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Research Article

Reliability analysis of shear strength equations of RC beams

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ABSTRACT

Shear strength of a reinforced concrete (RC) member should be larger than its flexural strength, in order to prevent the shear failure, which is sudden and brittle. The reliability of a RC beam against the shear failure is closely related to the reliability of the equation determining its shear strength. In this study, the reliabilities of shear strength equations of RC beams were investigated by constructing the performance function between prediction equations and experimental results using a second-moment approach. It is assumed that the random variables are statistically independent, and the correlation effects are not taken into account. It is observed from the reliability rankings that the equation of EN 1992:2004 yields the lowest failure probability while the equation of Zsutty is the highest failure probability.

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INTRODUCTION

The reliability or serviceability may be assured only in terms of probability that the available capacity (R) will be adequate to withstand the lifetime maximum load (S). The aim of the reliability analysis is to ensure the event ($R > S$) during the lifetime of the engineering system [1-8]. The difference between R and S is included in the member design through safety criteria used in the structural codes. As this must be accomplished under conditions of uncertainty, the assurances of performance is realistically possible only in terms of probability. In general, therefore, probability analyses will be necessary in the development of such probability-based designs [2]. The uncertainties inherent in the constituent material strengths and densities, the member geometry, the applied loads, and the errors in load and strength calculations give rise to uncertainties in the resistance of a RC member as well as in the loads that act on it. As a result, the nominal strength and loads computed by the designer differ from the actual ones. These safety criteria are provided either implicitly as those used for the working stress design format or explicitly as those for the ultimate strength design format.

In this study, the reliabilities of the shear strength equation of RC beams were investigated by constructing the performance function between those equations and experimental results using a second-moment approach [9-10]. Numerical analyses were conducted for iterative solution with non-normal distribution. In practice, non-normal distributions are transformed into equivalent normal distributions. Conversely, according to international statistical data, it is assumed that the material strengths are log-normal and, the other variables are normal [2,6-8,11-13]. Also, it is assumed that the random variables are statistically independent, and the correlation effects are not taken into account.

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FAILURE AND SURVIVAL PROBABILITIES

The performance of the structural system is represented by a performance function $Z = g(\mathbf{X}) = g(X_1, X_2, \dots X_n)$, where $\mathbf{X} = (X_1, X_2, \dots X_n)$ is a vector of basic random variables. These basic variables may be loads, material strengths, dimensions, etc. which are taken into account in the structural system [14]. The limit state of system may be defined as $Z = 0$. $Z > 0$ and $Z < 0$ are respectively survival and failure states, respectively. The probabilities of the failure p_F and the survival p_S can be determined with integral of the joint probability density function $f_{\mathbf{X}}(\mathbf{x})$, in spaces $Z < 0$ and $Z > 0$ [1,2,9,10]:

$$f_{\mathbf{X}}(\mathbf{x}) = f_{X_1,X_2,\dots,X_n}(x_1, x_2, \dots x_n) \tag{1}$$

$$p_F = \int_{Z<0} \dots \int f_{X_1,X_2,\dots,X_n}(x_1, x_2, \dots x_n)dx_1 dx_2 \dots dx_n = \int_{Z<0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{2}$$

$$p_S = 1 - p_F = \int_{Z>0} f_{\mathbf{X}}(\mathbf{x})d\mathbf{x} \tag{3}$$

The calculation of the probabilities of survival or failure requires the knowledge of the joint probability distribution. In practice, this information is often unavailable or difficult to obtain for reasons of insufficient data. For this reason, the failure and survival probabilities can be determined by the second-moment approach whose name is derived from the definition of variance [1,2,9,10]. In second-moment approach used for predicting the probabilities of failure and survival, performance function Z is expanded in the Taylor series. Then performance function Z is linearized by taking into account only first-order terms in the Taylor's series expansion.

Thus, the mean m_Z and standard deviation σ_Z of the linear function are determined. If the probability distributions of random variables are not normal, such distributions are transformed into equivalent normal distributions $N(m_Z, \sigma_Z)$. For the limit state $Z = 0$, the value of the standard normal distribution variable is $s = (0 - m_Z)/\sigma_Z = -m_Z/\sigma_Z$. The $\beta = m_Z/\sigma_Z$, is called as reliability index. The probability of survival, therefore, becomes $p_S = \Phi(\beta)$ and the corresponding probability failure is $p_F = \Phi(-\beta) = 1 - \Phi(\beta)$. In order to predict the probabilities of survival or failure using second-moment approach, the reliability index is determined. The cumulative density function of the standard normal distribution is $\Phi(.)$ [1,2,6-10].

In this study, it is assumed that the distributions of compressive strength of concrete (f_c), the yield strength of stirrups (f_{yw}), and the shear strength of RC beam (v_u) are log-normal. For this reason, the log-normal distribution is transformed into equivalent normal distribution. The mean value (λ_X) and standard deviation (ζ_X) of the log-normal distribution can be determined by using Equations 4 and 5 as follows:

$$\lambda_X = \ln m_X - 0.5\zeta_X^2 \tag{4}$$

$$\zeta_X^2 = \ln\left(1 + \frac{\sigma_X^2}{m_X^2}\right) \tag{5}$$

Then, the mean and standard deviation for the equivalent normal distribution of original variable can be expressed as [1,2]:

