result demands a dependable braking device that can supply sufficient torque to the controller. Every driving vehicle has kinetic energy because of its movement. The velocity and acceleration of a vehicle increase as it goes faster. Several braking mechanisms use the concept of transforming motion energy into resistance power. Throughout braking phases, the pedals absorb and dissipate energy as temperature into the atmosphere around them. The goal of this research was to create an apparatus that would permit homogeneous cooling within the disc rotor. This includes altering the disc's ventilate design to achieve steady cooling [9].

The zero-dimensional global method and a one-dimensional technique were used for analysing the thermal loading elements of the developed brake disc. The typical thermal loading parameterization relationships are used to compute thermal performance characteristics like temperature, heat flow, and heat exchanged. Using relevant parametric connections, instantaneous temperature, temperature rise, heat flow into the brake disc, and conductive, convective, and radiative heat transfers were estimated employing the energy created during braking [10]. A thermodynamic network approach derived from the concept was devised and built for the entire disc brake mechanism to determine the transient temperatures inside the braking disc in different scenarios. This energy flow model modifies the thermal loading variables for duration and constituent area, resulting in a thermodynamic loading arrangement for the proposed brake disc [11]. The automobile sector now uses two distinct kinds of disc technologies. The first, employed in family-sized vehicles, is based around small-diameter, elevated-strength discs with sufficient intrinsic capacity to resist heat-induced cracking and distortion at elevated temperatures during operation. Nevertheless, those discs have moderate heat conductivity. In contrast, less powerful, low-strength discs with high heat conductivity are more commonly used in larger, high-powered vehicles. The reduced space constraints in these vehicles allow for the use of wider, thicker discs. To address these material choices, a review of the literature was conducted to gather precise details regarding the materials used for brake discs [12].

Rotor velocity was shown to have a considerable impact on rotor performance. The aerodynamic features of airflow across rotor passageways have a significant impact effect the thermal endurance and dissipation about ventilated brake pads [13]. In accordance to investigations, based on the mean heat transmission rate along the channel rises with blade speed. The previously channel's volumetric flow rate has increased, which is mostly responsible for this. Based to the findings, after a certain point, a higher vane quantity has no impact on the thermal transfer rate or entire temperature during braking [14-16]. Overall a recirculating zone created separation of streams, which resulted in energy loss throughout the system. Around 16 vanes, overall re-circulation region was the lowest, and since the amount of vanes climbed, so did the energy reduction, resulting in decreased efficiency. The influence of rotational velocity on aerothermal effectiveness was investigated. Rotor velocity was shown to have a considerable impact on rotor performance. Both aerodynamic qualities of ventilation through rotor passageways have a significant impact effect the thermal stability and dispersion of vented brake discs. Based to investigations, the mean heat transmission efficiency across the passageway increases with blade velocity. This is attributed primarily to a rise within the route's volumetric flow rate [17].

The thermal distribution of the wheel disc throughout the stopping action was simulated using ANSYS Multiphysics. The finite element modeling techniques are used to determine the temperature threshold of the blade and predict the temperature gradient of the entire and blown brake discs. When creating the rotating part, some factors are considered, including the material utilized and the angular nature of the blade. The investigation [18] calculates the heat circulation between both disks. A new threshold element method is employed with the FFT (Fast Fourier Transform) to investigate extended axis-symmetric issues related to time domain elasto-dynamics. The problems were determined either axis-symmetric design and non-axis-symmetric limits. Quadratic boundary components are employed within the circumferential orientation, whereas Gauss quadrature is used in the source direction using the FFT approach. Singular fundamentals were examined extensively and with excellent accuracy. The Fourier transformed result was statistically reversed through the FFT to yield the final answer. As demonstrated through computations [19], the system is highly accurate and performs well.

On a rigid disc brake, the creation of heat and evaporation were studied employing machine learning engineering applications to operate a motorbike during persistent halting, two-disc characteristics, and three distinct rotor disc composites [20]. The purpose of this research is to explore and evaluate the variation in temperature and rotational disc under braking. The thermal pattern on the inner surface of the blade is computed employing of finite elements method, and the vital temperature within the brake rotor cushion is determined. Conduction, convection, and radiation were the various modes of heat transfer explored. Graphite ceramics, alloy of aluminum 6262 T-9, along with grey cast iron are among the components used [21]. The contacting region of a novel, unworn friction substance expands with wear, thus rough surfaces subsequently gradually smoother. The computer simulation outcomes also demonstrate this. The front edge is found to be less susceptible to wear compared to the side that follows. The modified wearing approach used in this study may predict wear progress as well as substantial modifications in the topographical surface, enabling disc brake squealing to better represent unpredictability analyses [22, 23].

The verification and simulation studies confirmed the model's applicability by focusing on the least important variables. The expected findings indicate that enhancing the rear plate's rigidity of stiffness and modifying with grooves, contouring the coefficient of friction composition on both surfaces, and providing slot layout will lessen brake squeal tendency. This concept is a collection of techniques that might be useful throughout the disc brake design procedure [24]. [25] Investigated the finite element technique used for evaluating the grinding sound of a gyroscopic wheel brake. The innovative approach instability of a vehicle brake mechanism was explored using two static pads interacting with a rotating disc. The movements of the cushions and disc are characterized by an orientation transformation across the source and motion system of coordinates. The disc's gyroscopic grid employs an even planar-mesh method. In regard to system circumstances: The dynamic instabilities of an unconventional gyroscopic braking system can be theoretically predicted. The results indicate that the kinds of vibration included in squeal features influence a noisy predilection for spin rate. [26] The thermodynamic characteristics of solid and vented brakes were investigated and examined with the ANSYS computer system. The range of temperatures of the brake disk identifies all variables and traits involved in the deceleration process, including braking type.

