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Research Article

Dynamics of a prey-predator-scavenger model with Holling type IV functional response

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ABSTRACT

In this paper, a non-linear system of differential equations is presented as a mathematical model to explain the interactions of three species in an ecosystem, which include prey, predator, and scavenger. The model takes into account the logistic growth of the prey population as well as the inter-species interactions. The paper uses local stability analysis to examine the system's characteristics, including positivity, the boundedness of the solution, and the prerequisites for the three populations' stable coexistence. The existence of limit cycles in the positive quadrant, a crucial component of the dynamics of ecological systems, and the system's persistence requirement are also examined in this work. The article also includes numerical simulations to support the theoretical study and provide a clearer picture of the ecosystem's long-term dynamics.

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INTRODUCTION

Ecology is a scientific field that focuses on examining the relationships between organisms and their environment, including how they interact with each other. To understand the dynamics of intricately interconnected populations within an ecosystem, mathematical modeling is widely used [1-4].The predator-prey model, also known as the Lotka-Volterra model, is a widely recognized and established model in the field of mathematical ecology. It uses two connected non-linear differential equations to represent the interactions between a single population of predators and prey [5-8]. Researchers have broadened the scope of the Lotka-Volterra model to encompass multiple species, yet very few investigations have examined scavengers within predator-prey models. Scavengers are essential to the cleansing of the environment. These critters are described as eating decaying plant or animal matter, animal remains, or both. Scavenger animals can therefore be herbivores or omnivores. Scavengers rarely hunt their own prey, despite having the ability to do so [9-12].

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Animals that are scavengers devour the remains of other animals that have either died naturally or have been killed by predators but have not been eaten by other animals. In essence, they are animals who consume corpses. Hyenas, vultures, and raccoons are a few well-known scavengers. By eating the left overs of animals killed by predators as well as plants or animals that have naturally passed away, scavengers serve a crucial part in the food chain. Decomposers break down the bones and other waste materials left behind after scavengers have cleaned the carcass. In essence, scavengers eat the leftover food scraps, which can include carrion, dead plants, diseased animals, and human-killed animals like those discovered as road kill or while hunting [13-15].

The classification of scavenger's animal contains a large number of mammals, fish, and insects which can be further broken down into smaller categories. There exist two primary categories of scavengers: obligate scavengers and facultative scavengers. Additionally, termites also fall into this class. Regardless of their type, scavenger animals can be found all over the world and can adapt to almost any type of ecosystem [16-18].

Obligate scavengers are animals that primarily depend on carrion, which refers to the meat and tissues of dead or decaying organisms, for their dietary needs. (Ex: vultures' species are the best example of obligate scavenges. While most scavengers search on the ground, vultures can glide overhead, which consumes little energy and gives them a better view of the area so that they may locate food more quickly. Accipitridae and Cathartidae are two other families of birds considered obligate scavengers [19].

Facultative scavengers are both scavengers and predators. Due to their hunting habits, they require higher amounts of energy, and hence their diet comprises of substantial quantities of food in a single meal. For example, a road kill animal is favorable for a facultative scavenger because the dead animal has not been eaten by another predator. In urban areas, raccoons and opossums also feed on trash left behind by humans [20,21].

As herbivores, termites play a crucial role in maintaining the ecosystem by eliminating dead plant matter. They act as decomposers when they feed on the waste and bones left behind. Termites construct their nests using decaying plant material, which facilitates the transfer of nutrients back into the soil. Blowflies feed on the remains of decaying plants [22].

Studies by Sadhukhan [1], Jansen and Van Gorder [2], and Ahmed and Bahlool [3] investigate the dynamics of prey, predators, and scavengers, considering factors such as harvesting, fear effects, and stability. Mellard et al. [4] and Alidousti [8] explore the ecological consequences of scavenging behavior, highlighting its impact on predation dynamics and system stability. Wilmers and Getz [9] and Colomer et al. [13] examine trophic interactions in ecosystems, including the flow of resources to scavengers and the role of top predators in facilitating scavenging. Studies by Gupta and Chandra [11], Selvam et al. [16], and Abdul Satar and Naji [10] employ various modeling techniques, such as quadratic harvesting and fractional-order modeling, to analyze prey-predator-scavenger systems. Empirical studies by Fodrie et al. [17], Manfrin et al. [21], and Brown et al. [22] provide insights into real-world scavenging dynamics, including dietary changes in predators, invasive species impacts, and reproductive costs in scavenger flies.

These works collectively underscore the complexity of three-species ecosystems and highlight the need for further research to elucidate the mechanisms driving prey-predator-scavenger interactions. Our study contributes to this body of literature by developing a novel mathematical model that integrates prey, predators, and scavengers, offering insights into the dynamics and stability of such ecosystems.

Several authors [23-25] have investigated the model under various parameters with scavenger, prey predator species. Ben Nolting and their team analyzed a population model comprising of a predator, its prey, and a scavenger, utilizing a three-species model. They incorporated an equation for the scavenger population into the traditional Lotka-Volterra equations. In order to show the interactions of prey, predator, and scavenger populations, a model that emphasizes stable cohabitation as a stable equilibrium point will be developed in this work. The study investigates positivity, boundedness, examination of local and global stability, and model persistence. Based on the analytical conclusions, numerical simulations are run to show that the system is in a stable state. The research also examines the relationship between the rate of scavenger assault and the rate of intrinsic growth of the prey [26-28].

The novelty of the current study lies in its exploration of a dynamic three-dimensional ecological system comprising prey, predators, and scavengers. It uniquely emphasizes the significance of intraspecies competition and the adaptive role of scavengers in mitigating food supply fluctuations. The study identifies scenarios leading to scavenger extinction due to mortality rate fluctuations and unveils the importance of insecure environmental conditions for species coexistence. Methodologically, advanced mathematical tools are employed to establish system stability and persistence, contributing to a deeper understanding of ecological dynamics [29,30].

Future research directions could involve deeper exploration of intraspecies competition mechanisms, investigation into scavenger adaptation to environmental changes, and examination of factors influencing stability and persistence in ecological communities. Additionally, empirical validation of theoretical findings, interdisciplinary collaborations, and innovative modeling approaches could further enhance our understanding of ecological dynamics and inform conservation efforts.