$$m_X^N = x^*(1 - \ln x^* + \lambda_X) \tag{6}$$

$$\sigma_X^N = x^*\zeta_X \tag{7}$$

In iterative second-moment approach, the point $\mathbf{x}'^* = (x_1'^*, x_2'^*, \dots, x_n'^*)$ on the failure surface with a minimum distance to the origin of reduced variates $X_i' = (X_i - m_{X_i})/\sigma_{X_i}$ is the most probable failure point. This distance equals to the reliability index $d_{min} = \beta$ [15,16]. The performance function $Z = g(\mathbf{X}) = g(X_1, X_2, \dots X_n)$ is expanded in the Taylor series at a point $\mathbf{x}^*$, which is on the failure surface $g(\mathbf{x}^*) = 0$, the partial derivatives are evaluated at $(x_1^*, x_2^*, \dots, x_n^*)$ and only first-order terms are taken into account. In result, the performance function Z becomes

$$Z \cong \sum_{i=1}^{n}(X_i - x_i^*)\left(\frac{\partial g}{\partial X_i}\right)_* \tag{8}$$

The mean value and variance of the Z are obtained by using Equations 9 and 10.

$$m_Z \cong -\sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i'}\right)_* x_i'^* \tag{9}$$

$$Var(Z) \cong \sigma_Z^2 \cong \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i'}\right)_*^2 \tag{10}$$

The dimensionless sensitivity coefficients α_i can be expressed as the ratio of the uncertainty σ_{Xi} in the random variable X_i to the total uncertainty σ_Z as follows:

$$\alpha_i = \left(\frac{\partial g}{\partial X_i}\right)\left(\frac{\sigma_{X_i}}{\sigma_Z}\right) \tag{11}$$

If the probability distribution of a random variable with mean m_{Xi}, standard deviation σ_{Xi} and coefficient of sensitivity α_i is normal, the value of standard normal distribution variable is $s = -\alpha_i\beta$ for a given reliability index β. Thus, the design value of the variable X_{id} can be determined by using Equation 12.

$$X_{id} = m_{X_i} - \alpha_i\beta\sigma_{X_i} \tag{12}$$

Properties of RC Beams

For slender RC beams $(a/d) \geq 2.5$, geometrical and material properties of RC beams with stirrups are shown in Table 1, where f_c is the concrete compressive strength, ρ_w is the stirrup ratio, f_{yw} is yield strength of stirrup reinforcement, ρ is longitudinal reinforcement ratio, a/d is the shear span-to-effective depth ratio, d is the effective depth, b_w is the web width, and v_u is the test shear strength. All the beams failed in shear. Moreover, the frequency distributions of the selected variables are illustrated in Figure 1.

Table 1. Geometrical and material properties of RC beams

References	Beam	f_c (MPa)	ρ_w (%)	f_{vw} (MPa)	ρ (%)	a / d	d (mm)	b_w (mm)	v_u (MPa)
[17]	A-1	24.06	0.10	325	1.80	3.92	466	307	1.63
[17]	A-2	24.27	0.10	325	2.28	4.93	464	305	1.73
[17]	B-1	24.75	0.15	325	2.43	3.95	461	231	2.09
[17]	B-2	23.17	0.15	325	2.43	4.91	466	229	1.88
[17]	C-1	29.58	0.20	325	1.80	3.95	464	155	2.17
[17]	C-2	23.79	0.20	325	3.66	4.93	464	152	2.29
[17]	CRB-1	23.65	0.15	340	2.28	4.01	457	229	1.65
[17]	1WCRA-1	26.34	0.10	350	1.71	4.01	457	305	1.54
[17]	1WCRB-1	23.17	0.15	340	2.26	3.99	459	229	1.94
[17]	1WCRC-1	26.75	0.20	350	1.69	3.98	460	152	2.05
[17]	1WCA-1	25.17	0.10	350	1.76	3.95	463	305	1.56
[17]	1WCB-1	26.48	0.15	340	2.34	3.97	460	231	1.90
[17]	1WCC-1	24.89	0.20	350	1.75	3.97	460	155	2.00
[17]	2WCA-1	26.34	0.10	350	1.77	3.96	461	305	1.72
[18]	29a-2	37.16	0.12	372	2.23	4.01	456	254	1.87
[18]	29f-2	41.78	0.12	372	2.23	4.01	456	254	2.03
[19]	R8	26.68	0.21	270	1.46	3.36	272	152	1.92
[19]	R9	29.58	0.41	280	1.46	3.36	272	152	2.52
[19]	R10	29.61	0.21	270	0.98	3.36	272	152	1.82
[19]	R11	26.20	0.21	270	1.95	3.36	272	152	2.16
[19]	R12	33.92	0.21	270	4.16	3.60	254	152	2.83
[19]	R13	32.27	0.41	280	4.16	3.60	254	152	3.86
[19]	R14	29.03	0.14	270	1.46	3.36	272	152	2.16
[19]	R15	29.86	0.41	280	4.16	3.60	254	152	3.61
[19]	R16	31.58	0.41	280	4.16	3.60	254	152	3.61
[19]	R17	12.76	0.21	270	1.46	3.36	272	152	1.69
[19]	R18	31.30	0.21	270	1.46	3.36	272	152	2.04
[19]	R19	30.27	0.41	280	1.46	3.36	272	152	2.89
[19]	R20	42.46	0.21	270	1.46	3.36	272	152	2.17
[19]	R21	48.13	0.42	280	4.16	3.60	254	152	3.86
[19]	R22	29.51	0.21	270	1.46	4.50	272	152	1.82
[19]	R24	30.89	0.21	270	4.16	5.05	254	152	2.38
[19]	R25	30.82	0.21	270	4.16	3.60	254	152	2.70
[19]	R28	31.58	0.83	270	4.16	3.60	254	152	4.63
[20]	C305-D0	25.99	0.23	354	2.61	3.00	315	150	2.28
[21]	E2l	30.40	0.41	370	2.47	2.78	270	190	3.32
[21]	E3l	28.20	0.42	388	2.47	2.78	270	190	3.63
[21]	E4l	30.40	0.59	261	2.47	2.78	270	190	3.65
[21]	E5l	30.40	0.58	278	2.47	2.78	270	190	3.68
[22]	C3	29.40	0.16	275	1.97	3.00	95	76	2.16
[22]	R3	29.40	0.38	208	1.97	3.00	95	76	2.51
[22]	J3	29.40	0.43	253	1.97	3.00	95	76	2.84
[22]	Y3	25.90	0.60	222	3.95	3.00	95	76	4.00
[23]	B50-7-3	39.90	0.12	292	3.36	3.60	298	152	2.07
[23]	B150-3-3	28.70	0.38	271	3.36	3.60	298	152	3.05
[23]	B100-7-3	47.10	0.26	269	3.36	3.60	298	152	2.66
[23]	B150-7-3	46.60	0.38	271	3.36	3.60	298	152	2.95
[24]	ST6	49.30	0.28	430	1.95	2.88	278	290	2.85
[24]	ST18	49.80	0.21	430	1.95	2.88	278	290	3.05

