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Research Article

An application of interval valued neutrosophic soft multisets in MCDM

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ABSTRACT

Interval valued neutrosophic soft multisets are the generalization of Interval valued neutrosophic soft sets, which is a new revelation for handling incomplete and imprecise data in multiple universes. In this paper, we propose new algorithms based on accuracy function for employing weighted sum method (WSM), weighted product method (WPM) and TOPSIS method to solve a decision making problem using Interval-valued Neutrosophic Soft Multisets. We adopt a real world personal selection problem and solve it by applying our proposed algorithms. Additionally, we make a comparison of Interval valued neutrosophic soft multisets with existing sets to illustrate the importance of Interval valued neutrosophic soft multisets. Furthermore, we compare and analyze the results obtained by WSM, WPM and TOPSIS.

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INTRODUCTION

Lotfi A. Zadeh [1] introduced Fuzzy sets and Fuzzy logic to study the nature of an object that cannot be described clearly. It assigns membership values ranging between 0 to 1 for those elements to characterize their nature. Thereafter, it can be extended to Interval valued fuzzy sets [2] which allocate intervals as membership values. In real world, there are innumerable situations based on both truth membership and false membership for proper description of an object in uncertain situations. To handle those situations Intuitionistic fuzzy sets [3] were developed. Later, it was extended to Interval valued intuitionistic fuzzy sets [4] by K. Atanassov. F. Smarandache [5] extended fuzzy sets, classical set theory and intuitionistic set theory in the name of Neutrosophic sets to talk about indeterminacy nature of an object through an indeterminacy membership function. Later, Wang et al. [6] extended neutrosophic sets to Interval valued neutrosophic sets which is more flexible to deal in case of real-life problems.

Molodtsov [7] initiated a new mathematical tool called Soft sets for handling uncertain information. It can easily handle objects with uncertain nature by the help of parameter sets which is one of the major benefits while using soft sets. Several researchers have studied soft sets. P.K. Maji gave an application of soft sets in a decision making problem with the aid of rough set theory [8]. Chen et al. [9] initiated the new concept called parameterization reduction of soft sets which reduced the attributes to enhance the application of soft sets. Çağman-Enginoğlu defined soft matrices to solve problems without the help of fuzzy soft sets or rough sets and they defined some operations on soft matrices [10]. Also, they presented a soft max-min decision making algorithm to select an optimal decision from the

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set of alternatives. Subsequently they defined the product of soft sets and presented a *uni-int* decision making method in [11]. Feng et al. [12] devised new decision making methods namely $uni\text{-}int^k$, $uni - int^t$, and $int^m - int^n$. Later, Fuzzy soft sets [13] and intuitionistic fuzzy soft sets [14] were introduced as an extension of fuzzy sets and intuitionistic fuzzy sets.

Later, Neutrosophic soft sets were introduced as an extension of neutrosophic sets by P.K. Maji [15]. It was a new mathematical approach for describing indeterminate and incomplete information through parameter sets and membership functions. I. Deli et al. [16] introduced Neutrosophic soft matrix and some operations on them and further solved a decision making problem using Neutrosophic soft matrices. I. Deli [17] proposed Interval valued neutrosophic soft sets as a generalization of soft sets, fuzzy soft sets, intuitionistic fuzzy soft sets, interval valued intuitionistic fuzzy soft sets and neutrosophic soft sets.

Research Gap and Motivation

Alkhazaleh et al. [18] introduced soft multisets to expand soft sets in multiple dimensions. Soft multisets are used to deal with multiple universes at a time. Many researchers have dealt with soft multisets ([19], [20]). Further, Alkhazaleh and Salleh [21] defined fuzzy soft multisets and solved a decision making problem for fuzzy soft multisets. A.E. Coskun [22] employs soft matrices on soft multisets to explore the use of soft matrices in decision making. I. Deli et al. [23] initiated Neutrosophic soft multisets that handle uncertain and indeterminate information in multiple universes. Neutrosophic vague soft multisets and Weighted neutrosophic soft multisets were introduced by A. Al-Quran & N. Hassan [24] and C. Granados et. Al [25] respectively as well as some real-life applications were discussed.

