



Research Article

Pricing for longevity risk in long-term care insurance

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ABSTRACT

The increase in life expectancy due to improved health conditions in recent years has led to the longevity risk. This situation has caused an increase in the demand for health insurance over time, and the importance of long-term care insurance for older people which is designed to cover the long-term care needs has increased. In this study, longevity risk is investigated for the long-term care insurance with classified degrees of dependency. The long-term care insurance model is constructed by using Monte Carlo Simulation method according to two different scenarios: static structure in which the mortality rate does not change over time and dynamic structure in which the mortality rate changes depending on the increasing future life expectancy. The duration of dependency for long-term care insurance is modeled under the Weibull distribution of the semi-Markov process, which is explained by the Cox proportional hazard model and the frailty model. Probabilities of death and transition from a healthy state to a need of care state (dependent) are used from Turkey and France. Under the dependency structure, premiums for long-term care insurance and the reserve required to be allocated are calculated and the change in premiums with the effect of dependency is examined and it is concluded that the longevity risk caused more liability to the insurer.

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INTRODUCTION

The risk of dependency is the possibility of needing other individuals' care as a result of limited physical mobility because of old age, illness, disability, and such.

Care insurance is a branch of the social insurance system reserved for the dependency risk of elderly people. It is very difficult to differentiate between health insurance and care insurance. Health insurance often fails to provide the protection and security needed for the dependent person. For this reason, the necessary expenses are covered by the

family of the dependent person. These expenses are generally higher than an average middle-income family can afford.

Long-term care insurance (LTCI) is often explained by multi-state models [1]. Discrete-time models express three states: healthy, dependent, or death.

In the simple LTCI model shown in Figure 1, not mentioning the levels of dependency results in unrealistic results in premium calculations for this insurance. Therefore, the dependency level should be clearly specified and the pricing should be calculated according to this

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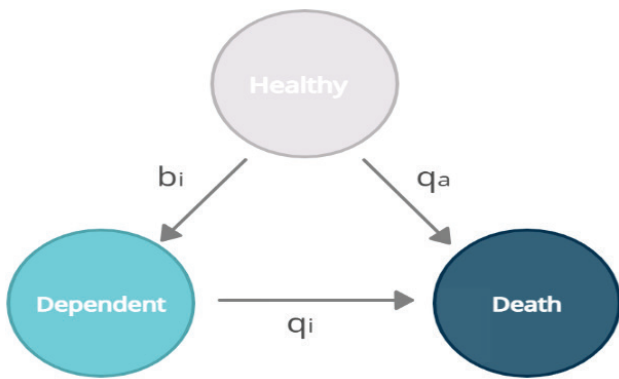


Figure 1. Basic model for LTCI.

data. A continuous-time and 10-transitions model of LTCI is used in this study. In France, dependency is specified in six levels. However, due to the very little need of care in the last two dependency levels, only four levels of dependency have been included in the LTCI. The four levels of dependency are calculated considering the level of health, and the probability of death. Given that the probability of recovery of the dependent person is very low, the transitions between dependency and health are considered one-sided in LTCI models [2].

The states that may occur during the life of a healthy individual are expressed in Figure 2. In the figure; b_i is the probability of a healthy individual becoming dependent, q_a is the probability of a healthy individual's death, and the Q_{ij} (as $i, j \in \{1,2,3,4\}$) are the semi-Markov kernels and

they indicate the probability of the transition. The levels of dependency increase from level 4 to level 1, and level 1 being the level with the highest need for care.

While modeling the time spent in dependency, explanatory variables that affect this period should also be included in the model. Therefore, two explanatory variables are added with the Cox proportional hazards model. These explanatory variables are the age and the gender of the dependent individuals. The Weibull distribution is used as the baseline hazard rate function.

Although the data from France includes the age and gender information of the individuals, it does not contain information about the types of their diseases. Individuals may have the same level of dependency due to different disease types. However, the time spent in dependency varies depending on the disease type. Diseases can be divided into two groups: cardiovascular and neurological. The frequency of these diseases directly relates to the increase in age. But the survival times for the two disease groups are different. For each level of dependency, the survival time of an individual with cardiovascular disease is shorter than an individual with a neurological disease. Since there are two different disease types, the Bernoulli distribution is used appropriately for determining the frailty levels. A generalized linear model with a logit link function is used for expressing the effect of age and gender on Bernoulli distribution parameters. With the parameter estimation results of this model in Biessy's paper [2] and the Monte Carlo simulation method, the premium and reserves for Turkey's LTCI are estimated.

