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Research Article

Statistical exploration for enlargement of reformed water cement ratio law for fly ash concrete

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ABSTRACT

In the last ten years, supplementary mineral admixture (SMA) for cement substitution became gradually practical because of their pozzolanic strength and durability characteristics. A crucial issue for SMA concrete is the strength change depending to the age of the binding ingredient. In order to preserve the pozzolanic reaction in SMA concrete, which aids in the development of strength in cementitious qualities, the time frame of water curing is crucial. In the present study, concrete specimens results from laboratories for various mix designs were evaluated, and the associated strengths were correlated with the Abrams law parameter. The Abrams law was developed for concrete having cement as binder content. Many times, this law has not workout where the alternative binder content to cement need to be used for producing concrete. In this work, statistical approaches were used to build mathematical models that relate compressive strength to many factors that impact it, such as cement content, fly ash content, fly ash to binder ratio, and water binder ratio. Variables are considered —f/cm, c, f, cm, cm/A, μ along with w/cm. The best equation after analysis is found as log (CS) = a0 + a1 (w/cm) + a2 (f/cm). These models might be helpful tools for changing Abrams laws to account for fly ash concrete. The current study effort has been focused on low to moderately strong ordinary concrete construction and the mix design process has been maintained as straightforward as possible. Fly ash was substituted for cement at 0% to 50% of the weight of the cementitious material. The water cementing substance ratios varied from 0.4 to 0.6. The amounts of cementitious materials varied for 300 kg/m^3, 375 kg/m^3, and 450 kg/m^3. This study is focused on adoptability of 54 mixes of fly ash combinations and their utility to common practitioners. As a result, it can be concluded that the influence of cement content to the strength of fly ash concrete is greatest at the beginning of the process, diminishes as the process progresses up to 56 days, and then becomes nearly constant. Also, it can be deduced that the strength of fly ash concrete achieves almost saturation at 56-90 days, and that the increment beyond this point is negligible.

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INTRODUCTION

The components of typical concrete include cement, water, and coarse and small particles. Additional elements, such as chemical or mineral admixtures, can be added to the basic ingredients to enhance the quality of fresh or cured concrete. It is quite difficult to choose suitable concrete components and their appropriate ratios in order to produce good concrete with the necessary strength, workability, and durability at the lowest practical cost. Researchers have been working on creating methods to identify the best strength prediction tool, which can lessen the laboratory effort required for specimen testing, over the past forty years. The cementitious mixtures affect the concrete specimens' quality [1].

The quality of concrete which designers admire the greatest is its strength. Concrete's compressive strength cannot be regarded as an inherent quality of concrete because it relies on a wide range of factors. In 1918, Duff A. Abrams proposed a law that states that for a given concrete material, the strength of workable concrete is solely dependent on the water-to-cement ratio; the size, grading, and quality of the cement are unimportant and only have an impact on the mix's workability [2-3]. For every venture, a controlled strength study is required to assess the level of supplementary mineral admixture (SMA) concrete quality [4]. Predicting concrete strength has long been a significant problem in the global field of concrete technology. In repair and distinct constructing activities, strength is a crucial factor in the concrete's service life [5]. Strength analysis has needed deliberate efforts when the alternative material is employed as a complement to conventional Portland cement. The SMA is essential to the environment because of global requirements and sustainability demands [6]. SMA is a reliable and promising constituent for performing various construction tasks including retrofitting, repairs, and large-scale building. The rheological and mechanical qualities of concrete are improved with SMA [7]. Fly ash is the influencing member for strength and the result might change as per the quality of source, and laboratory precautions [8].

Therefore, it is appropriate and reasonable to conduct studies on the establishment of a modified water-cement ratio regulation for concrete containing mineral admixture. The relationship between compressive strength and ratio is roughly linear. As a result, the cement-water ratio directly affects compressive strength [9]. In other words, when the water-cement ratio decreases, the compressive strength increases. Wider pore sizes in the healed concrete as a result of a high water-cement ratio reduce compressive strength. This is conceivable since the mortar process for ordinary concrete is when the majority of failures occurred.

Strength, one of concrete's key characteristics, serves as a yardstick for evaluating its total effectiveness. Any mix design should generally aim to attain the desired strength at the specified age, and when calculating mix designs for fly ash concrete, water-cementitious value must be taken into account. Relationships between strength development and pozzolanic admixture are also impacted by their compositions and contribution. It is feasible to use sustainable resources to their fullest extent if the most important aspect is taken into consideration and an attempt is made to design a strength model for concrete that incorporates minerals. It is challenging to create a model that can simultaneously take into consideration all of these factors and provide testable predictions in all situations (mixing procedures, curing settings, and testing environments). The effects of other variables can be eliminated by making an effort to keep the other variables constant. Statistics can also be used to assess if the studied factors have an effect on concrete strength or not [8, 27]. It is possible to argue that the discovery of the relationship between concrete's compressive strength and water cement ratio marked the start of modern concrete technology about 100 years ago. A medium-sized library might be filled with the amount of writing on this subject that has since been produced.

Experimental findings for parameter water-cement ratio (w/c ratio) confirmed the fundamental idea that the characteristics of concrete are regulated by the strength of its cementitious matrix. There are several more connections in diverse scientific domains that cause variations in experimental findings of one variable. The relationship between the various variables may be immediately analyzed together with these data and then subjected to statistical analysis. With this information, the value of the variable can be predicted based on the results of the study of the other components. As a consequence, the primary parameter may be further regulated and optimized by attempting to change the other variables. These simulated correlations may then be utilized to generate predictions since computational equations depicting relationships between variables can be created using quantitative methods [27].

One may at least make an educated initial guess about the mix ratio by first examining the most important strength-impacting qualities before deciding on the variables. It is important to remind the concrete scientist to take these elements into account while making concrete rather than assuming they have no bearing on its strength. The first step in doing this task is to identify the important factors that affect the strength of concrete in both standard and fly ash concrete, for example.

The followings are significant factors that determine strength as determined by the available reporting in the Table 1.