Multi-criteria decision making (MCDM) provides a method for decision making in a practical and familiar situation in which multiple criteria are taken into consideration. The aim of this technique is to help decision makers where a large number of alternatives exist. MCDM attempts to choose a best alternative among the set of given alternatives. There are lot of methods used to solve MCDM problems to name a few, AHP, TOPSIS, ELECTRE and PROMETHEE, but they are all unique in their own way. Many researchers used hybrid structures of fuzzy set theory, soft set theory, rough set theory and multiset theory to solve MCDM problems in uncertain environment ([26], [27], [28], [29]). Weighted Sum Method (WSM) and Weighted Product Method (WPM) are more frequently used in MCDM. In both the methods alternatives are being compared with one another according to weights and criteria. Weighted Sum Method (WSM) allocates weights to each criteria according to their importance then calculate weighted sum for each alternative and finally choose the best one. D. Handoko et al. [30] applied WSM method to determine the allocation of special funds to primary and secondary schools located in underdeveloped areas. In medical field, Z. Yong [31] used WSM method to analyze Breast cancer. C.A. Alban-Perez et al. [32] applied WSM technique in decision making to improve the structure of a complete street in Colombia. Weighted Product Method (WPM) is a simplified model of WSM which is used in multi-dimensional decision making problems. Particularly, Sathiyaraj et al. [33] used WPM technique to evaluate drinking water quality in Salem district. Many industrial decision making problems were dealt by WSM and WPM. Hwang and Yoon [34] defined one of the primary methods, called TOPSIS (Technique for Order Performance by Similarity to Ideal Solution). TOPSIS technique is to provide an optimal solution by measuring distance from PIS (Positive Ideal Solution) and NIS (Negative Ideal Solution) for each alternative. PIS is the most preferred solution and NIS is the least preferred solution as decided by decision makers. Triantaphyllou and Lin [35] used fuzzy arithmetic operator to define the fuzzy version of TOPSIS method, which shows fuzzy relative closeness. Chen [36] applied TOPSIS method to fuzzy group decision making by defining the crisp Euclidean distance between fuzzy numbers. Jahanshahloo et al. [37] extended the TOPSIS approach for decision making problems involving interval data. Chen and Tsao [38] expanded the TOPSIS method to solve MADM problems involving interval valued fuzzy data and compared the result by Hamming distance and Euclidean distance. M. Imtiaz et al. [39] used the accuracy function in the TOPSIS method to solve the MCDM problem in Octagonal intuitionistic fuzzy environment. Additionally, in the medical field TOPSIS, WSM and WPM techniques were used for selecting LASER as an efficient surgical instrument [40].

Contribution

The first section begins with a brief Introduction followed by the study framework of Interval valued neutrosophic soft multisets. In section 2, some fundamental definitions and concepts are discussed. In section 3, we propose new algorithms using accuracy function which employs WSM, WPM and TOPSIS techniques to solve problems using Interval valued neutrosophic soft multisets. In section 4, we make a comparison of some fundamental sets with IVNSMS and additionally the results of all three methods are compared and analyzed to find the best alternative in all given universes. Finally in section 5, we conclude our research work.

PRELIMINARIES

Definition 1 [5] Let U be a space of points (objects), with a generic element in U denoted by u. A neutrosophic set A in U is characterized by a truth-membership function T_A, an indeterminacy-membership function I_A and a falsity- membership function F_A. T_A(u); I_A(u) and F_A(u) are real standard or nonstandard subsets of [0, 1].

There is no restriction on the sum of T_A(u); I_A(u) and F_A(u), so $0 \leq supT_A(u) + supI_A(u) + supF_A(u) \leq 3$.

Definition 2 [6] Let U be a space of points (objects), with a generic element in U denoted by u. An interval valued neutrosophic set (IVN-set) A in U is characterized by truth-membership function T_A, an indeterminacy-membership function I_A and a falsity-membership function F_A. For each point $u \in U$; T_A, I_A and $F_A \subseteq [0,1]$.

Thus, an IVN-set over U can be represented by the set

$$A = \{\langle T_A(u)\ , I_A(u)\ , F_A(u)\rangle\ /u : u \in U\}.$$

Here, $(T_A(u),\ I_A(u),\ F_A(u))$ is called interval valued neutrosophic number for all $u \in U$ and all interval valued neutrosophic numbers over U will be denoted by $IVN(U)$.

Definition 3 [17] Let U be an initial universe set, $IVN(U)$ denotes the set of all interval valued neutrosophic sets of U and E be a set of parameters that describe the elements of U. An interval valued neutrosophic soft sets over U is a set defined by a set valued function Υ_K representing a mapping

$$\upsilon_K : E \to IVN(U).$$

It can be written a set of ordered pairs

$$\Upsilon_K = \{\ (x, \upsilon_K(x)) : x \in E\}.$$

Here, an interval valued neutrosophic set υ_K is called approximate function of the interval valued neutrosophic (ivn)- soft sets Υ_K. And $\upsilon_K(x)$ is called x-approximate value of $x \in E$.

Definition 4 [41] Let $x = ([T^L, T^U], [I^L, I^U], [F^L, F^U])$ be an Interval neutrosophic number and the accuracy function $a(x)$ of an Interval neutrosophic number can be defined as follows:

$$a(x) = \frac{T^L + T^U}{2} + 1 - \frac{I^L + I^U}{2} + \frac{F^L + F^U}{2}.$$

Definition 5 [18] Let $\{U_i : i \in U\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters, $U = \prod_{i\in I} P(U_i)$ where $P(U_i)$ denotes the powerset of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$.

A pair (I, A) is called a soft multiset over U given by the mapping $I : A \to U$.

Definition 6 [42] Let $\{U_i : i \in U\}$ be a collection of universes such that $\bigcap_{i\in I} U_i = \emptyset$ and let $\{E_{U_i} : i \in I\}$ be a collection of sets of parameters, $U = \prod_{i\in I} IVN(U_i)$ where $IVN(U_i)$ denotes the set of all Interval valued neutrosophic sets of U_i, $E = \prod_{i\in I} E_{U_i}$ and $A \subseteq E$.

An Interval valued neutrosophic soft multiset (IVNSMS) over U is the pair (I, A) given by the mapping I: $A \to U$. It can be represented by,

$(I, A) = \{(a_k, \langle[\inf T_I(u), supT_I(u)], [\inf I_I(u), supI_I(u)], [\inf F_I(u), supF_I(u)]\rangle) : a_k \in A \subseteq E, u \in U\}$.