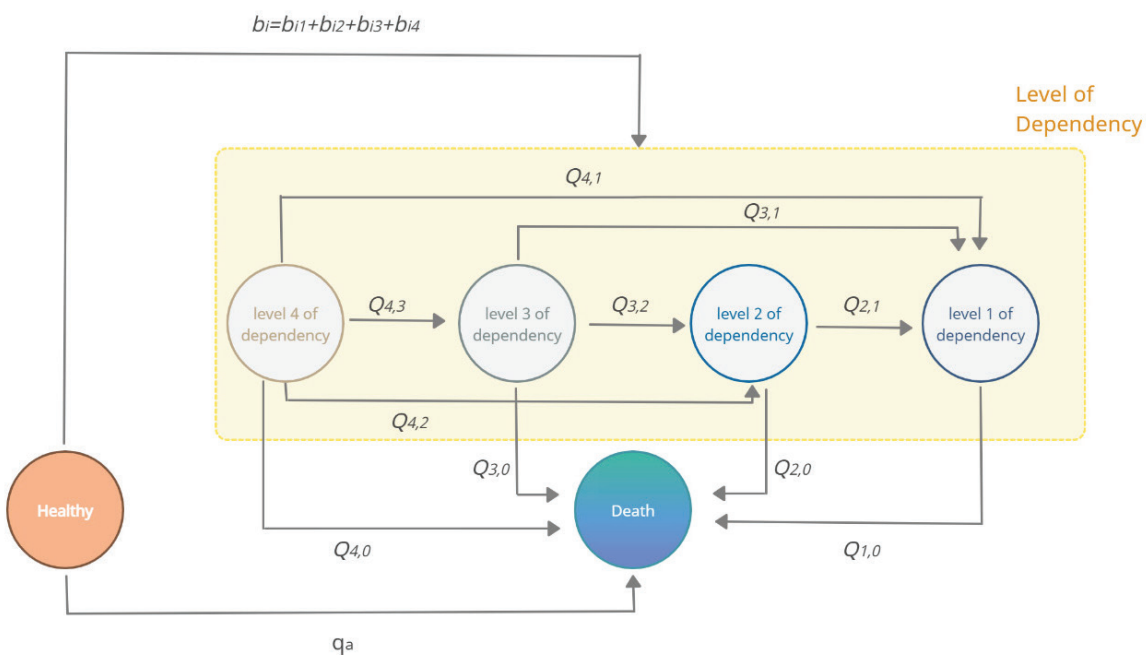


Figure 2. Multistate model for LTCI [2].

Beekman [3] has calculated the premium for LTCI by examining the time of first loss of daily living activities (ADL) as a random variable. Parker [4] has examined the different portfolios of policies, such as term, deferred, and endowment life policies, for stochastic death probabilities and interest rate. He has differentiated the investment risk and insurance risk with survival and interest rate conditions. Deleglise [5] has explained the life expectancy of patients with LTCI using multi-state Markov models. Gauzere [6] has used nonparametric approaches for corrected estimates of different transition densities with the multi-state model. Transition probabilities depend on the age of sick individuals with LTCI. These approaches are insufficient for severe patients with LTCI. Czado and Rudolph [7] examined the effect of observed damage, type of care, gender and severity of illness on the life curve. In the multi-state Markov model, the density of transition between states has been estimated using the Cox proportional hazards model. Helms [8] has obtained the transition probabilities by calculating actuarial values that give the required premium value and the long term care plans. Levante and Menziotti [9] have assessed the risks arising from future demographic trends and the uncertainty of impacts associated with the long-term care portfolio.

In this study, LTCI model is created for Turkey. The duration of dependency is explained by the semi-Markov process under the Weibull distribution with the Cox-proportional hazard rate and frailty model. The probabilities of entry into a dependency state and death are obtained from the data of Turkey and France. Assuming that all individuals are healthy, a model is created for individuals aged between [40,60] using the Monte Carlo simulation method. The simulation is done with both static and dynamic mortality probabilities. As a result of the simulation, it is observed that the increase in life expectancy caused an increase in the age of entering into a dependency state, but a shortening of the duration of dependency. In this case, the liability of the insurance company is uncertain in the future. Therefore, the liability of LTCI on the insurance company has been analyzed. It has been concluded that the increase in the number of dependent people, despite the shortening of the length of stay in need of care, causes more liability on the insurance company.

The insurance companies have a problem due to uncertainty, especially in the long-term insurance types. The liability of the insurance company increases with longevity risk. Therefore, we aimed to investigate the longevity risk for LTCI. We studied under two scenarios in order to better see the liability of the longevity risk for the insurer. The fact that the effect of longevity risk on LTCI, which classifies the level of dependency, is not examined in the literature, led us to conduct this study.

This study consists of five parts. In the second part, the models used for estimating the time spent in dependency are introduced and include the model and parameter estimates in Biessy's study. The Third part mentions

simulation, the probabilities required for simulation, and the interpretation of the results of the simulation. In the fourth chapter, LTCI premium and reserve are calculated for individuals aged between [40,60]. The fifth part consists of the conclusion.

MODEL

The models briefly are explained that are used to define the time spent in the dependency this section.

Markov Chain

Definition 1: Let the set of random variables $(X_n = n \in N)$, $E = (1, 2, \dots, m)$ be finite or countably infinite. If N is a state space with a set of non-negative integers, $X_n = i$ indicates that the process is in state i at time n . The Markov chain is defined below, with the states of the process $i_0, i_1, i_2, \dots, i_{n-1}, i, j$ and $n \geq 0$ [10].

$$\begin{aligned} P(X_{n+1} = j | X_0 = i_0, X_1 = i_1, \dots, X_{n-1} = i_{n-1}, X_n = i) \\ = P(X_{n+1} = j | X_n = i) = p_{ij} \end{aligned} \quad (1)$$

p_{ij} is defined as the probability of pass from state i to state j after a step.

Semi-Markov Process

The stochastic process with random time spent in the state that shows the Markov property of the process between states is called the semi-Markov process. The time spent in the given state before transitioning to another state is called the duration law.

Definition 2: Let $Y = (Y_n)_{(n \geq 0)}$ is a right-continuous stochastic process which the state space set $E = \{0, 1, 2, \dots\}$ be finite and countable integers. The transition time of Y from one state to another state is $0 \leq T_0 \leq T_1 \leq \dots \leq T_n \leq T_{n+1} \dots$ and the sequence of consecutive states X_0, X_1, X_2, \dots are expressed. For each $n \in N$, $t > 0$ and $j \in E$, $(X_n, T_n)_{n \in N}$ of the stochastic process is given by

$$\begin{aligned} P(X_{n+1} = j, T_{n+1} - T_n \leq t | X_0, T_0, \dots, X_n, T_n) \\ = P(X_{n+1} = j, T_{n+1} - T_n \leq t | X_n, T_n) \end{aligned} \quad (2)$$

Furthermore if the probabilities in the definition are independent from T_n , this process is called a homogeneous semi-Markov process.

