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Research Article

Antipodal resolving sets on algebraic graphs of finite groups

K. ARUNA SAKTHI[1,*], R. RAJESWARI[2]

[1]*A. P. C. Mahalaxmi College for Women, Thoothukudi, Manonmaniam Sundaranar University, Abhishekapatti, TamilNadu, 627012, India*

[2]*PG and Research Department of Mathematics, A. P. C. Mahalaxmi College for Women, TamilNadu, 628002, TamilNadu, India*

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ABSTRACT

In this research article concept of antipodal resolving sets have been introduced and the concepts have been analysed on some algebraic graphs like identity graphs and order prime graphs of finite groups. Also its dimensions have been found and compared with various antipodal sets like antipodal independent, pendant, cototal, equitable resolving sets. Finally those dimensions comparison have been elucidated as a theorem.

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INTRODUCTION

Resolving sets concept have been first introduced by Slater [1] and then joined work by Harary and Melter [2]. It is used to locate objects in graphs. Here, a restriction is made on the number of objects and cannot be more than the number of vertices of the graphs. Many resolving sets like independent, degree equitable, rational resolving sets have been introduced and studied by many mathematicians for various graphs [3-8]. Resolving sets have many real life applications in network discovery and verification, in chemistry and also in robot navigation etc [9]. The concept of antipodal graphs was introduced by Singleton [10] and developed by Mathematician like R.Aravamudhan, B.Rajendran [11,12] and E.Prisner [13]. In graph theory antipodal concepts has been used in many research areas like domination, steiner antipodal number etc. Inspiring all these antipodal concepts have been used for algebraic graphs and analyzed how the results work and found the dimensions between algebraic graphs like identity graphs [14], order prime graphs [15] of finite groups.

PRELIMINARIES

Definition: Resolving sets: A set of vertices S in a graph G is called a resolving set for G if, for any two vertices u, v there exists $x \in S$ such that the distances $d(u,x) \neq d(v,x)$. The minimum cardinality of a resolving set of G is called the dimension of G and is denoted $dim(G)$.

*Corresponding author.
*E-mail address: arunasakthi9397@gmail.com

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Definition: 1.2 Identity graphs: Let $\mathcal{g}$ be a group. The identity graph $G = (V, E)$ with vertices as the elements of group and two elements $x, y \in \mathcal{g}$ are adjacent or can be joined by an edge if $x.y = e$, where e is the identity element of $\mathcal{g}$ and identity element is adjacent to every other vertices in G.

Definition: 1.3 Order Prime graphs: Let Γ be a finite group. The order prime graph (Γ) of a group Γ is a graph with V ((Γ)) = Γ and two vertices are adjacent in (Γ) if and only if their orders are relatively prime in Γ.

ANTIPODAL RESOLVING SETS

In this section we have analyzed antipodal resolving set works on identity graphs of finite group and a comparison between various antipodal resolving sets have been worked out. Throughout this chapter set T is taken as a subset of $V(G)$.

Definition: 3.1Antipodal resolving sets : Let $G = (V, E)$ be a graph. Let $T \subseteq V$. For every ti , $tj \in V$ associated with a subset $T = \{t_1, t_2, \dots, t_k\}$ of V by $\Gamma(t_i/T)$={d(t_i, t_1), d(t_i, t_2),…,d(t_i, t_k)} where $d(t_i, t_j)$ is defined as distance between the vertex t_i and t_j. Then the subset T is said to be antipodal resolving sets if $d(t_i/T) \neq d(t_j/T)$ and the subset T should be antipodal set i.e if there exist vertices $t_i, t_j \in T$ such that $d(t_i, t_j) = diam(G)$. The minimum cardinality of T is called as antipodal dimension and it is denoted by ζ_{ad}.

Definition: 3.2 Antipodal Independent resolving sets: Let $G = (V, E)$ be a graph. Let $T \subseteq V$. Then the subset $T = \{t_1, t_2, \dots, t_k\}$ of V is called antipodal independent resolving set if subset T is antipodal resolving set and independent set i.e no two vertices in the set are adjacent. The minimum cardinality of T is called as antipodal independent dimension and it is denoted by ζ_{aid}.

Definition: 3.2 Antipodal Pendant resolving sets: Let $G = (V, E)$ be a graph. Let $T \subseteq V$. Then the subset $T = \{t_1, t_2, \dots, t_k\}$ of V is called antipodal pendant resolving set if subset T is antipodal resolving set and pendant set i.e if the induced subgraph $< T >$ contains atleast one pendant vertex. The minimum cardinality of T is called as antipodal pendant dimension and it is denoted by ζ_{apd}.