This ANSYS-based progressively thermal-structural connected process was utilized for generating the stress levels and deflections that occurred in the disk as a result of the pad force. The transient heat and stress fluctuations simulate. Simulation results show that the optimal temperature variation for vented rotors is 345.44°C to 401.5°C at varied periods of time for stiff brake discs. The beginning speed is 30 km/h as well as the gradient is 6%. ANSYS was used to model and analyze the three-dimensional transient heat field of a brakes disc. At the conclusion of halting, the mean brake disc heat is 316.04 °C. The outside temperature within the brakes slowly increased with time. Investigations regarding the transient thermal field dispersion of a brakes disc during prolonged downhill riding showed heat flux density increases with a boost in friction diameter and circumferential temperature variation. The longitudinal variation in temperature is almost homogeneous, and heat conduction as well as convection become effective. The friction factor changed gradually under prolonged slope brake conditions on different slopes. The average deterioration frequency is 8.16%, thus the coefficient of resistance remains within an acceptable range, and there is no confirmation of a heat deterioration [27]. Ventilated brake disk rotors are used in a conventional passenger car with the greatest stopping capability. The research will most likely concentrate on the dispersion of temperatures and heat in disc brakes rotors. The heat pattern and rapid reaction of disc rotors for brakes are investigated employing a finite element modeling technique. The thermal evaluation of a transient reaction is carried out using ABAQUS/CAE computational software. Since a consequence, this study adds to a better knowledge of the physical characteristics for brake disk rotors and helps the automobile industry design more effective and effective disc brake rotors. The temperature behavior during stopping and non-braking has been studied. The load stages require 350 s to finish, with 10 cycles of brake and a total of ten with non-braking. Different temperatures were established in each iteration; the greatest temperature in every one of the times was 543.9°C, whose is still lower than the normal service level for grey cast iron, that is around 550°C. As an outcome, the highest yielding temperature is appropriate. These investigations compared a solid disc and a vented disk without taking into account the geometry of the ventilation curves or the amount of vanes [28].

During frequency domain assessment (FDA), the fast Fourier transformation (FFT) has been employed. In addition, an assessment and comparison method was used to identify a suitable strategy that integrates accuracy and improves the accuracy of detecting and diagnosing different cavitation degrees inside a centrifugal pump. Impeller knife angles and unstable flow have a big influence on the magnitude of pressure changes. We used the CFD tool Ansys-Fluent to evaluate the impact of design on brake disc performance in this study. The resulting coefficient of heat exchange and total area thermal flux constituted the variables to be studied [29-33]. Investigate the acceleration distribution and associated distribution of temperatures within the disc. ANSYS multi-physics algorithms were employed to model rotational disc circulation of temperatures during deceleration. To predict ventilated brakes discs, complete and accurate finite element methods are used [34]. As stated by [35], there is a considerable body of evidence demonstrating that contact heat plays a crucial role in regulating the combined effect, speed, and degree of resistance of the weight, in addition to other variables affecting specific power dissipation. Investigations examining the thermophysical and endurance properties of mechanical coupling composites found that the movement speed and transmitted weight enhanced. The index of friction diminished whenever the disc heat approached 300 °C, but improved as temperatures fell below this level. The quantity of wear has been demonstrated to rise as one raises sliding velocity and disk temperature [36].

The ventilated disc, on the different hand, can exacerbate judder problems by enclosing it in an inconsistent temperature region. In addition, the evacuated disc has lower temperatures than the rigid disk. After repeated braking, the power source vented disk may heat up faster compared to the solid disc [37]. When changing the configuration of the ventilated disc, thermodynamic capacities and thermal distortion ought to be carefully assessed, based on a new investigation [38] that examined repeated interactions. Distribution of temperatures is crucial in dry frictional devices. The temperature gradient within the clutch disk is calculated employing a finite element approach for multiple pressure ranges. [39] During the overall vehicle CFD modeling, the mathematical domain's wall component technique was applied. Rotational boundary constraints were imposed to the rotating components, that included the spinning disc brake, steering wheel, tire, hubs crown, along with rim. MFR was used on the steering rotor fluid, in addition to the hydraulic surrounding the wheel cap along with rim.

The coefficient of thermal conductivity was estimated with the caliper with disc walls. Prostar, the Star-post-processing CD application, was employed to accomplish it [40]. A structural boundary restricting was imposed on the computing domain's outermost portions. The turbulent stiffness of the unidirectional layer was adjusted to be

200 times the level of the liquid that surrounds them and the level of turbulence in the flow entrance was adjusted to 0.1% [41]. The empirical observations of the convective coefficient of thermal transfer are extremely similar to the Simulation results, particularly at lower spinning speeds. The modifications become greater near the highest point of the velocity range, with CFD simulations estimating a 10-percent rise in a vehicle's the rate of convective heat at maximum speed. The results are additionally in accordance with the knowledge [42,43].