By conducting a meticulous analysis that combines theoretical insights with empirical simulations, we bridge the gap between mathematical abstraction and real-world applicability. While existing literature has predominantly explored local stability and basic dynamics, our work advances the discourse by incorporating essential elements that influence the long-term behavior of the ecosystem. This contribution is particularly vital in a landscape where holistic, dynamic models are lacking. The interplay of unpredictability and stability uncovered in our study presents a different picture of ecological resilience, shaping our understanding of how these ecosystems adapt to various challenges. The conclusions drawn from our study stands as a unique and impactful contribution to the field of ecological sciences.

MATHEMATICAL MODEL FORMULATION

$$\frac{dX}{d\tau} = RX\left(1 - \frac{A}{R}X - \frac{B}{R}Y\right) - \frac{XY}{1+X^2}$$
$$\frac{dY}{d\tau} = RY\left(-\frac{C}{R} + \frac{D}{R}X\right) + \frac{XY}{1+X^2} \qquad (1)$$
$$\frac{dZ}{d\tau} = RZ\left(-\frac{E}{R} + \frac{F}{R}X + \frac{G}{R}Y + \frac{H}{R}XY - \frac{I}{R}Z\right)$$

where
- R is the natural growth rate of the source prey.
- A is the intensity of competition among individuals of the same prey species.
- B is the rate of coefficient of predation, which quantifies the effect of predator attacks on the prey population.
- C is the intrinsic mortality rate of predators.
- D is the enhancement in the reproductive rate of predators for each prey consumed.
- E is the inherent mortality rate of scavengers.
- F is the efficiency of scavengers in benefiting from the corpses of prey that die due to natural causes.
- G is the efficiency of scavengers in benefiting from the corpses of predators that die due to natural causes.
- H is the effectiveness of scavengers in exploiting the carcasses of prey that have been killed by predators.
- I is the competition within the same species of scavengers.

Now, we transform

$$X = \frac{R}{A}x,\ Y = \frac{R}{B}y,\ Z = \frac{R}{I}z, \tau = Rt,$$

then the model (1) takes the following form

$$\frac{dx}{dt} = rx(1 - x - y) + \frac{\alpha xy}{a + r^2x^2}$$
$$\frac{dy}{dt} = ry(-c + dx) + \frac{\beta xy}{a + r^2x^2} \qquad (2)$$
$$\frac{dz}{dt} = rz(e + fx + gy + hxy - z)$$

where $c = \frac{C}{R}, d = \frac{D}{R}, e = \frac{E}{R}, g = \frac{G}{B}, h = \frac{FR}{AB}, f = \frac{F}{R}, g = \frac{G}{R}$

Positivity of the Solution

To demonstrate the biological significance and lucidity of model (2.1), it is imperative to ensure that all variables denoting populations are positive. This theorem guarantees that system (2.1) solutions are positive with regard to the specified beginning conditions.

Theorem

The spaceare $H_1 = \left\{\frac{(x,0,0)}{x} > 0\right\}$, $H_2 = \left\{\frac{(x,0,z)}{x} > 0, z > 0\right\}$, $H_3 = \left\{\frac{(x,y,0)}{y} > 0, z > 0\right\}$, $H_4 = \{(0,y,z)/y > 0,\ z > 0\}$ systemically invariant (1).

Proof

The phase space (1) is the positive octant $R_+^3 = \{(x,y,z)\epsilon R^3 : x \geq 0, y \geq 0, z \geq 0\}$ and since if $x = 0$, then $\dot{x}, \dot{y}, \dot{z} \geq 0$. Consequently, none of the system's trajectories can intersect the coordinate planes, making them invariant sets of system (1). When the initial value of the solution is positive, it will persist in the positive octant R_+^3 indefinitely, guaranteeing the presence of a non-negative solution with positive initial conditions [4, 10].

The three functions

$$f_1(x,y,z) = x(1 - x - y),$$
$$g_1(x,y,z) = y(-c + dx)$$
$$h_1(x,y,z) = z(-e + fx + gy + hxy - z$$

are continuous in the positive octant $R_+^3 = \{(x,y,z)\epsilon R^3 : x \geq 0, y \geq 0, z \geq 0\}$ and

$$\lim_{(x,y,z)\to(0,0,0)} f_1(x,y,z) = \lim_{(x,y,z)\to(0,0,0)=} g_1(x,y,z) =$$
$$\lim_{(x,y,z)\to(0,0,0)} = h_1(x,y,z) = 0$$
$$f_1(0,0,0) = g_1(0,0,0) = h_1(0,0,0) = 0\,.$$

As a result, the model's positive solution (1) is real and distinct.

BOUNDEDNESS OF SOLUTION

Theorem

All solutions of $(x(t), y(t), z(t))$ of (1) are bound with favorable initial conditions.

Proof

(i) Assume $x(0) = x_0 > 0, y(0) = y_0, and\ z(0) = z_0 > 0$, then from (1) we get

$$\frac{dx}{dt} = x(1-x-y) \leq x(1-x)$$

$$x(t) \leq \frac{1}{1+e^{-(t+C_1)}} \text{where } C_1 = ln\left(\frac{x_0}{1-x_0}\right)$$

$$x(t) \leq \frac{x_0}{x_0 + (1-x_0)e^{-t}}$$

$$\lim_{t\to\infty} x(t) \leq 1$$

Thus, $x(t)$ is bounded.

(ii) Considering any real numbers as "a" and "b," such that $a > b > 0$ and $d = \frac{D}{R} > 0$, then we get

$$adx'(t) + by'(t) = adx(t) - adx(t)y(t) - adx^2(t) - cby(t) + bdx(t)y(t)$$

Let $S(t) = adx(t) + by(t)$ and $0 < \gamma < c$, then

$$S'(t) + \gamma S(t) = ad(\gamma+1)x(t) + (-a+b)dx(t)y(t) - adx^2(t) + (\gamma - c)by(t) \leq ad(\gamma+1)$$

Hence S(t) is bounded. Since $x(t)$ is bounded, $y(t)$ is also bounded.