Definition: 3.3 Antipodal cototal resolving sets: Let $G = (V, E)$ be a graph. Let $T \subseteq V$. Then the subset $T = \{t_1, t_2, \dots, t_k\}$ of V is called antipodal cototal resolving set if subset T is antipodal resolving set and cototal set i.e if the induced subgraph $< V - T >$ is not an isolated vertex. The minimum cardinality of T is called as antipodal cototal dimension and it is denoted by ζ_{actd}.

Definition: 3.4 Antipodal equitable resolving sets: Let $G = (V, E)$ be a graph. Let $T \subseteq V$. Then the subset $T = \{t_1, t_2, \dots, t_k\}$ of V is called antipodal equitable resolving set if subset T is antipodal resolving set and equitable set i.e for any vertex $t_i \in V - T$ there exist a vertex $t_j \in T$ such that $t_i t_j \in E(G)$ $|\deg(t_i) - \deg(t_j)| \leq 1$. The minimum cardinality of T is called as antipodal equitable dimension and it is denoted by ζ_{aeqd}.

Theorem: 3.1 For any connected identity graphs of Z_n, $n > 3$ where n is odd has

a) $\zeta_{ad}(Z_n) = \frac{n-1}{2}, n > 3$

b) $\zeta_{ad}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{actd}(Z_n)$

c) $\zeta_{atd}(Z_n) = \zeta_{apd}(Z_n) = \zeta_{aeqd}(Z_n) = \frac{n+1}{2}, n \geq 3$

d) $\zeta_{ad}(Z_n) < \zeta_{atd}(Z_n)$

Proof: Let graph $G = (V, E)$. The vertex set of G is $V(G) = \{0,1,2, \dots n-1\} = \left\{t_0, t_1, t_2, \dots, t_{\frac{n-1}{2}}, t_{\frac{n+1}{2}}, \dots, t_{n-1}\right\}$. The edge set of G is $E(G) = \{t_0 t_i,\ t_1 t_{n-1}, t_2 t_{n-2}, \dots, t_{\frac{n-1}{2}}\ t_{\frac{n+1}{2}}\}$, $1 \leq i \leq n-1$. $|V(G)| = n$; $|E(G)| = \frac{3n-3}{2}$.

a) Let $T \subseteq V$. For the set T, the set of odd vertices like $\{t_1, t_3, t_5, \dots\}$ or set of even vertices like $\{t_2, t_4, t_6, \dots\}$ of degree 2 has been chosen. Using these vertices we can clearly see that each vertices receives distinct codes i.e distance between the vertex $t_i$to the set of vertices in the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2 for any Z_n. Also distance between any two vertices in the set T is also 2. Using the same subset of T we can clearly see that condition for the antipodal set has been satisfied i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G) = 2$. Therefore set T with minimum cardinality is an antipodal resolving set and its antipodal dimension ie. $\zeta_{ad}(Z_n) = \frac{n-1}{2}, n > 3$.

b) Using the same set of vertices of T condition of independent set i.e no two vertices in the set T is adjacent has been satisfied. Therefore set T is a antipodal independent resolving set and its antipodal independent dimension is $\zeta_{aid}(Z_n) = \frac{n-1}{2}, n > 3$. With the same set of vertices of T cototal resolving set condition is satisfied i.e every vertex in the set T-S contains only set of odd vertices or even vertices accordingly as set T is chosen and the vertex t_0. Therefore for every vertex $t_i \in T - S$ has no isolated vertex in the induced subgraph of $< T - S >$. Therefore set T is a antipodal cototal resolving set and its antipodal cototal dimension is $\zeta_{actd}(Z_n) = \frac{n-1}{2}, n > 3$. Comparing all the above dimensions it is conclude that $\zeta_{ad}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{actd}(Z_n)$.

c) The subset T_1 has been consider in such a way that set of odd vertices $\{t_1, t_3, t_5, \dots\}$ or set of even vertices $\{t_2, t_4, t_6, \dots\}$ of degree 2 and the vertex t_0. Clearly set T_1 is a resolving set (each vertices receives distinct codes) i.e distance between every vertex $t_i \in V(G)$ to the set of vertices in the set T_1 is distinct also the subset T_1 satisifies the condition of antipodal set. T_1 also satisfies the condition of total set i.e for every vertex $t_i \in V$ is adjacent to some vertex in the set T_1. Therefore set T_1 is an antipodal total resolving set and its dimension is $\zeta_{atd}(Z_n) = \frac{n+1}{2}, n \geq 3$. With this same set T_1pendant set condition also satisfies i.e induced graph T_1 contains atleast one pendant vertex. Therefore it is a antipodal pendant resolving sets and its dimension is $\zeta_{apd}(Z_n) = \frac{n+1}{2}, n \geq 3$. Equitable set condition is satisified i.e for every vertex of the subset $T_1 - S$ (contains either set of odd vertices or set of even vertices accordingly set T_1 has been chosen) there exist a vertex in the subset T_1 such that

$|\deg(t_i) - \deg(t_j)| \leq 1$ Therefore set T_1 is antipodal equitable resolving set and its dimension is $\zeta_{aeqd}(Z_n) = \frac{n+1}{2}, n \geq 3$. Comparing all these dimension with the same set T_1 we can conclude that $\zeta_{atd}(Z_n) = \zeta_{apd}(Z_n) = \zeta_{aeqd}(Z_n)$.

d) By above dimensions comparison it have been conclude that $\zeta_{ad}(Z_n) < \zeta_{atd}(Z_n)$.