The distinctive feature inherent in these statistical relationships is their capacity to serve as valuable instruments, offering primary support to fly ash concrete mix designers. They facilitate the identification of the most pivotal variables, allowing for vigilant monitoring to attain the targeted concrete mix. These equations, positioned as

Table 1. Important factors for strength parameter as per existing literature

Parameters	Researcher acknowledge reference no
Water-cementitious (w/cm) ratio	[11-13]
Cement-aggregates ratio	[23]
Grading, surface texture of aggregates	[16,19]
Fly ash content in the cementitious material	[14-15]
Air content	[18,26]
Cement content	[13, 25]
Water to cement ratio –(w/c), water to (cement+fly ash) ratio– w/(c+f), cement content (c), cement with fly ash (c+f), fly ash to cement ratio (f/c), fly ash (f), cement to fly ash ratio (c/f), slump	[20-21,28]
Capillary porosity, and w/c, Pore Size Distribution, Porosity	[13]
Role of FA/C and Blaine's - specific surface area	[17, 27]
Cement, water content, slump	[13,18, 22,29]
Water abosorption	[24]

straightforward guides, aid in the selection of foundational equations that incorporate crucial parameters or variables when crafting strength prediction models.

Research Significance

After studying a few of the most significant models for tying fly ash compressive qualities to the many variables that affect them, it can be said that the research in this area is rather extensive [14,15,17,27], even though a few critical subjects require more in-depth analysis, as follows: One of the most crucial factors affecting concrete strength is porosity, which is tougher to define but is strongly related to the water-cement ratio. The w/c regulates the capillary porosity of the concrete matrix and the voids in the transition zone. Abrams legislation has endured for more than a century as a result. However, the second most important factor affecting the strength of fly ash concrete after water-cement ratio has not yet been identified. The strength models considered a variety of elements. They were either all included at once in the models or each one was included to the primary variable one at a time. The most advantageous connection between them has not yet been thoroughly and scientifically investigated. It needs to be observed whether the various elements have any significant correlations or whether they each have unexplored higher order effects on strength. The relevance of these statistical connections comes from the possibility that fly ash concrete mix makers might utilize them as initial guidance in determining the most crucial elements and keeping an eye on them in order to produce the targeted concrete mix . The accompanying equations can be used as simple suggestions for building strength prediction models to choose the first-hand basic equation including the most crucial or important traits or factors.

MATERIALS AND METHODS

Utilizing strength information from fly ash concrete that was available in the literature, the hypothesized connections were investigated. Several additional experiments beyond the review of the current study were conducted to investigate the efficacy of the models. The detail of experimentation procedure was adopted as published self-research paper [8]. Therefore, all the repetitive reporting is neglected in this study. In brief, fly ash concrete compressive test results for various curing period are recorded with the help of compressive testing machine (CTM). The distinct easily measured parameters that are based on historical data and the principles of concrete technology have been selected. Regression analysis techniques were used to study various simulation approaches (empirical calculations) with considering curing age of concrete. For consideration of six different curing ages, in order to employ these variables as for each age group, the most effective and significant models were selected.

There are two types of variables used in statistics: predictor variables and response variables as per listed in Table 2.

The least square approach was found useful to analyze data and come to meaningful conclusions. The correlations between output and predictor factors considered for the

Table 2. Predictor variables and response variables

Predictor variables	Response variables
Input variables (regressor)	Output variables
X variables	Y variables
İndependent variables	Dependent variables

judgment. These parameters used in the regression model. The correlation's general framework is as follows:

Response variable (dependent variables) = Random error + Model function

The shape of the model function can be determined, and it contains both independent variables and elements that need to be assessed using data. The random error probability is typically seen as an independent normal population with a mean value of zero.

If Y is the dependent (response) variable, X is the independent (predictor) variable and the model function B_0 + B_1 (X), at such case linear first-order model can be written as

$$Y= B_0 + B_1 (X) + e \quad (1)$$

Where, B_0 , B_1 are the parameters of the model or constants that must be calculated based on data.

The linearity or non-linearity of the variable (B) determines whether a model is linear or non-linear. As an example of second order,

$$Y = B_0 + B_1 (X)+ B_1(X)^2 + e \quad (2)$$

There are times when researchers will use belief models that are considerably more general, in which a response variable (Y) is linked to several predictor variables, such as (X_1), (X_2), and so on.

For example,

$$Y = B_0 + B_1 (X_1)+ B_2(X_2) + e \quad (3)$$

In fact, several models are fitted just using the actual predictor variables (X_1), (X_2),....Xn in their own original incarnation; that is, they may be stated as

$$Y = B_0 + B_1 (X_1)+ B2(X_2) + \dots\dots\dots B_n(X_n) +e \quad (4)$$

There are many other common forms that can be used. The simplest basic form of linear model with variables (X_1), (X_2),....Xn may be stated as follows:

$$Y = B_0 (Z_0)+ B_1 (Z_1)+ B_2(Z_2) + \dots\dots\dots +Bp\text{-}1(Zp\text{-}1) +e \quad (5)$$

Z_0 = 1 is now a dummy variable with a constant value of one (Unity).

However, having a (Z_0) in the model might be mathematically advantageous. Z may include simply one X variable, several X variables, or a mix of the two.

To further explain the concept, take a simple model with four (4) predictor variables and $(2)^4$ = 16 potential regressions. For all potential regressions, the following Table 3 displays the combination of multiple variables.

The optimum equation to employ is then determined by analyzing the patterns seen. (X_1), (X_2), (X_3), (X_4) are the predictor variables in this case. There have been no transformations under this problem, hence Zi = Xi, and i = 1, 2, 3, 4. Y has been the response variable. There are a total of (2)4 = 16 potential regression equations using (X0) and (Xi), where i = 1, 2, 3, 4. The R2 and s parameters can be performed to analyze the different equations.

Y =B_0 + e is one of them. As a result, there are 15 regressing options that contain one or more of the (X)'s.

Table 3. Four-variable combination for all potential regressions.

