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Research Article

Analysis on equitable local edge antimagic coloring of certain graphs

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ABSTRACT

The study aims at assigning a bijective mapping ς that maps the vertices of a graph G to its own cardinality whose corresponding edge weights are defined by the sum of its end vertices such that it satisfies certain coloring conditions. A graph G is deemed to possess an equitable local edge antimagic coloring, that corresponds to the minimum number of colors if it meets both equitable edge and local edge antimagic coloring conditions. Ascertaining the equitable local edge antimagic connection number of any graph is considered to be a challenging NP-complete problem and the discussed results are proven to be helpful in its characterization among various graphs.

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INTRODUCTION

In this study, we will be examining a finite, connected, and undirected graph denoted by G. The set of vertices and edges in G will be represented by V(G) and E(G) respectively. The graph G will have a total of p vertices and q edges. Furthermore, we define the minimum degree of G as $\delta(G)$ and the maximum degree as $\Delta(G)$. Hartsfield and Ringel [14], introduced antimagic graphs, by which, antimagic labeling is the assignment of unique positive integers to each edge of the graph, such that the sum of the labels of the vertices adjacent to each other is always different. The chromatic index of a graph G is the minimum number of colors required to color the edges of a graph. An equitable chromatic index, (usually denoted by $\chi'_e(G)$ or $\chi'=(G)$ is a graph coloring problem that aims to assign minimum k-colors to the edges of a graph such that each color appears on an at most equal number of edges. In other words, for an undirected graph G = (V(G), E(G)), an equitable edge coloring is a coloring of the edges with k-colors such that for any two colors say, i and j, (for $1 \le i \le k$; $1 \le j \le k$), the number of edges colored with color i differs from the number of edges colored with color j by at most 1. The concept of equitable edge coloring was defined by Hilton and de Werra [1]. Finding an antimagic labeling and equitable edge coloring for a given graph is considered to be NP-complete and several algorithms have been proposed to solve this problem. Further few researchers namely, Kaliraj et. al. [10], Manikandan et. al. [11], Manickam et. al. [13], Sathiya et. al. [4], Sivaraman et. al. [12], Vivik et. al. [18- 20] and Zhang et. al. [21] developed the study of equitable edge coloring in certain graphs.

The local edge antimagic coloring ($\gamma_{lea}(G)$) was studied by *Agustin* [7], according to which a bijective function labels the vertices of G based on the cardinality of vertices of G. In 2018, *Agustin et al.* [8] studied the local edge

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antimagic coloring of comb product of graphs and in 2022, *S. Rajkumar et al.* [16] analysed the local edge antimagic chromatic number of friendship, wheel, fan, helm, flower and closed helm graphs. In 2017, *S. Arumugam et al.* [15] introduced the concept of pairing graph labeling and coloring, which inspired *Dafik et al.* [5] to propose the concept of rainbow antimagic coloring (**r**ac), combining the principles of antimagic labeling and rainbow coloring. The lower bounds of the **r**ac number for any connected graph G were determined by Septory et al. [2]. Further developments such as, rainbow antimagic coloring of ladder graphs [3], special graphs [6] and vertex amalgamation of graphs [9] are studied by Sulistiyono et al., Budi and Joedo respectively. Based on the findings, we have put forth a new coloring scheme called, equitable local edge antimagic coloring denoted by $\chi'_{eac}(G)$.

Vizing [17] proved that, for any graph G, chromatic index $(\chi'(G))$ lies in the range $\Delta(G)$ and $\Delta(G)+1$. For the graphs whose edge coloring equal to $\Delta(G)$ is called *class I* graphs and the graphs whose edge coloring equal to

$\Delta(G)+1$ is called *class II* graphs. Based on this, we categorize our results as *Type-I* when $\chi'_{eac}(G) \le \chi'_e(G)$, *Type-II* when $\chi'_{eac}(G) \le \chi'_e(G)+1$ and *Type-III* when $\chi'_{eac}(G) \le \chi'_e(G)+2$ in the upcoming sections.