AN APPLICATION OF INTERVAL VALUED NEUTROSOPHIC SOFT MULTISETS IN DECISION MAKING

In this section we try to solve a decision making problem using Interval valued neutrosophic soft multisets. If a person wants to purchase a personal computer, he has lot of choices for the same. By using the following algorithm and by setting required number of parameters, we shall arrive at a wise conclusion.

Algorithm 1: Weighted Sum Method

WSM technique is a frequently used technique in MCDM, which is strong in single dimension problems. Now, we propose a WSM algorithm to solve problem in multiple universes.

1. Input the $IVNSMS\ (I, A)$.
2. To use the accuracy function

$$a(I) = \left(\frac{\inf T_I(u) + supT_I(u)}{2}\right) + 1 - \left(\frac{\inf I_I(u) + supI_I(u)}{2}\right) + \left(\frac{\inf F_I(u) + supF_I(u)}{2}\right) \quad (1)$$

on every element of (I, A) to get the matrix $M = \left[d_{(i,j)k}\right]_{m\times n}$, $k = 1,2, \ldots n$ and $(i, j) = |U_1| + |U_2| + \cdots + |U_i|$.

3. Consider the i[th] part (U_i, the i[th] universe) of IVNSMS (I, A) in M.
4. Construct the normalized decision matrix R_i.

For beneficial attributes (criteria of benefits):

$$r_{(i,j)k} = \frac{d_{(i,j)k}}{d^{max}_{(i,j)k}} \quad (2)$$

For non-beneficial attributes (criteria of cost):

$$r_{(i,j)k} = \frac{d^{min}_{(i,j)k}}{d_{(i,j)k}} \quad (3)$$

5. Construct the weighted normalized decision matrix

$$W_i = \left[w_k . r_{(i,j)k}\right] \text{ and } \textstyle\sum_{k=1}^{n} w_k = 1. \quad (4)$$

6. Calculate the score of each alternative

$$S^{WSM}_{i,j} = \textstyle\sum_{k=1}^{n} w_k . r_{(i,j)k}. \quad (5)$$

7. Select $u_{i,j}$ (A_i) as the best alternative which have $S_{i,j} = max\left(S^{WSM}_{i,j}\right)$; $j = 1,2, \ldots, |U_i|$.
8. Continuing this procedure to all $IVNSM$-set parts (all universes), finally we obtain the optimal decision $(A_1, A_2, \ldots, A_n)$.

Algorithm 2: Weighted Product Method

WPM technique is a more efficient technique in MCDM. In this technique, alternatives are evaluated by multiplying the normalized data to the power of the corresponding weight criteria.

1. Input the $IVNSMS\ (I, A)$.
2. To use the accuracy function

$$a(I) = \left(\frac{inf\ T_I(u) + supT_I(u)}{2}\right) + 1 - \left(\frac{infI_I(u) + supI_I(u)}{2}\right) + \left(\frac{infF_I(u) + supF_I(u)}{2}\right) \quad (6)$$

on every element of (I, A) to get the matrix $M = \left[d_{(i,j)k}\right]_{m\times n}$, $k = 1,2, \dots n$ and $(i, j) = |U_1| + |U_2| + \dots + |U_i|$.

3. Consider the i^th^ part (U_i, the i^th^ universe) of (I, A) in M.
4. Construct the normalized decision matrix R_i.
   For beneficial attributes (criteria of benefits):

$$r_{(i,j)k} = \frac{d_{(i,j)k}}{d^{max}_{(i,j)k}}. \quad (7)$$

   For non-beneficial attributes (criteria of cost):

$$r_{(i,j)k} = \frac{d^{min}_{(i,j)k}}{d_{(i,j)k}}. \quad (8)$$

5. Construct the weighted normalized decision matrix

$$W_i = \left[r^{w_k}_{(i,j)k}\right] \text{ and } \textstyle\sum_{k=1}^{n} w_k = 1. \quad (9)$$

6. Calculate the score of each alternative

$$S^{WPM}_{i,j} = \textstyle\prod_{k=1}^{n} r^{w_k}_{(i,j)k}. \quad (10)$$

7. Select $u_{i,j}$ (A_i) as the best alternative which have $S_{i,j} = max\left(S^{WPM}_{i,j}\right)$; $j = 1,2, \dots, |U_i|$.
8. Continuing this procedure to all $IVNSM$-set parts (remaining universes), finally we obtain the optimal decision $(A_1, A_2, \dots, A_n)$

Algorithm 3: TOPSIS Method

The TOPSIS method is a straight-forward method in MCDM. The main idea of the TOPSIS method is to rank the optimal solution that is closer to PIS and far from NIS.

1. Input the $IVNSMS\ (I, A)$.
2. To use the accuracy function

$$a(I) = \left(\frac{inf\ T_I(u) + supT_I(u)}{2}\right) + 1 - \left(\frac{infI_I(u) + supI_I(u)}{2}\right) + \left(\frac{infF_I(u) + supF_I(u)}{2}\right) \quad (11)$$

on every element of (I, A) to get the matrix $M = \left[d_{(i,j)k}\right]_{m\times n}$, $k = 1,2, \dots n$ and $(i, j) = |U_1| + |U_2| + \dots + |U_i|$.

