Journal of Thermal Engineering
Web page info: https://jten.yildiz.edu.tr
DOI: 10.14744/thermal.0000830

Journal of Thermal Engineering

Research Article

New similarity method analysis of the transient thermal boundary layer on a horizontal flat plate: Strouhal and Prandtl numbers effect

Mohamed BACHIRI[1], Ahcene BOUABDALLAH[2], Lakhdar AIDAOUI[3,*], Yahia LASBET[3]

[1]*Laboratory of Materials for Applications and Valorization of Renewable Energies, Faculty of Sciences, Laghouat University, Laghouat, QRWX+CR2, Algeria*
[2]*Laboratory of Thermodynamic and Energy Systems, Faculty of Physics, USTHB, Bab Ezzouar-Algiers, 16111, Algeria*
[3]*Laboratory of Mechanics and Materials Development, Faculty of Sciences and Technology, Djelfa University, Djelfa, 17000, Alegria*

ARTICLE INFO

Article history
Received: 22 October 2021
Revised: 22 January 2022
Accepted: 30 March 2022

Keywords:
Horizontal Surface; Nusselt; Similarity Solution; Strouhal Number; Unsteady Heat Transfer

ABSTRACT

The present project highlights the behavior of the unsteady heat transfer phenomenon developing along a horizontal surface in terms of both Strouhal and Prandtl numbers. Based on the changes that occur in the gouverning equation of the studied problem, an adequate analytical law of the velocity is proposed to solve unsteady momentum equation. This result presented a good agreement with Rayleigh's exact solution and numerical solutions of Blasius and Williams–Rhyne for diferents values of Strouhal numbers adopted in this study. The obtained velocity expression is included in the unsteady energy equation in order to establish the temperature profile for all considered Strouhal and Prandtl numbers. Taking into account the existence of the velocity-temperature coupling in the heat boundary layer equation, the proposed formula is used to solve unsteady energy equation for all Strouhal and Prandtl numbers. As the main results, a new analytic expression of the local heat transfer coefficient for all Strouhal and Prandtl numbers is established and interesting curves are plotted to explain the heat transfer evolutions from diffusion flow to the convective flow.

Cite this article as: Bachiri M, Bouabdallah A, Aidaoui L, Lasbet Y. New similarity method analysis of the transient thermal boundary layer on a horizontal flat plate: Strouhal and Prandtl numbers effect. J Ther Eng 2024;10(3):737−745.

INTRODUCTION

The characterization of the boundary layer comportments of compressible and incompressible fluid flows along rigid wall has a great importance in various engineering applications including; aeronautic, space technology, hydrodynamics, mass and heat transfers and many others. It is well known that the resolutions of the dynamic and thermal equations are complicated mainly in the unsteady case, where different approaches can be used. The first theoretical investigation of the unsteady flow regime over a suddenly motion of a horizontal flat plate is given by Stewartson [1,2]. The main result from this work shows that the independence in time decreases exponentially. N. Tokuda [3] studied unsteady momentum equation of the stretching

*Corresponding author.
*E-mail address: laidaoui@gmail.com

This paper was recommended for publication in revised form by Regional Editor Liu Yang

Published by Yıldız Technical University Press, İstanbul, Turkey

surface. Results indicate that the flow regime varies from Raleigh's diffusif flow to Blasius' convective flow, and its solution is presented in a temporal polynomial. Watkins [4] examined the case of unsteady heat transfer developed near an impulsively motion of a horizontal surface via numerical description for many Prandtl numbers values.

Hall [5] and Dennis [6] used a numerical method to solve the unsteady dynamic equations of a surface suddenly in motion. In their studies, curves of the velocity variations and the shear stress values are illustrated in case of initial and steady flows. They also found that the decay to the steady flow is in exponential way. In the same wake, Williams and Rhyne [7] studied theoretically the unsteady laminar flow regime on a sudden start of the wedge surface. They used their own new similarity transformations and solved numerically the equations governing the dynamic phenomenon. The authors denoted that the boundary layer separation is far from being for a value less than or equal to -0.091 m. By using Williams-Rhyne's transformations and homotopy analysis method (HAM), Liao [8] was able to establish an analytical function of the shear stress rate valid for a wide time over a sudden start of a horizontal surface. Hang et al. [9] was interested in studying the case of electric fluid flow over a surface exposed to both sudden movement and thermal shock. The problematic was solved by HAM and given accurate polynomial solutions results of all physical quantities looked for. Simon et al. [10] studied theoretically heat transfer evolution over suddenly movement of the wall. An analytic solution is obtained for the concurrent variations of the energy and dynamic equations governing the unsteady phenomenon for wide values of m parameter. In the work of Revnic et al. [11], the dynamic and heat transfer boundary layers developed on a stretched surface are examined. They obtained an ordinary equation from partial differential equations by using an appropriate transformation. The main result outlines an enhancement of the heat rate with the increase of Prandtl number. Zheng and Ghate [12] studied unsteady flow and found an adequate solution along a horizontal wall by employing dimensionless parameters. The conclusions provided a general solution that was applicable to different types of flows. Hafidzuddin et al. [13] used an appropriate similarity variable in order to resolve momentum and energy equations on a permeable stretching/ shrinking surface problem. Via an appropriate transformation, authors changed governing equations in to simple equations and they demonstrated the stability of the obtained solution. Recently, an analytic approach is used by Nagler [14] to carry out a study of the sudden motion of a cylindrical shape. The obtained velocity profile is a polynomial in high order and it is in good agreement with the literature results. To solve the motion equation along blunt body, Bulgakov et al. [15] employed Paulhasen approach with appropriate variables in aim to calculate different thicknesses of the heat transfer and momentum boundaries layers.