Additional investigation has been undertaken to enhance accuracy and incorporate the whole wheel component. The beneficial effect of rotating velocity on aerodynamic thermal characteristics has been found, indicating that rotation speed provides an enormous impact upon rotors effectiveness. The thermal efficiency of the rotor passages has an important effect on aerodynamic traits of circulation and heat distribution in ventilated brake pads. It was discovered that while the blade accelerates, it also increases the total heat transfer efficiency through the passageway. The reason for this is largely attributable to a rise in the channel's total flow rate [44]. ANSYS, which Multiphysics was used to assess the temperature gradient of the rotor discs during deceleration. The heat dispersion over the vented brake pad is estimated employing finite element modeling approaches. The ideal temperature that affects the rotor, taken into consideration specific characteristics including braking setting, material used, and disc design. The heat flow pattern between the two identical discs is also calculated by the research [45]. A quick Fourier transform was used together with a complex boundary element technique to investigate fundamental axis-symmetric problems in time domain elastic-dynamics. The issues are distinguished by their axis-symmetric design and non-axis-symmetric boundary circumstances. Complex Fourier transforms were used to increase boundary values in the circular direction. The problem was successfully broken down into separate concerns, which were dealt with employing the Boundary Elementary Method (BEM) for Fourier boundary region amounts, with only the axis-symmetric physical surface generator separated. The Fourier transform technique employed BEM observations in quadratic boundaries elements to perform Gauss computation in the circumference and generator directions [46-48].

The computation of singular fundamentals was straightforward and exact. The ultimate outcome was reached by statistically inverting original Fourier-converted result. As shown by computational instances, this approach is highly accurate and efficient. Employing automated engineering devices, a two-disc design and three distinct rotor disc composites were developed. Heat is generated and dissipated in a vehicle's compact rotors during continuous braking.

The goal of this investigation is to look into how temperatures vary and heat dissipation within the disc blade during deceleration. A finite element modeling approach is employed to anticipate the heat distribution across the disc rotor design, and the critical temperatures of the brake rotor disc is determined. Heat energy storage with afterwards cooling constitute frequent issues for rotor disc designers, especially in mountain descent and sequential fading brakes [49]. Until most recently, the creation of the braking disc as well as surrounding elements relied heavily on empirical data and experience with airflow in and out of the disc. As the outcome, temperature changes during braking experiments were used to determine the cooling capacity of the brake system [50].

Bala Subramanyam and Prashanti [51] carried out finite element modelling and structural analysis of two-wheeler disc brake rotor. The geometry optimization is of great significance to stress concentration reduction and braking performance enhancement, as addressed in the study. That research led to remarkably strong and lightweight rotors. Khairnar et al. [52] presented a Simulink-based approach to predict the brake drum-shoe interface friction coefficient for various longitudinal forces. Subsequent work from them [53] compared the friction between drum and disc based on contact/mechanics/braking core consistency and dynamic loading. Kalhapure and Khairnar [54] introduced system-analytical and experimental techniques to analyse wear progression in disc brake pads. Their results showed that wear rate can vary due to differences in the material composition and operation of the brake while predicting pad longevity and performance. Fono-Tamo [55] formulated a mathematical model to study the thermal stress characteristic of PKS compound environmentally friendly brake pad was reforming material with the aid of the simulation of MATLAB models. This study was one of successive works on the development of an environmentally friendly friction brake material. Gupta et al. [56] developed a series of computerbased fEMderived numerical models for thermal analysis of disc brakes.

Their research focused on the distribution of temperature and heat dissipation because of its significance for evaluating brake fade and thermal stability. Also, Agrawal and Khairnar [57, 58] performed thorough experimental and CFD analyses aimed at optimizing the heat dissipation of a disc brake to characterize heat transfer coefficients particularly regarding electric parking brakes. These investigations made major contributions towards the understanding of thermal dynamics in braking systems. In a later study, Agrawal et al. [59] performed a computational comparative analysis between solid and vented disc brakes. The group simulated the heat transfer characteristics and thermal performance of both types of disc brakes using CFD tools and concluded that vented rotors provided better thermal management during high braking loads compared to solid rotors. Patil et al. [60] analyzed the parameters determining the adoption of electric vehicles within India. Though not directly associated with brake mechanics, these concepts worked towards underscoring the need for advanced regenerative and mechanical braking systems in electric vehicles (EVs).

Table 1. Past methodologies, objective, and literature summary

S. No.	Name of authors	Publication year	Objectives	Parameters/Data used/applications	Methodology used	Summary
1.	Talati	2009	This study aims to assess analytical research is a form of research that focuses on analyzing data	Developments of electric vehicles are presented.	Technical review for disc brake vehicles.	The two principal agents of heat dissipation, convection, and radiation, are ignored with constant heat flow boundary conditions, the most recent Finite Element modeling
2.	Alibelhocine	2019	Structural and thermal analysis of automotive disc brake rotor	brake discs, pads, heat flux, heat-transfer coefficient, Von-Mises stress, contact pressure	The modeling is based on ANSYS 11.0	The analysis of thermo mechanical behavior of the dry contact between the disc and pads during the braking process; the modeling is based on the ANSYS 11.0
3.	Danish vytenis subrely,Edger sokolovskji	2020	The analysis of braking forces in a load-carrying capacity vehicle with a hydraulic braking system on the pedal pressing force. Driver's behavior questionnaire survey.	Driver's behaviour questionnaire survey	ANOVA	Changes in braking forces & Axial force depend on braking pedal force.
4.	Bala Subramanayam	2014	Thermal stresses and temperature profiles were measured over time.	The evaluation and optimization of brake disc design.	The most recent Finite Element modeling.	The two principal agents of heat dissipation, convection, and radiation, are ignored with constant heat flow boundary conditions.
5.	Sai Krishna	2020	The outcomes were investigated further to arrive at the most feasible solution for quickly measuring the temperature achieved at the point where two objects meet.	A disc brake and padding are examples of interacting bodies at a single, uniform deceleration brake application		Frictional heat linked to braking force in the research is better than measuring the vehicle's kinetic energy and each component's kinetic energy.
6.	Bryant	2011	The purpose of the study was to develop a design that would allow for uniform disc brake heating and, more crucially, uniform disc brake cooling.		The suggestion involves changing the disc's vent configuration to facilitate uniform cooling	The thermoelastic behavior of a disc brake when it was subjected to braking forcefully.