3. Consider the i^th^ part (U_i, the i^th^ universe) of (I, A) in M.
4. Construct the normalized decision matrix $R_i = \left[r_{(i,j)k}\right]_{m\times n}$ by

$$r_{(i,j)k} = \frac{d_{(i,j)k}}{\sqrt{\sum_{j=1}^{|U_i|} d^2_{(i,j)k}}} \quad (12)$$

5. Construct the weighted normalized decision matrix

$$W_i = \left[w_k . r_{(i,j)k}\right] = \left[x_{(i,j)k}\right]_{m\times n} \text{ and } \textstyle\sum_{k=1}^{n} w_k = 1. \quad (13)$$

6. Determine the Positive Ideal Solution (PIS) and Negative Ideal Solution (NIS) for universe U_i as follows:

(i) Positive Ideal Solution $X_i{}^+ = \begin{cases} \underset{(i,j)}{max}(x_{(i,j)k}) \colon k \in K^+ \\ \underset{(i,j)}{min}(x_{(i,j)k}) \colon k \in K^- \end{cases}$ (14)

(ii) Negative Ideal Solution $X_i{}^- = \begin{cases} \underset{(i,j)}{min}(x_{(i,j)k}) \colon k \in K^+ \\ \underset{(i,j)}{max}(x_{(i,j)k}) \colon k \in K^-, \end{cases}$ (15)

where K^+ is the parameter set of benefit type and K^- is the parameter set of cost type.

7. Calculate the separation measure from PIS and NIS by

$$\tau^+_{(i,j)} = \sqrt{\textstyle\sum_{k=1}^{n} (x^+_{(i.j)k} - x_{(i.j)k})^2}\ , \quad j = 1,2, \dots, |U_i| \quad (16)$$

and

$$\tau^-_{(i,j)} = \sqrt{\textstyle\sum_{k=1}^{n} (x^-_{(i.j)k} - x_{(i.j)k})^2}\ , \ j = 1,2, \dots, |U_i|. \quad (17)$$

8. Relative closeness coefficient to ideal solution is calculated by

$$R_{(i,j)} = \frac{\tau^-_{(i,j)}}{\tau^+_{(i,j)} + \tau^-_{(i,j)}} \quad (18)$$

and alternatives get ranked by descending order.

9. Continuing the above steps 3 to 8 for all $IVNSM$-set parts (all universes), finally we obtain the optimal decision $(A_1, A_2, \dots, A_n)$.

Example 1.

Suppose that a person Mr. X wants to buy a computer, a printer and an UPS for his personal work within his budget. Let (I, A) be an $IVNSMS(U)$ which describes "computers," "printers" and "UPS for PC" respectively that Mr. X is considering a good branded computer, printer for document work and an UPS for battery backup. Let $U_1 = \{c_1, c_2, c_3, c_4\}$ be the universe for branded computers, $U_2 = \{p_1, p_2, p_3\}$ be the universe for printers and $U_3 = \{u_1, u_2, u_3\}$ be the universe for UPS. Let $\{E_{U1}, E_{U2}, E_{U3}\}$ be a collection of parameters which describes above universes, where

$E_{U1} = \{e_{U1,1}$ = Intel core i3 processor; $e_{U1,2}$ = 16GB RAM; $e_{U1,3}$ = AMD processor; $e_{U1,4}$ = SSD Storage; $e_{U1,5}$ = OS window 11$\}$

E_{U2} = {$e_{U2,1}$ = Inkjet; $e_{U2,2}$ = multi function; $e_{U2,3}$ = single function; $e_{U2,4}$ = built in wifi; $e_{U2,5}$ = good printing speed}

E_{U3} = {$e_{U3,1}$ = Output 600VA; $e_{U3,2}$ = Backup time upto 40mins; $e_{U3,3}$ = Output socket 3 nos}.

Let $U = \prod_{i=1}^{3} IVN(U_i)$, $E = \prod_{i=1}^{3} E_{U_i}$ and $A \subseteq E$ such that

$A = \{a_1 = (e_{U1,1}, e_{U2,1}, e_{U3,1}), a_2 = (e_{U1,3}, e_{U2,2}, e_{U3,2}), a_3 = (e_{U1,2}, e_{U2,1}, e_{U3,1}), a_4 = (e_{U1,4}, e_{U2,3}, e_{U3,3}), a_5 = (e_{U1,5}, e_{U2,5}, e_{U3,2})\}$.

Suppose that Mr. X wants to choose a combination of computer, printer and UPS for PC from the set of given objects with respect to the set of choice parameters.

WSM Algorithm:

Let Table 1. represents the Interval valued neutrosophic soft multiset (I, A).

Conversation of $IVNS$ to $IVNN$ by using accuracy function $a(I) = \left(\frac{\inf T_I(u)+\sup T_I(u)}{2}\right) + 1 - \left(\frac{\inf I_I(u)+\sup I_I(u)}{2}\right) + \left(\frac{\inf F_I(u)+\sup F_I(u)}{2}\right)$ to get matrix M.

Table 3 represents the weight criteria given by expert.

Table 3. The weighted criteria

a_1	a_2	a_3	a_4	a_5
0.3	0.3	0.1	0.1	0.2

Let us consider the U_1-part of M.