It appears difficult, for this problem, to come up with an accurate analytic solution for all Strouhal numbers. As a complementary investigation [16], an adequate analytic approach of the unsteady heat transfer along a horizontal wall for all Prandtl and Strouhal numbers over all time evolution which sweeping all regime flows from unsteady "diffusion flow" to steady "convective flow".

MATHEMATICAL FORMULATIONS

Consider a longitudinal flow over a horizontal wall. The fluid is at a cold temperature T_∞ and the plate is maintained at a hot temperature T_w, see Figure 1. The phenomenon is a boundary layer problem and the laws to be approached are the governing equations of the phenomenon. From t≥0, the thermal and dynamic transfer rates begin to manifest in a dynamic and thermal boundary layer where an initials and boundary conditions are mentioned in the system (4).

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Where T, t, ν and α denote the temperature profile, the time, the kinematic viscosity and the thermal diffusion coefficient, respectively.

$$\begin{aligned}
&t=0:\\
&u(x,y,0)=v(x,y,0)=0 \text{ and } T(x,y,0)=T_w\\
&t\geq 0:\\
&u(x,0,t)=v(x,0,t)=0 \text{ and } T(x,0,t)=T_w\\
&u(x,y\to\infty,t)=U_e \text{ and } T(x,y\to\infty,t)=T_\infty
\end{aligned} \tag{4}$$

It should be noted that the flow regime is governed mainly by the Strouhal number, $St_x=\frac{x}{U_e\,t}$. For St_x >>1, the flow is diffusive and it is called Rayleigh flow. Whereas small values of St_x, the flux is convective and it is known as Blasius flow.

By using a scaling analysis well suited to this flow type, Williams and Rhyne [7] found a new variable valid over the entire time interval and verifies well the similarity variable of Rayleigh "diffusion flow" and Blasius "convective flow":

$$\eta = \frac{y}{\nu^{1/2} g(x,t)} \quad \text{where} \quad g(x,t) = \sqrt{\frac{x}{u_e}\xi},$$
$$\xi = 1 - exp(-\frac{1}{St_x}), \; \psi = \nu^{1/2} u_e g(x,t) F(\xi,\eta) \tag{5}$$
$$\text{and} \quad \theta(\eta,\xi) = \frac{T - T_\infty}{T_w - T_\infty}$$

η and ξ are the similarity variables. Where ψ is the stream function, $F(\eta,\xi)$ is the stream function and $\theta(\eta, \xi)$ is the temperature profile.

Let's put Eq. (5) in Eqs. (1), (2) and (3), we get

$$\frac{\partial^3 F}{\partial \eta^3} + \frac{1}{2}\eta(1-\xi)\frac{\partial^2 F}{\partial \eta^2} + \frac{1}{2}\left[\xi + (1-\xi)\ln(1-\xi)\right]F\frac{\partial^2 F}{\partial \eta^2} + \left[\xi(1-\xi)\ln(1-\xi)\frac{\partial F}{\partial \xi}\right]\frac{\partial^2 F}{\partial \eta^2} - \xi(1-\xi)\left[1 + \frac{\partial F}{\partial \eta}\ln(1-\xi)\right]\frac{\partial}{\partial \xi}\left(\frac{\partial F}{\partial \eta}\right) = 0 \tag{6}$$

$$\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial \eta^2} + \left[\frac{1}{2}\eta(1-\xi) + \frac{1}{2}\left[\xi + (1-\xi)\ln(1-\xi)\right]F + \xi(1-\xi)\ln(1-\xi)\frac{\partial F}{\partial \xi}\right]\frac{\partial \theta}{\partial \eta} - \xi(1-\xi)\left[1 + \frac{\partial F}{\partial \eta}\ln(1-\xi)\right]\frac{\partial \theta}{\partial \xi} = 0 \tag{7}$$

The boundary conditions Eq. (3) reduce to

$$\eta \to 0: F(0,x) = F_\eta(0,x) = 0 \text{ and } \theta(0,\xi) = 1$$
$$\eta \to \infty: F_\eta(\infty,x) = 1 \text{ and } \theta(\infty,\xi) = 0 \tag{8}$$

Analytical Approach

Equation of motion

To know the Strouhal number impact on the flow regime, the equation of motion was solved by an approximate method using an adequate analytical function of the unsteady velocity profile. The boundary conditions are checked throughout the domain $0 \le \eta < \infty$. The precision of the expression obtained from the formula will be checked by evaluating the average residual deviation $\Delta\varepsilon$ of Eq. (6).