Preliminaries

In this section, the definitions of the coloring under study and some graphs are given. For a bijective function

$\varsigma: V(G) \to \{1,2,3,\ldots,|V(G)|\}$ whose corresponding edge weights defined by, $W_i(xy) = \varsigma(x) + \varsigma(y)$, is said to have equitable local edge antimagic coloring, if

(i) $\varsigma(e) \ne \varsigma(e')$ for any edges $e, e' \in E(G)$, incident on a common vertex $x \in V(G)$ and

(ii) $||C_{\varsigma(e)}| - |C_{\varsigma(e')}|| \le 1$, where $|C_{\varsigma(e)}|$ and $|C_{\varsigma(e')}|$ are the cardinalities of the color classes having some i^{th} and j^{th} edge weights and $i \ne j$.

A simple connected graph obtained from two copies of path connected by P_2 forms a *ladder graph* (L_p). Similarly, an *open ladder graph* (OL_p) is obtained by removing the first and last rungs of a ladder graph. A *slanting ladder graph* (ZL_p) is obtained from an open ladder graph by diagonally joining the two copies of path with P_2. In other words, the vertex and edge sets of ladder, open ladder and slanting ladder graphs are as follows: $V(L_p) = \{x_t, y_t : 1 \le t \le p\}$; $\{E\}(L_p) = \{x_t x_{t+1}, y_t y_{t+1} : 1 \le t \le p-1\} \cup \{x_t y_t : 1 \le t \le p\}$, $V(OL_p) = \{x_t, y_t : 1 \le t \le p\}$; $\{E\}(OL_p) = \{x_t x_{t+1}, y_t y_{t+1} : 1 \le t \le p-1\} \cup \{x_t y_t : 2 \le t \le p-1\}$ and $V(ZL_p) = \{x_t, y_t : 1 \le t \le p\}$; $E(ZL_p) = \{x_t x_{t+1}, y_t y_{t+1}, x_{t+1} y_t : 1 \le t \le p-1\}$. Here, t varies between $[1,p]$, x_t and y_t denotes each copy of path with order .

Lemma 2.1: [7,16] The local edge antimagic chromatic number of certain graphs (path, cycle, wheel, fan, friendship, complete and star graphs) are:

1. For $p \ge 3, \gamma_{lea}(P_p) = 2$
2. For $p \ge 3, \gamma_{lea}(\boldsymbol{C}_p) = 3$
3. For $p \ge 3, \gamma_{lea}(W_{1,p}) = \begin{cases} 5, & p = 3,4 \\ p, & p \ge 5 \end{cases}$
4. For $p \ge 2, \gamma_{lea}(F_{1,p}) = \begin{cases} p+1, & p = 2,3 \\ p, & p \ge 4 \end{cases}$
5. For $p \ge 1, \gamma_{lea}(\mathcal{F}_p) = \begin{cases} 3, & p = 1 \\ 2p, & p \ge 2 \end{cases}$
6. For $p \ge 3, \gamma_{lea}(K_p) = 2p-3$
7. For $p \ge 2, \gamma_{lea}(K_{1,p}) = p$

Bounds on Equitable Local Edge Antimagic Coloring

Proposition 3.1: For any connected graph G, $\chi'_{eac}(G) \ge \chi'_e(G)$, where $\chi'_e(G)$ is the equitable chromatic index of G.

Theorem 3.1: For any connected graph G, $\gamma_{lea}(G) \le max\{\chi'_e(G), \Delta(G)\} \le \chi'_{eac}(G)$.

Proof: Let G be a connected graph and for $u, v, w \in V(G)$, let $\varsigma : V(G) \to \{1,2,3,\ldots,|V(G)|\}$ be a bijective function that defines the antimagic vertex labeling, then $\varsigma(u) \ne \varsigma(v) \ne \varsigma(w)$. For $uv, vw \in E(G)$, their corresponding edge weights $W_i(uv) \ne W_j(vw)$. Hence, $\chi'_{eac}(G) \ge \Delta(G)$. Using this result and Proposition 3.1, $\chi'_{eac}(G) \ge max\{\chi'_e(G), \Delta(G)\}$. Since, $\chi'_e(G) \ge \chi'(G)$, it can be seen that, $\chi'_e(G) \ge \gamma_{lea}(G)$ and so, $\chi'_{eac}(G) \ge \gamma_{lea}(G)$.