(iii) Let $l \epsilon R$ such that $l > 0$ which satisfies

$$fx(t) + gy(t) + hx(t)y(t) \leq l,$$

$fx(t) + gy(t) + hx(t)y(t) \leq l$, for all $t\epsilon[0,\infty)$

$$\frac{dz}{dt} = z(-e + fx + gy + hxy - z) \leq (-e + l - z)$$

$$z(t) \leq \frac{am}{a+e^{-(mt)}}, a = \frac{z_0}{m - z_0}.$$

For $t \to \infty$, we have $z(t) \leq m$
Thus, $z(t)$ is bounded.

Existence of Equilibria

When population growth rates cease, the dynamical system reaches its equilibrium point. The fact that equilibrium circumstances are non-negative has biological significance.

(i) When $x = y = z = 0$, a trivial equilibrium point is denoted by $E_0(0, 0, 0)$.
(ii) In cases where the prey population x is non-zero, but the predator and scavenger populations y and z are both zero we get $x = 1$ which is denoted by $E_1(1, 0, 0)$.
(iii) When prey and predator population $x = y = 0$ but scavengerpopulation$z \neq 0$, we get $z = -e$. Thus, we have $(0, 0, -e)$. Yet, because the z coordinate is negative, this equilibrium position seems implausible.
(iv) When prey and scavenger population $x \neq 0, z \neq 0$ and predatorpopulation $y = 0$, we get $x = 1, z = h - e$ and is denoted by $E_2(1, 0, h - e)$ and it exists if $h - e > 0$.
(v) If the prey and predator populations x and y are non-zero, but the scavenger population z is zero, then

$$y = \frac{(x-1)(a+r^2x^2)}{\alpha - a - r^2x^2}, x > 1, \alpha - a - r^2x^2 > 0 \quad (3)$$

$$r^2dx^3 - cr^2x^2 + (ad+\beta)x - ac = 0$$
$$a_3x^3 + a_2x^2 + a_1x + a_0 = 0 \quad (4)$$

where, $a_3 = r^2d$, $a_2 = -cr^2$, $a_3 = ad + \beta$, $a_0 = -ac$

By Descarte's rule of sign, equation (4) has either two positive roots and one negative root or three positive roots depending on the condition $ad > -\beta$.

(vi) Finally, the co-existence equilibrium point $E(x^\star, y^\star, z^\star)$ exists if

$$z^\star = -e + fx^\star + gy^\star + hx^\star y^\star$$

$$y^\star = \frac{r^3x^3 - r^3x^{\star 2} + arx^\star - ar}{\alpha - r^3x^{\star 2} - ar}$$

and $x^\star$ is the positive root of the cubic equation

$$b_3x^{\star 3} + b_2x^{\star 2} + b_1x^\star + b_0 = 0$$

where $b_3 = r^3d$, $b_2 = -cr^3$, $b_1 = ard + \beta$, $b_0 = -acr$

ANALYSIS OF THE EQUILIBRIUM POINT STABILITY

Local Stability Analysis

This section examines the system's (2.2) local stability near its positive equilibrium point $E_0(0, 0, 0)$, $E_1(1, 0, 0)$, $E_2(1, 0, f - e)$, $E(x^\star, y^\star, z^\star)$.

The Jacobian matrix $J(x, y, z) = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{21} & D_{22} & D_{23} \\ D_{31} & D_{32} & D_{33} \end{pmatrix}$

where,

$$D_{11} = \frac{(r - 2rx - ry)(a + r^2x^2)^2 + 2\alpha r^2x^2y + 2ay + 2r^2x^2y}{(a + r^2x^2)^2},$$

$$D_{12} = \frac{\alpha x - ray - r^3x^2y}{a + r^2x^2}, \quad D_{13} = 0$$

$$D_{21} = \frac{dry(a + r^2x^2)^2 + 2\beta r^2x^2y + a\beta y + \beta r^2x^2y}{(a + r^2x^2)^2},$$

$$D_{22} = \frac{\beta x + (drx - cr)(a + r^2x^2)}{a + r^2x^2}, \quad D_{23} = 0$$

$$D_{31} = rzf + rzhy, D_{32} = rzg + rzhx,$$

$$D_{33} = re + rfx + rgy + rhxy - 2rz$$

(i) The Jacobian matrix at $E_0(0, 0, 0)$ is

$$J(E_0) = \begin{pmatrix} r & 0 & 0 \\ 0 & -rc & 0 \\ 0 & 0 & re \end{pmatrix}$$

The eigenvalues of the Jacobian matrix at E_0 are $\lambda_1 = -r, \lambda_2 = -rc, \lambda_3 = -re$

Therefore, the equilibrium point E_0 is an unstable saddle point.

(ii) The Jacobian matrix at $E_1(1, 0, 0)$ is

$$J(E_1) = \begin{pmatrix} -r & \frac{\alpha}{a + r^2} & 0 \\ 0 & \beta + r(d - c) & 0 \\ 0 & 0 & -re + rf \end{pmatrix}$$

The eigenvalues of the Jacobian matrix at E_1 are $\lambda_1 = -r, \lambda_2 = \beta + r(d - c), \lambda_3 = rf - rc$

Consequently, the equilibrium points E_1 along the axial direction is a stable node if d < c and $f < e$, which signifies the persistence of the prey population while the predator and scavenger populations will go extinct.

(iii) The Jacobian matrix at $E_2(1, 0, f - e)$ is

$$J(E_1) = \begin{pmatrix} -r & \frac{\alpha}{a + r^2} & 0 \\ 0 & \beta + r(d - c) & 0 \\ rf(f - e) & rg(f - e) & -re + rf - \alpha r(f - e) \end{pmatrix}$$

The eigenvalues of the Jacobian matrix at E_1 are

$$\lambda_1 = -r, \qquad \lambda_2 = \beta + r(d - c),$$

$$\lambda_3 = rf - rc - \alpha r(f - e)$$

Therefore, the equilibrium point E_2 is a stable node if $d < c$ and $f < e$

Theorem

The interior equilibrium point $E^\star(x^\star, y^\star, z^\star)$ is locally asymptotically stable if $C_1 > 0, C_3 > 0$, and $C_1C_2 - C_3 > 0$.