Table 4. Tabular representation of U_1 −part of M

U_2	a_1	a_2	a_3	a_4	a_5
c_1	1.85	1.25	1.6	1.6	1.95
c_2	2.05	1.25	1.75	1.4	2.15
c_3	1.25	1.7	1.75	1.5	1.8
c_4	1.45	1.65	1.65	1.45	1.8

Construct the normalized decision matrix for U_1 by using Equation (2).

Table 1. Tabular representation of (I, A).

U_i		a_1	a_2	a_3	a_4	a_5
U_1	c_1	[0.5,0.8],[0.2,0.3],[0.4,0.5]	[0.2,0.3],[0.9,1.0],[0.9,1.0]	[0.6,0.7],[0.5,0.5],[0.4,0.5]	[0.6,0.9],[0.3,0.5],[0.2,0.3]	[0.7,0.8],[0.2,0.3],[0.4,0.5]
	c_2	[0.7,0.9],[0.1,0.2],[0.3,0.5]	[0.2,0.3],[0.9,1.0],[0.9,1.0]	[0.7,0.9],[0.4,0.6],[0.4,0.5]	[0.5,0.8],[0.4,0.6],[0.2,0.3]	[0.8,1.0],[0.2,0.4],[0.5,0.6]
	c_3	[0.2,0.3],[0.9,1.0],[0.9,1.0]	[0.7,0.9],[0.2,0.3],[0.1,0.2]	[0.7,0.9],[0.4,0.6],[0.4,0.5]	[0.4,0.6],[0.7,0.8],[0.7,0.8]	[0.7,0.9],[0.2,0.5],[0.2,0.5]
	c_4	[0.4,0.6],[0.5,0.6],[0.5,0.5]	[0.6,0.8],[0.3,0.4],[0.2,0.4]	[0.6,0.7],[0.5,0.5],[0.4,0.6]	[0.2,0.3],[0.1,0.2],[0.3,0.4]	[0.2,0.5],[0.2,0.5],[0.7,0.9]
U_2	p_1	[0.7,0.9],[0.2,0.3],[0.4,0.6]	[0.6,0.8],[0.3,0.5],[0.5,0.6]	[0.8,1.0],[0.3,0.6],[0.2,0.4]	[0.2,0.4],[0.7,0.8],[0.7,0.8]	[0.6,0.7],[0.4,0.4],[0.3,0.4]
	p_2	[0.6,0.8],[0.4,0.5],[0.5,0.6]	[0.7,0.9],[0.4.0.6],[0.3,0.4]	[0.4,0.8],[0.4,0.6],[0.4,0.6]	[0.4,0.8],[0.5,0.5],[0.6,0.7]	[0.5,0.8],[0.3,0.6],[0.4.0.6]
	p_3	[0.3,0.4],[0.5,0.6],[0.5,0.7]	[0.5,0.7],[0.2,0.6],[0.5,0.8]	[0.5,0.7],[0.5,0.5],[0.3,0.5]	[0.9,1.0],[0.2,0.3],[0.2,0.3]	[0.7,0.9],[0.5,0.6],[0.3,0.4]
U_3	u_1	[0.7,0.9],[0.2,0.3],[0.4,0.6]	[0.6,0.9],[0.7,0.7],[0.3,0.5]	[0.6,0.6].[0.2,0.5],[0.1,0.5]	[0.3,0.5].[0.4,0.5],[0.2,0.3]	[0.4,0.7],[0.2,0.3],[0.5,0.6]
	u_2	[0.5,0.6],[0.4,0.6],[0.3,0.4]	[0.4,0.6],[0.5,0.7],[0.2,0.3]	[0.4,0.8],[0.1,0.2],[0.3,0.4]	[0.4,0.7],[0.6,0.9],[0.4,0.7]	[0.5,0.7],[0.3,0.4],[0.4,0.5]
	u_3	[0.3,0.6],[0.4,0.5],[0.3,0.4]	[0.2,0.8],[0.4,0.5],[0.4,0.5]	[0.5,0.7],[0.2,0.4],[0.3,0.5]	[0.7,0.8],[0.7,0.8],[0.2,0.3]	[0.5,0.6],[0.4,0.5],[0.5,0.6]

Table 2. Tabular representation of M

U_i		a_1	a_2	a_3	a_4	a_5
U_1	c_1	1.85	1.25	1.6	1.6	1.95
	c_2	2.05	1.25	1.75	1.4	2.15
	c_3	1.25	1.7	1.75	1.5	1.8
	c_4	1.45	1.65	1.65	1.45	1.8
U_2	p_1	2.05	1.85	1.75	1.3	1.6
	p_2	1.8	1.65	1.6	1.75	1.7
	p_3	1.4	1.85	1.5	1.95	1.6
U_3	u_1	2.05	1.45	1.55	1.2	1.85
	u_2	1.4	1.15	1.8	1.35	1.65
	u_3	1.35	1.5	1.7	1.25	1.65

Table 5. U_1 normalized decision matrix

U_2	a_1	a_2	a_3	a_4	a_5
c_1	0.90244	0.73529	0.91429	1	0.90698
c_2	1	0.73529	1	0.87500	1
c_3	0.60976	1	1	0.93750	0.83721
c_4	0.70732	0.97059	0.94286	0.90625	0.83721

Now, let us construct the weighted normalized decision matrix for U_1 and calculate the score of alternatives using Equations (4) and (5).