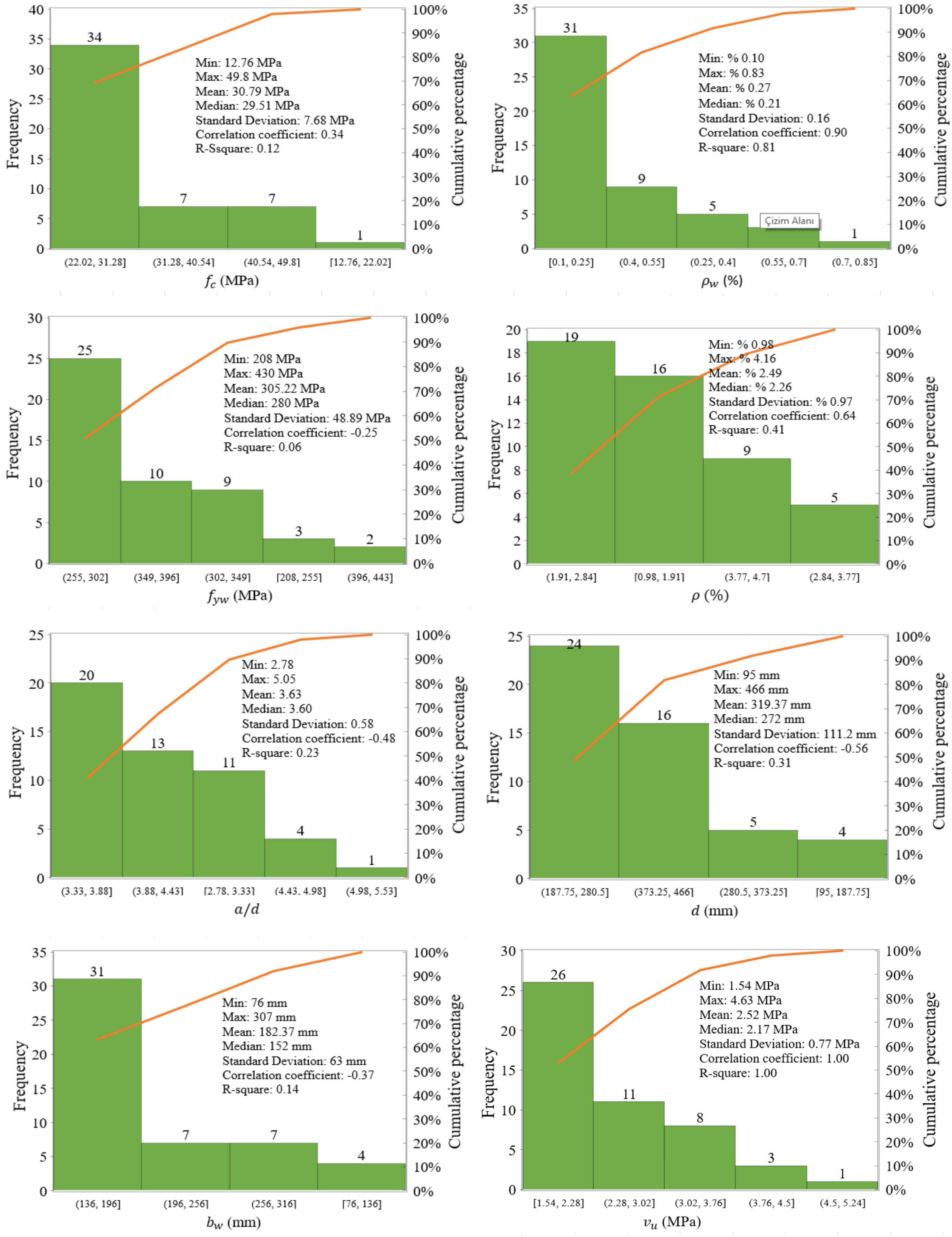

Figure 1. Frequency distributions of the variables.

