Sigma J Eng Nat Sci, Vol. 42, No. 6, pp. 1847−1855, December, 2024

Sigma Journal of Engineering and Natural Sciences
Web page info: https://sigma.yildiz.edu.tr
DOI: 10.14744/sigma.2024.00141

sigma Journal of Engineering and Natural Sciences
Mühendislik ve Fen Bilimleri Dergisi

Research Article

Computational method to solve Davey-Stewartson model and Maccari's system

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ARTICLE INFO

Article history
Received: 11 August 2023
Revised: 18 August 2023
Accepted: 29 September 2023

Keywords:
Computer Algebra Systems, Davey-Stewartson Equation; Maccari's System; Riccati-Bernoulli Sub-ODE Method

ABSTRACT

In this article, Computer algebra systems and the Riccati-Bernoulli sub-ODE method are efficiently utilized to solve Davey-Stewartson and Maccari's systems. We successfully obtained the set of new exact solutions for these systems using the computer algebra MAPLE system. For the validity of acquired solutions, the constraint conditions are given. To investigate the behavior of these solutions, graphical representations of the derived solutions are provided under suitable parameter values.

Cite this article as: Esen H, Özdemir N, Seçer A, Bayram M. Computational method to solve Davey-Stewartson model and maccari's system. Sigma J Eng Nat Sci 2024;42(6):1847−1855.

INTRODUCTION

Computers are generally thought of as number crunchers, but there is no reason why they cannot also be used as formula crunchers. Computer algebra systems allow us to manipulate formulae. A computer algebra system computes with symbols rather than numbers. Such systems are useful for manipulating formulae. A particularly useful technique is the calculation of the coefficients of the polynomials. We have used computer algebra systems in several ways to solve our problem [1-4].

Nonlinear partial differential equations (NLPDEs) are utilized in diverse areas to model notable phenomena. So, the solutions of NLPDEs have a significant role in the research of physics, engineering, and applied mathematics, containing population ecology, solid-state physics, plasma waves, plasma physics, optical fibers, quantum mechanics, fluid mechanics, heat flow, propagation of shallow waves and wave propagation phenomena. To acquire soliton and traveling wave solutions for NLPDEs, numerous computational approaches have been constructed. Some of these approaches are the new extended direct algebraic method [5-7], the extended Jacobi elliptic function expansion method [8], the generalized Kudryashov method [9], the generalized algebraic method [10], the extended generalized Riccati equation mapping method [11], Q-function method [12,13], extended tanh method [14].

This research article aims to examine the analytical solutions of the Maccari's system and Davey-Stewartson system with the aid of the Riccati-Bernoulli sub-ODE method [15-18].

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This paper was recommended for publication in revised form by Editor-in-Chief Ahmet Selim Dalkilic

Published by Yıldız Technical University Press, İstanbul, Turkey

The nonlinear complex Maccari's system (CNMS) [19] is described as:

$$\begin{aligned} iQ_t + Q_{xx} + RQ = 0, \\ iS_t + S_{xx} + RS = 0, \\ iN_t + N_{xx} + RN = 0, \\ R_t + R_y + (|Q + S + N|^2)_x = 0. \end{aligned} \tag{1}$$

The CNMS is a form of the complex nonlinear system modeling the motion of an isolated wave is localized in a small part of space and used in several fields such as plasma physics, nonlinear optic, hydrodynamic [20]. $Q(x, y, t)$, $S(x, y, t)$ and $N(x, y, t)$ are complex-valued functions and $R(x, y, t)$ is a real-valued function. In order to acquire novel traveling solutions of CNMS, various new methods have been presented. Some of these methods are the new extension of the (G'/G)-expansion method [19], the sine-Gordon expansion method [21], the modified exp $(-\phi(\eta))$-expansion function method [22], the modified F-Expansion method and the generalized projective Riccati equation method [23], the first integral method [24].

In this research article, the Riccati- Bernoulli sub- ODE method is also applied to the Davey Stewartson system given by:

$$\begin{aligned} iU_t + \gamma(U_{xx} + U_{yy}) + \mu|U|^2U - \alpha UV = 0, \\ V_{xx} + V_{yy} + \sigma(|U|^2)_{xx} = 0, \end{aligned} \tag{2}$$

where $U = U(x, y, t)$ and $V = V(x, y, t)$ are the complex wave envelope and the real forcing terms, respectively in [25]. γ, μ and σ are real constants. In order to acquire the solutions of various forms of the Davey Stewartson system, several mathematical techniques have been utilized, for example; the exponential function method [25], the extended sinh-Gordon equation expansion method [26], the exp $(-\Phi(\xi))$-expansion method, the first integral method and the Sine-Gordon expansion method [27], the Generalized Elliptic Equation Rational Expansion method [28], the direct similarity reduction method [29], the extended tanh method [30], the extended mapping method [31].

The paper's draft is formed as follows: the Riccati Bernoulli sub-ODE method is summarized in Section 2. The technique is utilized to solve the nonlinear Maccari's and Davey Stewartson systems in Section 3. Eventually, the conclusion of this paper is given in Section 4.