Theorem 3.2: Let $|C|$ be the maximum cardinality among all color classes in G. Then, $\chi'_{eac}(G) \le \Delta + \lceil \frac{e-\Delta}{|C|} \rceil$, where *e* is the number of edges in G.

Proof: In the proper equitable-edge coloring problem, the cardinalities of color classes in G do not play a crucial role except for satisfying the condition:

$$||C(u)| - |C(u')|| \le 1, for\ u, u' \in E(G). \quad (3.1)$$

However, in equitable local edge antimagic coloring, the antimagic labeling of vertices influences the cardinalities of color classes in G. To achieve a minimum coloring through local edge antimagic coloring, we analyse the cardinalities of color classes while ensuring that equation (3.1) holds.

According to *Vizing* [?], for any graph G, the chromatic index $\chi'(G)$ lies within the range $\Delta(G)$ *and* $\Delta(G)+1$.

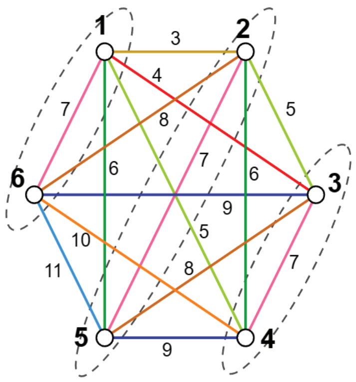

Figure 1. Illustration of K_6 not satisfying equitable local edge antimagic coloring.

Through proper edge coloring, there exist at least Δ colors, each with a minimum cardinality of at least 1. These cardinalities vary based on the parameters e and $\Delta(G)$ in G, where e represents the number of edges. Let $|C|$ be the maximum cardinality among all color classes in G.

- If the maximum cardinality is $|C| = 1$, then every edge in G recieves a unique color. On the other hand, if initially Δ colors are used, then $e - \Delta$ remaining edges can be colored with minimum cardinality $|C| = 1$, hence colors apart from Δ are required, i.e., $(e - \Delta)$ unique colors are added to the existing color list. Hence, if $|C| = 1, \Delta + (e - \Delta) = e$ is the upper bound.
- If the maximum cardinality is $|C| = 2$, then G will have Δ colors each with $|C| = 2$. The remaining edges can be colored with $\lceil\frac{(e-\Delta)}{2}\rceil$ with at most cardinality $|C| = 2$. Thus, $\Delta + \lceil\frac{(e-\Delta)}{2}\rceil$ is the upper bound if $|C| = 2$.

Proceeding similarly, we can deduce that for the maximum cardinality $|C|$, the minimum number of colors required for local edge antimagic coloring satisfies: $\chi'_{eac}(G) \leq \Delta + \lceil\frac{e-\Delta}{|C|}\rceil$.

To minimize the number of colors through local edge antimagic coloring, start analysing the graph by initially settingc$|C| = 2$ and increasing it incrementally until we reach proper minimum coloring through local edge antimagic coloring. By fixing the cardinality $|C| = \Delta - 1$, we can attain at most minimum colors for equitable local edge antimagic coloring and after $\Delta - 1$, the coloring either remains same or reaches $\chi'_e(G)$. If the minimum number of colors required through local edge antimagic coloring equals to $\chi'_e(G)$, then the upper bound is $\chi'_{eac}(G) \leq \chi'_e(G)$. Otherwise, the cardinality at which we achieve the minimum colors for local edge antimagic coloring determines the upper bound, i.e., $\chi'_{eac}(G) \leq \Delta + \lceil\frac{e-\Delta}{|C|}\rceil$.

The following theorem shows why the above Type classification of graphs does not exists for complete graphs K_p.