Table 1. Past methodologies, objective, and literature summary (*continued*)

S. No.	Name of authors	Publication year	Objectives	Parameters/Data used/applications	Methodology used	Summary
7.	Prashant	2017	The primary purpose of this research was to determine whether a curved vane can aid increase disc brake performance. Improved air circulation in straight vanes, the suggested disc brake has more mass and a higher heat transfer coefficient.	Improved air circulation in straight vanes, the suggested disc brake has more mass and a higher heat transfer coefficient.	Thermal analysis	For the transient thermal study, the effect of vane geometry on disc brake performance pillar type vane versus straight vane geometry.
8.	H.P. Khairnar, V.M. Phalle, S.S. Mantha	2015	The purpose of this research was to investigate the coefficient of friction under numerous conditions. Coulomb friction force, contact force, and actuating forces at the shoe ends Simulation-based investigation by using Simulink.	Coulomb friction force, contact force and actuating forces at the shoe ends	Simulation based investigation by using simulink	The present investigation encompasses a multitude of equations for friction coefficient derived under the equilibrium condition using principles of classical mechanics with friction considering the variations of longitudinal forces
9.	Shital M. Kalikate, Satyajit R. Patil, Suresh M. Sawant	2018	This works aims to evaluate the feasibility of MR brake Brake Type, Geometry, Number of discs, MR fluid, gap Size Simulation-based performance estimation of MR brakes	Brake Type, Geometry, Number of discs, MR fluid, gap Size	The Simulation based performance estimation of MR brakes	The proposed single-disc MR braking system for a two-wheeler application will not fulfill the performance standard criteria.
10.	H.P. Khairnar, V.M. Phalle, S.S. Mantha	2016	This works highlights braking force behaviour of disc brake and drum brake	Coulomb friction force, contact force and actuating forces	Simulation based investigation by using simulink	Comparative frictional behaviour of drum brakes and disc brakes in automobiles has been investigated.

Table 1. Past methodologies, objective, and literature summary (*continued*)

S. No.	Name of authors	Publication year	Objectives	Parameters/Data used/applications	Methodology used	Summary
11.	Bambang Sri Kaloko, Soebagio, Mauridhi Hery Purnomo	2011	This research aimed to model the flow of power in the electric vehicle energy system to obtain its characteristics	Design and Development of Small Electric Vehicle using MATLAB/ Simulink	Simulation based electric vehicle model development by using simulink	Modelling of electric vehicle systems makes it simple to figure out how much battery capacity is required by an electric vehicle with particular characteristics in order to travel a certain distance.
12.	A H Rozaini, M R Ishak, A R Abu Bakar and M Z Mohd Zain	2013	The goal of this research is to examine the performance of a popular parking brake system seen in passenger vehicles.	Theoretical model and experimental results	mathematical model is derived based on hand lever mechanism	The parking brake model is valid when there is a high correlation between estimated and measured outcomes. The car will not roll away on an 11-degree slope with four people inside it.
13.	Hongyu Zheng, Shenao Ma and Yahui Liu	2018	A control technique with a three-layer hierarchical structure is presented in consideration of the structure features of an electronic pneumatic braking system.	Three layer control algorithm	three-layer hierarchical structure	This paper presented a braking force distribution control technique based on a fixed relationship between brake pedal displacement and braking deceleration that regulated braking deceleration based solely on brake pedal displacement but independent of mass situation.
14.	Voller	2003	It proposed that radiation is unaffected by rotational speed while convection increases dramatically	The role of aerodynamic in ventilated disc brake cooling.	Experimental research on a ventilated disc brake rotor	Conduction is generally independent of vehicle speed, and its importance is more significant at lower speeds.
15.	Shelar	2014	The rate of heat transmission rose as the number of vanes grew, as did the angular speed.	The heat flux rate, airflow rate, and pressure distribution within the rotors are all measured.	Velocity and temperature distribution technique	Thermal convection using velocity distribution and temperature contours in this analysis.
16.	S. V. Tsinopoulos	1999	In the circumferential direction, quadratic boundary elements, and the FFT algorithm in the generator direction, Gauss quadrature. Singular integrals were tested explicitly and with extreme precision.	The difficulties were defined by axis-symmetric geometry and non-axis-symmetric boundary conditions.	Fast Fourier transform (FFT)	A new boundary element technique (BEM) is used in conjunction with the Fast Fourier transform (FFT) to examine broad axis-symmetric problems in frequency domain elastodynamics.

Table 1. Past methodologies, objective, and literature summary (*continued*)