Shear Strength Models

In the design of RC frame elements, it is necessary to prevent shear failure mechanisms. The question of what mechanisms of shear transfer will contribute most to the resistance of a particular beam is difficult to answer [25]. Most of the shear design equations provide a simple superposition of stirrup and concrete strength [26-30]. The following procedure outlines the guidelines recommended by ASCE-ACI 426 [31] in order to determine the shear strength of RC beams. The governing equation from ACI318 [26] states that the shear capacity must exceed the shear demand as shown in Equation 13.

$$\phi v_n \geq v_u \tag{13}$$

The nominal shear strength is derived from two components: concrete and stirrups. This relationship is given as follows:

$$v_n = v_c + v_s \tag{14}$$

in which v_c is the shear strength of concrete; and v_s is the shear strength of stirrup based on yield, respectively.

The shear strength of stirrup, v_s, in case of vertical stirrups can be derived from basic equilibrium considerations on a 45° truss model with constant stirrup spacing and effective depth [25]. Summary of different shear strength models for RC slender beams is shown in Table 2.

As shown in Table 2, in ACI318 [26], TS500 [27] and ENV 1992 [29], (a/d) was not taken into account in predicting of the shear strength of the beam, unlike the equation proposed by Zsutty [30]. The size effect as a variable is considered only in the EN 1992:2004 [28] equation. Contrary to the ACI318 [26], TS500 [27] and Zsutty [30] equations, the ENV 1992 [29] and EN 1992:2004 [28] equations have been predicted by reducing the contribution of stirrups to the shear strength.

Figure 2 compares the predicted shear strength with the experimental shear strength values obtained from tests. The experimental and predicted shear strengths for existing test data yield large scatter in the results, especially for increasing shear strength of the beams. The six different code requirements and researchers' predictions are compared with the test results of 49 beams with stirrups.

Probabilities of Failure and Survival with Second-Moment Approach

The survival or failure probabilities of the equations determining the shear strength of RC beams were calculated by the second-moment approach described in the previous section. In this study, R and S are the experimental load-carrying capacity and the estimate of the load carrying capacity of the beam, respectively. For this reason, the statistical evaluation has been conducted by considering the beams satisfying, $R - S > 0$.

Table 2. Summary of different shear strength models for RC slender beams

References	Equations (SI units)	Comments
ACI318 [26]	$v_n = \left(\frac{1}{6}\sqrt{f_c} + \rho_w f_{yw}\right)$	$(a/d) \geq 2.5$, $f_c < 70$ MPa
TS500 [27]	$v_n = (0.65 f_{ct} + \rho_w f_{yw}) \leq 0.22 f_c$	$f_{ct} = 0.35\sqrt{f_c}$
EN 1992:2004 [28]	$v_{rd,s} = 0.9\rho_w f_{yw}\cot\theta$	$1 \leq \cot\theta \leq 2.5$
	$v_{rd,max} = 0.9 v f_{cd}(\cot\theta/(1+\cot^2\theta))$	$v = 0.6$ for $f_{ck} \leq 60$MPa
		$v = 0.9 - f_{ck}/200 > 0.5$ for $f_{ck} \geq 60$ MPa
EN 1992 [29]	Standard Method $v_n = \min(v_{R_{d2}}, v_{R_{d3}})$	$v_{R_{d3}} = v_{R_{d1}} + 0.90\rho_w f_{yw}$ $v_{R_{d1}} = 0.035 f_c^{2/3} k(1.2 + 40\rho)$ $\rho \leq 0.02$, $k = 1.6 - d/1000 \geq 1$ $v_{R_{d2}} = 0.3 v_{f_c}$, $v = 0.7 - f_c/200 \geq 0.5$
ENV 1992 [29]	Variable Strut Inclination Method $(v_c = 0)$ $v_n = \min(v'_{R_{d2}}, v'_{R_{d3}})$	$v'_{R_{d2}} = 0.6 v_{f_c}/(\cot\theta + \tan\theta)$ $v'_{R_{d3}} = 0.9\rho_w f_{yw}\cot\theta$ $v = 0.7 - f_c/200 \geq 0.5$ $26.6^o < \theta < 45^o$ $\rho_w f_{yw} \leq 0.38 v_{f_c}$
Zsutty [30]	$v_n = 2.2(f_c \rho d/a)^{1/3} + \rho_w f_{yw}$	$(a/d) \geq 2.5$