Proof

The characteristic equation of the Jacobian matrix $J(E^\star)$ is given by

$$\lambda^3 + C_1\lambda^2 + C_2\lambda + C_3 = 0$$

where $C_1 = -c_{11} - c_{22}$

$$C_2 = c_{11}c_{22} - c_{23}c_{32} - c_{21}c_{12} - c_{13}c_{31}$$

$$C_3 = c_{11}c_{22}c_{32} + c_{13}c_{22}c_{31} - c_{12}c_{23}c_{31} - c_{13}c_{21}c_{32}$$

where,

$$c_{11} = \frac{(r - 2rx^\star - ry^\star)\left(a + r^2x^{\star 2}\right)^2 + 2\alpha r^2x^{\star 2}y^\star + 2ay^\star + 2r^2x^{\star 2}y^\star}{\left(a + r^2x^{\star 2}\right)^2},$$

$$c_{12} = \frac{\alpha x^\star - ray^\star - r^3x^{\star 2}y^\star}{a + r^2x^{\star 2}}, \qquad c_{13} = 0$$

$$c_{21} = \frac{dry^\star\left(a + r^2x^{\star 2}\right)^2 + 3\beta r^2x^{\star 2}y}{\left(a + r^2x^{\star 2}\right)^2},$$

$$c_{22} = \beta x^\star + (drx^\star - cr), \qquad c_{23} = 0$$

$$c_{31} = rz^\star f + rz^\star hy^\star, \qquad c_{32} = rz^\star g + rz^\star hx^\star,$$

$$c_{33} = -re + rfx^\star + rgy^\star + rhx^\star y^\star - \alpha rz^\star$$

According to Routh-Hurwitz criteria, all the roots of the characteristic equation are negative real numbers or complex roots with negative real parts if $C_1 > 0$, $C_3 > 0$, and $C_1C_2 > C_3$. Therefore, the interior equilibrium point $E^\star(x^\star, y^\star, z^\star)$ is locally asymptotically stable.

GLOBAL STABILITY ANALYSIS

In this part, we examine the equilibrium points' overall stability. Here, we build the appropriate Lyapunov function to demonstrate the system's (2) interior equilibrium point's global stability.

Furthermore, the Dulac function is employed to establish the non-existence of a limit cycle surrounding the equilibrium point.

Theorem

The interior equilibrium point $E^\star(x^\star, y^\star, z^\star)$ is globally asymptotically stable if

$\frac{y}{y^\star} > 1, \frac{x}{x^\star} > 1, \frac{z}{z^\star} > 1$

Proof

Let us choose a suitable Lyapunov function

$$V = (x - x^\star) - x^\star ln\frac{x}{x^\star} + (y - y^\star) - y^\star ln\frac{y}{y^\star} + \frac{1}{\alpha}\left[(z - z^\star) - z^\star ln\frac{z}{z^\star}\right]$$

Differentiating the above equation with respect to time, it is obtained that

$$\frac{dV}{dt} = \left(\frac{x - x^\star}{x}\right)\frac{dx}{dt} + \left(\frac{y - y^\star}{y}\right)\frac{dy}{dt} + \frac{1}{\alpha}\left(\frac{z - z^\star}{z}\right)\frac{dz}{dt}$$

Using the set of differential equations (2.2) and definition of equilibrium points, we have

$$\frac{dV}{dt} = (x - x^*)\left[rx(1 - x - y) + \frac{\alpha xy}{a + r^2x^2} - rx^*(1 - x^* - y^*) + \frac{\alpha x^* y^*}{a + r^2 x^{*2}}\right] + (y - y^*)\left[ry(-c + dx) + \frac{\beta xy}{a + r^2x^2} - ry^*(-c + dx^*) - \frac{\beta x^* y^*}{a + r^2 x^{*2}}\right] + (z - z^*)[rz(e + fx + gy + hxy - z) - rz^*(e + fx^* + gy^* + hx^*y^* - z^*)]$$

$$\frac{dv}{dt} = r\left[(x - x^*)^2 + \left(x^2 - x^{*2}\right)(x - x^*) + (xy - x^*y^*)(x - x^*) + \left(z^2 - z^{*2}\right)(z - z^*)\right] - rc[(y - y^*)^2 - (z - z^*)^2] + r[d(y - y^*)(xy - x^*y^*) + f(z - z^*)(xz - x^*z^*) + g(z - z^*)(yz - y^*z^*)] + rh(xyz - x^*y^*z^*)(z - z^*) + \frac{xy\left(a + r^2x^{*2}\right) + x^*y^*(a + r^2x^2)[\alpha(x - x^*) + \beta(y - y^*)]}{(a + r^2x^2)\left(a + r^2x^{*2}\right)}$$

$$\frac{dv}{dt} \le r\{\left[(x - x^*)^2 + \left(x^2 - x^{*2}\right)(x - x^*) + (xy - x^*y^*)(x - x^*) + \left(z^2 - z^{*2}\right)(z - z^*)\right] + c[(y - y^*)^2 - (z - z^*)^2] + [d(y - y^*)(xy - x^*y^*) + f(z - z^*)(xz - x^*z^*) + g(z - z^*)(yz - y^*z^*)] + h(xyz - x^*y^*z^*)(z - z^*)\}$$

If $\frac{y}{y^*} > \frac{x}{x^*} > 1$ and $\frac{z}{z^*} > 1$ holds, then $\frac{dv}{dt} \le 0$ if $\beta^2 < 4\alpha$

Persistence of the Model

If $x(0) > 0$ and $\lim_{t\to\infty} inf\, x(t) > 0$ which means that each population $x(t)$ remains positive over time, if each component of the system persists, the system is regarded as persistent. The population's long-term survival is referred to as persistence. Below, we demonstrate how system (2.2) endures.

To show that $y(t)$ persist, consider

$$\frac{dy}{dt} = ry(-c + dx)$$

$$\frac{dy}{dt} = -rcdt$$

$$y > e^{-rct + C_1}$$

$$= Ae^{-rct},$$

where $A = e^{C_1}$

But $y(0) > Ae^0 = A > 0$

and $lim \operatorname*{int}_{t\to\infty} y(t) \ge \lim_{t\to\infty} \frac{A}{e^{rct}} = 0$

Hence, y(t) continues.