Equation no.	(B_0)	Predictor variables-1 (X_1)	Predictor variables-2 (X_2)	Predictor variables-1 (X_3)	Predictor variables-1 (X_4)
1	Yes	-	-	-	-
2	Yes	-	-	-	Yes
3	Yes	-	-	Yes	-
4	Yes	-	-	Yes	Yes
5	Yes	-	Yes	-	-
6	Yes	-	Yes	-	Yes
7	Yes	-	Yes	Yes	-
8	Yes	-	Yes	Yes	Yes
9	Yes	Yes	-	-	-
10	Yes	Yes	-	-	Yes
11	Yes	Yes	-	Yes	-
12	Yes	Yes	-	Yes	Yes
13	Yes	Yes	Yes	-	-
14	Yes	Yes	Yes	-	Yes
15	Yes	Yes	Yes	Yes	-
16	Yes	Yes	Yes	Yes	Yes

Table 4. R^2 values for all potential regressions

SET (B)	SET (C)	SET (D)	SET (E)
For single variable & B_0	For 2 variable & B_0	For 3 variable & B_0	For 4 variable & B_0
0.675 (X_4)	0.979 (X_1,X_2)	0.98234 (X_1, X_2, X_4)	0.98237 (X_1, X_2, X_3, X_4)
0.666 (X_2)	0.972 (X_1,X_4)	0.98228 (X_1, X_2, X_3)	-
0.534 (X_1)	0.935 (X_3,X_4)	0.98128(X_1, X_3, X_4)	-
0.286 (X_3)	0.847 (X_2,X_3)	0.97282 (X_2, X_3, X_4)	-
-	0.680 (X_2,X_4)	-	-
-	0.548 (X_1,X_3)	-	-

X_1, X_2, X_3, X_4 are variables

Boundary the runs into 5 sets:

Set (A) not shown; consists of the run with only B_0 [model $E(Y) = B_0$]

Set (B) consists of the four 1 variable runs model [$E(Y) = B_0 + BiXi$]

Set (C) consists of all the 2 variable runs model [$E(Y) = B_0 + BiXi + BjXj$]

Set (D)consists of all the 3 variable runs (and so on ...) [$E(Y) = B_0 + BiXi + BjXj + BkXk$]

Set (E) consists of the run with 4 variable E(Y) = B0 + BiXi +BjXj + BkXk + BLXL

R^2 values for a hypothetical problem with four variables are provided below to demonstrate how R^2 may be used to evaluate a model as discussed by Draper and Smith [10]. The Set B, C, D, E and variables are noted in the following Table 4. The R^2. Values should be analyzed to check if the best equations for each set have a consistent trend of variables. It must be determined if the value of R^2 improves with the addition of more variables. In the case of the given problem, it is noted that following the introduction of two variables, the R^2 increase is minimal. When X_1 and X_2 or X_1 and X_4 were in the regression equation, adding more variables minimizes so little of the variance in the data in the response. When comparing Set C to Set D, this is easily noticeable. The increase in R^2 from Set D to Set E is negligible.

When looking at the preceding Table 4, it is evident that one of the models in Set C is definitely the best, but selecting it demands good judgment. There is considerable discrepancy if f(X1, X2) is used since the optimal single variable model incorporates X4. As a result, it is preferable to utilize f(X1, X4). Other information, such as knowledge about the product's qualities and the physical function of the X-variables, could also be required in order to make a conclusion.

Basic Terminology

In research, deploy a set of indispensable tools within the realm of regression analysis, crucial for unraveling intricate relationships between variables, gauging the aptness of the model, and fostering judicious decision-making rooted in empirical data. This arsenal includes the following key elements:

• Standard Error of the Regression (s or SE):

Function: Measures the precision of the regression model.

Insight: This metric quantifies the average error of the residuals, representing the disparities between observed and predicted values within the regression model. A diminutive standard error signifies an enhanced alignment of the model with the underlying data.

• F-Test (ANOVA):

Function: Assesses the overall significance of the regression model.

Insight: The F-test, synonymous with Analysis of Variance (ANOVA), scrutinizes the presence of a substantial relationship between independent variables and the dependent variable in the regression model. It quantifies the model's effectiveness by comparing the variability elucidated by the model against the unexplained variability.

• t-Test (t-Statistic):

Function: Examines the significance of individual independent variables in the model.

Insight: Deployed to scrutinize the statistical significance of individual coefficients (slope coefficients) for each independent variable in the regression, the t-test illuminates whether a specific independent variable exerts a noteworthy impact on the dependent variable. A heightened absolute t-value coupled with a diminished p-value underscores heightened significance.

RESULTS AND DISCUSSION

For the current investigation, the 28-day strengths of concrete ranged from 15 to 60 MPa. Specimens were examined for compressive strength at six age stages, namely 3, 7, 28, 56, 90, and 180 days. This record has been subjected to regression analysis in order to investigate the link between compressive strength and some of the major characteristics that influence the strength of fly ash concrete.

To propose an augmentation of the Abrams law for contemporary Fly Ash Concretes, a systematic mapping study must be conducted with the goal of identifying the most significant strength influencing parameters and then determining the most optimum combination that generates a correlation not only fitting the actual results but also showing better validation of exploratory data available in the available literature. The recent data analysis has been meticulously adjusted to suit the aforementioned specifications.

The water-cement ratio law of Abrams is expressed in the following Eq (6 to 8) as

$$CS=k_1/[\![k_2]\!]^\wedge(w/C) \tag{6}$$

$$log\ CS = log\ k_1 - w/C\ log\ k_2 \tag{7}$$

$$log\ CS = m_1 + m_2\,(w/c) \tag{8}$$

Where (s) is the strength and (m1) and (m2) are constants.