Example: 3.2

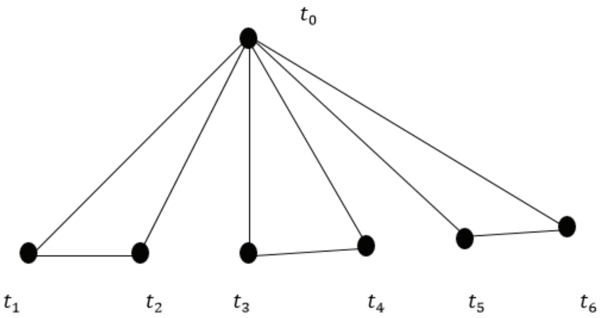

Figure 1. Identity graph of Z_7.

Here the subset T= $\{t_1, t_3, t_5\}$. Diameter of the graph is two. Therefore $d(t_i, t_j) = diam(G)$. Hence $\zeta_{ad}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{actd}(Z_n)$=3. Now choose T as $\{t_1, t_3, t_5, t_0\}$ which satisifies the condition of antipodal resolving set. Therefore $\zeta_{atd}(Z_n) = \zeta_{apd}(Z_n) = \zeta_{aeqd}(Z_n) = 4$. Hence $\zeta_{ad}(Z_n) < \zeta_{atd}(Z_n)$

Theorem: 3.3 For any connected identity graphs of Z_n, $n > 4$ where n is even has

a) $\zeta_{ad}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{actd}(Z_n)$ and $\zeta_{ad}(Z_n) = \frac{n-2}{2}, n > 3$

b) $\zeta_{atd}(Z_n) = \zeta_{apd}(Z_n) = \frac{n}{2}, n > 4$ and $\zeta_{aeqd}(Z_n) = \frac{n+2}{2}, n > 4$

c) $\zeta_{atd}(Z_n) < \zeta_{aeqd}(Z_n)$

d) $\zeta_{ad}(Z_n) < \zeta_{atd}(Z_n) < \zeta_{aeqd}(Z_n)$

Proof: Let $G = (Z_n, \oplus_n)$ be a graph for $n > 4$ even number.

$V(G) = \{0,1,2, \ldots n-1\} = \left\{t_0, t_1, t_2, \ldots, t_{\frac{n}{2}}, \ldots, t_{n-1}\right\}$.

$E(G) = \{t_0 t_i,\ t_1 t_{n-1}, t_2 t_{n-2}, \ldots, /\ \ 1 \leq i \leq n-1\}$.

$|V(G)| = n\ ; |E(G)| = \frac{3n-3}{2}$.

a) Let $T \subseteq V(G)$. For the subset T, set of odd vertices like $\{t_1, t_3, t_5, \ldots, t_{n-2}\}$ or set of even vertices like $\{t_2, t_4, t_6, \ldots\}$ of degree 2 has been chosen. Using these vertices we can clearly see that each vertices receives distinct codes i.e distance between the vertex $t_i$to the set of vertices in the set T is distinct. Therefore set T is a resolving set. Now checking the condition for antipodal resolving set. Here diameter of the graph is 2 for any Z_n. Also distance between any two vertices in the set T is also 2. Using the same subset of T we can clearly see that condition for the antipodal set has been satisfied i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore set T with minimum cardinality is antipodal resolving set and its antipodal dimension ie. $\zeta_{ad}(Z_n) = \frac{n-2}{2}, n > 4$. With the same set of vertices of T it satisfies the condition of independent set i.e no two vertices in the set T is adjacent and antipodal set. Therefore set T is a antipodal independent resolving set and its antipodal independent dimension is $\zeta_{aid}(Z_n) = \frac{n-2}{2}, n > 4$. Also condition for the cototal resolving set is satisfied i.e every vertex $t_i \in T - S$ has no isolated vertex in the induced subgraph of $< T - S >$. Therefore for Therefore set T is a antipodal cototal resolving set and its antipodal cototal dimension is $\zeta_{actd}(Z_n) = \frac{n-2}{2}, n > 4$. From the above dimensions it is concluded that $\zeta_{ad}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{actd}(Z_n)$.