If x(t) persists, then consider

$$\frac{dx}{dt} = rx(1 - x - y) > rx(-x - y) = -rx(x + y)$$

Because y(t) is constrained, we have $y(t) \le M$ for some $M > 0$

$$x + y \le x + M$$

$$-x(x + y) \ge -rx(x + M)$$

$$\frac{dx}{dt} > -rx(x + y) \ge -rx(x + M)$$

$$\frac{x}{x + M} > e^{-Mrt + MC_2}$$

$e^{MC_2}e^{-Mrt} = Be^{-Mrt}$ where $B = e^{MC_2}$

$$x > \frac{BMe^{-Mrt}}{1 - Be^{-Mrt}} = \frac{BM}{e^{Mrt} - B}$$

$x(0) > \frac{BM}{1-B} > 0$ for $0 \le B < 1$

$$\lim_{t\to\infty} inf x(t) \ge \lim_{t\to\infty} \frac{BM}{e^{Mrt} - B} = 0$$

Thus $x(t)$ persists if $0 \le B < 1$

To prove $z(t)$ persists:

$$\frac{dz}{dt} = rz(-e_1 + hxy + fx + gyz) > rz(-e_1 - z)$$

$$= -rz(e_1 + z)$$

$$\frac{dz}{z(e_1 + z)} > -rdt$$

$$ln\frac{z}{e_1 + z} > -re_1t + e_1C_2$$

$$\frac{z}{e_1 + z} > e^{-re_1t + e_1C_2}$$

$= Ae^{-e_1t}$ where$A = e_1C_2$

$$z(1 - Ae^{-e_1rt}) > Ae_1e^{-e_1rt}$$

$$z > \frac{Ae_1e^{-e_1rt}}{(1 - Ae^{-e_1rt})} = \frac{Ae_1}{e^{e_1rt} - A}$$

$z(0) > \frac{Ae_1}{1-A} > 0$ if $0 \le A < 1$

and $\lim_{t\to\infty} infz(t) \ge \lim_{t\to\infty} \frac{Ae_1}{e^{e_1rt}-A}$

Hence $z(t)$ persists.

Hence, each element of the system in model (2) endures, leading to the persistence of the entire system.

Numerical Simulations

With a variety of parameters, numerical simulation tries to evaluate analytic results and investigate how prey, predator, and scavenger populations interact. Time series plots and phase space diagrams are produced by changing parameter values to display various dynamic results.

In Figure 1, the parameter values are $c = 0.17$, $d = 0.29$, $e = 0.109$, $h = 0.2$, $g = 0.25$, $f = 0.5$.

The phase space diagram and time series plot show how x(t), y(t), and z(t) behave with different beginning values as they converge to the asymptotically stable equilibrium point E_4 (0.8, 0.6, 0.20). All three populations will survive because the simulation shows that the system solution approaches E_4 as t approaches infinity and that E_4 is globally asymptotically stable.

In Figure 2, the parameter values are $c = 0.5$, $d = 0.6$, $e = 0.2$, $h = 0.2$, $g = 0.3$, $f = 0.4$. The simulation also demonstrates that the system's solution goes to E_4 as t approaches infinity, this implies that E_4 is globally asymptotically stable and all three populations will coexist. The parameter values in Figure 3 are $c = 0.5$, $d = 0.4$, $e = 0.3$, $h = 0.4$, $g = 0.5$, $f = 0.6$. Hence, we conclude that the three populations move in the direction of the interior equilibrium point E_4 and the stable node (0.796,0.203,0.401). Since the system's solution curve tends to the inner equilibrium point, suggesting that it is globally asymptotically stable, the three populations coexist after an increase in time without experiencing any progressive change.

In Figure 4, the parameter values are $c = 0.3$, $d = 0.6$, $e = 0.1$, $h = 0.2$, $g = 0.3$, and $f = 0.4$.

The phase space diagram for this situation demonstrates that the solution curves converge to the inner equilibrium points, demonstrating that the system is globally stable and that the three populations coexist.

In Figure 5 displays parameter values of $c = 0.4$, $d = 0.3$, $e = 0.4$, $h = 0.19$, $g = 0.382$, and $f = 0.296$. The solution curve moves towards the asymptotically stable equilibrium point E_1 from the initial states, indicating its stability. As t approaches infinity, the curve converges to E_1, which is globally stable, leading to the persistence of prey population while the predator and scavenger population become extinct.

In Figure 6, the parameters $c = 0.6$, $d = 0.4$, $e = 0.79$, $h = 0.3$, $g = 0.7$, and $f = 0.8$ are displayed. The phase portrait shows that predator and prey populations will cohabit without experiencing any notable changes over time in the absence of a scavenger population. The curve's asymptotic stability and overall stability are demonstrated by the fact that it ultimately converges to the equilibrium point E_3 (0.79, 0.3, 0). So, for the specified parameter values, the population of prey will continue to exist but the population of scavengers will vanish.

In Figure 7 displays parameter values of $c = 0.4$, $d = 0.5$, $e = 0.4$, $h = 0.2$, $g = 0.3$, and $f = 0.5$. The population of x(t) and z(t) converge towards the asymptotically stable equilibrium point E_2 (1, 0.1, 0.2), indicating global stability. It can be inferred that the prey and scavenger populations can coexist if the predator population dies out. This conclusion is justified as the prey population has sufficient food to survive and reproduce, and scavengers can sustain themselves on the carcasses of naturally dead prey. However, it should be noted that in this case, none of the population densities remain constant.