By introducing a new independent variable several at a time, the connections have been constructed using Abrams law. As a result, the following connections have been considered:

$$log\ (CS) = a(0) + a(1)\ (w/cm) + a(2)\ (f/cm) \quad (1\text{-}I)$$

$$log\ (CS) = a(0) + a(1)\ (w/cm) + a(2)\ (c) \quad (2\text{-}I)$$

$$log\ (CS) = a(0) + a(1)\ (w/cm) + a(2)\ (f) \quad (3\text{-}I)$$

$$log\ (CS) = a(0) + a(1)\ (w/cm) + a(2)\ (cm) \quad (4\text{-}I)$$

$$log\ (CS) = a(0) + a(1)\ (w/cm) + a(2)\ (cm/A) \quad (5\text{-}I)$$

$$log\ (CS) = a(0) + a(1)\ (w/cm) + a(2)\ (\mu) \quad (6\text{-}I)$$

where

(CS): compressive strength of concrete in MPa at a given age, w/cm, c, f, cm, cm/A.

w/cm: the water binder ratio, c: cement content (kg/m^3), f: fly ash content (kg/m^3), cm: binder (kg/m^3), cm/A: binder aggregate ratio, and μ : the slump of concrete in mm, a(0), a(1), a(2) are constants of regression equations

The strength data of 54- concrete mixes at each of the six age levels were used in a multiple regression analysis. Tables 5 provide the outcomes of the analyses (ANNOVA), containing regression constants, R and S values, F test results, and t test results, as well as their critical values. The value of the F statistic estimated from the present data at the age of 28 days for equation 1 examined with 54 number of strength results is 289.55. (ANOVA). The number of independent variables on which strength is dependent is 2 in equation 1, and the number of degrees of freedom is 54-(2+1)=51. At a 5% level of significance, the critical value of F for these values is 3.175. The null hypothesis is incorrect because F calculated is bigger than F crucial.

The results are plotted in the following Figures. 1 to 7.

The parameters considered for verification of w/cm ratio as it is an important factor for final outcome. From the Figures 8, 9, and 10, it's clear that porosity of concrete decreases with the incorporation of fly ash percentage. All the three cement binder content namely 300kg/m^3, 375kg/m^3, and 450 kg/m^3 are found same nature of graph.

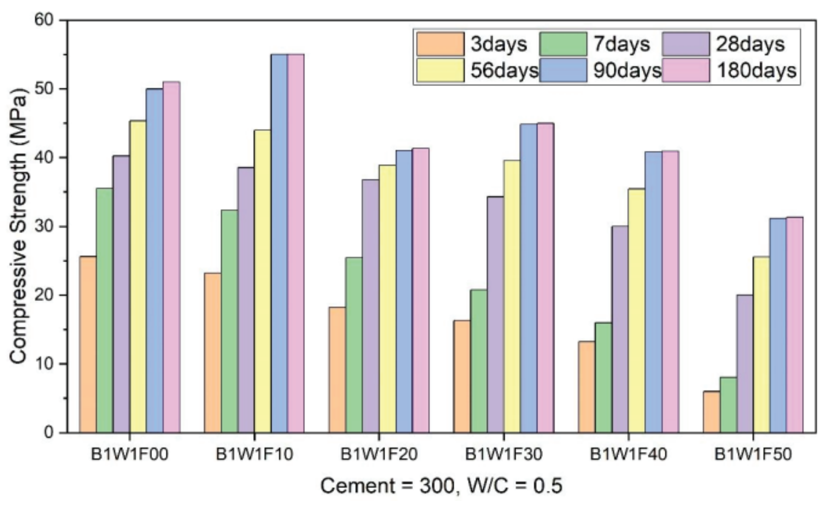


Figure 1. Compressive strength of concrete for binder = 300 kg/m^3, w/c 0.5.

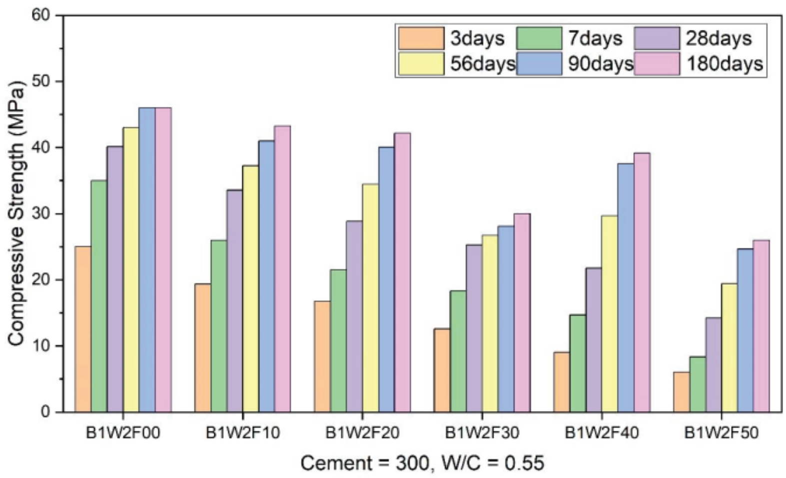


Figure 2. Compressive strength of concrete for binder = 300 kg/m^3, w/c 0.55.

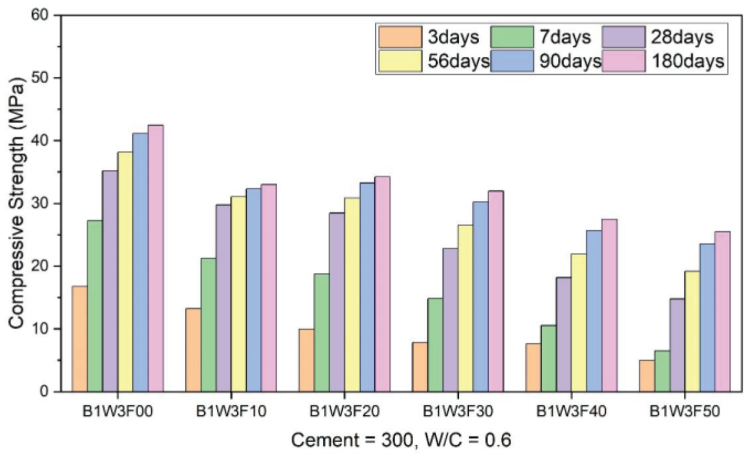


Figure 3. Compressive strength of concrete for binder = 300 kg/m^3, w/c 0.6.