Theorem 3.3: The equitable local edge antimagic connection number of a complete graph does not exist unless $p \leq 5$ and $\chi'_{eac}(K_p) = 2p - 3$.

Proof: Let us consider $p \geq 6$ first. K_6 is *5-regular* graph, so start labeling the vertices in cyclic order.

The cardinalities of $|C_3| = |C_4| = |C_{10}| = |C_{11}| = 1$ and $|C_5| = |C_6| = |C_8| = |C_9| = 2$, but $|C_7| = 3$, which fails to satisfy equitable edge coloring condition. On the other hand, for $p \leq 5$, label the vertices of K_p in cyclic order, which produces the following set of colors $\{3,4,5, \ldots ,2p - 1\}$ with cardinalities, $|C_3| = |C_4| = |C_{2p-2}| = |C_{2p-3}| = 1$ and $|C_5| = \cdots = |C_{2p-3}| = 2$, for $p = 4,5$.

Theorem 3.4: The equitable local edge antimagic connection number of a k-regular graph for $2 \leq p \leq 9$ is $2k - 1 \leq \chi'_{eac}(G^k) \leq k + p - 2$, only if $1 \leq k \leq 5$.

Proof: Let G^k be a k-regular graph of order p. The cases are analysed based on the values of k.

- For $k = 1$: A *1*-regular graph is a set of disconnected path graphs P_2, which is only possible for even values of p. Since each component consists of a single edge, an equitable local edge antimagic labeling with minimum coloring will assign the same label to each edge. Hence, the equitable local edge antimagic labeling with minimum coloring will assign the same label to each edge. Hence, the equitable local edge antimagic connection number is given by:$\chi'_{eac}(G^1) = 1$ and its cardinality is $\frac{p}{2}$.
- For $k = 2$: A *2*-regular graph consists of cycles. For $3 \leq p \leq 9$, the cycle graph C_p is known to satisfy
- $\chi'_{eac}(C_p) = 3$. The other *2*-regular graphs of order $3 \leq p \leq 9$ are either disjoint cycles or isomorphic to itself. The equitable local edge antimagic connection number satisfies: $3 \leq \chi'_{eac}(G^k) \leq k + p - 2$.
- For $k = 3$, 4: The *3*-regular graphs of order $p \geq 4$ and *4*-regular graphs of order $p \geq 5$ yields: $5 \leq \chi'eac\ (G^3) \leq k + p - 2$ and $7 \leq \chi'eac\ (G^4) \leq k + p - 2$ respectively.
- For k-regular graphs where $k \geq 5$, the equitable local edge antimagic connection number exists if and only if the graph is not a complete graph (i.e., $G^k \neq K_p, for\ p \geq 6$).

The below figure illustrates the equitable local edge antimagic connection number of 3 and 4 regular graphs for p=6.

From the below table one can verify the equitable local edge connection number of any regular graphs for $2 \leq k \leq 5$ and $2 \leq p \leq 9$.

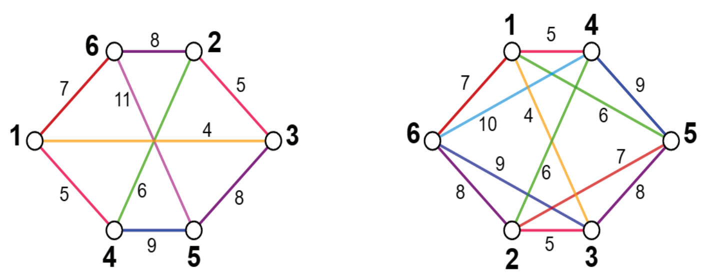

Figure 2. Illustration equitable local edge connection number of 3-regular and 4-regular graphs for p = 6.

Table 1. Equitable local edge connection number of regular graphs for $2 \leq k \leq 5$ and $2 \leq p \leq 9$.

p	2	3	4	5	6	7	8	9
$k = 2$	-	3	3	3	3 & 4	5	4 & 5	5
$k = 3$	-	-	5	No graph	7	No graph	9	No graph
$k = 4$	-	-	-	Does not exist	7	9	9	11
$k = 5$	-	-	-	-	Does not exist	No graph	9	No graph

Theorem 3.5: The equitable local edge antimagic connection number of trees for $p \leq 6$ are *Type-I* graphs.