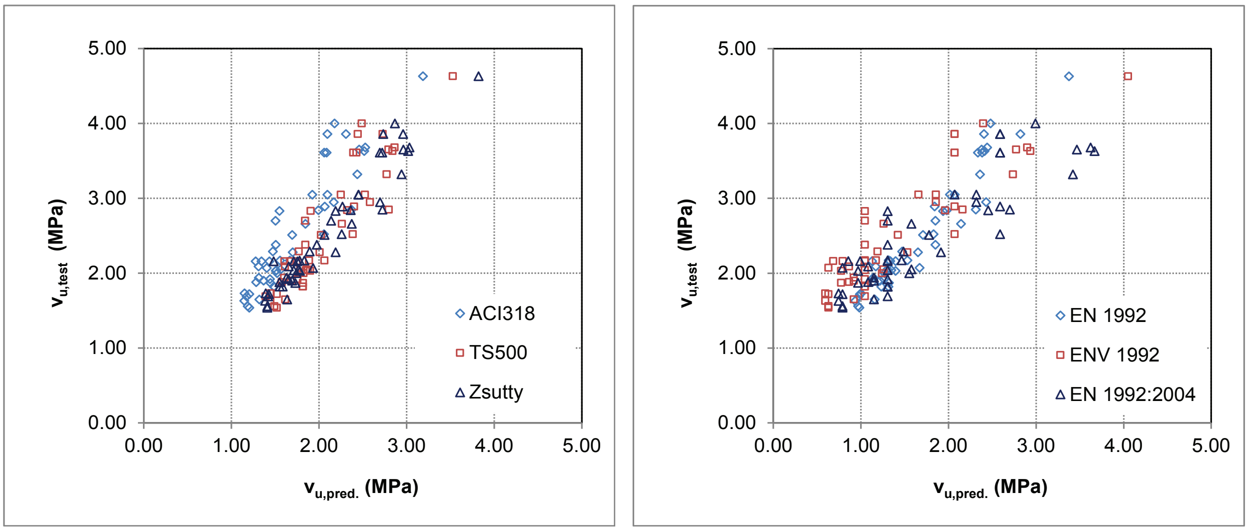

Figure 2. Predicted shear strength values versus test shear strength values.

In this reliability assessment, the equation resulting in the largest difference between R and S will be the most reliable one. In other words, the equation resulting the smallest failure probability will be the most reliable one. In the probabilistic evaluation, primarily, the performance function $Z = R - S > 0$ has been established. Therefore, the basic random variables included in the performance function are f_c, f_{yw}, ρ_w, ρ, d, a/d and v_u, which are used in the shear strength equations proposed by ACI318 [26], TS500 [27], EN 1992:2004 [28], EN 1992 [29], ENV1992 [29] and Zsutty [30].

Uncertainties of Random Variables

In general, the variations in the properties of RC beams depend on the construction quality control and environmental conditions. Both shear strengths obtained through experiments and equations were modeled as random variables to perform a probability-based analysis. In modeling those parameters as random variables, the values of standard deviations were determined based on the studies available in the literature and codes, and are summarized in Table 3.

The standard deviation of concrete compressive strength, σ_{fc} , under average construction quality control usually depends on the f_c and varies in between $0.10f_c$ and $0.21f_c$ through the literature. According to TS500 [32], σf_c ranges from 3.1 MPa to 6.25 MPa depending on the f_c.

ACI318 [26] recommends to increase the value of σ_{fc} in case that the number of samples is less than 30, and proposes 5.2MPa for $f_c < 21$ MPa, 6.34MPa for 21MPa $\leq f_c \leq 35$ MPa and a value larger than 6.34 MPa based on the f_c.

The σ_{fc} is taken as $0.10f_c$ by Nowak and Szerszen [33] and Ribeiro and Diniz [34], $0.11f_c$ by Hao et al. [35], $0.12f_c$ by Neves et al. [36], $0.13f_c$ by Val et al. [37], $0.15\,f_c$ by Mirza [38], Mirza et al. [39], Mirza and MacGregor [40], Mirza and MacGregor [41], $0.16f_c$ by Val and Chernin [42], $0.20f_c$ by Melchers [43] and $0.21f_c$ by Ellingwood [44]. It is taken as $0.10f_c$ and $0.15f_c$ for cases 1 and 2, respectively, in the present study to model variations of f_c.

Table 3. Standard deviations of the variables

Variable	Standard deviation (σ)		Distribution
	Case 1	Case 2	
f_c	$0.10f_c$	$0.15f_c$	Log-normal
f_{yw}	$0.10f_{yw}$		Log-normal
ρ_w	$0.15\rho_w$		Normal
ρ	0.10ρ		Normal
d	$0.03d$		Normal
a/d	$0.03(a/d)$		Normal
v_u	$0.04v_u$		Log-normal

The standard deviation of reinforcement yield strength σ_{fy} ranges from $0.05f_y$ to $0.15f_y$. The σ_{fy} are taken as $0.05f_y$ by JCSS [45], $0.08f_y$ by Val et al. [37] and Low and Hao [46], $0.08f_y - 0.11f_y$ by Ostlund [47], MacGregor et al. [48] and $0.15f_y$ by Mirza [38], Mirza et al. [39], Mirza and MacGregor [40], Mirza and MacGregor [41]. It is taken as $0.10f_{yw}$ in the present study to model variations of σ_{fyw}.

Although the reinforcement ratios depend on the structural dimensions, in this study they are assumed to be statistically independent from each other and from the other random structural parameters. In Hao et al. [35] study, it is assumed that standard deviation of longitudinal reinforcement ratio, σ_ρ, is 0.10ρ and standard deviation of stirrup ratio, $\sigma_{\rho W}$, is $0.15\sigma_{\rho W}$. In the present study, σ_ρ and $\sigma_{\rho W}$ are taken as 0.10ρ and $0.15\sigma_{\rho W}$, respectively.

In Enright and Frangopol [49] studies, it is assumed that the standard deviation of effective depth, σ_d, is $0.03d$.