When $St_x >> 1$, Equation. (6) transforms to the type of Rayleigh equation and its exact solution is written as: $F_\eta(\eta,0) = erf(\frac{\eta}{2})$. And When $St_x << 1$ ($t \to \infty$), Eq. (6) suits the equation of Blasius.

It can be seen from Equation (6), if ξ augments in the range 0 to 1, the velocity profile $F_\eta(\eta,\xi)$ evolves from the initial flow "Rayleigh's solution" until the steady flow "Blasius' solution". From mathematical and Phenomenogically point of view, when $St_x >> 1$ the diffusion flow differs is essentially diverse at $St_x << 1$ when the flow is convective. In our previous work Bachiri and Bouabdallah [16], we proposed an adequate formula of the profile of the non-dimensional velocity taking into account the boundary-layer evolution from diffusion flows to convective flows. The analytical approach permits us to give analytical expressions of both velocity profile and skin friction coefficient on the surface in all time and for all Strouhal numbers. So, the solution is expressed as:

-For the velocity profile:

$$F_\eta(\eta,\xi) = Erf\left(\frac{\eta}{2}\exp(-0.53007\xi^{9/5})\right) + \xi^6\left(0.00749\eta^3\exp(-0.1394\eta^2)\right) \tag{9}$$

-For the skin friction coefficient on the surface:

$$C_f = 2\,Re_x^{-1/2}\,\xi^{-1/2}\left(\frac{1}{\sqrt{\pi}} exp(-\frac{1}{2}\xi^{9/5})\right) \tag{10}$$

$Re_x = U_e x/\nu$ is the local Reynolds number

Energy equation

Thereafter, the analytical approximation of the unsteady heat transfer is made on the assumption that the plate is brought instantaneously up to a constant temperature T_w by imposing a suitable heat flow.

When $St_x >> 1$, the general equation reduces to

$$\frac{\partial T}{\partial t} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{11}$$

with $T(y = 0, t = 0^+) = 1$ and $T(y \to \infty, t) = 0$

Eq. (11) is independent of the velocity profile and expresses the diffusion heat transfer.

If we pose $\eta^* = \frac{y}{2\sqrt{\alpha t}}$, the exact solution is

$$T(\eta^*) = erfc(\eta^*) = 1 - erf(\eta^*) \tag{12}$$

Based on the Rayleigh's solution of the diffusion flow, the ratio thicknesses of energy and dynamic boundary layers can be expressed as follows:

$$\frac{\delta_{Diffusion}}{\delta_T} = \frac{\sqrt{\nu t}}{\sqrt{\alpha t}} = \Pr^{1/2} \tag{13}$$

So we can evaluate the Nusselt number in the transition regime at $y = 0$

$$Nu = -x\left.\frac{\partial T}{\partial y}\right|_{y=0} \tag{14}$$

From solution of Eq. (12), we find

$$Nu_0 = \frac{Pr^{1/2}}{\sqrt{\pi}}\frac{x}{\sqrt{\nu t}} \tag{15}$$

As we know that $\frac{x}{\sqrt{\nu t}} = Re_x^{1/2}\,St_x^{1/2}$,

The local Nusselt number expression in just the transient case between the wall and the fluid current is given by the formula

Table 1. Nusselt number calculation for wide Prantdl number values in case of St<<1

Pr	0.01	0.1	0.7	0.8	1	10	100
$Nu_x/Re_x^{1/2}$ numerical results	0.051	0.140	0.292	0.307	0.332	0.728	1.571
$Nu_x/Re_x^{1/2}$ Present approach	0.051	0.140	0.292	0.307	0.332	0.727	1.571

$$Nu_0 = \frac{1}{\sqrt{\pi}} Pr^{1/2}\, Re_x^{1/2}\, St_x^{1/2} \tag{16}$$

We note that the Nusselt number have maximum values when (t=0$^+$), St_x>>1. However, over time, the Nu number decreases in a similar way to the skin friction coefficient.

when ξ=0, which corresponds to St_x>>1, Eq. (7) reduces to Rayleigh type equation

$$\frac{1}{Pr}\frac{d^3\theta}{d\eta^3} + \frac{1}{2}\eta\frac{d^2\theta}{d\eta^2} = 0 \tag{17}$$

with the boundary conditions

$$\theta(\eta=0,0) = 1 \text{ and } \theta(\eta\to\infty,0)=0 \tag{18}$$

The exact solution of Eq. (34) is

$$\theta(\eta) = 1 - erf\left(\frac{Pr^{1/2}}{2}\eta\right)$$

and at ξ= 1, corresponds to St_x<<1, Eq. (7) reduces to Blasius type equation

$$\frac{1}{Pr}\frac{d^3\theta}{d\eta^3} + \frac{1}{2}F\frac{d^2\theta}{d\eta^2} = 0 \tag{19}$$

with the same precedent boundary conditions.