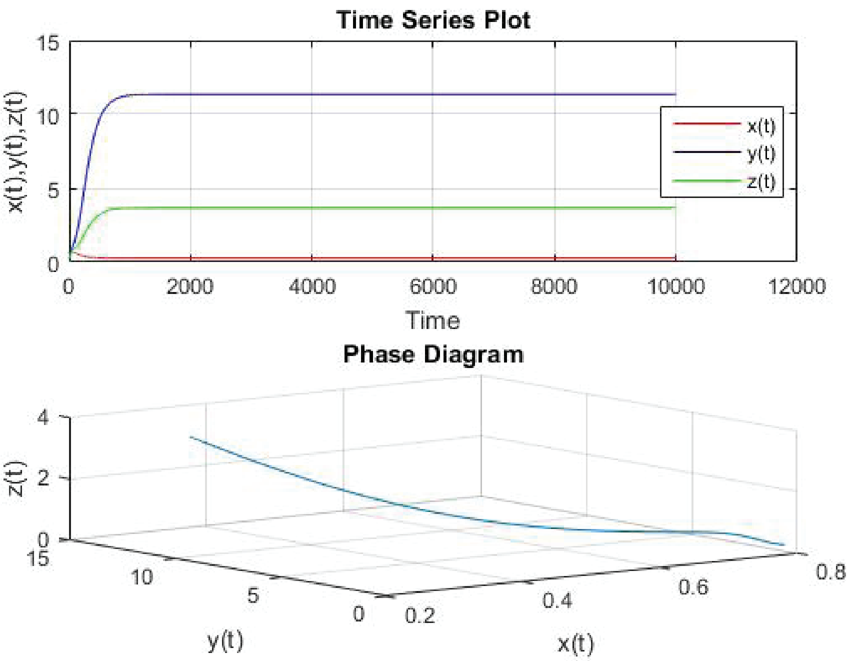

Figure 1. The parameter values are $c = 0.17$, $d = 0.29$, $e = 0.109$, $h = 0.2$, $g = 0.25$, $f = 0.5$. Here, the phase space diagram and time series plot for the specified parameters x(t), y(t), and z(t) are shown.

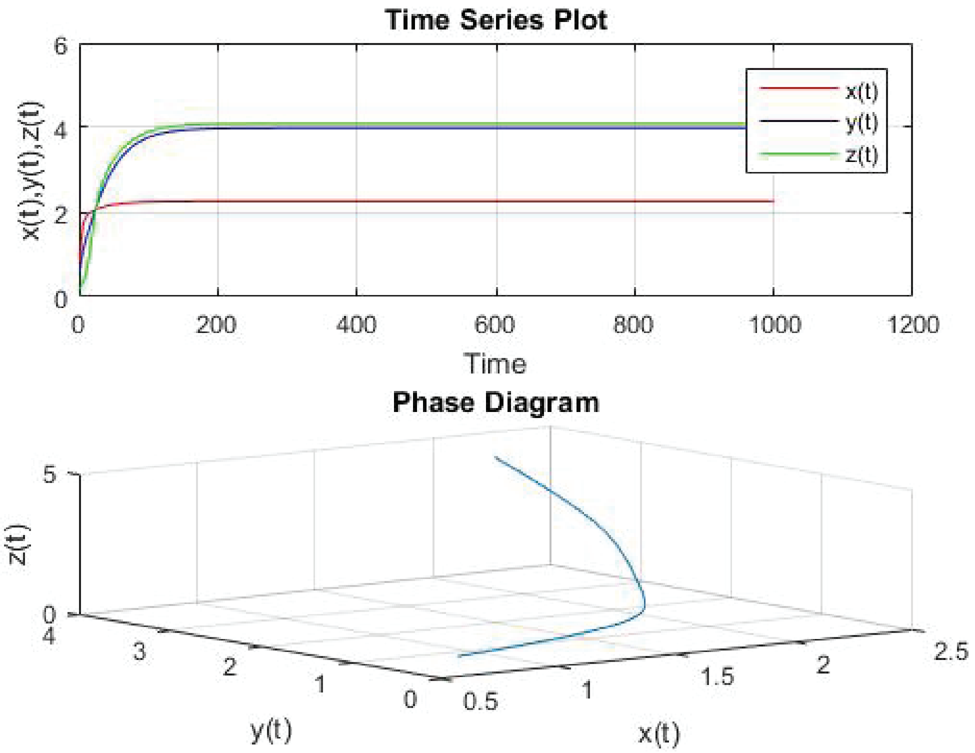

Figure 2. The parameter values in Figure 2 are c = 0.5, d = 0.6, e = 0.2, h = 0.2, g = 0.3, and f = 0.4. The simulation also reveals that the system's solution goes to E_4 as t tends to infinity, suggesting that E_4 is asymptotically stable universally.

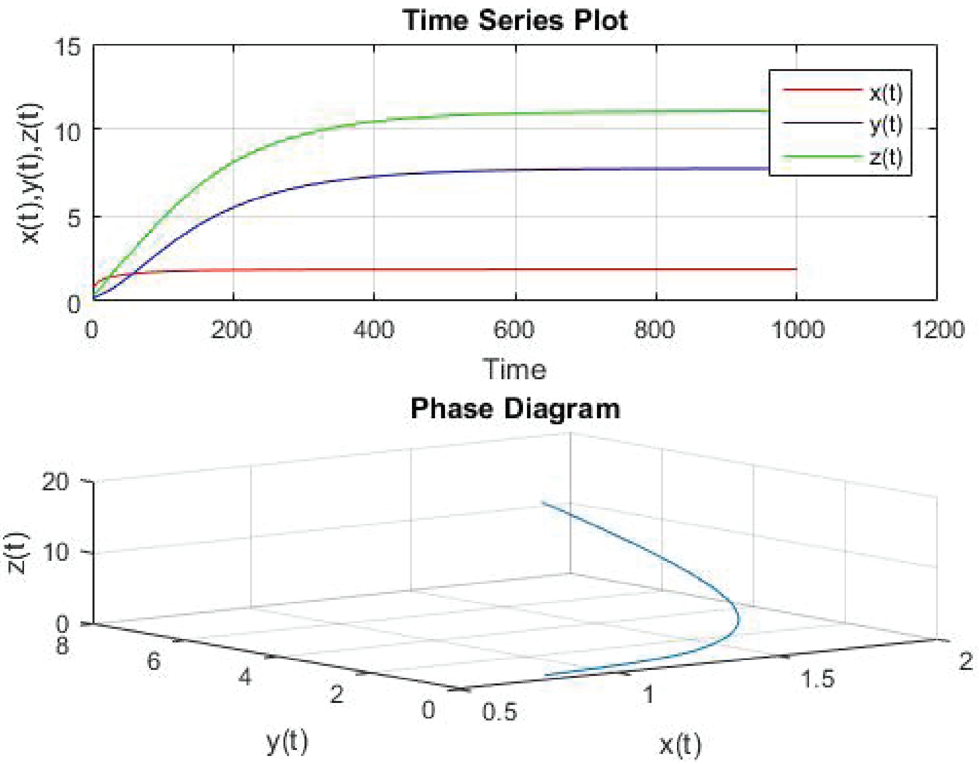

Figure 3. The parameter values in Figure 3 are c = 0.5, d = 0.4, e = 0.3, h = 0.4, g = 0.5, and f = 0.6. Here, we draw the conclusion that the three populations move towards the stable node and interior equilibrium point E_4 (0.796,0.203,0.401).