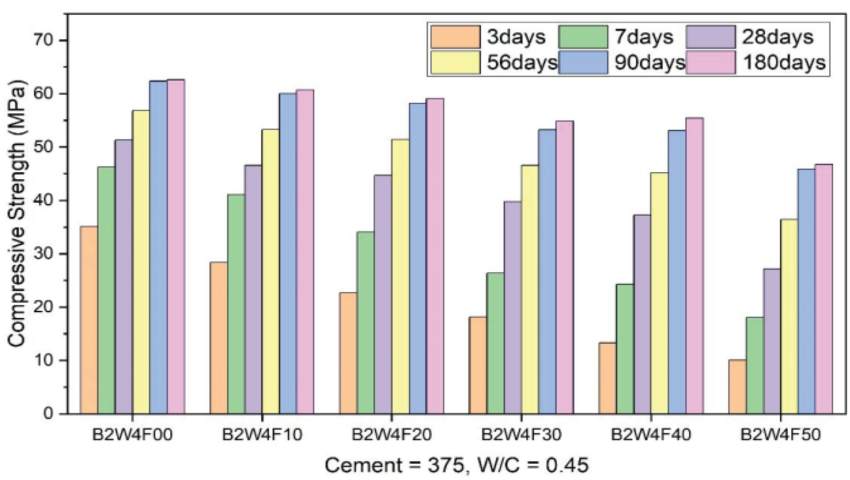


Figure 4. Compressive strength of concrete for binder = 375 kg/m^3, w/c 0.45 .

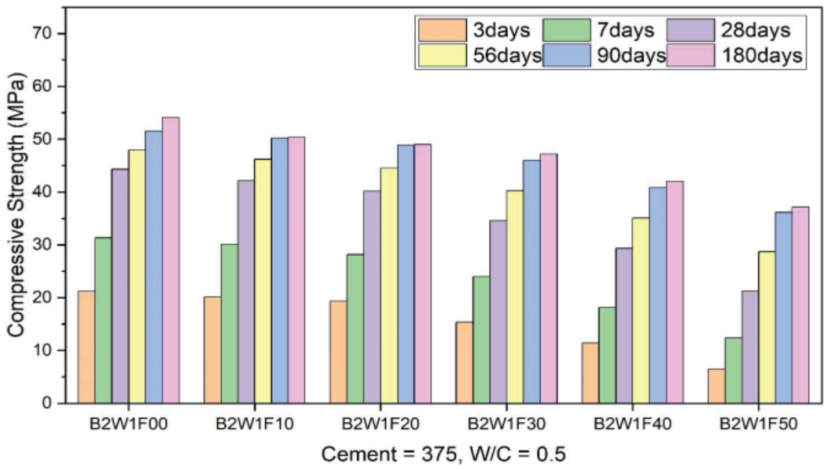


Figure 5. Compressive strength of concrete for binder = 375 kg/m^3, w/c 0.5.

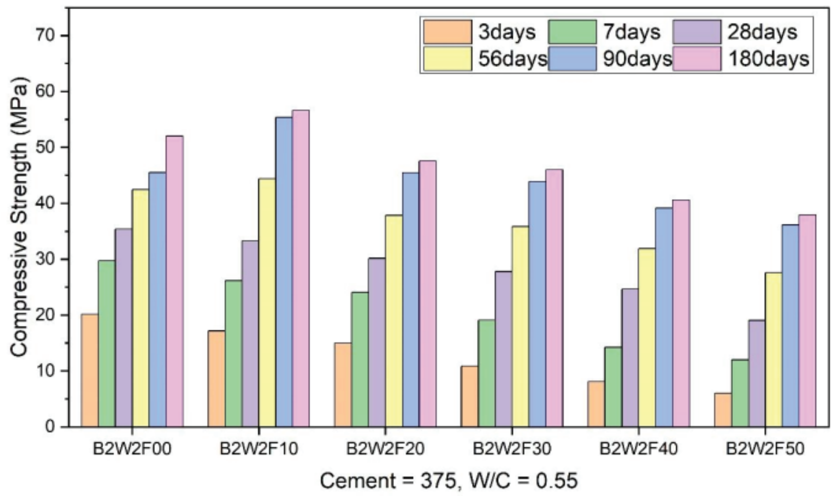


Figure 6. Compressive strength of concrete for binder = 375 kg/m^3, w/c 0.55.

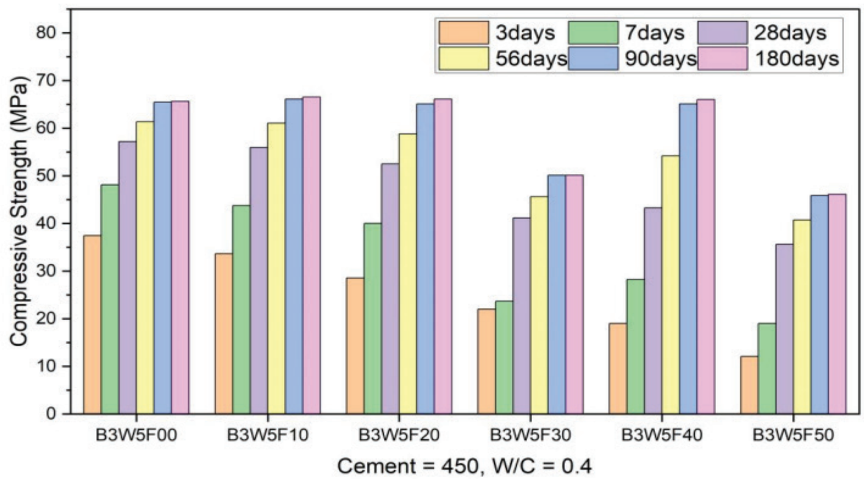


Figure 7. Compressive strength of concrete for binder = 450 kg/m^3, w/c 0.4.