Definition 3: Let Y be the homogeneous semi-Markov process. For, $\forall i, j \in E$, $\forall t \geq 0$ semi-Markov Kernel define by

$$Q_{ij}(t) = P(X_{n+1} = j, T_{n+1} - T_n \leq t | X_n = i) \quad (3)$$

The semi-Markov Kernel is the probability that a process known to be in state i will pass to state j in at most t time. $\forall i, j \in E$, $\forall t \geq 0$ for semi-Markov Kernel is given by

- $\sum_{j \in E} Q_{ij}(\infty) = 1,$
- $Q_{ij}(t) \geq 0,$
- $Q_{ij}(0) = 0,$
- $Q_{ii}(t) \neq 0$

The transition probability p_{ij} has a discrete-time Markov chain structure. It satisfies the following equation for $\forall i, j \in E$

$$p_{ij} = \lim_{t \rightarrow \infty} Q_{i,j}(t) \tag{4}$$

For $\forall i, j \in E, \forall t \geq 0,$ the distribution function of the duration in state i before entering state j is defined below [2], [11].

$$F_{ij}(t) = P(T_{n+1} - T_n \leq t | X_n = i, X_{n+1} = j) = \frac{Q_{ij}(t)}{p_{ij}} \tag{5}$$

Weibull Distribution

Weibull distribution frequently uses for survival analysis because it is flexible. The distribution has two parameters: shape (v) and scale (σ). The probability density function of Weibull is by

$$f(t) = \sigma v t^{v-1} e^{-\sigma t^v}, \quad \sigma, v > 0 \tag{6}$$

Cox Hazard Rate Model

It is a flexible and popular model that does not assume a specific probability distribution for the time until failure occurs [12]. This model includes life expectancy and independent variables that affect this period.

Let the vector of variables $\mathbf{x}^T = (x_1, \dots, x_k)$ and the hazard of individuals at time t be defined as $\lambda(t|\mathbf{x})$. The \mathbf{x}^T vector is the transposition of the x column vector. Cox regression model is defined by [13],

$$\lambda(t|\mathbf{x}) = \lambda_0(t)g(\mathbf{x}) \tag{7}$$

$\lambda_0(t)$ is the baseline hazard rate function. In the paper, it assumes that all individuals have the same baseline hazard rate. It contains the relevant primary parameters $g(\mathbf{x}) = g(\beta, \mathbf{x})$

$$g(\mathbf{x}) = \exp\left(\sum_{j=1}^k \beta_j x_j\right) = e^{\beta^T \mathbf{x}} \tag{8}$$

$\beta^T = (\beta_1, \dots, \beta_k)$ is defined as the vector of regression parameters. $g(\mathbf{x})$ is not time-dependent, it is a function of explanatory variables.

The baseline hazard rate function is considered as Weibull distribution in the study. Gender and age affect the

duration of dependency. So, gender ($g \in 1,2$ {1 = "female", 2 = "male"}) and age ($s > 0$) are explanatory variables. The parameters of gender and age are expressed with α and β , respectively. (6) and (7) from equations Cox proportional hazard rate function is by:

$$\lambda(t|\mathbf{x}) = \sigma v t^{v-1} e^{\alpha g + \beta s} \tag{9}$$

Frailty Model

If Z is a non-negative random variable, the frailty model is defined by

$$\lambda(t|Z) = Z\lambda(t) \tag{10}$$

$\lambda(t)$ is expressed as the Cox proportional hazard model $\lambda_0(t)e^{\beta^T \mathbf{x}}$ and Z value is expressed as $Z = \exp(u)$ [14]. u , is the random effect that varies depending on the observation.

The disease types are another explanatory variable that affects the duration of dependency. There is no data about this information. To explain heterogeneity inflicted from the disease types, the frailty model is used. Disease types can be divided into two groups: cardiovascular and neurological. Thus, the random variable u is assumed to have a Bernouilli distribution. Let the distribution parameter be expressed as $\eta(g,s)$. The generalized linear model was used to express the effect of gender and age on the probability $\eta(g,s)$. The random variable of u changes depending on the explanatory variables of g and s and is by:

$$u \sim \text{Bernouilli} \left(\frac{e^{\eta_0 + \eta_1 g + \eta_2 s}}{1 + e^{\eta_0 + \eta_1 g + \eta_2 s}} \right) \tag{11}$$

The parameters of the disease type's effect are expressed as γ . (9) and (10) from equations, for $\gamma > 0$, hazard rate function of duration of dependency is by:

$$\lambda(t|g, s, u) = e^{\gamma u} \lambda(t|g, s) = \begin{cases} \lambda(t|g, s) & \text{if } u = 0, \\ e^{\gamma u} \lambda(t|g, s) & \text{if } u = 1 \end{cases} \tag{12}$$

Estimation of parameters

g being the gender, s being the age, u being the effect of the disease type, and v and σ being the shape and scale parameters of the Weibull distribution, respectively. For the transition time from dependency state i to dependency state j , (the duration in state i) the hazard rate function is defined as below (12) from equation.

$$\lambda_{ij}(t|g, s, u) = \lim_{h \rightarrow \infty} \frac{P(t \leq T_{n+1} - T_n \leq t + h | X_{n+1} = j, X_n = i, T_n = s)}{h} = \sigma_{ij} v_{ij} t^{v_{ij}-1} e^{\alpha_{ij} g + \beta_{ij} s + \gamma_{ij} u} \tag{13}$$

For each transition state $v, \sigma, \alpha, \beta, \gamma$ values should be determined. This variable will have a different value for each transaction state [2]. For each transition to dependency

Table 1. Parameter estimations for each transition [2]

Transition	σ_{ij}	ν_{ij}	α_{ij}	β_{ij}	γ_{ij}
4 → 3	0.0107	1.43	-0.23	0.044	0.13
4 → 2	0.0043	1.43	-0.15	0.046	0.62
4 → 1	0.0005	1.65	-0.11	0.070	1.17
4 → 0	0.0413	1.39	-0.90	0.039	3.09
3 → 2	0.0375	1.43	-0.12	0.029	0.57
3 → 1	0.0136	1.59	-0.22	0.044	0.22
3 → 0	0.0439	1.23	-0.73	0.037	2.95
2 → 1	0.1279	1.49	0.06	0.008	0.21
2 → 0	0.0515	1.23	-0.82	0.037	3.38
1 → 0	0.0711	1.14	-0.61	0.036	3.64

Table 2. Parameter estimations for frailty effect [2]

η_0	η_1	η_2
0.93	-0.06	-0.04

state the constants η_0, η_1, η_2 will change accordingly to the frailty parameters of age and gender. In addition, this equation can be solved by using Yalçın and Çelik [15].