Riccati Bernoulli Sub-Ode Method

In this section, we express the elementary steps of the Riccati Bernoulli sub-ODE method. Any NLPDE can be taken in the following form:

$$R(\varphi, \varphi_x, \varphi_t, \varphi_{xx}, \varphi_{tt}, \varphi_{xt}, \dots) = 0, \tag{3}$$

where R is a polynomial that consists $\varphi(x, t)$ and its partial derivatives.

Step 1: In order to acquire the solitary wave solution of Equation 1 and Equation 2, we utilize the traveling wave transformation,

$$\varphi(x, t) = \varphi(\eta), \eta = k(x \pm vt), \tag{4}$$

where $\varphi(x, t) = \varphi(\eta)$ is an unknown function to be found, k is defined as the width of the traveling wave and v is identified as the velocity of the soliton. Then, the Equation 3 is turned into the following ODE:

$$P(\varphi, \varphi', \varphi'', \dots) = 0, \tag{5}$$

in which $\varphi' = \frac{d\varphi}{d\eta}$, $\varphi'' = \frac{d^2\varphi}{d\eta^2}$ and so on.

Step 2: Assume that Equation 5 is the solution of the Riccati-Bernoulli equation of the form:

$$\varphi' = a_1\varphi + a_2\varphi^{2-m} + a_3\varphi^m, \tag{6}$$

in which a_1, a_2, a_3 and m are constants. Utilizing from the Equation 6, we acquire

$$\begin{aligned} \varphi'' = \varphi^{-1-2m}(a_2\varphi^2 + a_3\varphi^{2m} + a_1\varphi^{1+m})(-a_2(m-2)\varphi^2 \\ + a_3 m\varphi^{2m} + a_1\varphi^{1+m}), \end{aligned} \tag{7}$$

and

$$\begin{aligned} \varphi''' = \varphi^{-2(1+m)}(a_1\varphi + a_2\varphi^{2-m} + a_3\varphi^m)\big(a_2^2(-2+m)\big)(-3+2m)\varphi^4 \\ + a_3^2\, m(-1+2m)\varphi^{4m} + a_1a_2(-3+m)(-2+m)\varphi^{3+m} \\ + (a_1^2 + 2a_2a_3)\varphi^{2+2m} + a_1a_3m(1+m)\varphi^{1+3m}. \end{aligned} \tag{8}$$

The other derivatives of the function φ can be similarly acquired.

Remark 1. Equation 6 is reduced to the Riccati equation when $a_1a_2 \neq 0$ and $m = 0$. Additionally, Equation 6 is reduced to the Bernoulli equation when $a_2 \neq 0$, $a_3 = 0$, and $m \neq 1$. The solutions of Equation 6 are as follows:

Set 1: For $m = 1$, Equation 6 has the following solution

$$\varphi(\eta) = Ce^{(a_1+a_2+a_3)\eta}. \tag{9}$$

Set 2: For $m \neq 1$, $a_1 = 0$ and $a_3 = 0$, Equation 6 has the following solution

$$\varphi(\eta) = \big(a_2(m-1)(\eta + C)\big)^{\frac{1}{m-1}}. \tag{10}$$

Set 3: For $m \neq 1$, $a_1 \neq 0$ and $a_3 = 0$, Equation 6 has the following solution

$$\varphi(\eta) = \left(Ce^{a_1(m-1)\eta} - \frac{a_2}{a_1}\right)^{\frac{1}{m-1}}. \tag{11}$$

Set 4: For $m \neq 1$, $a_2 \neq 0$ and ${a_1}^2 - 4a_2a_3 < 0$, Equation 6 has the following solution

$$\varphi(\eta)=\left(-\frac{a_1}{2a_2}+\frac{\sqrt{4a_2a_3-a_1^2}}{2a_2}tan\left[\frac{(1-m)\sqrt{4a_2a_3-a_1^2}}{2}(\eta+C)\right]\right)^{\frac{1}{1-m}}, \quad (12)$$

and

$$\varphi(\eta)=\left(-\frac{a_1}{2a_2}-\frac{\sqrt{4a_2a_3-a_1^2}}{2a_2}cot\left[\frac{(1-m)\sqrt{4a_2a_3-a_1^2}}{2}(\eta+C)\right]\right)^{\frac{1}{1-m}}. \quad (13)$$

Set 5: For $m \neq 1$, $a_2 \neq 0$ and $a_1{}^2-4a_2a_3>0$, Equation 6 has the following solution

$$\varphi(\eta)=\left(-\frac{a_1}{2a_2}-\frac{\sqrt{a_1^2-4a_2a_3}}{2a_2}tanh\left[\frac{(1-m)\sqrt{a_1^2-4a_2a_3}}{2}(\eta+C)\right]\right)^{\frac{1}{1-m}}, \quad (14)$$

and

$$\varphi(\eta)=\left(-\frac{a_1}{2a_2}-\frac{\sqrt{a_1^2-4a_2a_3}}{2a_2}coth\left[\frac{(1-m)\sqrt{a_1^2-4a_2a_3}}{2}(\eta+C)\right]\right)^{\frac{1}{1-m}}. \quad (15)$$

Set 6: For $m \neq 1$, $a_2 \neq 0$ and $a_1{}^2-4a_2a_3=0$, Equation 6 has the following solution

$$\varphi(\eta)=\left(\frac{1}{a_2(m-1)(\eta+C)}-\frac{a_2}{a_1}\right)^{\frac{1}{1-m}}, \quad (16)$$

in which C is a constant.