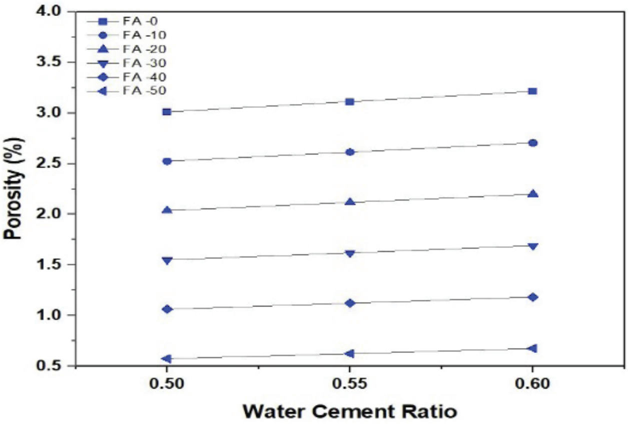


Figure 8. Binder content 300kg/m^3, w/c vs Porosity.

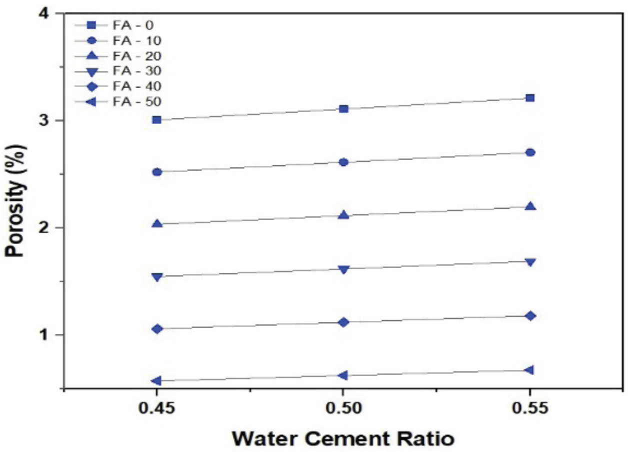


Figure 9. Binder content 375kg/m^3, w/c vs Porosity.

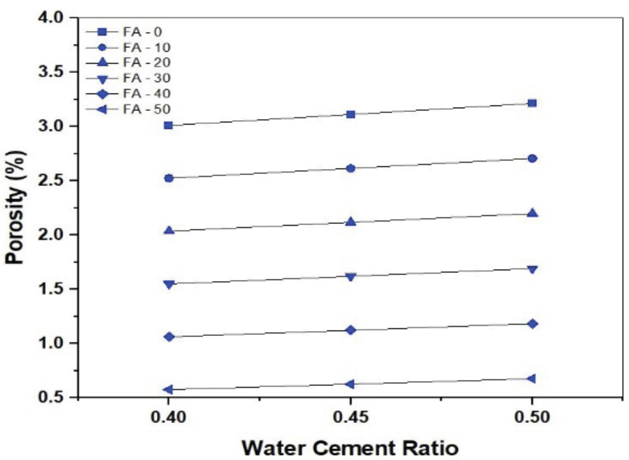


Figure 10. Binder content 450kg/m^3, w/c vs Porosity.

The t test was used to evaluate all of the characteristics in the models at the 5% significant level with [n-(m+1)] degrees of freedom, (where n=number of strength observations and m=number of independent variables). The absolute values of the t statistics for the regression coefficients corresponding to water-cementitious material ratio and fly ash cementitious material ratio for equation 1 examined with 54 number of strength findings are -17.27 and -16.75, respectively. For this equation, the number of degrees of freedom is equal to 51 [54-(2+1)]. The critical value of t at a 5% significant level corresponds to this amount of degrees of freedom: 2.006. The null hypothesis is incorrect since the estimated absolute values of t are larger than t crucial for all of the variables (including the intercept, a0), and they may all be deemed statistically meaningful. Regression statistics of strength models of method "the most significant strength regulating variable following w/c or w/cm among the several criteria influencing the basic composition of concrete".

Because all regression equations have been investigated in this study, only the models that pass the t test are important. The best final equations found are as follows Table 6.

Every one of the equations' a(0) values are positive at all six ages. The value of the intercept a(0) has increased from 3 to 7 days, but after this age, the values have been steadily decreasing up to 365 days. For Equations (9) and (10), a nearly same trend has been found. The curve's slope is sharper at the beginning, then flattens off and almost becomes horizontal as it progresses as shown in Fig 11. Equations (9) and (11) are comparable since they deal with comparable variables regarding fly ash.

Equation (10) is about cement and has a distinct character to it. Unlike the previous equations, the value of the intercept increases over time and becomes horizontal beyond 90 days. At all six ages, the results of a(1) of the three equations are negative, showing a similar tendency. The negative partial regression co-efficient for w/cm implies that at a certain age and with a steady fly ash binder ratio, increasing w/cm leads to a decrease in strength, which is consistent with Abrams w/c rule. The influence of w/cm on strength is greatest at early ages, when the pace of strength growth is rapid. The slope of the curve changes from steeper to flatter as one gets older. This pattern is analogous to the growth in concrete compressive strength, which is fast in the beginning but gradually decreases as it progresses.

When the deviation in numerical values of a(1) at various age levels is calculated (as an indicator of the slope of the a(1) vs. age curve), it is discovered that the discrepancy at successive age levels is quite significant up to 90 days, but the difference among 90 and 180 days is considerably less. This result is comparable to a(0). It may be deduced that the impact of this factor on concrete strength after 90 days is negligible. According to researcher [14], hydration is complete in 90 days, hence strength gains far beyond point are irrelevant. The current findings support the previous assertion.