Proof: According to the classification, *Type-I* graphs are $\chi'_{eac}(G) \leq \chi'_e(G)$. Typically, the path and star graphs are *Type-I* graphs. The following figure is an example of some trees classified under *Type-I* for $p \leq 6$.

Equitable Local Edge Antimagic Coloring of Some Ladder Graphs

Theorem 4.1: For $p \geq 2$, the equitable local edge antimagic connection number of a ladder graph is $\chi'_{eac}(L_p) = 3$.

Proof: Let L_p be a ladder graph with vertex set $V(L_p) = \{x_t, y_t: 1 \leq t \leq p\}$ and edge set $E(L_p) = \{x_t x_{t+1}, y_t y_{t+1}: 1 \leq t \leq p - 1\} \cup \{x_t y_t: 1 \leq t \leq p\}$. The minimum and maximum degrees are: $\delta(L_p) = 2$ and $\Delta(L_p) = 3$ respectively. The lower bound follows from Theorem 3.1. To prove the upper bound define a bijective function $\varsigma: V(L_p) \rightarrow \{1,2,3, \ldots ,2p\}$:

1. Label the vertices as:
   - When $p = odd$:

$$x_t = \begin{cases} t, & for\ t = 1,3,5, \ldots , p \\ 2p - t + 1, & for\ t = 2,4,6, \ldots , p - 1 \end{cases}$$

$$y_t = \begin{cases} 2p - t + 1, & for\ t = 1,3,5, \ldots , p \\ t, & for\ t = 2,4,6, \ldots , p - 1 \end{cases}$$