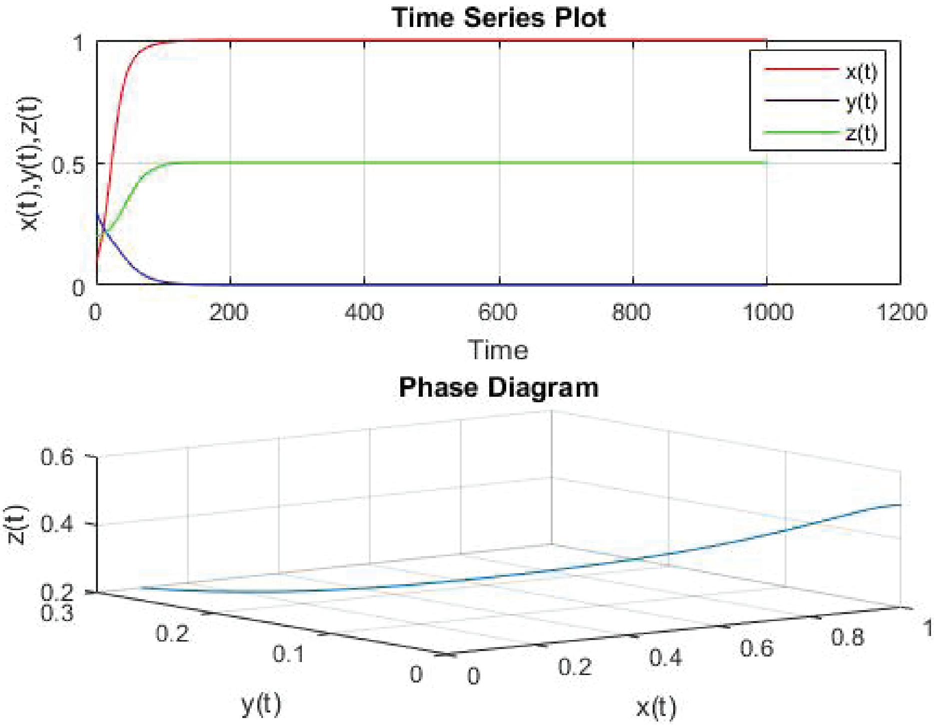

Figure 4. The parameter values in Figure 4 are $c = 0.3$, $d = 0.6$, $e = 0.1$, $h = 0.2$, $g = 0.3$, and $f = 0.4$. The phase space diagram in this case demonstrates that the solution curves converge to the internal equilibrium points, demonstrating that the system is globally stable.

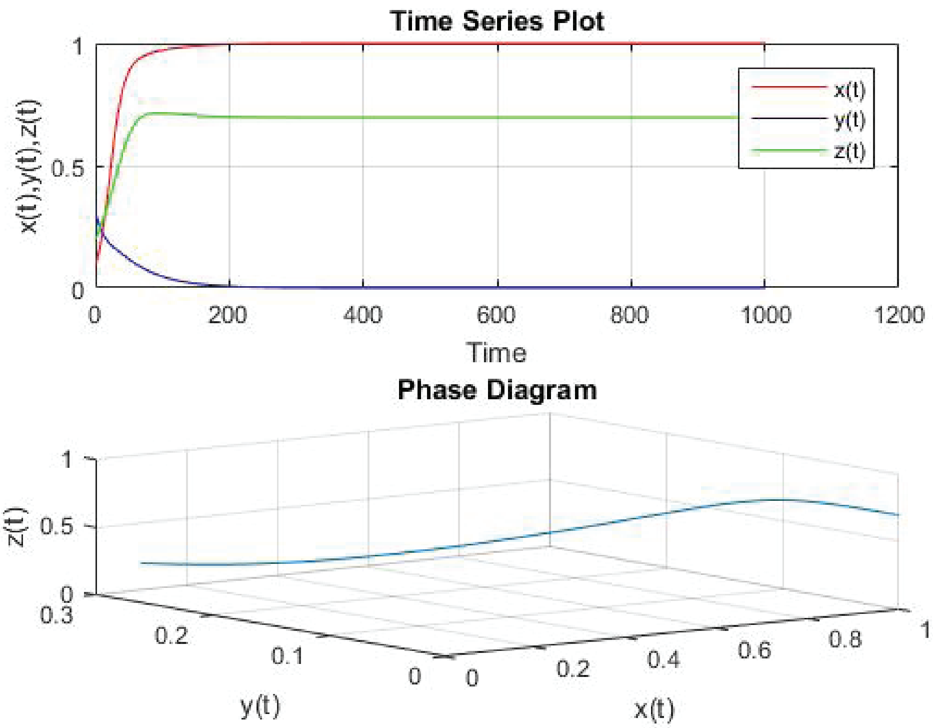

Figure 5. The parameter values in Figure 5 are $c = 0.4$, $d = 0.3$, $e = 0.4$, $h = 0.19$, $g = 0.382$, and $f = 0.296$.The solution curve at the starting states in this figure travels in the direction of the axial equilibrium point E_1, indicating that it is asymptotically stable.