In Biessy’s article, the parameters were estimated using the Nelder-Mead algorithm, and the results shown in Tables 1 and 2 were reached [2].

According to the results of this parameter estimation, at a later age, the transition from level 3 of dependency to level 2, (which is a more severe dependency level) increases, and the time spent in level 3 dependency decreases. Besides, a male who is in a level 3 dependency state spends a shorter time in this state than a female in the same situation does. The duration in the state is shorter for males due to the disease type effect. At later ages, the impact of the type of disease on the length of staying at a dependency level diminishes for both males and females.

Markov Transition probabilities in Biessy’s paper are shown in Table 3 [2].

SIMULATION

Information Needed for Simulation

For simulation, some probability values must be obtained. It is necessary to produce healthy individuals’

gender ratio, mortality rate, probability of entering a dependency state, and the distribution probability for the levels of dependency.

- To determine the gender ratio, the number of people in Turkey in 2015 is used. However, this data includes both healthy and dependent people. The data for the number of dependent people in 2015 is unknown. That is why the research of population and housing by TÜİK [16] in 2011 is referred. It is assumed that the ratio between dependent people and the whole population had not changed in between. Thus, the number for dependent people in 2015 is estimated. Also, the gender ratio is estimated based on these numbers.
- The mortality rate is obtained from TÜİK’s 2015 study [17].
- The paper of Cohen, Miller, and Ingoldsby [18] is used for estimating the probability of entering into the dependency state.
- The distribution of dependency levels for the first entry into dependency state is evaluated using Biessy’s paper [2].
- Probabilities of the longevity risk are obtained from the paper of M. Sucu at all [19].

Required Parameters for Simulation

Some probabilities according to gender and age should be obtained. Using the static and dynamic mortality rates of different ages and gender, along with the probabilities of entry into dependency state, the following probabilities are obtained. In all probabilities, g and s respectively indicate the gender and age.

- $q_a^g(s)$, is the probability of death of a healthy person.

Table 3. Markov transition probabilities

Transition	4→3	4→2	4→1	4→0	3→2	3→1	3→0	2→1	2→0	1→0
p_{ij}	0.27	0.34	0.03	0.36	0.43	0.05	0.52	0.13	0.87	1.0

- $b_i^g(s)$, is the probability of entry into dependency state (incidence rate).
- $p_i^g(s, t)$, is the probability that a person aged s becoming dependent at the age $s + t$.
- $p_a^g(s, t)$, is the probability that a person aged s will die at age $s + t$ without becoming dependent.
- $p_i^g(s)$, is the probability of a person aged s being dependent.
- $p_a^g(s)$, is the probability of death without becoming dependent for a person aged s .
- $p_{|i}^g(s, t)$, is the probability that someone who is known to become dependent in the future being dependent at the age of $s + t$.
- $p_{|a}^g(s, t)$, is the probability of death at the age of $s + t$ knowing that person will never be dependent in their lifetime.

A healthy person will die either after they are dependent or without ever being dependent. The death of a healthy person ($q_a^g(s)$) and incidence rate ($b_i^g(s)$) are known. Others are calculated using these probabilities. Therefore $p_i^g(s) + p_a^g(s) = 1$ for $s \in N$. The following probabilities obtain for a person aged s for $s, t \in N$ [2].

$$p_i^g(s, t) = \left(\prod_{k=0}^{t-1} (1 - b_i^g(s+k))(1 - q_a^g(s+k)) \right) b_i^g(s+t) \quad (14)$$

$$p_a^g(s, t) = \left(\prod_{k=0}^{t-1} (1 - b_i^g(s+k))(1 - q_a^g(s+k)) \right) (1 - b_i^g(s+t))q_a^g(s+t) \quad (15)$$

$$p_i^g(s) = \sum_{t=0}^{\infty} p_i^g(s, t) \quad (16)$$

$$p_a^g(s) = \sum_{t=0}^{\infty} p_a^g(s, t) \quad (17)$$

$$p_{|i}^g(s, t) = \frac{p_i^g(s, t)}{p_i^g(s)} \quad (18)$$

$$p_{|a}^g(s, t) = \frac{p_a^g(s, t)}{p_a^g(s)} \quad (19)$$

These probabilities are obtained in these two scenarios.

Simulation

The survival time for a k individual at the starting age of $s_k^0 \in [40,60]$ and with the gender $g_k \in \{1,2\}$ is simulated. The simulation is created in the same way for the two

scenarios. Before starting the simulation, some limits are given below.

- The number of states the individual gets in during his survival time show with n_k . ($2 \leq n_k \leq 6$).
- The set of visited states denote by $X^k = (X_m^k)_{1 \leq m \leq n_k}$.
- The set of times transition from one state to another state express as $h^k = (h_m^k)_{1 \leq m \leq n_k}$.
- If the age of entry into the dependency state is less than 60, it is expressed with $(X_m^k) = 7$, and the simulation for this individual stops.

The simulation flow chart under the limits mentioned above is in Figure 3. The simulation is repeated 1,000,000 times for each age.

The distribution function of the dependency time is obtained from equation (13) and defined as:

$$F(t|g, s, u) = 1 - \exp(-t^v \sigma e^{\alpha g + \beta s + \gamma u}) \quad (20)$$

The inverse of the distribution function is

$$\left[\ln \left(\frac{1}{1-x} \right) \frac{e^{-(\alpha g + \beta s + \gamma u)}}{\sigma} \right]^{-\frac{1}{v}} = t \quad (21)$$

x refers to a random number generated from a uniform distribution on $[0,1]$. If $X_2^k = i$, $X_3^k = j$, the time spent (duration) in the state i is,

$$t_{ij} = \left[\ln \left(\frac{1}{1-x} \right) \frac{e^{-(\alpha_{ij} g + \beta_{ij} s + \gamma_{ij} u)}}{\sigma_{ij}} \right]^{-\frac{1}{v_{ij}}} \quad (22)$$

Simulation Results

According to the simulation results, as seen in Figure 3, in both scenarios generally, male death happens at an earlier age than females. Contrary to this generally, females enter the dependency at an earlier age than men. With the static mortality rate, the ages of death and entry to dependency for males are concentrated around 78, while it accumulates around 82 for females. As a result of a simulation that uses dynamic mortality rates, the ages of death and entry to dependency for males are 80, while it is 84 for females. According to the dynamic and static mortality rates, the age of entry into dependency or death is quite different in females but similar in males. In the simulation with the dynamic mortality rate, the probability of death and entry into dependency are higher than static in the grand old ages.