This last equation admits numerical solutions or approximate analytical solutions because it is difficult to deduce an explicit analytical solution he solution needs the knowledge of the steady state dimensionless stream function $F(\eta,1)$, [16]:

$$F(\eta,1) = \eta erf(0.29428\eta) + \frac{1}{0.29428\sqrt{\pi}} exp(-(0.29428)^2\eta^2) - \frac{a}{2b}\left(\frac{1}{b}+\eta^2\right)exp(-b\eta^2) \tag{20}$$

The solution of Eq. (19) gives the dimensionless temperature profile

$$\theta(\eta,1) = 1 - \theta_\eta(0,1)\int_0^\eta \exp\left[-\frac{1}{2}\Pr\int_0^{\eta'} F(\eta'',1)d\eta''\right]d\eta' \tag{21}$$

Where $\theta_\eta(0,1) = \left\{\int_0^\infty \exp\left[-\frac{1}{2}\Pr\int_0^{\eta'} F(\eta'',1)d\eta''\right]d\eta'\right\}^{-1}$

We represent these last results in the Figures 1-2, they highlight the temperature variations according to the Prandtl number Pr for two extremes of the St values, valid in the entire interval $\eta \in [0,\eta_\infty]$.

In the convective flow, the heat performances are written as:

$$\frac{Nu_x}{\mathrm{Re}_x} = -\theta_\eta(0,1) = \left\{\int_0^\infty \exp\left[-\frac{1}{2}\Pr\int_0^{\eta'} F(\eta'',1)d\eta''\right]d\eta'\right\}^{-1} \tag{22}$$

Which agree well with the numerical results.

Table below gives the evaluation of Nusselt number for different Prandtl number values.

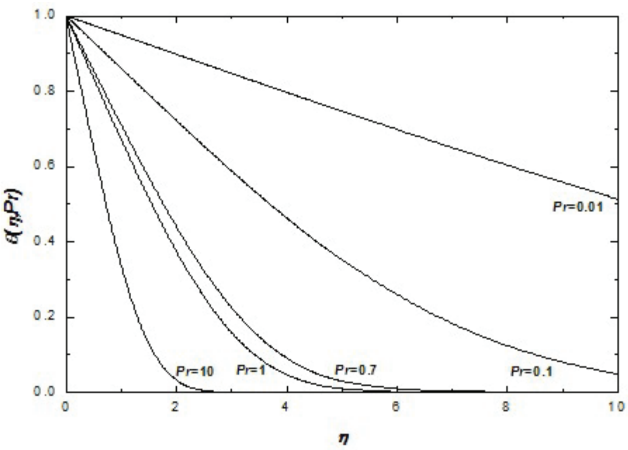

Figure 1. Temperature evolution $\theta(\eta,0)$ for a wide values of Pr ; case of St<<1.

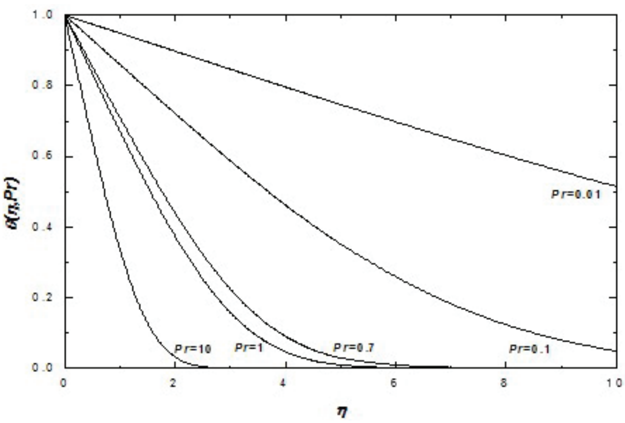

Figure 2. Non dimensional temperature profile $\theta(\eta,1)$ for a wide Prandtl values, Pr; case of St>>1.