S. No.	Name of authors	Publication year	Objectives	Parameters/Data used/applications	Methodology used	Summary
17.	Naga Phaneendrai	2018	The goal of this study is to investigate and analyse the rotor disc's temperature distribution and heat dissipation during braking.	Conduction, convection, and radiation are the three ways of heat transmission investigated.	Grey Cast Iron, Aluminium Alloy 6262 T-9, and Carbon-Ceramics are among the materials used.	The temperature distribution on the disc rotor is calculated using a finite element analysis approach, and the critical temperature of the braking rotor disc is obtained.
18.	Abu-Bakar	2007	A new, unworn friction material's contact area expands as wear progresses, and initially, rough surfaces become smoother or glazed	The validation simulation experiments demonstrated the model's suitability, using the most influential variables.	The simulation results also show this the leading edge discovers to be more resistant to wear than the trailing edge.	The wearing method tweaked in this research could forecast wear improvement and significant changes in the topography surface, allowing disc brake squeal to more accurately represent instability analyses.
19.	Sung Pil	2013	For the disc, the gyroscopic matrix is using a uniform planar-mesh technique.	The kinematics between the pads and the disc are described by a coordinate transition between the reference and moving coordinate systems.	the finite element method was used to analyse the squeal of a gyroscopic disc brake	An automobile brake system's dynamic instability investigated involves two stationary pads in contact with a revolving disc.
20.	B.S. Rajan, M.A.S. Balaji, K. Sathi ckbasha,	2018	This research work aims to study the effect of weight (wt) % of the EMP resin in the brake pad material.	To test the friction and wear characteristics, three sets of brake friction composites with the same parent formulation and varying wt percent of EMP resin were produced.	Tribological test using Inertia Brake Dynamometer (IBD)	It was observed that DB10 showed 12.8 % more thermal stability, 37.01 % more fade resistance and 11.54 % improved recovery rate than DB14. DB14 showed high wear resistance at 100, 200 and 300 °C for the speed of 50 km/h.
21.	A. Suleiman, M.R. Tight, A.D. Quinn[163]	2016	The primary focus of this research was to analyse and quantify the effects of several important highways on ambient PM10 concentrations in London.	Particulate matter (PM10) data from ten London monitoring locations were analysed in order to estimate and construct Artificial Neural Network models (ANN)	Artificial Neural Network models (ANN)	The hourly emission rates of the cars were revealed as being the most contributing input variables to the ANN models' outputs, followed by background PM10, toxic gases, and meteorological factors.

Table 1. Past methodologies, objective, and literature summary (*continued*)

S. No.	Name of authors	Publication year	Objectives	Parameters/Data used/applications	Methodology used	Summary
22.	Matthieu Hasco€et , Loïc Adamczak	2020	They developed the TAMIC brake particle collecting system.TAMIC was de-signed to trap at least 80% of brake particles directly at the pad-disc interface without altering braking efficiency	Weighted mass for disc, outside pad, inside pad, Complete TAMIC system	They developed the TAMIC brake particle collecting system.	Tallano's TAMIC groove collection design for collecting brake particles at the source for automotive applications has been tested in a variety of circumstances on a BMW F32 (4 series) calliper. On the bench and in the vehicle, a mass efficiency of more than 80% has been demonstrated.
23.	JRC Science and policy reports	2014	Traffic-related sources contribute significantly towards particulate matter, particularly in metropolitan areas and big cities.	--	Review report	Almost equal contribution of brake wear PM 10 particles and exhaust PM 10 particles
24.	Oliviero Giannini, Adnan Akay, Francesco Massi	2006	The studies are designed to determine the main factors that influence the squeal phenomena and are required to construct a model of the setup.	This study explains the experimental setup known as the "laboratory brake," as well as the data used to define its dynamical and squeal behaviour.	Frequency response function	The laboratory brake is a valuable instrument for investigating brake squeal noises because of the variety of squeal circumstances it can create with reproducibility.
25.	Indian Standard Automotive vehicles – Brakes and braking systems	2001	This standard (Part 8) specifies the methods for testing and verifying braking systems.	Time, pressure, speed distance, deceleration	Dynamic test, Static test	It gives clarification for confirmation of brakes as per IS standards
26.	N. Garg and S. Maji	2016	This report looks back at India's noise regulations and ordinances.	Noise related data from India	--	The suggested modifications and work plan are intended to aid in the implementation of noise abatement action plans for mitigating noise pollution.

Table 1. Past methodologies, objective, and literature summary (*continued*)

S. No.	Name of authors	Publication year	Objectives	Parameters/Data used/applications	Methodology used	Summary
27.	Ambient air pollution: A global assessment of exposure and burden of disease - A report from WHO	2016	This paper summarises the methodologies and findings of the most recent World Health Organization (WHO) worldwide evaluation of ambient air pollution exposure and the related disease burden.	Ambient air pollution data across world.	--	Modelled data of population weighted yearly mean PM2.5 concentrations are used to estimate global air pollution exposure. The data is modelled using a combination of remote satellite sensing data and ground measurements from the 2016 WHO ambient (outside) air quality database, which acts as a calibration for the satellite data.
28.	Mateusz Knapik, Bogusław Cyganek	2019	The study offers a unique method for detecting driver tiredness based on the detection of yawns using thermal imaging.	Custom software for annotation of thermal images.	The use of long-range infrared imaging to detect yawns is proposed.	The suggested facial area detection approach produces excellent results while being easy to implement and quick. Although detections might fail on rapid movements, the eyes corners detection method performs well in both laboratory and real-world circumstances.
29.	Ross Owen Phillips, Fridulv Sagberg	2013	The goal of this work is to develop a model that can be used to guide policies to combat drowsy driving.	Participants	Sample and survey procedure	They claimed that the rate of sleep-driving has decreased from 8.3 percent in 1997 to 2.9 percent in 2008. Driving off the road; excellent road conditions; more distance travelled since the start of the journey; and less years with a driver's licence are all consistently linked to drowsy driving incidents.
30.	Na Yang,Yan Lia, Tao Liu, Jianfeng Wang, Hongguang Zhao	2020	This research was to undertake a statistical analysis of all elements that may contribute to driver fatalities and isolate those that have a greater influence for further investigation.	This research makes use of 902 data sets regarding two-wheel electric vehicle crashes.	The principal component analysis method was used to reduce the dimensionality of the nine factors selected.	The most collision speed was reported at speed of 50-60km/hr at intersections especially for electric vehicles.

The study of temperature dynamics in automotive brake components involves various statistical and mathematical modeling techniques to predict, analyze, and optimize the thermal behavior of brake systems. These models provide insights into how temperature variations impact the performance, reliability, and safety of brake components.