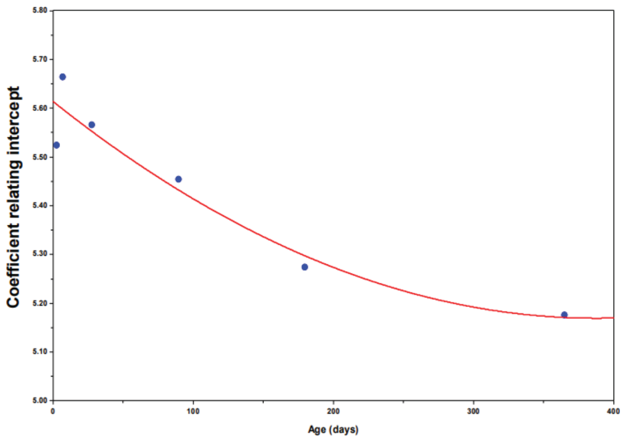
Validation of the Proposed Models of compressive strength as per Equation 1 for 28 days curing age as noted in Table 7.

Table 5. Four Regression statistics of strength models

Coefficient					t statistics			tcr=2.007, fcr=3.175		
Eq No	**Age**	**a(0)**	**a(1)**	**a(2)**	**t(o)**	**t(1)**	**t(2)**	**F(obs)**	**R**	**S**
Eq 1	3	5.524	-4.261	-1.985	44.26	-17.63	-20.95	375.06	0.967	0.121
Eq 1	7	5.663	-3.925	-1.849	47.74	-14.94	-17.95	272.91	0.956	0.131
Eq 1	28	5.565	-3.326	-1.264	55.98	-17.27	-16.75	289.55	0.958	0.096
Eq 1	56	5.454	-2.844	-0.737	66.39	-17.87	-11.82	229.63	0.948	0.08
Eq 1	90	5.274	-2.376	-0.739	63.98	-14.88	-11.82	180.66	0.935	0.08
Eq 1	180	5.175	-2.106	-0.661	63.95	-13.43	-10.76	148.24	0.922	0.078
Eq 2	3	3.334	-0.036	0.0037	10.89	-6.1	8.62	77.16	0.865	0.238
Eq 2	7	3.576	-2.749	0.0036	12.51	-5.92	8.87	78.29	0.866	0.222
Eq 2	28	4.073	-2.472	0.00263	22.47	-8.39	10.14	120.33	0.907	0.141
Eq 2	56	4.569	-2.334	0.00157	36.86	-11.59	8.86	147.89	0.922	0.096
Eq 2	90	4.303	-1.804	0.0017	44.1	-11.37	12.63	202.2	0.941	0.076
Eq 2	180	4.284	-1.576	0.0016	48.9	-11.07	13.02	204.11	0.942	0.068
Eq 3	3	5.829	-4.907	-0.00513	44.51	-19.79	-20.55	361.35	0.966	0.123
Eq 3	7	5.918	-4.505	-0.0045	34.89	-14.03	-14.24	177.46	0.934	0.159
Eq 3	28	5.731	-3.716	-0.00309	44.69	-15.3	-12.67	175.75	0.933	0.12
Eq 3	56	5.545	-3.066	-0.00176	55.38	-16.17	-9.24	157.11	0.926	0.094
Eq 3	90	5.357	-2.59	-0.0017	51.47	-13.16	-8.71	111.87	0.9	0.097
Eq 3	180	5.243	-2.295	-0.0015	51.17	-11.83	-7.7	89.62	0.88	0.096
Eq 4	3	5.629	-4.757	-0.0011	7.59	-5.41	-1.03	17.32	0.632	0.368
Eq 4	7	5.383	-4.104	-0.00039	7.58	-4.87	-0.38	15.62	0.613	0.352
Eq 4	28	5.085	-3.231	0.00021	10.36	-5.55	0.29	23.39	0.688	0.243
Eq 4	90	5.067	-2.707	0.00030	16.51	-7.44	0.68	43.89	0.792	0.152
Eq 4	180	4.55	-1.989	0.00086	15.28	-0.563	2.003	34.37	0.755	0.148
Eq 4	360	4.396	-1.66	0.00099	16.43	-5.23	2.56	34.78	0.757	0.133
Eq 5	3	5.673	-4.789	-2.107	10.14	-6.06	-1.66	18.71	0.647	0.362
Eq 5	7	5.562	-4.238	-1.248	10.31	-5.56	-1.02	163.3	0.621	0.349
Eq 5	28	5.297	-3.389	-0.255	14.09	6.38	-0.29	23.39	0.688	0.243
Eq 5	56	5.226	-2.826	0.069	22.14	-8.47	0.13	43.28	0.79	0.153
Eq 5	90	4.82	-2.192	0.736	20.74	-6.67	1.39	32.16	0.744	0.15
Eq 5	180	4.683	-1.875	0.92	22.29	6.31	1.93	31.84	0.742	0.136
Eq 6	3	4.859	-3.973	-0.0006	12.28	-4.87	-0.83	17.009	0.629	0.369
Eq 6	7	5.096	-3.797	-0.00027	13.5	-4.87	-0.38	15.62	0.613	0.352
Eq 6	28	5.25	-3.427	0.00021	20.15	-6.37	0.44	23.49	0.689	0.243
Eq 6	56	5.295	-2.965	0.00025	32.54	-8.82	0.84	44.2	0.794	0.152
Eq 6	90	5.193	-2.707	0.00068	33.34	-8.42	2.41	36.31	0.763	0.145
Eq 6	180	5.121	-2.449	0.00071	36.34	-8.42	2.76	35.87	0.761	0.132

Table 6. Final Equations for computing compressive strength of fly ash concrete

Curing Age	Equation form	Eq. No
3 days	$\log(CS) = 5.524 - 4.261\,(w/cm) - 1.985\,(f/cm)$	(9)
7 days	$\log(CS) = 5.663 - 3.925\,(w/cm) - 1.849\,(f/cm)$	(10)
28 days	$\log(CS) = 5.565 - 3.326\,(w/cm) - 1.264\,(f/cm)$	(11)
56 days	$\log(CS) = 5.454 - 2.844\,(w/cm) - 0.737\,(f/cm)$	(12)
90 days	$\log(CS) = 5.247 - 2.376\,(w/cm) - 0.739\,(f/cm)$	(13)
180 days	$\log(CS) = 5.175 - 2.106\,(w/cm) - 0.661\,(f/cm)$	(14)