Table 6. Weighted normalized decision matrix for U_1

U_1	a_1	a_2	a_3	a_4	a_5	Score
c_1	0.27073	0.22059	0.09143	0.10000	0.18140	0.86414
c_2	0.30000	0.22059	0.10000	0.08750	0.20000	0.90809
c_3	0.18293	0.30000	0.10000	0.09375	0.16744	0.84412
c_4	0.21219	0.29118	0.09429	0.09063	0.16744	0.85572

Hence, the ranking order of U_1 alternatives is $c_2 > c_1 > c_4 > c_3$ and c_2 is the best alternative in the universe of computers.

Now, consider the U_2 part in M.

Table 7. Tabular representation of U_2-part of M

U_2	a_1	a_2	a_3	a_4	a_5
p_1	2.05	1.85	1.75	1.3	1.6
p_2	1.8	1.65	1.6	1.75	1.7
p_3	1.4	1.85	1.5	1.95	1.6

The normalized decision matrix for U_2 is constructed in the following table.

Table 8. U_2 normalized decision matrix

U_2	a_1	a_2	a_3	a_4	a_5
p_1	1	1	1	0.66667	0.94118
p_2	0.87805	0.89189	0.91429	0.89744	1
p_3	0.68293	1	0.85714	1	0.94118

Table 9. Weighted normalized decision matrix for U_2

U_2	a_1	a_2	a_3	a_4	a_5	Score
p_1	0.30000	0.30000	0.10000	0.06667	0.18824	0.95491
p_2	0.26342	0.26757	0.09143	0.08974	0.20000	0.91216
p_3	0.20488	0.30000	0.08571	0.10000	0.18824	0.87883

For the U_2-part, the weighted normalized decision matrix is constructed and the score of each alternative is calculated as given below.

Hence, $p_1 > p_2 > p_3$ is the ranking order of U_2 alternatives and p_1 is the best alternative in the universe of printers.

Next, let us consider the U_3 part in M.

The normalized decision matrix for U_3 is computed as follows.

Repeating the procedure as above for U_3-part, we get the score of alternatives as shown below.

Here, the ranking order of U_3 alternatives is $u_1 > u_3 >$

Table 10. Tabular representation of U_3-part of M

U_3	a_1	a_2	a_3	a_4	a_5
u_1	2.05	1.45	1.55	1.2	1.85
u_2	1.4	1.15	1.8	1.35	1.65
u_3	1.35	1.5	1.7	1.25	1.65

u_2, where u_1 is the best alternative in the universe of UPS.

Thus, from the above rankings Mr. X can choose the combination (c_2, p_1, u_1). (i.e) Mr. X chooses computer c_2, printer p_1 and UPS u_1 for his personal work.

Next, we use WPM algorithm to solve the above Example 1.

Table 11. U_3 normalized decision matrix

U_3	a_1	a_2	a_3	a_4	a_5
u_1	1	0.96667	0.86111	0.88889	1
u_2	0.68293	0.76667	1	1	0.89189
u_3	0.65854	1	0.94444	0.92593	0.89189

Table 12. Weighted normalized decision matrix for U_3

U_3	a_1	a_2	a_3	a_4	a_5	Score
u_1	0.30000	0.29000	0.08611	0.08889	0.20000	0.96500
u_2	0.20488	0.23000	0.10000	0.10000	0.17838	0.81326
u_3	0.19756	0.30000	0.09444	0.09259	0.17838	0.86297

Table 13. Tabular representation of Normalized decision matrix

U_i		a_1	a_2	a_3	a_4	a_5
U_1	c_1	0.90244	0.73529	0.91429	1	0.90698
	c_2	1	0.73529	1	0.87500	1
	c_3	0.60976	1	1	0.93750	0.83721
	c_4	0.70732	0.97059	0.94286	0.90625	0.83721
U_2	p_1	1	1	1	0.66667	0.94118
	p_2	0.87805	0.89189	0.91429	0.89744	1
	p_3	0.68293	1	0.85714	1	0.94118
U_3	u_1	1	0.96667	0.86111	0.88889	1
	u_2	0.68293	0.76667	1	1	0.89189
	u_3	0.65854	1	0.94444	0.92593	0.89189

WPM Algorithm

Conversation of $IVNS$ to $IVNN$ by using accuracy function is identical to WSM method. Next, we use Equation (7) and construct the normalized decision matrix and display it in Table 13.

Now, we use Equation (9) to construct the weighted normalized decision matrix for U_1 and calculate the score of alternatives. The resultant matrix is given below.

Hence, the ranking order of U_1 alternatives is $c_2 > c_1 > c_4 > c_3$, where c_2 is the best alternative in the universe of computers.

Now, construct the weighted normalized decision matrix for U_2 and calculate the score of alternatives as executed in U_1-part.

The ranking order of U_2 alternatives is $p_1 > p_2 > p_3$, where p_1 is the best alternative in the universe of printers. We shall follow the same steps for U_3 and calculate the score of alternatives and determine the ranking order.

Hence, $u_1 > u_3 > u_2$ is the ranking order of U_3 alternatives and u_1 being the best alternative in the universe of UPS.