Table 4. β values for case 1 $\sigma_{fc} = 0.10f_c$

Reference	Beam	ACI318	TS500	EN 1992:2004	EN 1992	ENV 1992	Zsutty
[17]	A-1	4.76	1.83	11.67	6.42	11.67	2.30
[17]	A-2	5.53	2.65	12.12	6.71	12.12	3.08
[17]	B-1	5.45	3.38	10.40	6.68	10.40	2.98
[17]	B-2	4.42	2.28	9.61	5.79	9.61	2.40
[17]	C-1	3.97	1.76	8.62	5.08	8.62	2.36
[17]	C-2	4.77	3.16	9.01	6.03	9.01	2.37
[17]	CRB-1	2.62	0.30	8.32	4.06	8.32	0.30
[17]	1WCRA-1	3.30	0.24	10.67	5.10	10.67	1.46
[17]	1WCRB-1	4.51	2.47	9.51	5.84	9.51	2.43
[17]	1WCRC-1	2.91	1.08	7.66	4.45	7.66	2.07
[17]	1WCA-1	3.66	0.68	10.77	5.41	10.77	1.64
[17]	1WCB-1	3.90	1.64	9.35	5.17	9.35	1.67
[17]	1WCC-1	2.82	1.04	7.48	4.34	7.48	1.87
[17]	2WCA-1	4.75	1.84	11.51	6.30	11.51	2.84
[18]	29a-2	3.24	0.32	10.30	4.13	10.30	1.40
[18]	29f-2	3.84	0.86	10.92	4.48	10.92	2.14
[19]	R8	3.40	1.27	8.67	4.56	8.67	1.04
[19]	R9	1.89	0.59	5.60	2.89	5.60	0.31
[19]	R10	2.49	0.14	8.28	4.27	8.28	1.07
[19]	R11	4.76	2.83	9.54	5.26	9.54	1.79
[19]	R12	7.11	5.29	11.57	7.10	11.57	2.72
[19]	R13	5.68	4.71	8.64	6.23	8.64	2.93
[19]	R14	6.87	4.30	12.67	7.67	12.67	4.25
[19]	R15	5.17	4.19	8.15	5.76	8.15	2.36
[19]	R16	5.09	4.08	8.15	5.65	8.15	2.27
[19]	R17	3.78	2.30	7.74	5.25	7.74	1.12
[19]	R18	3.63	1.38	9.12	4.63	9.12	1.38
[19]	R19	3.11	1.92	6.56	4.08	6.56	1.55
[19]	R20	3.39	0.81	9.57	4.05	9.57	1.39
[19]	R21	4.95	3.69	8.49	5.15	8.49	2.13
[19]	R22	2.50	0.16	8.28	3.61	8.28	0.85
[19]	R24	5.42	3.43	10.26	5.54	10.26	1.70
[19]	R25	6.82	5.04	11.21	6.96	11.21	2.38
[19]	R28	2.93	2.30	5.19	3.56	5.19	0.69
[20]	C305-D0	3.18	1.64	7.30	3.94	7.30	1.06
[21]	E2l	2.64	1.70	5.58	3.26	5.58	1.89
[21]	E3l	3.05	2.23	5.72	3.71	5.72	2.75
[21]	E4l	3.36	2.49	6.14	4.00	6.14	0.81
[21]	E5l	3.14	2.30	5.89	3.78	5.89	0.98
[22]	C3	5.95	3.57	11.50	5.02	11.50	2.44
[22]	R3	4.08	2.51	8.25	4.00	8.25	0.04
[22]	J3	3.32	2.11	6.81	3.48	6.81	0.61
[22]	Y3	5.26	4.49	7.84	5.69	7.84	1.27
[23]	B50-7-3	5.49	2.24	12.92	4.69	12.92	0.69
[23]	B150-3-3	4.37	3.21	7.74	5.05	7.74	1.45
[23]	B100-7-3	4.30	2.15	9.58	4.19	9.58	0.69
[23]	B150-7-3	3.17	1.55	7.50	3.42	7.50	0.20
[24]	ST6	1.81	0.22	6.14	2.05	6.14	1.84
[24]	ST18	4.15	2.39	8.65	4.20	8.65	2.30
Mean		4.10	2.22	8.84	4.87	8.84	1.76
Minimum		1.81	0.14	5.19	2.05	5.19	0.04
Maximum		7.11	5.29	12.92	7.67	12.92	4.25