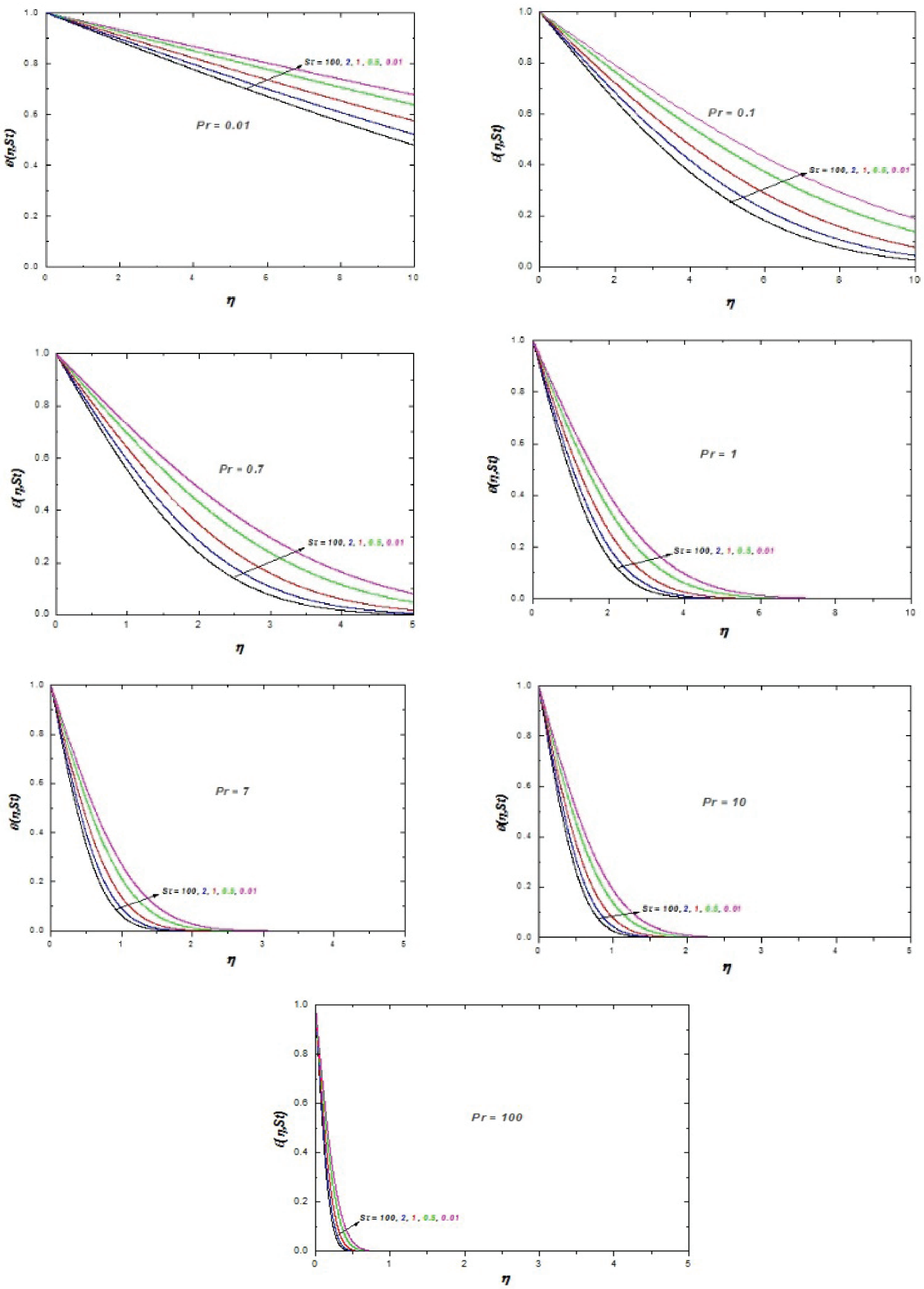

Figure 3. Temperature profiles $\theta(\eta,St)$ distribution for different values of St number for cases of Pr = 0.01, 0.1, 0.7, 1, 7, 10 and 100.