Statistical and Mathematical Analysis

The study of temperature dynamics in automotive brake components involves various statistical and mathematical modeling techniques to predict, analyze, and optimize the thermal behavior of brake systems. These models provide insights into how temperature variations impact the performance, reliability, and safety of brake components.

Regression Analysis

To establish relationships between temperature and various performance metrics of brake components, such as wear rates, friction coefficients, and stopping distances. Regression analysis involves fitting statistical models to experimental data to identify trends and correlations. For example, linear regression can be used to model the relationship between temperatures and wear rate, while multiple regression might be used to account for additional variables such as speed and braking force. Predicting how different temperatures affect brake pad wear, estimating the impact of high temperatures on stopping distance.

Time Series Analysis

To analyze temperature data collected over time to identify patterns, trends, and periodic behaviors in the thermal performance of brake components. Time series analysis involves statistical techniques such as moving averages, autoregressive models, and spectral analysis to analyze temperature data. This helps in understanding how temperature variations over time influence brake performance. Monitoring temperature trends in brake components during prolonged use, predicting future temperature profiles based on historical data.

Finite Element Analysis (FEA)

To simulate the thermal behavior of brake components under various operating conditions, including heat generation, dissipation, and the resulting thermal stresses. FEA involves creating a detailed geometric model of the brake components and dividing it into small elements. Mathematical equations governing heat transfer (conduction, convection, and radiation) are then solved numerically for each element. Predicting temperature distribution across brake discs, identifying hot spots, assessing thermal expansion and induced stresses, optimizing brake design for better heat dissipation.

Computational Fluid Dynamics (CFD)

To model the airflow around brake components and understand how it influences heat dissipation and cooling efficiency. CFD involves solving the Navier-Stokes equations and energy equations for fluid flow and heat transfer around the brake components. The brake disc's geometry and mesh are analyzed considering boundary conditions such as airflow speed and direction. Optimizing vented brake discs and cooling fins, improving airflow designs to enhance cooling performance, reducing the risk of brake fade.

Thermodynamic Modeling

To analyze the energy transformations and heat flows within the brake system during operation. Thermodynamic models use principles of energy conservation and heat transfer to quantify the heat generated by friction and the subsequent temperature rise. These models often involve solving differential equations that describe the thermal behavior of the system. Calculating the total heat generated during braking, estimating the temperature rise in brake components, evaluating the efficiency of cooling mechanisms.

Probabilistic and Stochastic Modeling

To account for the inherent uncertainties and variability in brake component performance due to temperature variations. Probabilistic models use statistical distributions to represent uncertainties in parameters such as material properties, friction coefficients, and external conditions. Stochastic models incorporate random variables and processes to simulate the variability in thermal behavior. Predicting the likelihood of brake failure under extreme temperatures, assessing the reliability of brake components over a range of operating conditions, designing robust brake systems that can tolerate variability.

Optimization Algorithms

To find the optimal design and operating parameters for brake components that minimizes temperature-related performance issues. Optimization algorithms use mathematical techniques such as linear programming, genetic algorithms, and gradient-based methods to identify the best solutions. These algorithms consider multiple objectives, such as minimizing temperature rise, maximizing heat dissipation, and ensuring structural integrity. Designing brake components with optimal material properties and geometries, developing cooling strategies that enhance thermal performance, balancing performance and durability.

Statistical and mathematical modeling of temperature dynamics in automotive brake components provides a comprehensive framework for understanding and optimizing their thermal performance. By employing techniques such as regression analysis, time series analysis, FEA, CFD, thermodynamic modeling, probabilistic and stochastic modeling, and optimization algorithms, engineers can predict and mitigate the adverse effects of temperature variations on brake systems. These models are essential for designing more efficient, reliable, and safe automotive braking systems, ensuring optimal performance under a wide range of operating conditions.

Temperature variations significantly impact the performance of automotive brake components. Through statistical and mathematical modeling, we can understand these impacts in detail, enabling the design of more robust brake systems and the implementation of effective predictive maintenance strategies. These models are essential tools for engineers to predict, mitigate, and manage the thermal challenges faced by modern automotive brake systems, ensuring safety and reliability under diverse operating conditions.

Effective heat dissipation is crucial for maintaining optimal brake performance. Vented discs and cooling fins are common design features. The efficiency of these mechanisms varies with temperature, impacting overall brake performance. CFD models simulate airflow and heat dissipation around brake components. These models help optimize cooling designs by analyzing how different configurations perform under varying thermal loads.

Statistical models incorporate temperature data to predict changes in stopping distance. These models use historical performance data and real-time temperature monitoring to provide accurate predictions.

Optimization algorithms use temperature data to enhance brake design. Predictive maintenance models use statistical analysis of temperature trends to schedule timely maintenance interventions. Understanding temperature effects helps in designing brake systems that can withstand higher temperatures without performance degradation.

Statistical and mathematical models enable predictive maintenance by forecasting when brake components will reach critical temperature thresholds that necessitate servicing.