b) Let $T_1 \subseteq V(G)$. The subset T_1 contains set of odd vertices $\{t_1, t_3, t_5, \ldots, t_{n-2}\}$ or set of even vertices $\{t_2, t_4, t_6, \ldots\}$ of degree 2 and the vertex t_0. Clearly set T_1 is an antipodal resolving The same set T_1 satisfies the condition of total set i.e for every vertex $t_i \in V$ there is some vertex adjacent to the set T_1. Therefore set T_1 is an antipodal total resolving set and its dimension is $\zeta_{atd}(Z_n) = \frac{n}{2}, n > 4$. With this same set T_1 pendant set condition also satisfies i.e induced graph T_1 contains atleast one pendant vertex. Therefore it is a antipodal pendant resolving sets and its dimension is $\zeta_{apd}(Z_n) = \frac{n}{2}, n > 4$.

c) Now for the antipodal equitable resolving set subset T has chosen in such a way that set of odd vertices $\{t1, t3, t5, \ldots, tn-1\}$ or set of even vertices $\{t2, t4, t6, \ldots\}$ of degree 2 and the vertex $t0$. Equitable set condition has been satisifies i.e for every vertex of the subset $T - S$ (contains either set of odd vertices till $n-1$ or set of even vertices accordingly set T has been chosen) there exist a vertex in the subset T such that $|\deg(t_i) - \deg(t_j)| \leq 1$. Therefore set T is antipodal equitable resolving set and its dimension is $\zeta_{aeqd}(Z_n) = \frac{n+2}{2}, n > 4$. Thus $\zeta_{atd}(Z_n) = \zeta_{apd}(Z_n) = \frac{n}{2}, n > 4$ and $\zeta_{aeqd}(Z_n) = \frac{n+2}{2},\ n > 4$. Comparing the dimensions of antipodal total resolving set and equitable resolving sets $\zeta_{atd}(Z_n) < \zeta_{aeqd}(Z_n)$.

d) By all the above dimension comparison it is concluded that $\zeta_{ad}(Z_n) < \zeta_{atd}(Z_n) < \zeta_{aeqd}(Z_n)$.

Example: 3.4

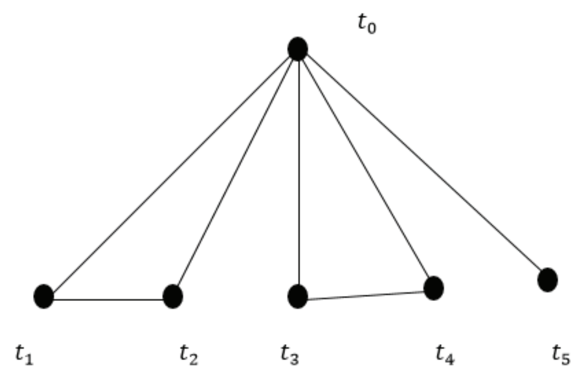

Figure 2. Identity graph of Z_6.

Here the subset T= $\{t_1, t_4\}$. Diameter of graph is two. Here $\zeta_{ad}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{actd}(Z_n) = 2$. Now choose T as $\{t_1, t_3, t_0\}$. Therefore $\zeta_{atd}(Z_n) = \zeta_{apd}(Z_n) = \zeta_{aeqd}(Z_n) = 3$.

For $\zeta_{aeqd}(Z_n)$ choose T as $\{t_1, t_3, t_5, t_0\}$. Therefore $\zeta_{aeqd}(Z_n) = 4$. So $\zeta_{ad}(Z_n) < \zeta_{atd}(Z_n) < \zeta_{aeqd}(Z_n)$.

Theorem: 3.3 For any identity graph of klein-4 group has $\zeta_{ad}(K_4)$ =3 also $\zeta_{ad}(K_4) = \zeta_{aid}(K_4) = \zeta_{apd}(K_4) = \zeta_{atd}(K_4) = \zeta_{aeqd}(K_4)$.