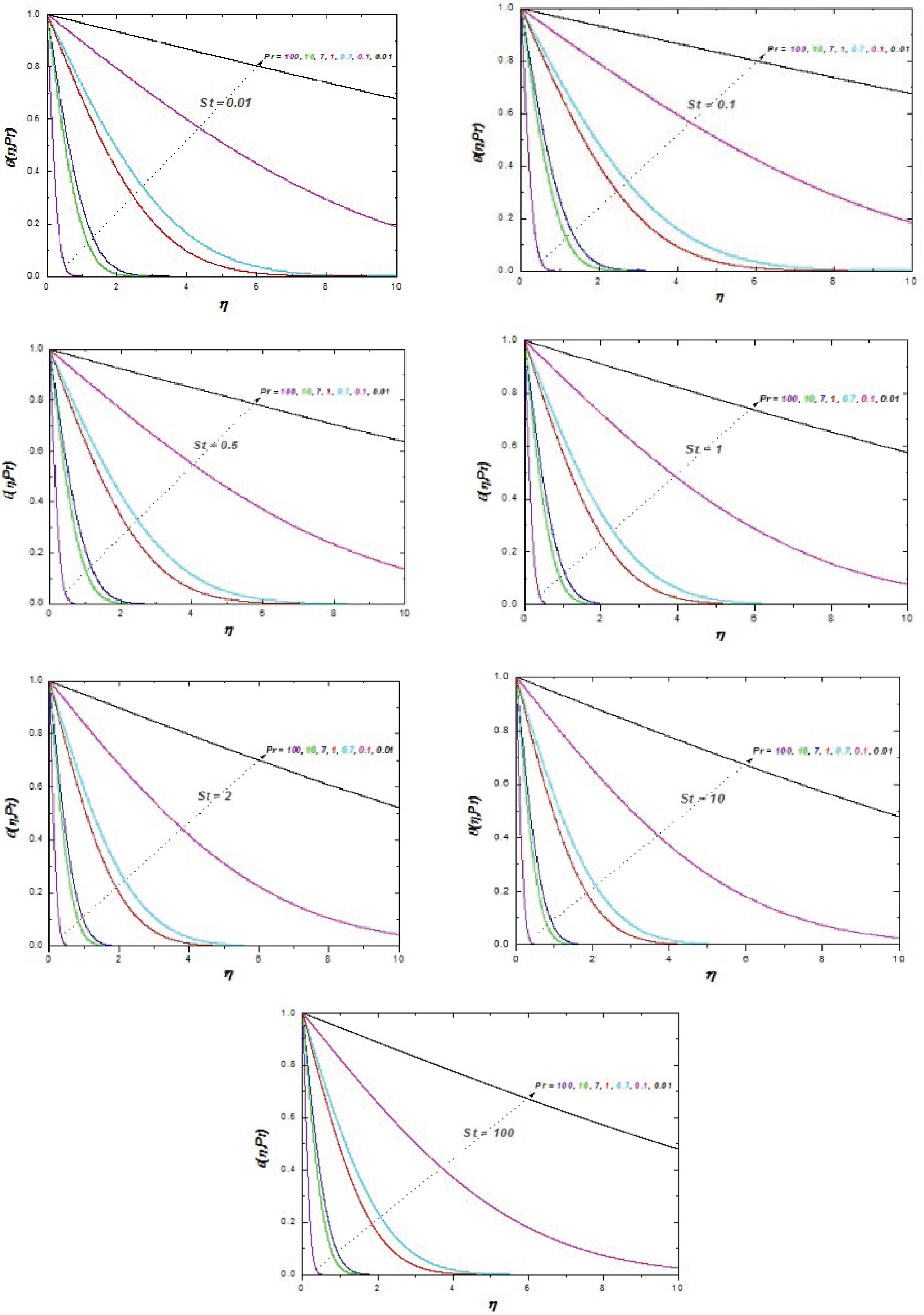

Figure 4. Temperature profiles *θ(η,St)* distribution for different values of *Pr* number for cases of *St* = 0.01, 0.1, 0.5, 1, 2, 10 and 100.

It looks From Eqs. (1) and (2) the similarity between the dynamic and heat transfer boundaries layers. This property permits us to solve by the same method as previously the Eq. (8) for a wide values of the Strouhal *St* and Prandtl *Pr* numbers. Indeed, the proposed formula of the dimensionless temperature profile must check the boundary conditions, diffusion and steady solutions and the unsteady energy equation in the whole domain $0 \leq \eta < \eta_\infty$

The interesting point of the proposed analytic function can be measured by the calculation $\Delta\varepsilon$ of equation (7)

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2}+\left[\frac{1}{2}\eta(1-\xi)+\frac{1}{2}\left[\xi+(1-\xi)\ln(1-\xi)\right]F+\xi(1-\xi)\ln(1-\xi)\frac{\partial F}{\partial\xi}\right]\frac{\partial\theta}{\partial\eta}$$
$$-\xi(1-\xi)\left[1+\frac{\partial F}{\partial\eta}\ln(1-\xi)\right]\frac{\partial\theta}{\partial\xi}=\Delta\varepsilon \tag{23}$$

The appropriate analytical formula of the temperature profile which converge approximately this last equation can be expressed as

$$\theta(\eta,\xi) = 1 - erf(Pr^{1/2}\frac{A(\xi)}{2}\eta) \tag{24}$$

Where $A(\xi) = exp(-0.53007\xi^{9/5})$

It found that the unsteady temperature profile confirms the equation energy equilibrium, with the residual errors $\Delta\varepsilon$ does not exceed 1% for all time thus for all Strouhal numbers in all space domain. Therefore, the simple formula of $\theta(\eta,\xi)$ converges well the unsteady heat transfer boundary layer equation along a horizontal surface for all chosen values of Strouhal and Prandtl numbers.