Sigma J Eng Nat Sci, Vol. 42, No. 1, pp. 141–152, February, 2024

Table 5. β values for case 2 $\sigma_{fC} = 0.15f_c$

Reference	Beam	ACI318	TS500	EN 1992:2004	EN 1992	ENV 1992	Zsutty
[17]	A-1	4.36	1.59	11.67	5.61	11.67	2.17
[17]	A-2	5.05	2.29	12.12	5.81	12.12	2.90
[17]	B-1	5.21	3.09	10.40	6.18	10.40	2.85
[17]	B-2	4.22	2.08	9.61	5.38	9.61	2.31
[17]	C-1	3.57	1.64	8.62	4.82	8.62	2.30
[17]	C-2	4.65	2.99	9.01	5.80	9.01	2.31
[17]	CRB-1	2.49	0.29	8.32	3.77	8.32	0.30
[17]	1WCRA-1	2.99	0.24	10.67	4.48	10.67	1.38
[17]	1WCRB-1	4.33	2.27	9.51	5.46	9.51	2.34
[17]	1WCRC-1	2.82	1.03	7.66	4.27	7.66	2.03
[17]	1WCA-1	3.33	0.61	10.77	4.77	10.77	1.55
[17]	1WCB-1	3.71	1.50	9.35	4.76	9.35	1.60
[17]	1WCC-1	2.74	0.99	7.48	4.18	7.48	1.83
[17]	2WCA-1	4.33	1.61	11.51	5.52	11.51	2.67
[18]	29a-2	2.97	0.30	10.30	3.62	10.30	1.33
[18]	29f-2	3.49	0.76	10.92	3.88	10.92	2.01
[19]	R8	3.26	1.18	8.67	3.05	8.67	1.02
[19]	R9	1.87	0.58	5.60	2.82	5.60	0.31
[19]	R10	2.36	0.15	8.28	3.99	8.28	1.05
[19]	R11	4.59	2.62	9.54	4.83	9.54	1.75
[19]	R12	6.87	4.87	11.57	6.32	11.57	2.61
[19]	R13	5.63	4.62	8.64	6.08	8.64	2.90
[19]	R14	6.39	3.76	12.67	6.62	12.67	4.08
[19]	R15	5.12	4.11	8.15	5.63	8.15	2.33
[19]	R16	5.05	3.99	8.15	5.50	8.15	2.25
[19]	R17	3.70	2.21	7.74	5.12	7.74	1.11
[19]	R18	3.45	1.27	9.12	4.24	9.12	1.35
[19]	R19	3.07	1.87	6.56	3.99	6.56	1.54
[19]	R20	3.17	0.74	9.57	3.59	9.57	1.35
[19]	R21	4.88	3.57	8.49	4.91	8.49	2.09
[19]	R22	2.37	0.16	8.28	3.32	8.28	0.84
[19]	R24	5.20	3.15	10.26	4.98	10.26	1.65
[19]	R25	6.60	4.66	11.21	6.27	11.21	2.29
[19]	R28	2.93	2.29	5.19	3.53	5.19	0.69
[20]	C305-D0	3.11	1.58	7.30	3.77	7.30	1.04
[21]	E2l	2.63	1.68	5.58	3.21	5.58	1.87
[21]	E3l	3.03	2.21	5.72	3.67	5.72	2.73
[21]	E4l	3.34	2.46	6.14	3.94	6.14	0.81
[21]	E5l	3.13	2.27	5.89	3.73	5.89	0.97
[22]	C3	5.60	3.18	11.50	4.31	11.50	2.34
[22]	R3	3.98	2.39	8.25	3.72	8.25	0.05
[22]	J3	3.28	2.05	6.81	3.35	6.81	0.61
[22]	Y3	5.23	4.44	7.84	5.59	7.84	1.26
[23]	B50-7-3	4.95	1.90	12.92	3.91	12.92	0.66
[23]	B150-3-3	4.31	3.13	7.74	4.91	7.74	1.44
[23]	B100-7-3	4.10	1.97	9.58	3.75	9.58	0.68
[23]	B150-7-3	3.10	1.48	7.50	3.21	7.50	0.21
[24]	ST6	1.78	0.22	6.14	1.95	6.14	1.80
[24]	ST18	4.03	2.25	8.65	3.87	8.50	3.94
Mean		3.93	2.09	8.84	4.49	8.84	1.70
Minimum		1.78	0.15	5.19	1.95	5.19	0.05
Maximum		6.87	4.87	12.92	6.62	12.92	4.08

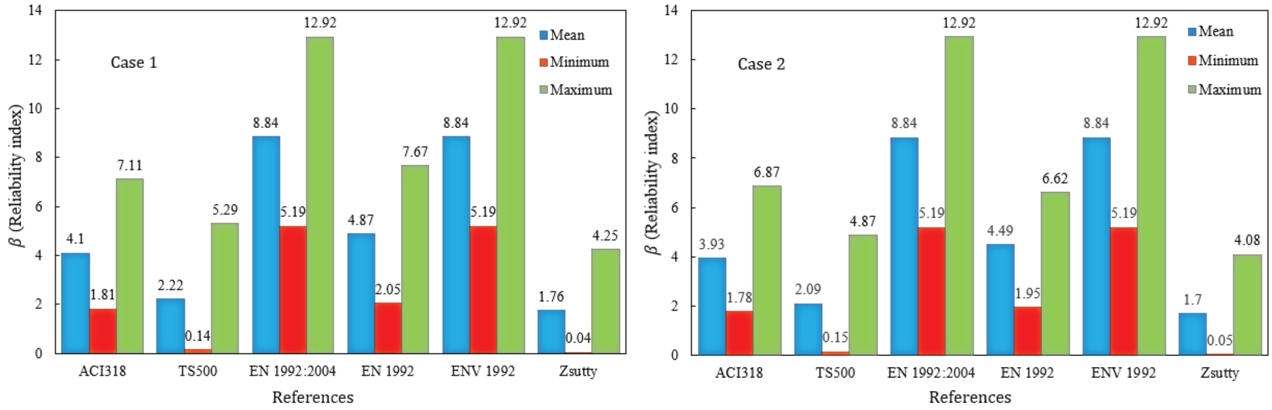

Figure 3. Reliability index β of the predicted equations for cases 1 and 2 .

Table 6. Reliability rankings according to the probability of failure (p_F)

Ranking	Case 1	Case 2	
	Mean, Minimum, Maximum	**Mean, Maximum**	**Minimum**
1	EN 1992:2004; ENV 1992	EN 1992:2004; ENV 1992	EN 1992:2004; ENV 1992
2	EN 1992	EN 1992	ACI318
3	ACI318	ACI318	EN 1992
4	TS500	TS500	TS500
5	Zsutty	Zsutty	Zsutty

The standard deviations of structural dimensions are taken as $0.03d$ by Low and Hao [46] and Hao et al. [35], $0.03d$ by Ribeiro and Diniz [34]. In the present study, σ_d and $\sigma_{a/d}$ are taken as $0.03d$ and $0.03(a/d)$, respectively. In Hognestad [50] and Mirza [38] studies, it is assumed that the standard deviation of strength, σ_{vu}, due to test procedure is $0.04v_u$, this value is used in this study..