Proof: Let graph $G = I(K_4)$. $V(G) = \{t_0, t_1, t_2, t_3\} = \{e, a, b, ab\}$. $E(G) = \{t_0t_i/1 \leq i \leq 3\}$. Let $T \subseteq V$ .The subset T has been chosen in such a way that $\{t_1, t_2, t_3\}$ each vertices are of degree one. Using these vertices we can clearly see that each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2. Also distance between any two vertices in the set T is also 2. For the subset of T there exist vertices $t_it_j \in T$ such that $d(t_i, t_j) = diam(G)$ the condition for the antipodal set has been satisfied which is of minimum cardinality of antipodal resolving set and its antipodal dimension is three ie.$\zeta_{ad}(K_4) = 3$. Also using same set of vertices independent set condition is satisfied i.e no two vertices in the set T is adjacent. Therefore set T is also a antipodal independent resolving set Now for the antipodal pendant resolving set subset T_1 has chosen in such a way that $\{t_0, t_1, t_2\}$. Clearly the consider set T is a antipodal set because there exist vertices $t_1t_2 \in T$ such that $d(t_1, t_2) = diam(G) = 2$. Using this subset T_1, the induced subgraph $<T_1>$ contains atleast one pendant vertex. And with the same subset T_1 total antipodal resolving set condition is satisfied i.e every vertex in the set $V(G)$ is adjacent to some vertex in the subset T_1. Therefore antipodal total resolving set is 3. Also equitable set condition is satisifies i.e for every vertex of the subset $T_1 - S$ (contains either set of odd vertices till $n-1$ or set of even vertices accordingly set T_1 has been chosen) there exist a vertex in the subset T_1 such that $|\deg(t_i) - \deg(t_j)| \leq 1$. Therefore set T_1 is antipodal equitable resolving set and antipodal pendant resolving set. Therefore $\zeta_{ad}(K_4) = \zeta_{aid}(K_4) = \zeta_{atd}(K_4) = \zeta apd(K4) = \zeta aeqd\ (K4) = 3$.

Remark: For the identity graphs of Klein-4 group antipodal cototal resolving set is not possible because we need minimum three vertices for the resolving set. Obviously subset T-S contains only one vertex however the subset is chosen which fails the condition for the cototal set i.e the induced graphs of <T-S> should not contains isolated vertex.

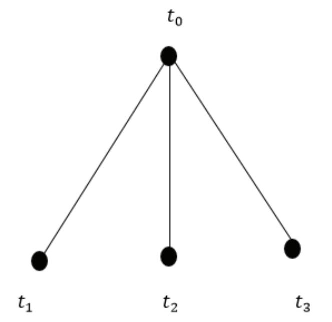

Figure 3. Identity graph of Klein-4 group.

Example: 3.4

Choose T as $\{t_1, t_2, t_3\}$. Diameter of the grah is two. With this set of vertices condition of antipodal resolving set has been satisified. Hence $\zeta_{ad}(K_4) = \zeta_{aid}(K_4) = \zeta_{atd}(K_4) = \zeta_{apd}(K_4) = \zeta_{aeqd}(K_4) = 3$.

Theorem: 3.4 For the identity graph of Quaternion group Q_8,

a) $\zeta_{ad}(Q_8) = 3$,
b) $\zeta_{ad}(Q_8) = \zeta_{aid}(Q_8) = \zeta_{actd}(Q_8)$
c) $\zeta_{atd}(Q_8) = \zeta_{apd}(Q_8) = 4$ and $\zeta_{aeqd}(Q_8) = 5$
d) $\zeta_{atd}(Q_8) < \zeta_{aeqd}(Q_8)$
e) $\zeta_{ad}(Q_8) < \zeta_{atd}(Q_8) < \zeta_{aeqd}(Q_8)$

Proof: Let graph G =identity graph of Q_8. $V(G) = \{t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$. $E(G) = \{t_0t_i, t_2t_3, t_4t_5, t_6t_7 : 1 \leq i \leq 7\}$. This graph $G \cong identity\ graph\ of\ Z_8$. From the Theorem: 3.2 by substituting n=8 we have obtained $\zeta_{ad}(Q_8)$ = 3, ii) $\zeta_{ad}(Q_8) = \zeta_{aid}(Q_8) = \zeta_{actd}(Q_8)$, iii) $\zeta_{atd}(Q_8) = \zeta_{apd}(Q_8) = 4$ and $\zeta_{aeqd}(Q_8) = 5$, iv) $\zeta_{atd}(Q_8) < \zeta_{aeqd}(Q_8)$, v)$\zeta_{ad}(Q_8) < \zeta_{atd}(Q_8) < \zeta_{aeqd}(Q_8)$.

ANTIPODAL RESOLVING SETS ON ORDER PRIME GRAPHS OF FINITE GROUP

In this section results have been observed for the various antipodal resolving sets on order prime graphs of finite group.

Theorem: 4.1 For the order prime graphs of $Z_{n,} n = 2p$ where p is prime satisfies

a) $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = 2p - 3, p \geq 3$
b) $\zeta_{aeqd}(Z_n) = \zeta_{apd}(Z_n) = 2p - 2, p \geq 3$
c) $\zeta_{ad}(Z_n) \leq \zeta_{aeqd}(Z_n)$ and $\zeta_{actd}(Z_n) \leq \zeta_{apd}(Z_n)$

Proof: Let $G=OP(\Gamma(Z_n))$ be a graph and $n = 2p, p > 3$ and p is a prime number.