Table 14. Weighted normalized decision matrix for U_1

U_1	a_1	a_2	a_3	a_4	a_5	Score
c_1	0.96967	0.91188	0.99108	1	0.98066	0.85939
c_2	1	0.91188	1	0.98674	1	0.89978
c_3	0.86208	1	1	0.99357	0.96509	0.82663
c_4	0.90133	0.99108	0.99413	0.99020	0.96509	0.84865

Table 15. Weighted normalized decision matrix for U_2

U_2	a_1	a_2	a_3	a_4	a_5	Score
p_1	1	1	1	0.96027	0.98795	0.94869
p_2	0.96174	0.96626	0.99108	0.98924	1	0.91109
p_3	0.89189	1	0.98470	1	0.98795	0.86766

Table 16. Weighted normalized decision matrix for U_3

U_3	a_1	a_2	a_3	a_4	a_5	Score
u_1	1	0.98988	0.98516	0.98829	1	0.96377
u_2	0.89189	0.92338	1	1	0.97738	0.80493
u_3	0.88221	1	0.99430	0.99233	0.97738	0.85077

Thus, from the above rankings Mr. X can choose the combination (c_2, p_1, u_1).

Now, we shall apply TOPSIS method and solve Example 1.

TOPSIS Method

Conversation of $IVNS$ to $IVNN$ by using accuracy function is identical to WSM method and the same was presented in Table 2. Let us consider the U_1-part of M and construct the normalized decision matrix using Equation (12). The resulting matrix is given in Table 17.

Table 17. U_1 normalized decision matrix

U_1	a_1	a_2	a_3	a_4	a_5
c_1	0.55059	0.42288	0.47372	0.53715	0.50509
c_2	0.61011	0.42288	0.51813	0.47001	0.55689
c_3	0.37202	0.57512	0.51813	0.50358	0.46624
c_4	0.43154	0.55820	0.48853	0.48679	0.46624

Constructing the weighted normalized decision matrix for U_1 by using weight vector for each criteria, we get the following matrix.

Table 18. Weighted normalized decision matrix for U_1

U_1	a_1	a_2	a_3	a_4	a_5
c_1	0.16518	0.12686	0.04737	0.05372	0.10102
c_2	0.18303	0.12686	0.05181	0.04700	0.11138
c_3	0.11161	0.17253	0.05181	0.05036	0.09325
c_4	0.12946	0.16746	0.04885	0.04868	0.09325

Now, let us determine PIS and NIS as follows:

$X^+ = \{0.18303, 0.17253, 0.05181, 0.05372, 0.11138\}$

and

$X^- = \{0.11161, 0.12686, 0.04737, 0.04700, 0.09325\}$

Using the PIS and NIS, let us calculate the separation measures and closeness coefficient and represent it in Table 19.

Table 19. Table of relative closeness coefficient

U_1	$\tau^+_{(1,j)}$	$\tau^-_{(1,j)}$	$R_{(1,j)}$
c_1	0.05032	0.05455	0.52017
c_2	0.04616	0.07383	0.61528
c_3	0.07377	0.04601	0.38412
c_4	0.05708	0.04441	0.43755

Hence, the ranking order of U_1 alternatives based on relative closeness coefficient is $c_2 > c_1 > c_4 > c_3$ and c_2 is the best alternative in the universe of computers.

Now, consider the U_2-part of M and construct the normalized decision matrix as done in U_1-part.

Table 20. U_2 normalized decision matrix

U_2	a_1	a_2	a_3	a_4	a_5
p_1	0.66855	0.59810	0.62371	0.46720	0.56533
p_2	0.58702	0.53344	0.57025	0.53908	0.60067
p_3	0.45657	0.59810	0.53461	0.70080	0.56533

The weighted normalized decision matrix for U_2 is constructed by using weight vector for each criteria. Hence, we get the following matrix.

Table 21. Weighted normalized decision matrix for U_2

U_2	a_1	a_2	a_3	a_4	a_5
p_1	0.20056	0.17943	0.06237	0.04672	0.11307
p_2	0.17611	0.16003	0.05702	0.05391	0.12013
p_3	0.13697	0.17943	0.05346	0.07008	0.11307

The PIS and NIS are determined as follows:

$X^+ = \{0.20056, 0.17943, 0.06237, 0.07008, 0.12013\}$

and

$X^- = \{0.13697, 0.16003, 0.05346, 0.04672, 0.11307\}$

The separation measures and closeness coefficient are calculated using PIS and NIS.

Table 22. Table of relative closeness coefficient

U_2	$\tau^+_{(2,j)}$	$\tau^-_{(1,j)}$	$R_{(2,j)}$
p_1	0.02441	0.06708	0.73323
p_2	0.03556	0.04057	0.53288
p_3	0.06460	0.03036	0.31973

Here, the ranking order of U_2 alternatives based on relative closeness coefficient is $p_1 > p_2 > p_3$, where p_1 is the best alternative in the universe of printers.

We shall repeat the same procedure for U_3-part of M and obtain the following matrices.