Step 3: Finally, if φ and its derivatives are substituted into Equation 5, we can get a set of algebraic equations consisting of the powers of φ. Assuming the coefficients of each power of φ equal to zero, we acquire a system of algebraic equations for a_1, a_2, a_3, k and v. When the parameters are substituted into Equations 9-16, the traveling wave and other solutions of the Equation 3 are acquired.

APPLICATIONS OF THE METHOD TO GOVERNING SYSTEMS

The Maccari's System

To construct analytical solutions of the system in Equation 1, we assume,

$$\begin{aligned}Q(x,y,t)&=u(x,y,t)e^{i(ax+by+ct+d)},\\ S(x,y,t)&=v(x,y,t)e^{i(ax+by+ct+d)},\\ N(x,y,t)&=w(x,y,t)e^{i(ax+by+ct+d)},\end{aligned} \quad (17)$$

in which a, b, c and d are constants to be calculated later. Substituting Equation 17 into the system in Equation 1, we acquire

$$\begin{aligned}i(u_t+2au_x)+u_{xx}-(1+a^2)u+uR&=0,\\ i(v_t+2av_x)+v_{xx}-(1+a^2)v+vR&=0,\\ i(w_t+2aw_x)+w_{xx}-(1+a^2)w+wR&=0,\\ R_t+R_y+(u^2)_x&=0.\end{aligned} \quad (18)$$

Utilizing the following transformation to reduce the Equation 1,

$$u=U(\eta),\ v=V(\eta),\ w=W(\eta),\ R=R(\eta),\ \eta=x+\beta y-2at, \quad (19)$$

in which β is constant, the system in Equation 1 is rewritten as,

$$\begin{aligned}U''-(c+a^2)U+UR&=0,\\ V''-(c+a^2)V+VR&=0,\\ W''-(c+a^2)W+WR&=0,\\ (\beta-2a)R'+((U+V+W)^2)'&=0.\end{aligned} \quad (20)$$

Integrating the fourth equation of the Equation 20 with respect to η and assuming the integration constant as zero, we acquire

$$R=-\frac{1}{\beta-2a}(U+V+W)^2. \quad (21)$$

Replacing Equation 21 into the other equations of Equation 20, we obtain

$$\begin{aligned}U''-(c+a^2)U-\frac{1}{\beta-2a}(U+V+W)^2U&=0,\\ V''-(c+a^2)V-\frac{1}{\beta-2a}(U+V+W)^2V&=0,\\ W''-(c+a^2)W-\frac{1}{\beta-2a}(U+V+W)^2W&=0.\end{aligned} \quad (22)$$

To solve the system in Equation 22, we can give the following relations

$$V=c_1U, \qquad W=c_2U, \quad (23)$$

in which c_1 and c_2 are constants. Replacing Equation 23 into the system in Equation 22, we acquire

$$U''-(c+a^2)U-\frac{(1+c_1+c_2)^2}{\beta-2a}U^3=0. \quad (24)$$

If U and its derivatives are substituted into Equation 24, and we take $m = 0$, then we acquire the following equation

$$\begin{aligned}&\left(2a_2^2-\frac{(1+c_1+c_2)^2}{\beta-2a}\right)U^3+3a_1a_2U^2\\ &+(a_1^2-c-a^2+2a_2a_3)U+a_1a_3=0.\end{aligned} \quad (25)$$

If we collect all the coefficients of $u^j (j = 0, 1, 2, 3)$ and assuming each to equal zero in Equation 25, the following system is acquired.

***U^0* coefficient:**

$$a_1a_3=0, \quad (26)$$

***U^1* coefficient:**

$$a_1^2 - c - a^2 + 2a_2a_3 = 0, \tag{27}$$

***U^2* coefficient:**

$$3a_1a_2 = 0, \tag{28}$$

***U^3* coefficient:**

$$2a_2^2 - \frac{(1+c_1+c_2)^2}{\beta - 2a} = 0. \tag{29}$$

Solving the system consisting of Equations 26-29, we acquire the following families:

Family 1:

$$a_1 = 0, a_2 = -\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}, c = -a^2 - 2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}. \tag{30}$$

When substituting parameters in Equation 30 into Equation 18, we get the following equations:

$$u_{1,1}(x,y,t) = -\frac{\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,tan\left(\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{31}$$

$$u_{1,2}(x,y,t) = \frac{\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,cot\left(\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{32}$$

in which 2β - $4a > 0$ and $a_3(1 + c_1 + c_2) < 0$.