$V(G) = \{0,1,2, \ldots, n-1\} = \{t_0, t_1, t_2, \ldots, t_{n-1}\}$.

$E(G) = \{t_0t_i,\ t_{\frac{p}{2}}t_{\frac{p}{2}-1},\ t_{\frac{p}{2}}t_{\frac{p}{2}+1}\ /1 \leq i \leq n-1\}$.

$|V(G)| = n$ and $|E(G)| = n + 1$.

a) Let $T \subseteq V(G)$. For the subset T vertices has been chosen in such a way that $2p - 5$ vertices are of degree one and the vertices $\{t_{\frac{p}{2}-1}, t_{\frac{p}{2}}\}$. Using these vertices we can clearly see that each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2. Also the distance between two vertices in the subset T is also 2. Using the subset of T we can clearly see that condition for the antipodal set has been satisfied i.e there exist vertices $t_i\ t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore subset T is of minimum cardinality satisfying antipodal resolving set and its antipodal dimension ie. $\zeta_{ad}(Z_n) = 2p - 3$. Also using same set of vertices cototal set condition is satisfied i.e the subset T-S contains only $\{t_0, t_{\frac{p}{2}+1}, t_{n-1}\}$. Therefore for every vertex $t_i \in T - S$ has no isolated vertex in the induced subgraph of $< T - S >$. Therefore set T is a antipodal cototal resolving set and its antipodal cototal dimension is $\zeta_{actd}(Z_n) = 2p - 3, n \geq 3$. Now for the antipodal independent resolving set subset T has been chosen in such a way that 2p-5 vertices are of degree one and the vertices $\{t_{\frac{p}{2}-1}, t_{\frac{p}{2}+1}\}$. Using these vertices we can clearly see that each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2. Also the distance between two vertices in the subset T is also 2. Using the subset of T we can clearly see that condition for the antipodal set has been satisfied i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore set T with minimum cardinality is antipodal resolving set and its antipodal dimension ie.$\zeta_{ad}(Z_n) = 2p - 3$. Also every vertex in the set T is not adjacent therefore antipodal independent resolving set and its dimension is $\zeta_{aid}(Z_n) = 2p - 3$. Therefore we can conclude that $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = 2p - 3, p \geq 3$.

b) Let $T \subseteq V(G)$. For the set T vertices has been chosen in such a way that 2p-5 vertices are of degree one and the vertices $\{t_0, t_{\frac{p}{2}-1}, t_{\frac{p}{2}}\}$. Each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set.Here diameter of the graph is 2. Also distance between two vertices in the set T is also 2. Using the subset of T we can clearly see that condition for the antipodal set has been satisfied i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore set T with minimum cardinality is antipodal resolving set and its antipodal dimension ie.$\zeta_{ad}(Z_n) = 2p - 2$. Also equitable set condition is satisifies i.e for every vertex of the subset $T - S$ there exist a vertex in the subset T such that $|\deg(t_i) - \deg(t_j)| \leq 1$. Therefore set T is antipodal equitable resolving set. Also using subset T pendant set condition also satisfies i.e induced graph T contains atleast one pendant vertex. Therefore it is a antipodal pendant resolving sets and its dimension is $\zeta_{apd}(Z_n) = 2p - 2, p \geq 3$.

c) Comparing the dimensions from part a) and b) we can conclude that $\zeta_{ad}(Z_n) \leq \zeta_{aeqd}(Z_n)$ and $\zeta_{actd}(Z_n) \leq \zeta_{apd}(Z_n)$.

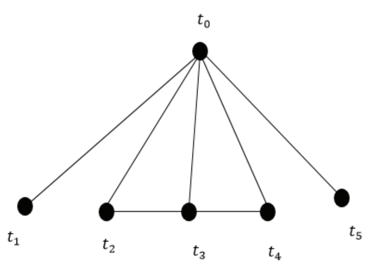

Figure 4. Order prime graph of Z_6.

Example:

Choose T as $\{t_1, t_2, t_3\}$. Diameter of the graph is two. This set of vertices satisfies $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = 3$. Now choose T as $\{t_1, t_2, t_3, t_0\}$ which satisfies$\zeta_{aeqd}(Z_n) = \zeta_{apd}(Z_n) = 4$. Therefore $\zeta_{ad}(Z_n) \leq \zeta_{aeqd}(Z_n)$ and $\zeta_{actd}(Z_n) \leq \zeta_{apd}(Z_n)$.

Theorem:4.2 For the order prime graphs of $Z_n, n = 3p$ where p is prime and p>3 satisfies

a) $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = 3p - 4, p > 3$
b) $\zeta_{aeqd}(Z_n) = \zeta_{apd}(Z_n) = 3p - 3, p > 3$
c) $\zeta_{ad}(Z_n) \leq \zeta_{aeqd}(Z_n)$ and $\zeta_{actd}(Z_n) \leq \zeta_{apd}(Z_n)$

Proof: Let $G=OP(\Gamma(Z_n))$ and $n = 3p$, $p > 3$ and is a prime number.