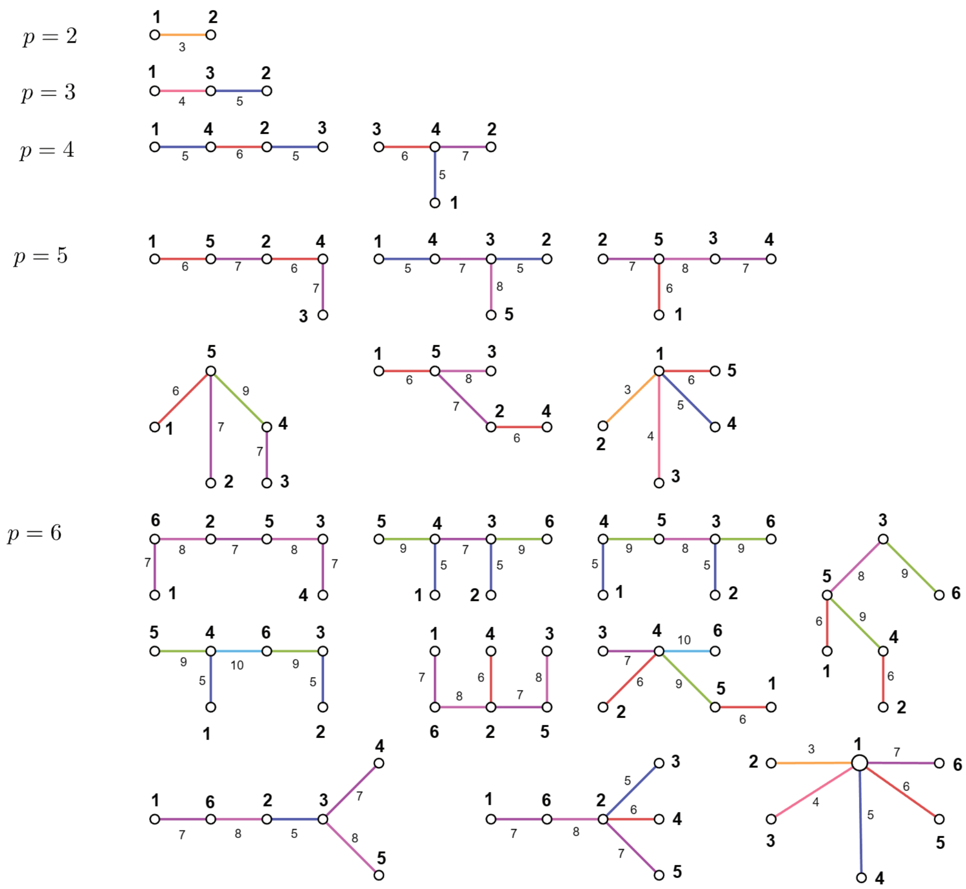

Figure 3. Illustration of $\chi'_{eac}(G^T)$ for $p \leq 6$.

$Edge\ Weights$	$Cardinalities$		
		$p = odd$	$p = even$
$\varsigma(x_t x_{t+1}) = 2p$	$t = odd$	$\frac{p-1}{2}$	$\frac{p}{2}$
$\varsigma(y_t y_{t+1}) = 2p$	$t = even$	$\frac{p-1}{2}$	$\frac{p-2}{2}$
$\varsigma(x_t x_{t+1}) = 2p+2$	$t = even$	$\frac{p-1}{2}$	$\frac{p-2}{2}$
$\varsigma(y_t y_{t+1}) = 2p+2$	$t = odd$	$\frac{p-1}{2}$	$\frac{p}{2}$
$\varsigma(x_t y_t) = 2p + 1$	$1 \le t \le p$	p	p

- When $p = even$:

$$x_t = \begin{cases} t, & for\ t = 1,3,5,\ldots,p-1 \\ 2p-t+1, & for\ t = 2,4,6,\ldots,p \end{cases}$$

$$y_t = \begin{cases} 2p-t+1, & for\ t = 1,3,5,\ldots,p-1 \\ t, & for\ t = 2,4,6,\ldots,p \end{cases}$$

2. The corresponding edge weights and their cardinalities are:

The cardinalities of color classes are, $|C_{2p}| = p - 1$; $|C_{2p+2}| = p - 1$ and $|C_{2p+1}| = p$. Inorder to verify that this satisfies the equitable edge coloring condition take,

(i) $||C_{2p}| - |C_{2p+1}|| \le |p - 1 - p\ | = 1$

(ii) $||C2p| - |C2p{+}2|| \le |p - 1 - p + 1\ | = 0$

(iii) $||C_{2p+1}| - |C_{2p+2}|| \le |p - p + 1\ | = 1$

Thus (i), (ii) and (iii) satisfies the equitable edge coloring condition and the total number of edge weights required is 3.

Corollary 4.1: For $p \ge 3$, the equitable local edge antimagic connection number of an open ladder graph is $\chi'_{eac}(OL_p) = 3$.

Proof: Since, an open ladder graph is obtained by deleting the first and last edges of a ladder graph, the result is obvious. By Theorem 3.1, we have the lower bound to be, $\chi'_{eac}(OL_p) \ge max\{3,3\} \ge 3$. To prove the upper bound, define a bijective function $\varsigma : V(OL_p) \to \{1,2,3, \ldots ,2p\}$ and just label the vertices as in Theorem 4.1 and calculate the edge weights by neglecting the first and last rungs.

$Edge\ Weights$	$Cardinalities$		
		$p = odd$	$p = even$
$\varsigma(x_t x_{t+1}) = \varsigma(y_t y_{t+1}) = 2p + 1$	$t = odd$	$\frac{p-1}{2}$	-
$\varsigma(x_t x_{t+1}) = \varsigma(y_t y_{t+1}) = 2p + 3$	$t = even$	$\frac{p-1}{2}$	-
$\varsigma(x_{t+1} y_t) = 2p + 2$	$1 \le t \le p-1$	$p - 1$	-
$\varsigma(x_t x_{t+1}) = \frac{3p+2}{2}$	$t = odd$	-	$\frac{p}{2}$
$\varsigma(x_t x_{t+1}) = \frac{3p+4}{2}$	$t = even$	-	$\frac{p-2}{2}$
$\varsigma(y_t y_{t+1}) = \frac{5p+2}{2}$	$t = odd$	-	$\frac{p}{2}$
$\varsigma(y_t y_{t+1}) = \frac{5p+4}{2}$	$t = even$	-	$\frac{p-2}{2}$
$\varsigma(x_{t+1} y_t) = p + 1$	$t = odd$	-	$\frac{p}{2}$
$\varsigma(x_{t+1} y_t) = 3p + 2$	$t = even$	-	$\frac{p-2}{2}$