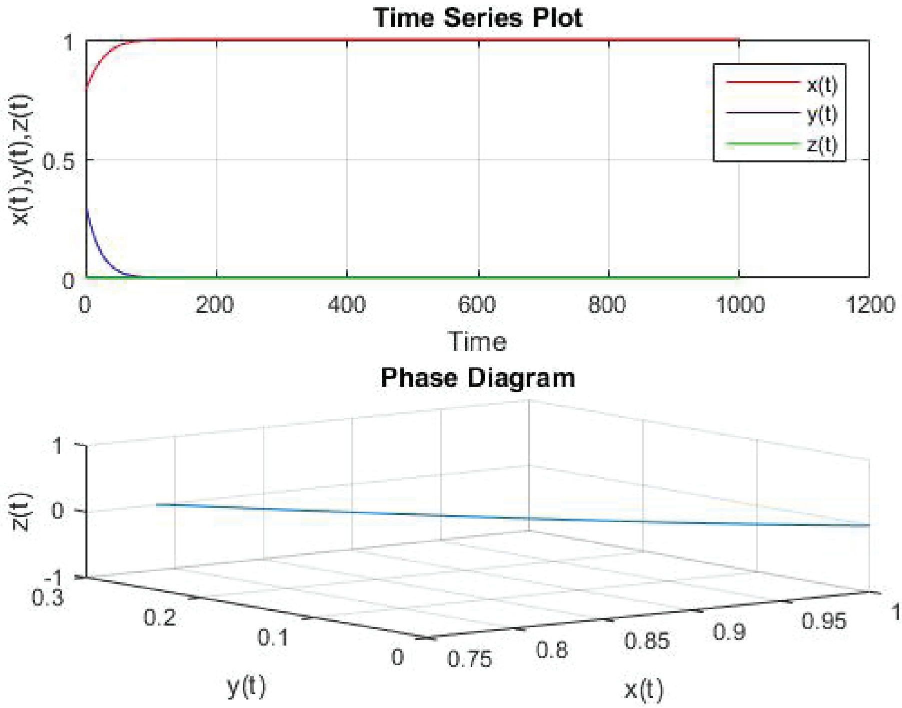

Figure 6. The parameter values in Figure 6 are $c = 0.6$, $d = 0.4$, $e = 0.79$, $h = 0.3$, $g = 0.7$, and $f = 0.8$. The phase portrait in this case indicates that if the scavenger population goes extinct, there will be a coexistence of the predator and prey with no gradual change over time. With the system being asymptotically stable and the equilibrium point being globally stable, the curve converges to the equilibrium point E_3 (0.79,0.3,0).

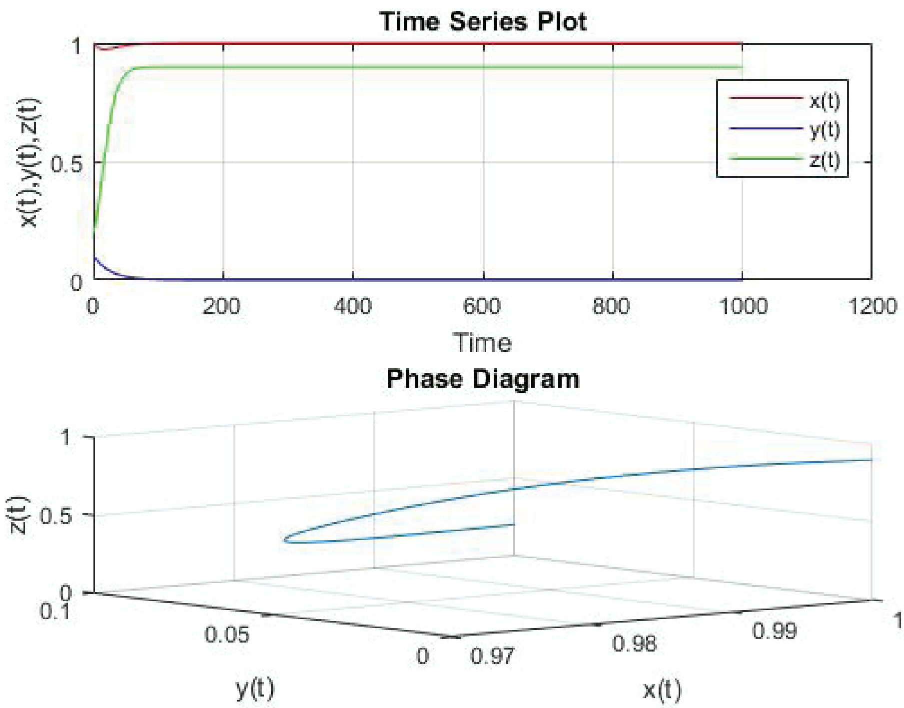

Figure 7. The parameter values in Figure 7 are $c = 0.4$, $d = 0.5$, $e = 0.4$, $h = 0.2$, $g = 0.3$, and $f = 0.5$. The population x(t) and z(t) in this figure travel in the direction of equilibrium point E_2 (1,0.1,0.2), suggesting that the equilibrium point is stable in the asymptotic sense and, as a result, the curve is asymptotically stable throughout.

RESULTS AND DISCUSSION

The non-linear system of differential equations introduced in this study provides comprehensive insights into the dynamics of prey, predators, and scavengers within an ecosystem. By integrating the logistic growth of the prey population and inter-species interactions, the model elucidates crucial aspects of the system's behavior, including positivity, boundedness of solutions, and prerequisites for stable coexistence. Local stability analysis identifies the existence of limit cycles in the positive quadrant, underscoring their significance in ecological dynamics. Moreover, the study rigorously examines the persistence requirement of the model, shedding light on the conditions necessary for long-term viability. Numerical simulations corroborate theoretical findings, enhancing our understanding of the ecosystem's long-term dynamics. Importantly, the research highlights the adaptive role of scavengers in mitigating food supply fluctuations, while also identifying scenarios that may lead to scavenger extinction due to mortality rate fluctuations. These findings deepen our understanding of ecosystem dynamics and have implications for conservation efforts, emphasizing the importance of considering intraspecies competition and environmental uncertainties in ecological modeling.

CONCLUSION

The current study looks at a three-dimensional dynamic system that includes populations of prey, predators, and scavengers. The stability of the system is heavily reliant on competition within species. Scavengers help in dealing with fluctuations in the food supply in the environment. Due to fluctuations in the mortality rate of the scavenger population, the species exhibits complex behavior. The scavenging species can go extinct as a result of a rise in mortality. Insecure circumstances permit coexistence of the three populations. Using Lyapunov functions for inner equilibrium points and Dulac's criterion for equilibrium points, it is determined for some conditions that the limit cycle of the model system does not exist. The solution is shown to be positive and bounded, and the absence of periodic solutions establishes the system's overall stability. The study also determines the persistence of the three populations. The species' unpredictable and irregular behavior may be stable under certain parameters.

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

We declare that we have no conflicts of interest in relation to the research and publication of this paper. We have no financial or personal relationships that could influence our work.

ETHICS

There are no ethical issues with the publication of this manuscript.

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