$V(G) = \{0,1,2, \dots, n-1\} = \{t_0, t_1, t_2, \dots, t_{n-1}\}$.

$E(G) = \left\{t_0 t_i,\ t_{\frac{p}{3}} t_j, t_{\frac{2p}{3}} t_k / 1 \leq i \leq n-1\ ;\ \frac{pj}{3} \cong 0mod3p\ ; \frac{2pk}{3} \cong 0mod3p\right\}$. $|V(G)| = n$ and $|E(G)| = n + 1$.

a) Let $T \subseteq V(G)$. For the subset T vertices has been chosen in such a way that $3p - 6$ vertices of degree one, and $\{t_p, t_{2p}\}$ vertex. Using these vertices we can clearly see that each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2. Also distance between two vertices in the set T is also 2. Using the subset of T we can clearly see that condition for the antipodal set has been satisfied i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore subset T is of minimum cardinality satisfyings antipodal resolving set and its antipodal dimension ie. $\zeta_{ad}(Z_n) = 3p - 4$. Also using same set of vertices cototal set condition is satisified. In the set T-S contains only $\{t_0, t_j, t_k\}$ accordingly as set T has been chosen. Therefore for every vertex $t_i \in T - S$ has no isolated vertex in the induced subgraph of $< T - S >$. Therefore set T is a antipodal cototal resolving set and its antipodal cototal dimension is $\zeta_{actd}(Z_n) = 3p - 4, p > 3$. Using these vertices it is observed that every vertex in the subset T are not adjacent therefore antipodal independent resolving set and its dimension is $\zeta_{aid}(Z_n) = 3p - 4$. Therefore, $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = 3p - 4, p > 3$.

b) Let $T \subseteq V(G)$. For the set T vertices has been chosen in such a way that $3p - 6$ vertices of degree one, and the vertices $\{t_0, t_p, t_{2p}\}$. Each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2. Also there exist distance between two vertices in the set T is also 2. Using the subset of T the condition for the antipodal set has been satisfied i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore set T with minimum cardinality is antipodal resolving set and its antipodal dimension ie.$\zeta_{ad}(Z_n) = 3p - 3$. Also equitable set condition is satisifies i.e for every vertex of the subset $T - S$ there exist a vertex in the subset T such that $|\deg(t_i) - \deg(t_j)| \leq 1$. Therefore set T is antipodal equitable resolving set. Also using subset T pendant set condition also satisfies i.e induced graph T contains atleast one pendant vertex. Therefore it is a antipodal pendant resolving sets and its dimension is $\zeta_{apd}(Z_n) = 3p - 3, p > 3$.

c) Comparing the dimensions from part a) and b) we can conclude that $\zeta_{ad}(Z_n) \leq \zeta_{aeqd}(Z_n)$ and $\zeta_{actd}(Z_n) \leq \zeta_{apd}(Z_n)$.

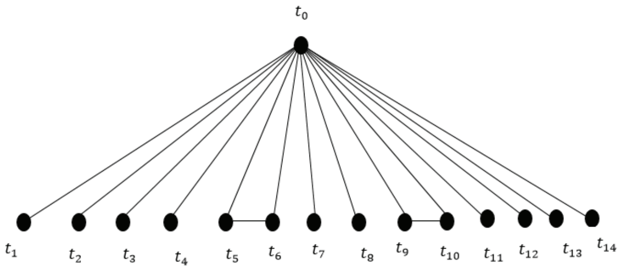

Figure 5. Order prime graph of Z_{15}.

Example:

Choose T as $\{t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_{10}, t_{11}, t_{12}, t_{13}\}$. Diameter of the graph is two. The condition for the antipodal resolving set has been satisfied. Hence $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = 11$. Now to check other conditions of antipodal resolving set choose T as $\{t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_{10}, t_{11}, t_{12}, t_{13}\}$ which satisifies $\zeta_{aeqd}(Z_n) = \zeta_{apd}(Z_n) = 12$. Hence $\zeta_{ad}(Z_n) \leq \zeta_{aeqd}(Z_n)$ and $\zeta_{actd}(Z_n) \leq \zeta_{apd}(Z_n)$.

Theorem: 4.3 The $OP(\Gamma(Z_n)), n \neq 2p$ and $n \neq 3p$ $n \geq 4$ under addition modulo n $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{apd}(Z_n) = n - 2, n \geq 4$.