Theorem 4.2: For $p \geq 3$, the equitable local edge antimagic connection number of a slanting ladder graph is

$$\chi'_{eac}(ZL_p) = \begin{cases} 3, & if\ p \equiv 1\ (mod\ 2) \\ 6, & if\ p \equiv 0\ (mod\ 2) \end{cases}$$

Proof: Let ZL_p be a slanting ladder graph whose vertex set is $V(ZL_p) = \{x_t, y_t : 1 \leq t \leq p\}$ and edge set is $E(ZL_p) = \{x_t x_{t+1}, y_t y_{t+1}, x_{t+1} y_t : 1 \leq t \leq p-1\}$. The minimum and maximum degrees are: $\delta(ZL_p) = 1$ and $\Delta(ZL_p) = 3$ respectively. The lower bound follows from Theorem 3.1. To prove the upper bound, define a bijective function $\varsigma : V(ZL_p) \to \{1,2,3,\ldots,2p\}$:

1. Label the vertices as:
   - When $p = odd$:

$$\varsigma(x_t) = \begin{cases} t, & for\ t = 1,3,5,\ldots,p \\ 2p, & for\ t = 2,4,6,\ldots,p-1 \end{cases}$$

$$\varsigma(y_t) = \begin{cases} t+1, & for\ t = 1,3,5,\ldots,p \\ 2p-t+1, & for\ t = 2,4,6,\ldots,p-1 \end{cases}$$

   - When $p = even$:

$$\varsigma(x_t) = \begin{cases} p + \frac{t+1}{2}, & for\ t = 1,3,5,\ldots,p-1 \\ \frac{p-t}{2} + 1, & for\ t = 2,4,6,\ldots,p \end{cases}$$

$$\varsigma(y_t) = \begin{cases} \frac{p+t+1}{2}, & for\ t = 1,3,5,\ldots,p-1 \\ 2p - \frac{t}{2} + 1, & for\ t = 2,4,6,\ldots,p \end{cases}$$

2. The corresponding edge weights and their cardinalities are:

The cardinalities of color classes are, $|C_{2p+1}| = p-1$; $|C_{2p+3}| = p-1$ and $|C_{2p+2}| = p-1$. Inorder to verify that this satisfies the equitable edge coloring condition take,

(i) $||C_{2p+1}| - |C_{2p+2}|| \leq |p-1-p+1| = 0$

(ii) $||C_{2p+1}| - |C_{2p+3}|| \leq |p-1-p+1| = 0$

(iii) $||C_{2p+2}| - |C_{2p+3}|| \leq |p-1-p+1| = 0$

Thus, (i), (ii) and (iii) satisfies the equitable edge coloring condition and the total number of edge weights required is 3, when $p = odd$. For $p = even$, the difference in the cardinalities of edge weights results in 0 and 1, thus satisfying the equitable edge coloring condition. Further, the total number of edge weights required is 6, when $p = even$.

CONCLUSION

The study of various graph coloring and local edge antimagic coloring has already proven to be successful. So, the proposed equitable local edge antimagic coloring, combining both the aspects is a new and innovative approach to this field of research, which is currently wide open for exploration. The paper provides some bounds and optimal existence of χ'_{eac} number for certain graphs such as path cycle, wheel, trees and regular graphs. Having these as a foundation, one can determine the existence of equitable local edge antimagic coloring for higher order graphs and other graph families, which is still an open problem.

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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