$$u_{1,3}(x,y,t) = \frac{\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,tanh\left(\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{33}$$

$$u_{1,4}(x,y,t) = \frac{\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,coth\left(\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{34}$$

in which 2β - $4a > 0$ and $a_3(1 + c_1 + c_2) > 0$ for proper solutions.

$$v_{1,1}(x,y,t) = -\frac{c_1\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,tan\left(\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{35}$$

$$v_{1,2}(x,y,t) = \frac{c_1\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,cot\left(\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{36}$$

in which 2β - $4a > 0$ and $a_3(1 + c_1 + c_2) < 0$.

$$v_{1,3}(x,y,t) = \frac{c_1\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,tanh\left(\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{37}$$

$$v_{1,4}(x,y,t) = \frac{c_1\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,coth\left(\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{38}$$

in which 2β - $4a > 0$ and $a_3(1 + c_1 + c_2) > 0$ for proper solutions.

$$w_{1,1}(x,y,t) = -\frac{c_2\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,tan\left(\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{39}$$

$$w_{1,2}(x,y,t) = \frac{c_2\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,cot\left(\sqrt{-a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{40}$$

in which 2β - $4a > 0$ and $a_3(1 + c_1 + c_2) < 0$.

$$w_{1,3}(x,y,t) = \frac{c_2\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,tanh\left(\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{41}$$

$$w_{1,4}(x,y,t) = \frac{c_2\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}\sqrt{2\beta-4a}\,coth\left(\sqrt{a_3\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}}(x+\beta y-2at+C)\right)}{1+c_1+c_2}, \tag{42}$$

in which 2β - $4a > 0$ and $a_3(1 + c_1 + c_2) > 0$ for proper solutions. For the solutions $u_{1,2}(x, y, t)$, $v_{1,2}(x, y, t)$ and $w_{1,2}(x, y, t)$, we yield the dark optical solutions,

$$Q(x,y,t) = \frac{\sqrt{-a_3\Phi}\sqrt{2\beta-4a}\,cot\left(\sqrt{-a_3\Phi}(x+\beta y-2at+C)\right)}{1+c_1+c_2}e^{I\left(ax+by+\left(-a^2-2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}\right)t+d\right)},$$

$$S(x,y,t) = \frac{c_1\sqrt{-a_3\Phi}\sqrt{2\beta-4a}\,cot\left(\sqrt{-a_3\Phi}(x+\beta y-2at+C)\right)}{1+c_1+c_2}e^{I\left(ax+by+\left(-a^2-2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}\right)t+d\right)},$$

$$N(x,y,t) = \frac{c_2\sqrt{-a_3\Phi}\sqrt{2\beta-4a}\,cot\left(\sqrt{-a_3\Phi}(x+\beta y-2at+C)\right)}{1+c_1+c_2}e^{I\left(ax+by+\left(-a^2-2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}\right)t+d\right)},$$

where $\Phi = \frac{1+c_1+c_2}{\sqrt{2\beta-4a}}$. For the solutions $u_{1,3}(x, y, t)$, $v_{1,3}(x, y, t)$ and $w_{1,3}(x, y, t)$, we yield the dark optical solutions,

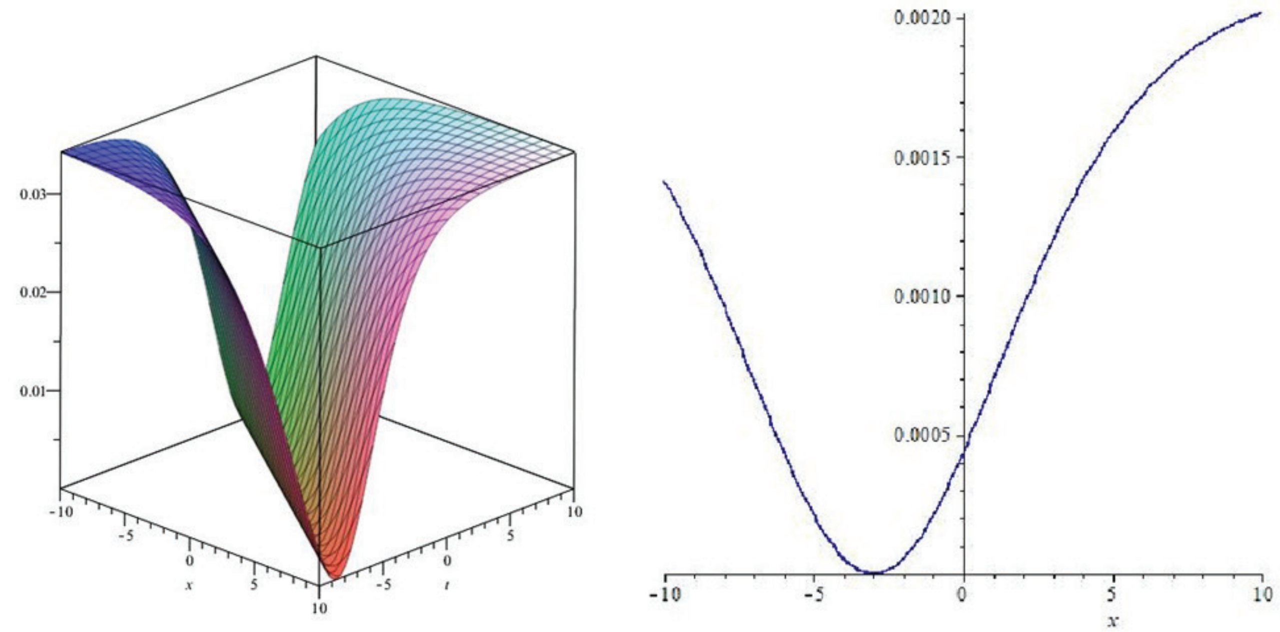
$$Q(x,y,t) = \frac{\sqrt{a_3\Phi}\sqrt{2\beta-4a}\,tanh\left(\sqrt{a_3\Phi}(x+\beta y-2at+C)\right)}{1+c_1+c_2}e^{I\left(ax+by+\left(-a^2-2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}\right)t+d\right)},$$