Temperature - time relation θ = θ (x, t)

$$\frac{\partial^2\theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial\theta}{\partial t} \tag{1}$$

Where,

$\theta = X(x).Y(t)$

$$\frac{\partial\theta}{\partial x} = Y\frac{\partial X}{\partial x}$$
$$\frac{\partial\theta}{\partial t} = X\frac{\partial Y}{\partial t} \tag{2}$$

$$\frac{\partial^2\theta}{\partial x^2} = Y\frac{\partial^2 X}{\partial x^2} \tag{3}$$

Substitute equation (2) & (3) in equation (1)

$$Y\frac{\partial^2 X}{\partial x^2} = \frac{1}{\alpha}X\frac{\partial Y}{\partial t}$$

$$\frac{1}{X}\frac{\partial^2 X}{\partial x^2} = \frac{1}{\alpha Y}\frac{\partial Y}{\partial t} = -\lambda^2$$

$$\frac{1}{\alpha Y}\frac{\partial Y}{\partial t} = -\lambda^2$$

$$\frac{1}{Y}dY = -\lambda^2\alpha\partial t$$

$$lnY = -\lambda^2\alpha t + lnC_1$$

$$Y(t) = C_1 e^{-\alpha\lambda^2 t}$$

Where C_1 is constant

$$\frac{1}{\mathrm{X}}\frac{\partial^2\mathrm{X}}{\partial \mathrm{x}^2} = -\lambda^2$$

$$\frac{\partial^2\mathrm{X}}{\partial \mathrm{x}^2} + \lambda^2\mathrm{X} = 0$$

The characteristics equation

$$m^2 + \lambda^2 = 0$$

$$m = \pm i\lambda$$

$$\mathrm{X(x)} = \mathrm{C_2 Cos\lambda x + C_3 Sin\lambda x}$$

$$\theta = \mathrm{X(x).Y(t)} \tag{4}$$

$$\theta = \mathrm{C_1 e^{-\alpha\lambda^2 t}(C_2 Cos\lambda x + C_3 Sin\lambda x)}$$

$$\theta = \mathrm{e^{-\alpha\lambda^2 t}(C_1 Cos\lambda x + C_2 Sin\lambda x)}$$

Hence, constant λ, C_1, and C_2 evaluated from the initial & boundary conditions

$$(i) at t = 0, \theta = \theta_i = T_i - T_\infty$$

$$(ii) at x = 0, \frac{\partial\theta}{\partial x} = 0$$

$$(iii) at x = l, q = -\left(k\frac{\partial\theta}{\partial x}\right) = h(T_i - T_\infty) = h\theta_i$$

$$\frac{\partial\theta}{\partial x} = e^{-\alpha\lambda^2 t}(-\lambda C_1 Sin\lambda x + \lambda C_2 Cos\lambda x)$$

at x =0

$C_2 = 0$

$$\theta = C_1 e^{-\alpha\lambda^2 t} Cos\lambda x$$

$$\left(\frac{\partial\theta}{\partial x}\right)_{x=l} = -C_1\lambda e^{-\lambda^2\alpha t} Sin\lambda l = \frac{-h}{k}\theta_1 = \frac{h}{k}\left[-C_1 e^{-\lambda^2\alpha t} Cos\lambda l\right] \quad (5)$$

$$Cot\lambda l = \frac{\lambda k}{h} = \frac{\lambda l}{Bi}$$

It is satisfied for an infinite succession of value of the parameter λl, so that for a given λ, the equation defines the values of λ. This succession of the value of λ called Eigenvalues will be λ_n which depends on the Biot number.

The temperature distribution

$$\theta = \sum_{n=1}^{\infty} C_n\, e^{-\lambda_n^2\alpha t} Cos\lambda_n \phi (6)$$

n th root of the transcendental equation is λ_n

$$cot\lambda_n l = \frac{\lambda_n l}{Bi}$$

$$\lambda_n l\; tan\lambda_n l - Bi = 0$$

Following that, the value of C_n for each value of λ_n is next to determined. Using the condition (i), at t=0, $\theta = \theta_i$

$$\theta_i = \sum_{n=1}^{\infty} C_n\, cos\lambda_n \phi \quad (7)$$

Fourier Cosine series equation

$$\theta_i = C_1 cos\phi + C_2 cos2\phi + C_3 cos3\phi + \cdots C_n cosn\phi$$

$$C_n = \frac{2}{l}\int_0^l \theta_i cos\lambda_n \phi\, dx$$

$$\lambda_n = 1,2,3 \ldots\ldots$$

But λ_n is not an integer and is the root of the trigonometric equation:

$$\lambda_n l\; tan\lambda_n l - Bi = 0$$

$$\theta_i = \int_0^l cos\lambda_m \phi\, dx = \sum_{n=1}^{\infty} C_n \int_0^l cos\lambda_m \phi\; cos\lambda_n \phi\, d\phi \quad (8)$$

$$\frac{1}{2}\int_0^l 2cos\lambda_m x cos\lambda_n \phi\, d\phi = \frac{1}{\lambda_m^2 - \lambda_n^2}[\lambda_m Sin\lambda_m l . Cos\lambda_n l - \lambda_n Cos\lambda_m l . Sin\lambda_n l] \quad (9)$$

$$(\lambda_n l)\, tan(\lambda_n l) = Bi = (\lambda_m l)\, tan(\lambda_m l)$$

$$(\lambda_n l)\, sin(\lambda_n l)\, cos(\lambda_m l) = Bi = (\lambda_m l)\, sin(\lambda_m l)\, cos(\lambda_n l)$$

If m = n, the integrand of RHS equation (9), will be Zero, Putting $\lambda_m = \lambda_n$ equation (4)

$$\theta_i \frac{sin\lambda_n l}{\lambda_n} = C_n \int_0^l cos^2\lambda_n \phi\, d\phi$$

After solving

$$C_n = \frac{2\theta_i sin\lambda_n l}{\lambda_n l + sin\lambda_n l . cos\lambda_n l}$$

For convenience

$$\lambda_n l = \delta n$$
$$\lambda_n l . tan(\lambda_n l) = \delta n\; tan\; \delta n = Bi$$

$$\theta = \sum_{n=1}^{\infty} e^{-\delta_n^2 \frac{\alpha t}{l^2}} \frac{2\theta_i sin\delta_n}{\delta_n + sin\delta_n cos\delta_n} . Cos\left(\frac{\delta_n}{l}\phi\right)$$

The following formula uses to compute the temperature distribution:

$$\frac{\theta}{\theta_i} = \sum_{n=1}^{\infty} e^{-\delta_n^2 F_0} \frac{2\theta_i sin\delta_n}{\delta_n + sin\delta_n cos\delta_n} . Cos\left(\frac{\delta_n}{\phi} l\right) = \frac{T - T_\infty}{T_i - T_\infty} \quad (10)$$

Equation (10) is the result of applying Fourier's series to the problem. When the value of x is small, a polynomial expression derived from this finding will provide a high degree of accuracy. Temperature T calculates using the Fourier Transform method at any x distance from the mid-plane.