In Table 4, life expectancy in Turkey is compared with the simulation results. There are two reasons why results differ: i) using the data from France for the probability of entering into a dependent state and ii) the use of the general mortality rates (total mortality rate of the healthy and dependent person). There is no different mortality rate table of the healthy or dependent person in Turkey. So, we

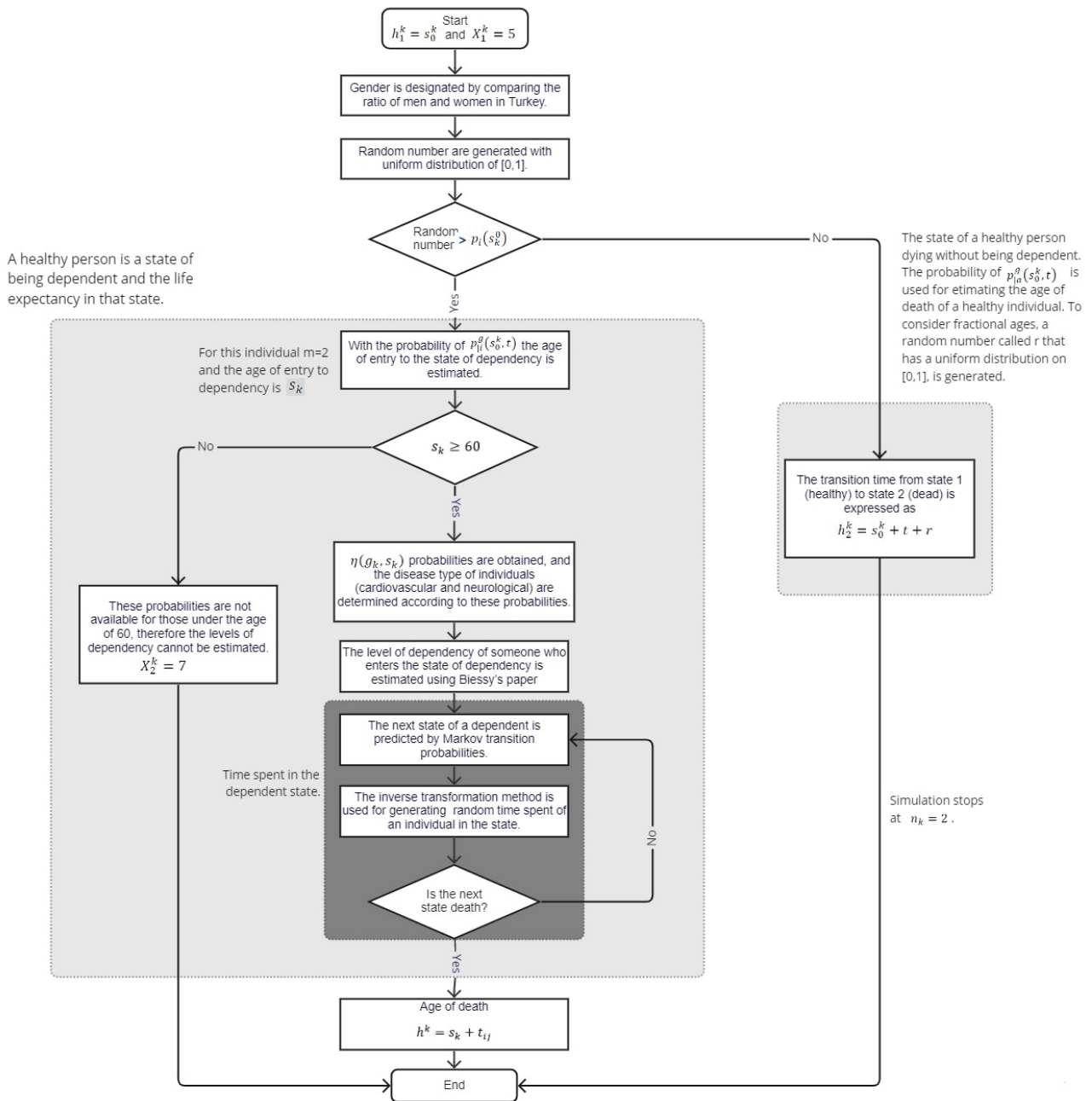


Figure 3. Simulation flow chart for one individual.

had to use a mortality table including both dependent and healthy people. Life expectancy with dynamic mortality rate is higher than the one with static mortality rate. Besides this is an expected result, it is seen as a proof that the simulation gives consistent results.

Table 5 shows the average time spent in the dependency state of an individual. This time is longer for females. In level 1 of dependency, the time spent is shorter compared to the other levels. In the table, the ages of entry into dependency are ignored. The time spent in the states could vary according to the age of entry to dependency. As the age of entry to

dependency gets higher, the time spent in dependency state would decrease. The time spent in the dependency state is shorter when calculated with dynamic mortality rate due to the fact that individuals enter the dependent state at a later age compared to the static mortality rate results.

Even if Table 5 shows the data for 60 years old individuals, the mean and variance values are similar for every entrance to dependency age levels. Individuals with 4th and 2nd level of dependency have a longer amount of time spent in dependency state and have a bigger value for variance. The reason for that is the higher probability for people from



Figure 4. Rates of death and entry into dependency based on age and gender. (Static: result of simulation with static mortality rate. Dynamic: result of simulation with dynamic mortality rate.)