$$S(x,y,t) = \frac{c_1\sqrt{a_3\Phi}\sqrt{2\beta-4a}\,tanh\left(\sqrt{a_3\Phi}(x+\beta y-2at+C)\right)}{1+c_1+c_2}e^{I\left(ax+by+\left(-a^2-2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}\right)t+d\right)},$$

$$N(x,y,t) = \frac{c_2\sqrt{a_3\Phi}\sqrt{2\beta-4a}\,tanh\left(\sqrt{a_3\Phi}(x+\beta y-2at+C)\right)}{1+c_1+c_2}e^{I\left(ax+by+\left(-a^2-2\frac{a_3+a_3c_1+a_3c_2}{\sqrt{2\beta-4a}}\right)t+d\right)}.$$

Family 2:

$$a_1 = 0, a_2 = \mp\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}, a_3 = \pm\frac{(c+a^2)\sqrt{2\beta-4a}}{1+c_1+c_2}. \tag{43}$$



(a) **(b)**

Figure 1. The (a) 3D and (b) 2D graphs of $Q(x, y, t)$ for the solution $u_3(x, y, t)$ under $c_1 = 0.5$, $c_2 = 0.25$, $a_3 = 0.03$, $\beta = 0.5$, $a = -0.75$, $b = 0.02$, $d = 0.04$, $C = 3$ and $y = 0$.

When we substitute parameters in Equation 43 into Equation 18, we get the following equations:

$$u_{2,1}(x,y,t) = -\frac{\sqrt{2c+2a^2}\sqrt{2\beta-4a}\,\tan\left(\frac{\sqrt{2c+2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}, \quad (44)$$

$$u_{2,2}(x,y,t) = \frac{\sqrt{2c+2a^2}\sqrt{2\beta-4a}\,\cot\left(\frac{\sqrt{2c+2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}, \quad (45)$$

in which $c + a^2 > 0$ and $\beta - 2a > 0$ for valid solutions.

$$u_{2,3}(x,y,t) = \frac{\sqrt{-2c-2a^2}\sqrt{2\beta-4a}\,\tanh\left(\frac{\sqrt{-2c-2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)} \quad (46)$$

$$u_{2,4}(x,y,t) = \frac{\sqrt{-2c-2a^2}\sqrt{2\beta-4a}\,\coth\left(\frac{\sqrt{-2c-2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}, \quad (47)$$

in which $c + a^2 < 0$ and $\beta - 2a > 0$ for valid solutions. Since $v_{2,3}(x, y, t) = c_1 u_{2,3}(x, y, t)$ and $w_{2,3}(x, y, t) = c_2 u_{2,3}(x, y, t)$, we acquire the following solutions for the solutions $u_{2,3}(x, y, t)$, $v_{2,3}(x, y, t)$ and $w_{2,3}(x, y, t)$.

$$Q(x,y,t) = \frac{\sqrt{2c+2a^2}\sqrt{2\beta-4a}\,\tanh\left(\frac{\sqrt{2c+2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}\,e^{I(ax+by+ct+d)},$$

$$S(x,y,t) = \frac{c_1\sqrt{2c+2a^2}\sqrt{2\beta-4a}\,\tanh\left(\frac{\sqrt{2c+2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}\,e^{I(ax+by+ct+d)},$$

$$N(x,y,t) = \frac{c_2\sqrt{2c+2a^2}\sqrt{2\beta-4a}\,\tanh\left(\frac{\sqrt{2c+2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}\,e^{I(ax+by+ct+d)}.$$

We acquire the following solutions for the solutions $u_{2,3}(x, y, t)$, $v_{2,3}(x, y, t)$ and $w_{2,3}(x, y, t)$.