Proof: Let graph G=$OP(\Gamma(Z_n))$ and $n \neq 2p$ and $n \neq 3p$ $n > 4$ where p is a prime number $V(G) = \{0,1,2, \ldots, n-1\} = \{t_0, t_1, t_2, \ldots, t_{n-1}\}$. $E(G) = \{t_0 t_i \,/ 1 \leq i \leq n-1\}$. $|V(G)| = n$ and $|E(G)| = n - 1$. Let $T \subseteq V(G)$. The vertices in the set T has been chosen in such a way that n −2 vertices of degree one. Clearly each vertices receives distinct codes i.e distance between each vertex in set V(G) to the set T is distinct. Therefore set T is a resolving set. Here diameter of the graph is 2. Also there exist distance between two vertices in the set T is also 2. The subset of T satisfies the condition for the antipodal set i.e there exist vertices $t_i t_j \in T$ such that $d(t_i, t_j) = diam(G)$. Therefore set T with minimum cardinality is antipodal resolving set and its antipodal dimension ie. $\zeta_{ad}(Z_n) = n - 2$. Also using same set of vertices cototal set condition is satisified. In the set T-S contains only $\{t_0, t_{n-1}\}$. Therefore for every vertex $t_i \in T - S$ has no isolated vertex in the induced subgraph of $< T - S >$. Therefore set T is a antipodal cototal resolving set and its antipodal cototal dimension is $\zeta_{actd}(Z_n) = n - 2, n \geq 4$. Every vertex in the set T are not adjacent therefore antipodal independent resolving set and its dimension is $\zeta_{aid}(Z_n) = n - 2$. Also using subset T pendant set condition also satisfies i.e induced graph T contains atleast one pendant vertex. Therefore it is a antipodal pendant resolving sets and its dimension is $\zeta_{apd}(Z_n) = n - 2$. Therefore we can conclude that $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = n - 2, n \geq 4$.

Example:

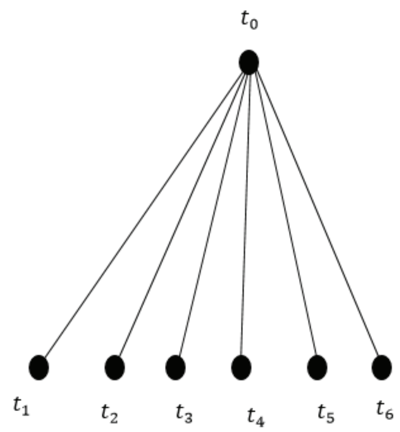

Figure 6. Order prime graph of Z_7.

Here Choose T as $\{t_1, t_2, t_3, t_4, t_5\}$. Diameter of the graph is two which satisifies $\zeta_{ad}(Z_n) = \zeta_{actd}(Z_n) = \zeta_{aid}(Z_n) = \zeta_{apd}(Z_n) = 5$.

Theorem: 4.4 For the $OP(\Gamma(K_4))$, $\zeta_{ad}(K_4) = \zeta_{actd}(K_4) = \zeta_{aid}(K_4) = \zeta_{apd}(K_4) = 2$.

Proof: Let graph $G = Order\ prime\ graph$ of Klein-4 group. $V(G) = \{t_0, t_1, t_2, t_3\} = \{e, a, b, ab\}$. $E(G) = \{t_0 t_i / 1 \leq i \leq 3\}$. This graph $G \cong order\ prime\ of\ Z_4$. Using the Theorem: 4.3 we can conclude that $\zeta_{ad}(K_4) = \zeta_{actd}(K_4) = \zeta_{aid}(K_4) = \zeta_{apd}(K_4) = 2$.

Theorem:4.5 The $OP(\Gamma(Q_8))$, under composition has $\zeta_{ad}(Q_8) = \zeta_{actd}(Q_8) = \zeta_{aid}(Q_8) = \zeta_{apd}(Q_8) = 6$.

Proof: Let graph $G = OP(\Gamma(Q_8))$ under composition. $V(G) = \{-1, 1, i, -i, j, -j, k, -k\} = \{t_0, t_1, t_2, t_3, t_4, t_5, t_6, t_7\}$. $E(G) = \{t_0 t_i$: $1 \leq i \leq 8\}$. $|V(G)| = 8$ and $|E(G)| = 7$. This graph $G \cong OP(\Gamma(Z_8))$. Using the Theorem: 4.3 substitute $n = 8$ we obtain the dimensions as $\zeta_{ad}(Q_8) = \zeta_{actd}(Q_8) = \zeta_{aid}(Q_8) = \zeta_{apd}(Q_8) = 6$.

CONCLUSION

In this article concept of antipodal resolving have been observed for the algebraic graphs of finite group and also its dimension have been compared with various antipodal resolving sets. In future this work will be carried out for the network graphs and result will be compared with algebraic graphs.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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