Table 3. Life expectancy based on the simulation

Age	TUIK life expectancy (2013-2015)		Life expectancy with static mortality rate, based on simulation		Life expectancy with dynamic mortality rate, based on simulation	
	Male	Female	Male	Female	Male	Female
40	37.5	42.4	35.9	40.2	37.1	42.3
45	32.8	37.5	31.3	35.5	32.4	37.5
50	28.3	32.8	26.8	30.9	27.7	32.7
55	23.9	28.2	22.6	26.4	23.3	28.0
60	19.9	23.7	18.2	21.6	19.2	23.4

different age groups to enter those states. In addition, with the dynamic mortality rate, the duration of dependency is shorter in simulation.

PREMIUM FOR LONG TERM CARE INSURANCE

Calculating the Premium

In LTCI, it is assumed that the benefits are paid at the end of the period and the premiums are received at the

beginning of the period. In addition, a certain amount of financial assistance is provided to the person in need of care in the first month and no assistance is provided for the next two months. At the end of three months, if the individual is still in need of care, the benefit begins to be paid. In this study, LTCI is considered different from the general practice. LTCI is not implemented in Turkey, only the state assists people with disabilities. For this reason, we have developed a model based on the following assumptions,

Table 4. The means and variances of the time spent in dependency for individuals moving to a dependent status at the age of 60

Female				
Level of dependency	Simulation results with static mortality rate		Simulation results with dynamic mortality rate	
	Mean	Variance	Mean	Variance
Level 4	3.9565	8.4024	3.8692	7.9821
Level 3	3.1308	6.3812	3.0707	6.1007
Level 2	3.4877	8.3649	3.4102	7.8427
Level 1	2.3623	3.8009	2.3185	3.634
Male				
Level of dependency	Simulation results with static mortality rate		Simulation results with dynamic mortality rate	
	Mean	Variance	Mean	Variance
Level 4	3.2457	6.01466	3.2146	5.9604
Level 3	2.3981	2.8009	2.3981	2.7757
Level 2	2.3197	2.8836	2.9888	2.8190
Level 1	1.6693	1.6393	1.6773	1.6335

taking into account Turkey’s socio-economic structure and services for the disabled.

- Individuals between the ages of 40 and 60 can enter the LTCI.
- Premiums are considered to be paid at the beginning of the period and benefits to be paid at the end of the period.
- Benefit is paid to those who are in a dependent state from the age of 65 and over.
- The premiums must be paid until the age of 65. If the individual does not pass to a state of dependency, he must continue paying the premiums until the age of 70. The benefit is paid at the end of each month, according to the level of dependency.
- The level 4 of dependency: $T_4= 400$ TL
- The level 3 of dependency: $T_3= 600$ TL
- The level 2 of dependency: $T_2= 1000$ TL
- The level 1 of dependency: $T_1= 1500$ TL

In the study base on the assumption of inflation being equal to zero, the cash flow net present value of premium expresses as NPV_p , and the cash flow net present value of the benefit as NPV_T . The monthly effective interest rate is determined as 0,2%. The index k represents the individuals in the simulation. The premium is calculated with the simulation conclusions mentioned in section 4 and with the following limitations.

- The premiums paid by an individual who dies or becomes dependent before the age of 65 are refunded.
- M_k is time spent as healthy for the k individual. The premium payments of an annuity due of 1 unit in each month for M_k periods are

$$NPV_p^k = \frac{(1 - v^{M_k})}{d} \tag{23}$$

(i is the effective interest rate, d is the discount rate, and v is the discount factor.)

- n is the number of repetitions of the simulation. The premium is

$$\widehat{p}^n = \frac{E(NPV_T)}{E(NPV_p)} = \frac{\frac{1}{n} \sum_{k=0}^n NPV_T}{\frac{1}{n} \sum_{k=0}^n NPV_p} \tag{24}$$

The estimated premium is calculated using the law of large numbers. With a higher number of simulation repetition $n \rightarrow \infty, \widehat{p}^n \rightarrow p$ will move closer to the real premium value.

Premium and Confidence Interval

The simulation is repeated 1,000,000 times for each age of entry between 40 and 60. Figure 4 shows simulation results for the age of 60. Repeating the simulation 10.000 times was observed to be sufficient to obtain consistent results. However, examining ages different than 60, it is seen that the simulation should be repeated at least 20.000 times for reaching consistent results.

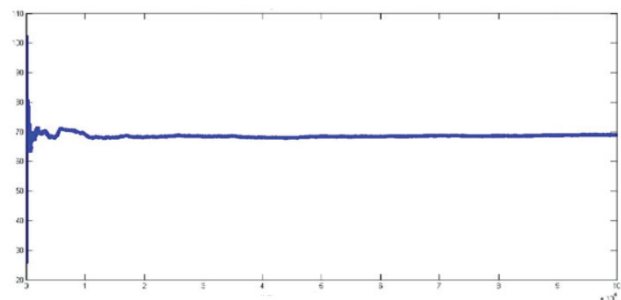


Figure 5. Premium values calculated for 60 year-old individuals.

Table 5. The premium for different ages with static and dynamic mortality rates

Monthly Premium Values Calculated with Static Mortality Rate					
Age	40	45	50	55	60
Monthly premium value	16.08 TL	20.96 TL	28.51 TL	41.67 TL	68,84 TL
Confidence interval %	±0.0755	±0.0979	±0.1322	±0.1908	±0.3088
Monthly Premium Values Calculated with Dynamic Mortality Rate					
Age	40	45	50	55	60
Monthly premium value	18.29 TL	23.59 TL	31.87 TL	45,99 TL	74.80 TL
Confidence interval %	±0.0778	±0.1006	±0.1355	±0.1946	±0.314

The premium takes different values for each age between the ages of 40 and 60. The net single premium value is calculated for both male and female individuals of the same age. As an example, the premium value and the confidence interval of the premium with $\alpha = 0,05$ significance level for various ages are shown in Table 6.