$$Q(x,y,t) = \frac{\sqrt{-2c-2a^2}\sqrt{2\beta-4a}\,\cot\left(\frac{\sqrt{-2c-2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}\,e^{I(ax+by+ct+d)},$$

$$S(x,y,t) = \frac{c_1\sqrt{-2c-2a^2}\sqrt{2\beta-4a}\,\cot\left(\frac{\sqrt{-2c-2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}\,e^{I(ax+by+ct+d)},$$

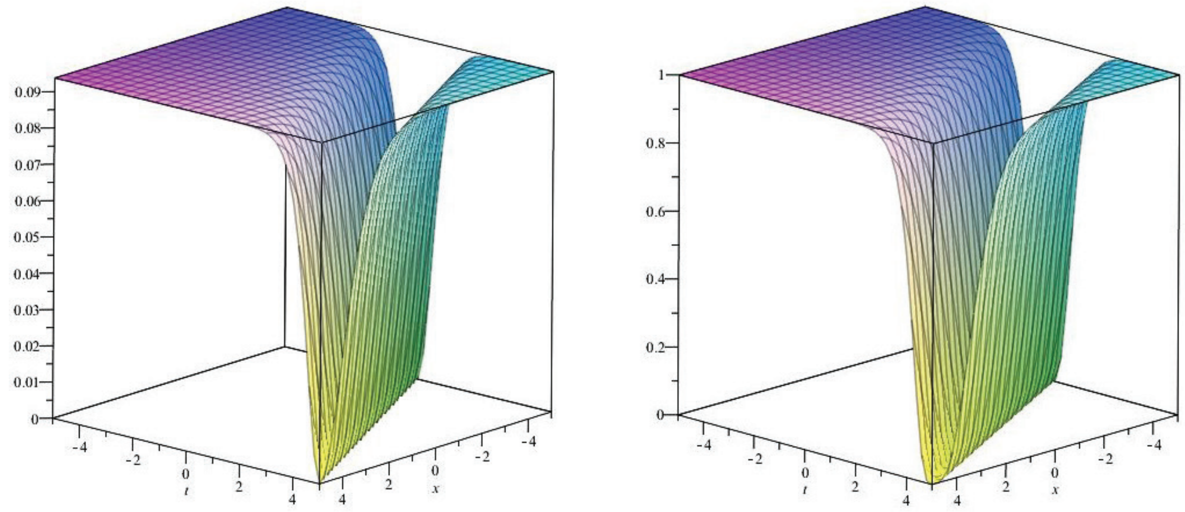
$$N(x,y,t) = \frac{c_2\sqrt{-2c-2a^2}\sqrt{2\beta-4a}\,\cot\left(\frac{\sqrt{-2c-2a^2}\,(x+\beta y-2at+C)}{2}\right)}{2(1+c_1+c_2)}\,e^{I(ax+by+ct+d)},$$

Family 3:

$$a_1 = 0, a_2 = \mp\frac{1+c_1+c_2}{\sqrt{2\beta-4a}}, a_3 = 0, c = -a^2. \quad (48)$$

Inserting the parameters in Equation 48 into Equation 18, we get the following equations:

$$u(x,y,t) = \frac{\sqrt{2\beta-4a}}{(1+c_1+c_2)(x+\beta y-2at+C)}, \quad (49)$$



(a) **(b)**

Figure 2. The 3D graph of (a) $Q(x, y, t)$ and (b) $R(x, y, t)$ for the solution $u_{2,3}(x, y, t)$ under $c_1 = 1$, $c_2 = 2$ $c = -2$, $\beta = 0.5$, $a = 1$, $b = 2$, $d = 0$, $C = 5$, $y = 0$.

$$v(x,y,t) = \frac{c_1\sqrt{2\beta - 4a}}{(1 + c_1 + c_2)(x + \beta y - 2at + C)}, \tag{50}$$

$$w(x,y,t) = \frac{c_2\sqrt{2\beta - 4a}}{(1 + c_1 + c_2)(x + \beta y - 2at + C)}, \tag{51}$$

in which β - $2a > 0$ for proper solutions. For the solutions in Equations 49-51, we get

$$Q(x,y,t) = \frac{\sqrt{2\beta - 4a}\ e^{I(ax+by-a^2t+l)}}{(1 + c_1 + c_2)(x + \beta y - 2at + C)},$$
$$S(x,y,t) = \frac{c_1\sqrt{2\beta - 4a}\ e^{I(ax+by-a^2t+l)}}{(1 + c_1 + c_2)(x + \beta y - 2at + C)},$$
$$N(x,y,t) = \frac{c_2\sqrt{2\beta - 4a}\ e^{I(ax+by-a^2t+l)}}{(1 + c_1 + c_2)(x + \beta y - 2at + C)},$$
$$R(x,y,t) = -\frac{2}{(x + \beta y - 2at + C)^2}.$$