Table 23. U_3 normalized decision matrix

U_3	a_1	a_2	a_3	a_4	a_5
u_1	0.72546	0.60867	0.53063	0.54630	0.62126
u_2	0.49544	0.48274	0.61622	0.61459	0.55409
u_3	0.47775	0.62966	0.58198	0.56906	0.55409

Table 24. Weighted normalized decision matrix for U_3

U_3	a_1	a_2	a_3	a_4	a_5
u_1	0.21764	0.18260	0.05306	0.05463	0.12425
u_2	0.14863	0.14482	0.06162	0.06146	0.11082
u_3	0.14332	0.18890	0.05820	0.05691	0.11082

The PIS and NIS are as follows:

$X^+ = \{0.21764, 0.18890, 0.06162, 0.06146, 0.12425\}$ and

$X^- = \{0.14332, 0.14482, 0.05306, 0.05463, 0.11082\}$

Now, we have the following table using PIS and NIS.

Table 25. Table of relative closeness coefficient

U_3	$\tau^+_{(3,j)}$	$\tau^-_{(3,j)}$	$R_{(3,j)}$
u_1	0.01263	0.08444	0.86989
u_2	0.08298	0.01217	0.12789
u_3	0.07573	0.04443	0.36976

The ranking order of U_3 alternatives based on relative closeness coefficient is $u_1 > u_3 > u_2$, and u_1 being the best alternative in the universe of UPS.

Thus, from the above rankings Mr. X can choose the combination (c_2, p_1, u_1).

COMPARISON

The decision making problem in Example 1 is converted to $IVNSMS$ and solved by proposed WSM, WPM and TOPSIS methods. By using WSM, WPM and TOPSIS techniques we calculate the rank of all alternatives in all universes. Now, let us compare the results obtained by WSM, WPM and TOPSIS. In this comparison, rank of all alternatives in each universe are presented in the following table.

From the comparative study, each alternative in the available universe achieves the same rank in all the methods. According to the results, the combination (c_2, p_1, u_1) is the first choice for Mr. X. Otherwise he goes with the second choice (c_1, p_2, u_3) and the worst choice is either (c_3, p_3, u_2) or (c_4, p_3, u_2).

In this section, we compare the theory of IVNSMS with other existing theories like Soft sets (SS), Neutrosophic soft sets (NSS), Interval valued neutrosophic soft sets (IVNSS), Soft multisets (SMS) and Neutrosophic soft multisets (NSMS) to show that IVNSMS are more adaptable and generalized than other existing sets. The comparison analysis is developed by their characters like domain, co-domain, universe, and membership functions. In Table 27, a comparison analysis of the IVNSMS with other above-mentioned sets has been executed.

Now, we observe that the set IVNSMS is a generalized form of all above mentioned sets. The main advantage of IVNSMS is that it describes real world problems involving multiple universes, one at a time. Another merit of the set is that it can address information ranges from minimum to maximum with the help of interval membership functions. The application of IVNSMS can assist people pursue a right choice out of capable choices in uncertain and incomplete data conditions.

Table 26. Alternative rank comparison using WSM, WPM and TOPSIS

Universes	Elements	WSM	Rank	WPM	Rank	TOPSIS	Rank
U_1	c_1	0.86414	2	0.85939	2	0.52017	2
	c_2	0.90809	1	0.89978	1	0.61528	1
	c_3	0.84412	4	0.82663	4	0.38412	4
	c_4	0.85572	3	0.84865	3	0.43755	3
U_2	p_1	0.95491	1	0.94869	1	0.73323	1
	p_2	0.91216	2	0.91109	2	0.53288	2
	p_3	0.87883	3	0.86766	3	0.31973	3
U_3	u_1	0.96500	1	0.96377	1	0.86989	1
	u_2	0.81326	3	0.80493	3	0.12789	3
	u_3	0.86297	2	0.85077	2	0.36976	2

Table 27. Comparison of IVNSMS with existing sets

Sets	Multiple Universe	Domain	Co-domain	Membership Functions		
				Truth	Indeterminate	False
SS [7]	×	E	U	×	×	×
NSS [15]	×	E	$N(U)$	✓	✓	✓
IVNSS [17]	×	E	$IVN(U)$	✓	✓	✓
SMS [18]	✓	$A \subseteq E = \prod_{i \in I} E_{U_i}$	$U = \prod_{i \in I} P(U_i)$	×	×	×
NSMS [23]	✓	$A \subseteq E = \prod_{i \in I} E_{U_i}$	$U = \prod_{i \in I} NS(U_i)$	✓	✓	✓
IVNSMS [42]	✓	$A \subseteq E = \prod_{i \in I} E_{U_i}$	$U = \prod_{i \in I} IVN(U_i)$	✓	✓	✓

CONCLUSION

In this paper, we have proposed WSM, WPM and TOPSIS methods to solve MCDM problems for Interval valued neutrosophic soft multisets and a decision making problem is illustrated. We have demonstrated the significance of IVNSMS by comparing it with other relevant models. In addition, we have compared the results and concluded that three methods prefer the same combination to Mr. X. These techniques were more adaptable and effective in solving decision making problems and were very useful to rank the alternatives in multiple universes at the same time. These algorithms can be used in many practical problems like decision making and personal selection problems. The major advantage of IVNSMS is that it can be used to maximize the benefits and minimize the cost by considering only choice parameters. In future work, we will provide many applications of these methods and propose a distance based TOPSIS technique to handle MCDM problems for Interval valued neutrosophic soft multisets.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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