Evaluation of Failure Probability

By taking into account the standard deviations and mean values of the variables, the performance function Z is separately formed for each shear strength equation [26-30]. Then, the mean value mZ and standard deviation σZ of the performance function Z are determined. Thus, the reliability index β is a function of the ratio m_Z/σ_Z. The probability of survival, therefore, becomes $p_S = \Phi(\beta)$ and the corresponding probability of failure is $p_F = 1 - \Phi(\beta)$. As shown in Tables 4-5, numerical analysis was carried out for non-normal solution with iteration. Log-normal distributions are transformed into equivalent normal distributions as explained in the previous section. Mean, minimum and maximum reliability indexes β of the predicted equations are shown Figure 3 for cases 1 and 2, respectively.

Mean, minimum and maximum probabilities of failures, obtained for each beam were evaluated and sorted from minimum to maximum, as shown in Table 6. As can be clearly seen in Table 6, EN 1992:2004 equation yields the lowest probability of failure and Zsutty equation has the highest probability of failure.

For case 1 $\sigma_{fc} = 0.10f_c$, the targets β of RC beams for ultimate states are calculated as 4.10 (range 1.81-7.11) according to ACI318, 2.22 (range 0.14-5.29) according to TS500, 8.84 (range 5.19-12.92) according to EN 1992:2004 and ENV1992, 4.87 (range 2.05-7.67) according to EN 1992, 1.76 (range 0.04-4.25) according to Zsutty. For case 2 $\sigma_{fc} = 0.15f_c$, the targets β of RC beams for ultimate states are calculated as 3.93 (range 1.78-6.87) according to ACI318, 2.09 (range 0.15-4.87) according to TS500, 8.84 (range 5.19-12.92) according to EN 1992:2004 and ENV1992, 4.49 (range 1.95-6.62) according to EN 1992, 1.70 (range 0.05-4.08) according to Zsutty. Hence, there are considerable differences in β defined by current codes. These differences show that defining a range of target β is reasonable for a code instead of defining a target value.

CONCLUSION

Considering the limited data collected from literature, the following conclusions can be drawn from the results of this study:

- It can be said that EN 1992:2004 equation yields the lowest probability of failure and Zsutty equation has the highest probability of failure based on the reliability rankings provided by non-normal iterative solution.

- By comparing the reliabilities of shear strength equations of RC beams for two different cases($\sigma_{fC} = 0.10f_c$ and $\sigma_{fC} = 0.15f_c$), there were no significant changes in the reliability rankings.
- For case 1 ($\sigma_{fC} = 0.10f_c$), the mean target values of β for ultimate states are calculated as 4.10 according to ACI318, 2.22 according to TS500, 8.84 according to EN 1992:2004 and ENV1992, 4.87 according to EN 1992, 1.76 according to Zsutty. For case 2 ($\sigma_{fC} = 0.15f_c$), the mean target values of β for ultimate states are calculated as 3.93 according to ACI318, 2.09 according to TS500, 8.84 according to EN 1992:2004 and ENV1992, 4.49 according to EN 1992, 1.70 according to Zsutty. Hence, there are considerable differences in β defined by current codes. These differences show that defining a range of target β is reasonable for a code instead of defining a target value.
- It can be observed that although the number of shear failure probabilities calculated for normal strength RC beams, with different materials and geometric properties, is sufficient, the shear failure probabilities calculated for high strength concrete beams have not been studied. In order to make a more reliable evaluation, the determination of failure probabilities for a greater number of beams with different material and geometric properties should be realized.

NOMENCLATURE

a/d	Ratio of the shear span to effective depth of beam
b_w	The web width (mm)
d	The effective depth (mm)
f_c	Compressive strength of concrete (MPa)
$f_X(x)$	Probability density function,
f_y	Yield strength of longitudinal reinforcement (MPa)
f_{yw}	Yield strength of stirrup reinforcement (MPa)
$g(X)$	Performance function
m	Mean
m_{Xi}^N	Mean value of equivalent normal distribution
p_F	Failure probability
p_S	Survival probability
R	Capacity
S	Demand
s	The standard normal variable
V	Coefficient of variation
v_c	The shear strength of concrete
v_{cr}	Cracking shear strength of beam
v_n	Nominal shear strength
v_s	The shear strength of stirrup
X_i	Random variable
X'	Reduced variates
$\mathbf{x}^*$	Most probable failure point

Greek symbols

α	Sensitivity coefficient
β	Reliability index
ϕ	Strength reduction factor
λ_R, λ_S	Capacity, demand. Mean value of lognormal distribution
ρ	Longitudinal reinforcement ratio (%)
ρ_w	Stirrup ratio (%)
σ	Standard deviation
σ_R, σ_S	Capacity, demand. Standard deviation of normal distribution
σ_{Xi}^N	Standard deviation of equivalent normal distribution
ζ_R, ζ_S	Capacity, demand. Standard deviation of lognormal distribution

Subscripts

c	Refers to concrete
cr	Refers to crack
n	Refers to nominal
s	Refers to steel
w	Refers to web
y	Refers to yield

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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