According to Table 6, as the age of entering into dependency gets higher, the premium value to be paid also gets higher due to the fact that there will be fewer premium instalments to be paid. With higher entry age, there is a higher death probability. Due to the uncertainty of the

premium amount to be collected in a shorter time, the confidence interval is wider. Monthly premium payment amounts are lower when calculated with static mortality rates, compared to the one with dynamic mortality rates. The premium increases in proportion to the rise in life expectancy.

Reserve

Technical reserve calculations in LTCI consist of two components: the premium and the benefit. In this paper, reserve is the difference between the accumulated values of premium and benefit.

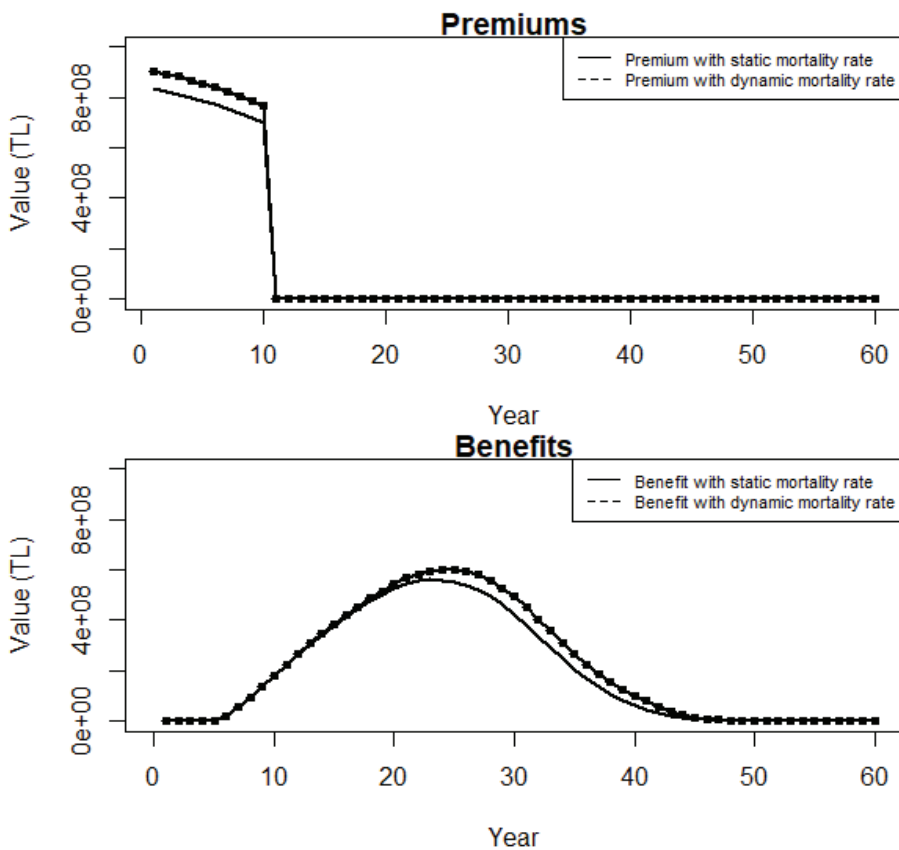


Figure 6. The accumulated value of the monthly premiums to be paid and monthly benefits to be received after each age period for 60 years old.

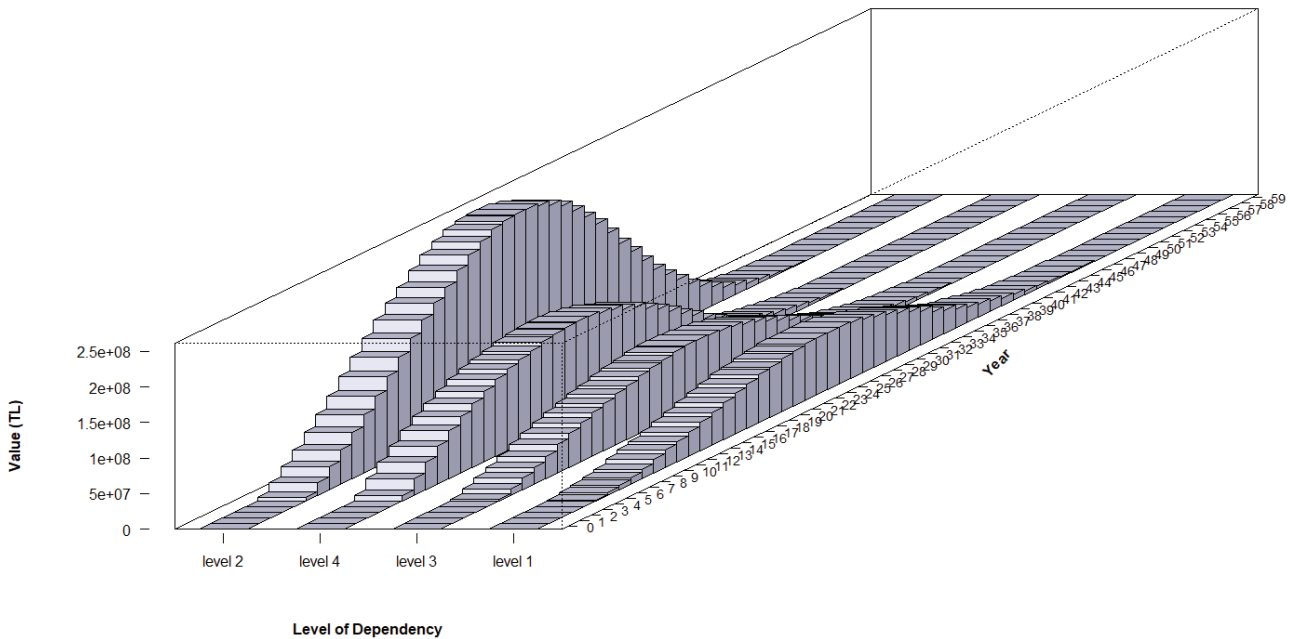


Figure 7. The accumulated value of the benefits according to the dependency levels.