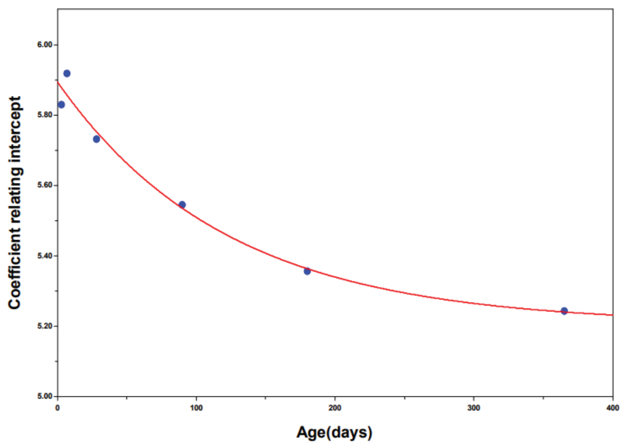


Figure 11. The curve's slope pattern of Coefficient a(0).

Table 7. Validation of the Proposed Models of compressive strength with literature

Researcher	w/cm	f/cm	Age	FPredicted	Experimental (Fexp)	% Difference
Self	0.42	0.25	28	37.01	48.38	23.50
Self	0.42	0.35	28	32.81	40.49	18.98
Self	0.52	0.25	28	27.04	35.32	23.45
Self	0.52	0.35	28	23.97	30.25	20.77
Researcher [30]	0.57	0.26	28	23.19	31.64	26.70
Researcher [30]	0.54	0.21	28	27.07	35.58	23.92
Researcher [30]	0.52	0.17	28	30.25	38.43	21.29
Researcher [30]	0.48	0.17	28	34.30	41.72	17.80

The efficacy of the proposed models in predicting strength across diverse fly ash concrete blends necessitates scrutiny. To authenticate these models, several mixes beyond the current study's purview were formulated, and their strengths at 28 days was ascertained. Intentionally opting for water-binder ratios, binder contents, and fly ash replacement percentages at intermediary levels, distinct from the prevailing protocol, served as a deliberate strategy to assess the robustness of the strength equations. Additionally, strength data gleaned from existing literature has been incorporated to forecast the strength of the fly ash concrete blends. The anticipation does not extend to the prospect that these equations, laden with multiple regression constants, could aptly predict outcomes for every conceivable variant of fly ash concrete mixes. However, the inherent value of these equations lies in their capacity to offer utility; specifically, the identified parameters and equation types prove instrumental in the prediction of strength. Moreover, it's noteworthy that anyone well-versed in basic computational methods can derive the values for the regression constants.

CONCLUSION

The set of equations are judged important after examining 36 models using Regression analysis with the first one being the best.

log (CS) = a0 + a1 (w/cm) + a2 (f/cm)

Both other equations (i.e. 2 and 3) are, meanwhile, also true.

log (CS) = a0 + a1 (w/cm) + a2 (c)

log (CS) = a0 + a1 (w/cm) + a2 (f)

The following conclusions may be taken from the research:

(1) To propose an augmentation of the Abrams law for contemporary Fly Ash Concretes, a systematic mapping study must be conducted with the goal of identifying the most significant strength influencing parameters and then determining the most optimum combination that generates a correlation fitting the actual results. While the F and t tests confirm the validity of Abrams' law for fly ash concretes but outcome of R observed that some adjustments are imperative to enhance its efficacy.

(2) The regression statistics reveal that the relationship between age and the strength models is not uniform across

equations. Some equations show stronger relationships and model fits with age, while others exhibit weaker associations. The specific age-dependent trends in the coefficients, t statistics, F(obs) values, and R values should be carefully considered when interpreting these strength models in the context of different age groups. Every one of the equations' a(0) values are positive at all six ages. Whereas a(1) values were found negative and positives. The value of the intercept (a0) has increased from 3 to 7 days, but after this age, the values have been steadily decreasing up to 365 days.

(3) The value of the intercept increases over time and becomes horizontal beyond 90 days.

(4) Among of a number of elements impacting the compound structure of concrete, the fly ash-binder ratio (f/cm) has been found as the second most noteworthy strength modifying factor after w/cm.

(5) It is seen that the numbers of a(2) are positive at all ages in Equation (10), that relates with w/cm and c. This means that when the cement content of concrete increases, the strength of the concrete increases at any given age and w/cm. It was observed that increasing the cement content at a fixed w/cm leads to an increase in concrete strength up to a particular level of cement, referred to as the optimal cement content. The total cementitious material content in this study ranged from 300 to 450 kg/m^3, with cement content varying from 150 kg/m3 to 450 kg/m^3.

(6) However, the influence of cement content on fly ash concrete strength decreases with age, from 3 to 56 days, indicating that the influence of cement content on fly ash concrete strength decreases with age. However, the value of a2 does not change much after 56 days. As a result, it can be concluded that the influence of cement content to the strength of fly ash concrete is greatest at the beginning of the process, diminishes as the process progresses up to 56 days, and then becomes nearly constant. As a result, it can be deduced that the strength of fly ash concrete achieves almost saturation at 56-90 days, and that the increment beyond this point is negligible.

[7] Through an exhaustive statistical scrutiny, delving into a myriad of potent variables influencing concrete strength, our analysis unveils a paramount revelation. Amidst the intricate fabric of concrete composition, it becomes evident that the second most impactful factor shaping concrete strength is the ratio of fly ash binder (f/cm).

By opting for uncomplicated, user-friendly, and easily measurable variables in the strength modeling process, we anticipate their extensive applicability in fly ash mix proportioning. This not only fosters confidence in the material among the general populace but also contributes to an augmented utilization of fly ash in concrete. The validation of these proposed relationships has been undertaken by juxtaposing them with strength results extracted from existing literature on fly ash concrete. Moreover, supplementary experiments, surpassing the confines of the current study, have been conducted to scrutinize the effectiveness of the proposed models.

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AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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