Evaluation of the Nusselt number

The effectiveness of the heat transfer is measured by means of Nusselt number given by

$$Nu_x = -\xi^{-1/2}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}\mathrm{Re}_x^{1/2} \tag{25}$$

The local heat Transfer coefficient corresponding to $\xi \in]\,0, 1]$ and for all Strouhal numbers values St_x is given by the following relation:

$$\frac{Nu_x}{Re_x^{1/2}} = \xi^{-1/2}\left(\frac{Pr^{1/2}}{\sqrt{\pi}}exp(-0.53007\xi^{9/5})\right) \tag{26}$$

In Figures 3-4, we represent the dimensionless heat profiles distributions corresponding to wide values of the Strouhal *St* and Prandtl *Pr* numbers. We note that the temperature profiles variations inside the heat boundary layer are much faster for high Prandtl numbers. This is due mainly to the viscosity effects on the heat transfer from surface to fluid flow. In addition, the temperature profiles evolutions are also very fast for Strouhal number values higher than unit $St_x > 1$ when we compare them with $St_x < 1$. We explain it by the fact that the diffusion forces due to the heat transfer between surface and fluid flow dominate the flow. Moreover, we interpret the great shift between the values of Strouhal number related to the Figure 2 by the fact that the distributions, inside the thermal boundary layer, are identical for $St_x > 100$ and for $St_x < 0.01$. From Figure 4 we note an interesting property; the temperature profiles present a quasi-identical distributions for all $St_x > 1$ and $St_x < 1$, and this at any Prandtl number values.

RESULTS AND DISCUSSION

The results concerning the determination of the heat transfer coefficient are well summarized on the Figures 5 and 6. We represent in Figure 5 the corresponding variations of the Nusselt number according to all Strouhal numbers, and for various Prandtl number values such as: 0.01, 0.1, 1, 10, 100. The corresponding curves reveal two linears domains of variations, in which the Nusselt number variations are more important for $St_x > 1$. and when *Pr* increases. Furthermore, we notice a significant increase Nusselt number variations when Prandtl number increases.

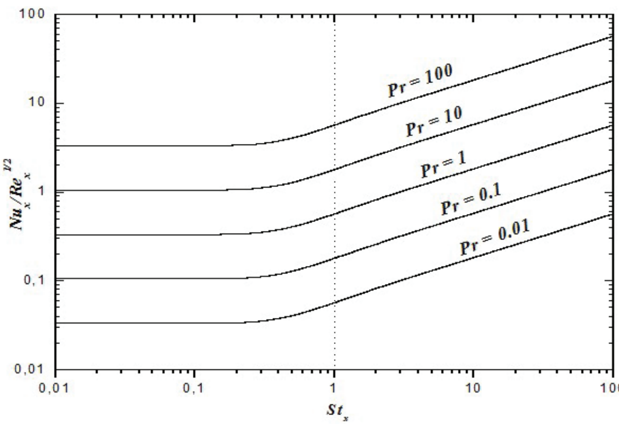

Figure 5. Nusselt number evolution depending to Strouhal and Reynolds numbers for different values of Prandtl number.

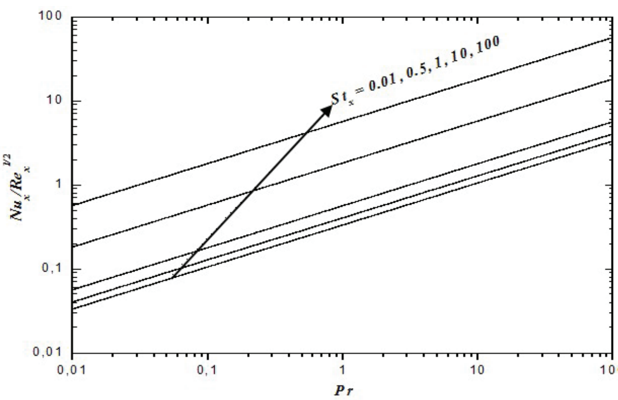

Figure 6. Nusselt number evolution depending to Prandtl and Reynolds numbers for different values of Strouhal number *St*.

Figure 6 recapitulates the main results and, since the curves are parallel, highlights the same evolution law of the Nusselt number. The increase of *Nu* according to *Pr* is sensitive of a curve to other, so when the Strouhal number *St* increases substantially from 1 to 10 initially then from 10 to 100.

CONCLUSION

In this contribution, an analytic approach is employed to study the Strouhal number effects on heat transfer evolution from Rayleigh flow to Blasius flow of a horizontal surface for various Prandtl numbers. Relevant analytic functions of the velocity and temperature profiles are proposed to approximately converging the unsteady dynamic and energy equations for all time intervals. Firstly, the realistic and powerful character related to the precision of the velocity formula is established. The obtained results made it possible to find the Rayleigh, Blasius and Williams-Rhyne solutions and to carry out the checking of the unsteady momentum equation valid for all time. Consequently, analytic expression of the skin friction coefficient is achieved for different values of Strouhal numbers. Subsequently, basing on the same reasoning as previous, an analytic temperature profile is proposed for converging the unsteady energy equation. An interesting result highlighted the impact of the flow regime on the heat transfer phenomenon near a surface were obtained, particularly, the establishment of an analytical law of the heat transfer coefficient for all Strouhal and Prandtl numbers. Depending on the Prandtl number, the Nusselt number *Nu* is almost constant in the case of diffusion flow $St < 1$ (due to the conduction transfer) while it increases linearly in both the transition regime (conduction-convection transfer) and during the convective flow $St > 1$ (convection transfer). In the same way, it is confirmed that, whatever the flow regime, the Nusselt number evolutions also increase when the Prandtl number increases.