The Davey-Stewartson System

To construct new analytical solutions of the system, utilizing the following wave transformation

$$U(x,y,t) = e^{i(\lambda x + ny + rt)}\phi(\xi), \quad V(x,y,t) = \vartheta(\xi), \quad \xi = x + y + \kappa t, \tag{52}$$

the following reduced ODEs are acquired:

$$\left(-r - \gamma(n^2 + \lambda^2)\right)\phi + \mu\phi^3 - \alpha\phi\vartheta + 2\gamma\phi'' = 0, \tag{53}$$

$$-\sigma(\phi')^2 - \sigma\phi\phi'' + \vartheta'' = 0, \tag{54}$$

from the real part and the relation is derived as:

$$\kappa = -2\gamma(n + \lambda). \tag{55}$$

Integrating Equation 54, we have

$$\vartheta = \frac{\sigma}{2}\phi^2 . \tag{56}$$

Substituting Equation 56 into Equation 53, we have

$$2\left(-r - \gamma(n^2 + \lambda^2)\right)\phi + (2\mu - \alpha\sigma)\phi^3 + 4\gamma\phi'' = 0 . \tag{57}$$

If ϕ and ϕ'' are substituted into Equation 57 and setting $m = 0$, then the following equation is produced:

$$(8a_2^2\gamma - \alpha\sigma + 2\mu)\phi^3 + 12\gamma a_1 a_2\phi^2 + \left(4\gamma(2a_2a_3 + a_1^2) - 2r - 2\gamma(n^2 + \lambda^2)\right)\phi + 4\gamma a_1 a_3 = 0. \tag{58}$$

Compiling all the coefficients of ϕ^s (s = 0, 1, 2, 3) and assuming each to equal zero in Equation 58, the following algebraic system is derived:

ϕ^0 coefficient:

$$\gamma a_1 a_3 = 0, \tag{59}$$

ϕ^1 coefficient:

$$4\gamma(2a_2a_3 + a_1^2) - 2r - 2\gamma(n^2 + \lambda^2) = 0, \tag{60}$$

ϕ^2 coefficient:

$$\gamma a_1 a_2 = 0 , \tag{61}$$

ϕ^3 coefficient:

$$8a_2^2\gamma - \alpha\sigma + 2\mu = 0. \tag{62}$$

Solving this algebraic system from Equation 59 to Equation 62, we obtain the following families.

Family 1:

$$a_1 = 0, a_2 = \frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}}{4\gamma}, a_3 = \frac{\gamma n^2 + \gamma\lambda^2 + r}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}}. \tag{63}$$

Substituting parameters in Equation 63 into Equation 52, we acquire the following soliton solutions

$$U_{1,1}(x,y,t) = \frac{2\sqrt{\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}\gamma\, tan\left[\frac{1}{2}\sqrt{\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}}e^{((\lambda x + ny + rt)i)}, \tag{64}$$

$$V_{1,1}(x,y,t) = -\frac{\sigma(\gamma n^2 + \gamma\lambda^2 + r)\, tan\left[\frac{1}{2}\sqrt{\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]^2}{-\alpha\sigma + 2\mu}, \tag{65}$$

$$U_{1,2}(x,y,t) = -\frac{2\sqrt{\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}\gamma\, cot\left[\frac{1}{2}\sqrt{\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}}e^{((\lambda x + ny + rt)i)}, \tag{66}$$

$$V_{1,2}(x,y,t) = -\frac{\sigma(\gamma n^2 + \gamma\lambda^2 + r)\, cot\left[\frac{1}{2}\sqrt{\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]^2}{-\alpha\sigma + 2\mu}, \tag{67}$$

where $\gamma(\gamma n^2 + \gamma\lambda^2 + r) > 0$ for existence of obtained solutions and the following dark soliton solutions:

$$U_{1,3}(x,y,t) = -\frac{2\sqrt{-\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}\gamma\, tanh\left[\frac{1}{2}\sqrt{-\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}}e^{((\lambda x + ny + rt)i)}, \tag{68}$$

$$V_{1,3}(x,y,t) = \frac{\sigma(\gamma n^2 + \gamma\lambda^2 + r)\, tanh\left[\frac{1}{2}\sqrt{-\frac{\gamma n^2 + \gamma\lambda^2 + r}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]^2}{-\alpha\sigma + 2\mu}, \tag{69}$$

where $\gamma(\gamma n^2 + \gamma\lambda^2 + r) < 0$ for existence of solutions $U_{1,3}(x, y, t)$ and $V_{1,3}(x, y, t)$.

Family 2:

$$a_1 = 0, a_2 = \frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}}{4\gamma}, r = a_3\sqrt{-2\gamma(-\alpha\sigma + 2\mu)} - \gamma n^2 - \gamma\lambda^2 \tag{70}$$