Figure 5 shows the cumulative values of premiums and benefits realized in one year for individuals who entered the insurance plan at the age of 60. The accumulative premium value is higher when it's calculated using dynamic mortality rate. The benefit payments start after the age of 65, therefore the first five years' values are equal to zero. The benefits for the two scenarios are similar for the first years, these values begin to differ in the following years.

Figure 6 shows the distribution of the accumulated value of benefits according to the levels of dependency. For both scenarios, the distribution of benefits in the level of dependency is similar. Insured individuals may be in several different levels of dependency. Considering the Markov transition probabilities, the probability of transition from level 4 or level 3 of dependency to the level 2 of dependency is higher than the transition between other levels. Therefore, the total benefit amount for level 2 of dependency is the highest. Moreover, the age and gender of entry to this dependency state causes the total benefit at this level to be higher than in other levels. Since the probability of level 1 of dependency to occur is low, the total cumulative benefit per person is the highest value among all the levels of dependency, while the total benefit amount remains the lowest.

Figure 7 shows the annual reserve for all individuals with insurance starting age of 60. The reserve amount increases during the first decade because of two reasons, i) the benefit is not paid during the first five years, and ii) the probability of transition into dependency is lower in younger ages. As the number of individuals entering into dependency state increases in the following years, the annual reserve amount decreases.

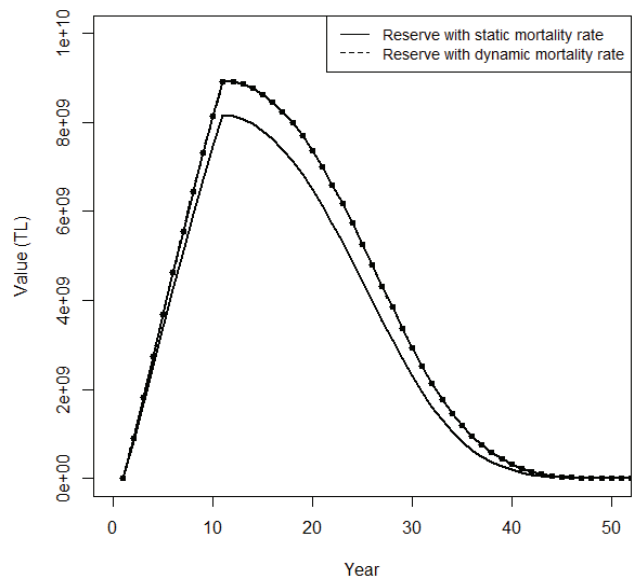


Figure 8. Reserve for 60 years old.

Since dynamic and static mortality rates at an earlier age give similar values, the number of people who pay premiums are very similar. However, in the dynamic mortality rate, the number of people continuing living in older ages are higher, therefore also the total benefit is higher. Thus, with the static mortality rate, the annual reserve is less than the one with the dynamic mortality rate.

Sensitive Analysis

The effect of the probability of transition from dependency state to death and from healthy to death on premium

Table 6. Sensitivity of the monthly premium with static mortality rate for 60 years old

	Probability of moving from a healthy state to a dependent state		Static mortality rate of healthy individuals	
	-% 10	+% 10	-% 10	+% 10
Rate of Change	-% 10	+% 10	-% 10	+% 10
Premium	63.27 TL	74.19 TL	71.93 TL	66.11 TL
Relevant Change	-%8.1	% 7.8	% 4.5	-% 4

Table 7. Sensitivity of the monthly premium with dynamic mortality rate for 60 years old

	Probability of moving from a healthy state to a dependent state		Dynamic mortality rate of healthy individuals	
	-% 10	+% 10	-% 10	+% 10
Rate of Change	-% 10	+% 10	-% 10	+% 10
Premium	63.86 TL	80.5 TL	77.84 TL	72.05 TL
Relevant Change	-%8	% 7.6	% 4.1	-% 3.6

payments are examined. Each probability is changed by 10%, and the effect of this change on the premium payments are observed.

Changes in dynamic and static mortality rates affects the premiums in the same direction but with different intensity. Tables 7 and 8 show that the probability of entering into dependency has a stronger effect on the premium than the probability of death has. The reason for that is the correlation between the benefit and the speed of entry into dependency. As a result, as the number of dependent individuals increases, the premiums should also increase. On the contrary, with a higher speed of death rates the premiums would diminish. Detailed information about conclusions can be found in the study of Lazoğlu Ç. [20].

CONCLUSION

In this paper, we aimed to investigate longevity risk for LTCI that classifies the level of dependency. LTCI model for Turkey has been constructed. The parameters for the duration of dependency in Biessy's study, and the mortality rate table, gender rate, and projections for longevity risk in Turkey, and probability of entry into the dependency in France have been used. The death probabilities and the probabilities of entry to a dependent state have been examining to estimate the life expectancy and health conditions.

Calculating the present value of the benefit in LTCI requires many mathematical operations. Therefore, it was found appropriate to obtain the current values of the premium and benefit through the simulation method. The simulation was repeated 1,000,000 times for each age between 40 and 60. LTCI premium and reserves have been calculated according to both the time-invariant death probabilities (static mortality rate) and the time-varying death probabilities (dynamic mortality rate).

As a result of the simulations with the static and dynamic mortality rates, the life expectancy of simulation

with the dynamic mortality rate is higher. This increment gives rise to entry into the dependency of more people and as a result an increase in premium. Also, it can be observed that LTCI premium payments increase proportionally with higher ages due to the fact that the higher the age of entry to the system is, the fewer Premium installments are paid in total. The confidence interval of the premium becomes wider with the increase of age due to the rise in the age of entering the system and the high number of entries and exits to the insurance system in a short time period.

It can be observed that the premium is directly proportional to the probability of transition from a healthy state to a dependency state and the duration in dependency, and inversely proportional to the probability of transition from a healthy state to death. Dynamic mortality rates change every year due to the longevity risk, therefore also the number of people who move into dependency state increases. However, as the age of entry into dependency increases, the time spent in the dependent state will decrease. Even so, the longevity risk puts higher liability on the insurer.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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