NOMENCLATURE

English symbols

A	Dimensionless function
a	Constant
B	Dimensionless function
b	Constant
C_f	Skin friction coefficient
F	Dimensionless stream function
F_η	Dimensionless velocity profile
g	Function depends on x and t
Nu	Nusselt number
Nu_o	Transient Nusselt number
Pr	Prandtl number
Re	Reynolds number
St	Strouhal number
T	Temperature in (°C)
T_W	Wall temperature in (°C)
T_∞	Ambient temperature in (°C)
t	Time (s)
U_e	Ambient velocity (m.s^{-1})
u	Parallel velocity to the flow (m.s^{-1})
v	Normal velocity to the flow (m.s^{-1})
x	Parallel coordinates to the flow (m)
y	Normal coordinates to the flow (m)

Greek symbols

α	Thermal diffusivity (m^2. s^{-1})
β, γ	Constants
δ_c	Convective boundary layer thickness (m)
δ_d	Diffusive boundary layer thickness (m)
Δε	Residual error
η	Dimensionless variable
θ	Dimensionless temperature profile
μ	Dynamic viscosity (kg. m^{-1}.s^{-1})
ν	Kinematic viscosity (m^2. s^{-1})
ξ	Dimensionless variable
ϕ	Dimensionless function
Ψ	Stream function (m^2.s^{-1})

Subscripts

η	Differentiation with respect to *η*

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Stewartson K. On the impulsive motion of a flat plate in a viscous fluid, Part-I. Quart J Mech Appl Math 1951;4:182−198. [CrossRef]

[2] Stewartson K. On the impulsive motion of a flat plate in a viscous fluid, Part-II. Quart J Mech Appl Math 1973;26:143−152. [CrossRef]

[3] Tokuda N. On the impulsive motion of a flat plate in a viscous fluid. J Fluid Mech 1968;33:657−675. [CrossRef]

[4] Watkins CB. Heat transfer in the boundary layer over an impulsively started flat plate. ASME J Heat Trans 1975;97:282−484. [CrossRef]
[5] Hall MG. The boundary layer over an impulsively started flat plate. Proc Roy Soc Lond 1969;310:401−414. [CrossRef]
[6] Dennis SC. The motion of a viscous fluid past an impulsively started semi-infinite flat plate. J Inst Math Appl 1972;10:105−117. [CrossRef]
[7] Williams JC, Rhyne TH. Boundary layer development on a wedge impulsively set into motion. SIAM J Appl Math 1980;38:215−224. [CrossRef]
[8] Liao SJ. An analytic solution of unsteady boundary-layer flows caused by an impulsively stretching plate. Commun Nonlinear Sci Numer Simul 2006;11:326−339. [CrossRef]
[9] Hang X, Liao SJ, Pop I. Series solutions of unsteady three-dimensional MHD flow and heat transfer in the boundary layer over an impulsively stretching plate. Eur J Mech B Fluids 2007;26:15−27. [CrossRef]
[10] Simon DH, Ingham B, Pop I. Unsteady heat transfer in impulsive Falkner-Skan flows: Constant wall temperature case. Eur J Mech B Fluids 2002;21:447−468. [CrossRef]
[11] Revnic C, Grosan T, Pop I. Unsteady boundary layer flow and heat transfer over a stretching sheet. AIP Conf Proc 2008;1046:119−125. [CrossRef]
[12] Zheng ZC, Ghate AS. A solution of two-parameter asymptotic expansions for a two-dimensional unsteady boundary layer. Appl Math Comput 2015;270:90−104. [CrossRef]
[13] Hafidzuddin MEH, Roslinda N, Norihan MA, Ioan P. Unsteady flow and heat transfer over a permeable stretching/shrinking sheet with generalized slip velocity. Int J Numer Method 2018;28:1457−1470. [CrossRef]
[14] Nagler J. Higher order solution of Boundary layer formation as a result of impulsive start of motion. Zamm J Appl Math Mech 2019;99:e201800124. [CrossRef]
[15] Bulgakov VN, Kotenev VP, Ozhgibisova IuS. Analytical study of laminar boundary layer near blunted bodies. Math Mod Comput Sim 2019;31:82−94.
[16] Bachiri M, Bouabdallah A. Analytical approach of unsteady boundary-layer flows over a semi-infinite plate for all strouhal numbers. J Appl Mech 2011;78:024501. [CrossRef]