Substituting parameters in Equation 70 into Equation 52, we acquire the following soliton solutions:

$$U_{2,1}(x,y,t) = \frac{2\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}\gamma\, tan\left[\frac{1}{2}\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}} \times e^{\left(\lambda x + ny + \left(a_3\sqrt{-2\gamma(-\alpha\sigma + 2\mu)} - \gamma n^2 - \gamma\lambda^2\right)t\right)i}, \tag{71}$$

$$V_{2,1}(x,y,t) = \frac{\sigma\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3\, tan\left[\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]^2}{-\alpha\sigma + 2\mu}, \tag{72}$$

$$U_{2,2}(x,y,t) = -\frac{2\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}\gamma\, cot\left[\frac{1}{2}\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}} \times e^{\left(\lambda x + ny + \left(a_3\sqrt{-2\gamma(-\alpha\sigma + 2\mu)} - \gamma n^2 - \gamma\lambda^2\right)t\right)i}, \tag{73}$$

$$V_{2,2}(x,y,t) = -\frac{\sigma\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3\, cot\left[\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]^2}{-\alpha\sigma + 2\mu}, \tag{74}$$

where $\gamma(-\alpha\sigma + 2\mu) < 0$ and $a_3\gamma > 0$ for existence of obtained solutions and the following dark soliton solutions:

$$U_{2,3}(x,y,t) = -\frac{2\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}\gamma\, tanh\left[\frac{1}{2}\sqrt{\frac{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}a_3}{\gamma}}(x + y - 2\gamma(n + \lambda)t + J)\right]}{\sqrt{-2\gamma(-\alpha\sigma + 2\mu)}} \times e^{\left(\lambda x + ny + \left(a_3\sqrt{-2\gamma(-\alpha\sigma + 2\mu)} - \gamma n^2 - \gamma\lambda^2\right)t\right)i}, \tag{75}$$

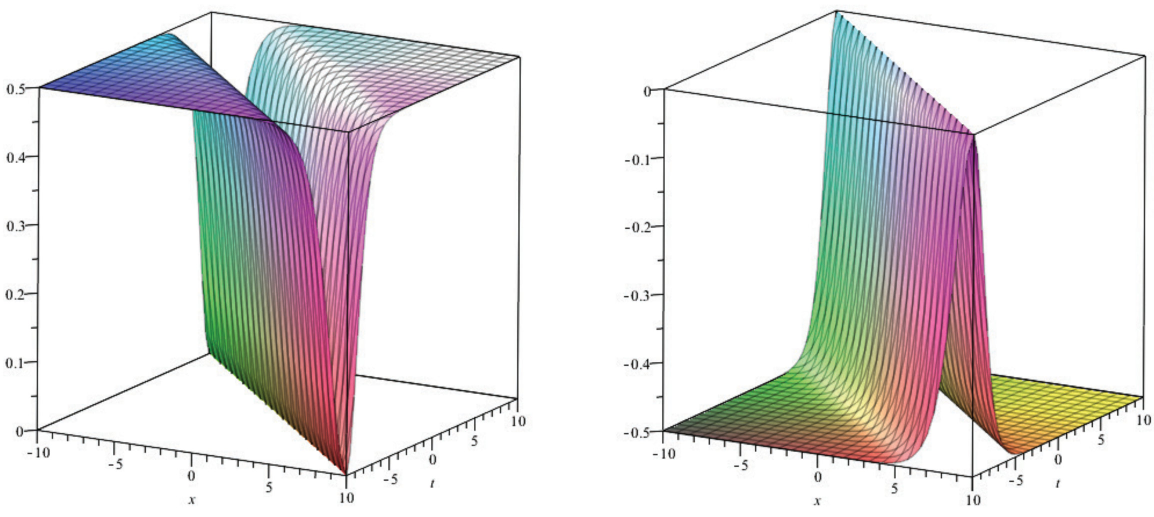


Figure 3. The 3D graph of (a) $U_{1,1}(x, y, t)$ and (b) $V_{1,1}(x, y, t)$ with, $r = 1$, $\gamma = -0.5$, $n = 0.5$, $\sigma = 0.5$, $\mu = 0.5$, $\alpha = 1$, $\beta = 2$, $J = 0$.

$$V_{2,3}(x,y,t) = -\frac{\sigma\sqrt{-2\gamma(-\alpha\sigma+2\mu)}a_3 \tanh\left[\sqrt{-\frac{\sqrt{-2\gamma(-\alpha\sigma+2\mu)}a_3}{\gamma}}(x+y-2\gamma(n+\lambda)t+J)\right]^2}{-\alpha\sigma+2\mu}, \quad (76)$$

where $\gamma(-\alpha\sigma + 2\mu) < 0$ and $a_3\gamma < 0$ for existence of solutions $U_{2,3}(x, y, t)$ and $V_{2,3}(x, y, t)$.

CONCLUSION

In this research article, we have seen that computer algebra is a powerful technique to solve all complicated problems in mathematical sciences. Using the method, we obtained the new exact wave and soliton solutions of the complex nonlinear systems Davey-Stewartson and Maccari's systems. All the solutions verify the models in this study. We also describe the 2D, and 3D graphs of some of the acquired solutions in this study. Some of the reported solutions in this paper have important physical meanings, for instance, the hyperbolic tangent arises in the calculation of magnetic moment and rapidity of special relativity, and the hyperbolic cotangent arises in the Langevin function for magnetic polarization [32]. All the computations in this study are carried out with the aid of Maple. The acquired solutions are new in the literature. The proposed method can also be used to solve various NLPDEs in mathematical physics.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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