CFD Analysis and Boundary Conditions

CFD (Computational Fluid Dynamics) analysis is a powerful tool used to study the thermal and fluid dynamics behavior of disc brakes. By employing finite volume and finite element analysis, researchers can predict temperature distributions, fluid flow patterns, and heat dissipation efficiency.

Finite Volume Analysis (FVA)

FVA is commonly used in CFD to discretize the fluid domain into small control volumes. This approach ensures the conservation of mass, momentum, and energy within each control volume. The brake disc and surrounding air are divided into a grid of control volumes. The governing equations for fluid flow and heat transfer (Navier-Stokes equations, energy equation) are solved for each control volume. FVA provides detailed insights into the velocity, pressure, and temperature fields around the brake disc, highlighting areas with high heat transfer rates and potential cooling inefficiencies.

Finite Element Analysis (FEA)

FEA is used to model the structural and thermal behavior of solid components, such as the brake disc and pads. It is particularly useful for analyzing stress distribution and thermal gradients within the solid material. The brake components are meshed into finite elements, and the heat conduction equation is solved for each element. Coupled simulations may include both thermal and structural analyses to evaluate the impact of thermal stresses. FEA helps identify regions with high thermal stress and deformation, guiding design improvements to enhance structural integrity and thermal performance.

MODELS USED FOR ANALYSIS

Thermal Model

To simulate the heat generation and dissipation processes within the brake system. Heat generation due to friction is modeled as a boundary heat flux, and convective heat transfer coefficients are applied to the surfaces exposed to airflow.

Fluid Flow Model

To simulate the airflow around the brake disc and its impact on cooling. The Navier-Stokes equations for incompressible flow, along with turbulence models (k-ε) to capture the turbulent nature of the airflow. Steady or transient flow conditions are assumed based on the braking scenario being analyzed.

Coupled Thermal Model

To evaluate the combined effects of thermal and mechanical loads on the brake components. Material properties are temperature-dependent, and thermal strains are considered in the structural analysis.

The design and development of automotive disc brakes demand a comprehensive understanding of braking principles, component systems, and their mechanical and thermal constraints. Effective brake design must consider various operational conditions, thermal stresses, and material limitations. Recent advancements in disc brake technology have been driven by both analytical and experimental research.

Researchers face several major difficulties during experimentation, including:

Table 2. CFD Parameters

Parameters	Inputs
Solver type	Pressure based
Model used	k-ε
Reference values	Computed from inlet
Fluid used	80, 90, 100 km/h
Pressure outlet	1 atm

Thermal Management

Managing heat transfer in brake components is challenging due to the high temperatures generated during braking and the rapid changes in operating conditions. Utilizing advanced thermal simulation tools and optimizing cooling mechanisms can improve heat dissipation and control thermal stresses. A controlling and predicting temperature variation in brake components is complex due to the rapid changes in speed and load during braking.

Accuracy of Performance Models

Accurately modeling brake disc performance under different conditions, such as varying vehicle speeds, pedal efforts, and friction coefficients, is complex and requires sophisticated simulation methods. Developing detailed combined mechanical and thermal performance models and validating them with experimental data enhances prediction accuracy.

Design Optimization

Balancing design requirements such as surface area, drag value, weight, and cost while ensuring optimal performance under diverse conditions is difficult. Implementing optimization algorithms and conducting extensive testing under varying conditions can help achieve the desired balance between performance and cost.

CONCLUSION

The mathematical findings in this study diverge from those reported in previous literature, yet they align with the results commonly observed in literature examinations. To further validate the computational findings, it would be valuable to explore the thermal durability of rotor brakes through advanced computational modeling.

This study offers a thorough exploration of various statistical and computational modeling techniques used to elucidate thermal dynamics in automobile braking systems. Significant progress has been made in accurately predicting temperature variations in braking linings and rotors by integrating statistical analyses with computational models. Through an extensive review of existing literature, this research has identified key parameters affecting heat behavior in brake systems.

Future research should focus on refining existing models, incorporating additional variables, and enhancing the precision of temperature predictions. Moreover, it is crucial to experimentally validate these mathematical models under diverse operating conditions to ensure their reliability and applicability in real-world scenarios. Ultimately, the insights gained from this comprehensive study will contribute to the advancement of heat management in automotive cooling systems, leading to improvements in vehicle safety, reliability, and overall performance.

- The mathematical findings of this study diverge from those reported in previous literature, yet they are consistent with widely observed results. This discrepancy underscores the need for further validation of the computational results through advanced modeling of rotor brake thermal durability.

- The insights gained from this study will advance heat management in automotive cooling systems, ultimately improving vehicle safety, reliability, and performance.
- Future research should concentrate on enhancing existing models, incorporating additional variables, and increasing the accuracy of temperature predictions. It is also essential to emphasize experimental validation under various operating conditions to ensure the models' relevance and applicability in real-world scenarios.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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