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Research Article

IInsight of boundary layer structure with heat transfer through a diverging porous channel in Darcy-Forchheimer porous material with suction/injection: A study of separation control

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ABSTRACT

Separation control and formation of boundary layer Newtonian flow in a diverging permeable channel in Darcy-Forchheimer porous material having suction/injection are discussed. Self-similar equations from governing equations are acquired and existence conditions for boundary layer structure are derived using nature of velocity gradient inside boundary layer. It reveals that if sum of Darcy permeability parameter and twice of Forchheimer parameter exceeds 2, then the boundary layer flow always exists with all type of mass suction/injection and even without suction/injection. Also, if mass suction parameter goes beyond $2\sqrt{2}$, then there is no matter what are the values of Darcy permeability parameter and Forchheimer parameter, a boundary layer exists inside the divergent channel. In addition, obtained numerical solutions are exhibited graphically. It reveals that thicknesses of velocity and thermal boundary layers reduce with Darcy and non-Darcy resistances of porous medium and fluid temperature also diminishes. The velocity and temperature reduce with increment of mass suction and contrary results are found for mass injection.

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INTRODUCTION

Formation of Newtonian boundary layer flow in divergent channels is not a normal phenomenon and in contrary, flow separation is an expected outcome. Boundary layer formation near the channel walls may be possible under certain conditions, which should have the capability to stop the back flow construction. Pioneering works on converging/diverging channels were discussed by Jeffery [1] and Hamel [2], where channel walls are taken as stationary

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and in the intersection of two walls a source/sink of fluid mass is causing steady flow. While stability of divergent flow under same small disturbance was discussed by Dean [3]. Rosenhead [4] reported solutions for two-dimensional radial converging/diverging flow in form of elliptic function. Wang and Price [5] demonstrated creeping flow of inside convergent channel ignoring the pressure gradient. James and Saringer [6] studied flow of non-Newtonian polyethylene oxide solution through a converging channel. Bariş [7] investigated second-grade visco-elastic converging flow. Magyari [8] explained backward boundary layer in convergent channel flow with power-law temperature of channel walls. Maranzoni et al. [9] discussed lateral outflow over side weir in converging channel experimentally as well as numerically.

If Reynolds number goes beyond certain fixed value, then boundary layer divergent flow in a channel has no similarity solution and there should occur regions of back-flow. There was a belief that for large Reynolds numbers, divergent channel flow is not separable into boundary layer and free-stream similar to convergent flow. But, Holstein [10] clarified that if substantial suction is implemented in diverging flow with large Reynolds number then only boundary layer is possible. Gersten and Körner [11] reported some characteristics of heat transfer for Holstein[10] problem. Eagles [12] discussed stability of many Jeffery-Hamel solutions using quasi-parallel approximation. Kamel [13] investigated the diverging micropolar flow. Drazin [14] explained the flow instability through a diverging channel with bifurcation theory. Sadeghy et al. [15] performed a theoretical study for exploring influence of implemented magnetic field in regulating flow separation inside Jeffrey-Hamel flow for viscoelastic fluids. Esmaeilpour and Ganji [16] obtained the Jeffery-Hamel flow solution in rigid flat walls with an angle 2α by OHAM. Bhattacharyya and Layek [17] discussed the possibility of MHD boundary layer diverging flow of dilatant power-law fluid. Sheikholeslami et al. [18] described Jeffery-Hamel MHD flow of electrically conducting nanofluid using ADM. Layek et al. [19] conferred the steady-state MHD flow in a divergent channel and they explored the possibility of MHD flow with any suction/blowing when magnetic field strength is chosen suitably. Gerdroodbary et al. [20] described effects of radiation on MHD Jeffery-Hamel flow with stretchable/shrinkable channel walls. Gepner and Floryan [21] revealed various aspects of mixing in converging–diverging channel flow depending the geometry of the channel. Some important characteristics of divergent/convergent flows may also be found existing in literature [22-24]. Darcy and non-Darcy porous media are important physical aspects in fluid flow having several applications in engineering and industries processes. Recently, Hoseinzadeh et al. [25-28], Ashrafi et al. [29], Ahmadi et al. [30] and Xu et al. [31] considered flow through porous medium in their study. Jabbary et al. [32] presented a 3D numerical procedure to solve the governing equation based on finite volume method, whereas numerical investigation of rectangular thermal energy storage units with multiple phase change materials was done by Hoseinzadeh et al. [33]. A dual-phase-lag (DPL) transient non-Fourier heat transfer analysis of functional graded cylindrical material under axial heat flux was done by Ghasemia et al. [34]. Some important studies related to porous medium and divergent structures may be found in literature[35-39].

The existence of boundary layer in divergent channel flow by suppressing separation phenomenon, i.e., the back flow is an important aspect and also the consideration of non-Darcian porous medium in the flow field. So, we are very much motivated to the investigation of the boundary layer structure inside diverging porous channel and heat transfer analysis in Darcy-Forchheimer porous material having suction/injection. In this problem, we investigate the impact of resistance (in the form of frictional forces) offered by non-Darcy porous medium on velocity and temperature fields along with existence of boundary layer flow inside divergent channels in presence of suction/injection. In the investigation, the roles of two types of resistive forces in the form Darcy permeability related force and Forchheimer resistance force in prevention of the flow separation, i.e., the role of forces in the existence of boundary layer solution have been explored. This is the main objective and novelty of the present analysis. Self-similar nonlinear ODEs(ordinary differential equations) are achieved and several existence conditions for boundary layer by controlling flow separation and back flow are achieved. Also, the converted self-similar ODEs are numerically solved. The obtained findings are expressed through figures to understand controls of parameters involved on flow and heat transfer properties.

FLOW CONSTRUCTION

Consider steady state boundary layer 2D flow of a viscous incompressible Newtonian fluid inside diverging channel in Darcy-Forchheimer porous material. Fixed channel walls are taken as permeable and through those channel walls suction/injection is assumed. Here, flow near one channel wall is discussed, because identical situation will be found near other channel wall. The x- and y-axes are considered along and perpendicular to channel wall and intersection of those two walls is taken as origin. Boundary layer equations for flow inside non-Darcy porous material are written as [19,35]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \upsilon\frac{\partial^2 u}{\partial y^2} - \frac{\upsilon}{k}\varphi u - c\varphi u^2 , \tag{2}$$

$$\text{and } 0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} , \tag{3}$$

where u, v are velocity components in x-, y-directions, respectively, ρ is constant fluid density, $k(= k_0 x^2)$ is variable permeability of porous material, φ is porosity, $c(= c_0/x)$ is variable Forchheimer inertia coefficient in second-order resistance. Using equation (3), it may be confirmed that p is function of x only.

Corresponding boundary conditions are [19]

$$\left.\begin{array}{l} u=0,\ v=-v_w(x) \text{ at } y=0, \\ u\to U(x) \text{ for } x>0 \text{ as } y\to\infty. \end{array}\right\} \tag{4}$$

The central-line velocity U is specified by[19]:

$$U=\frac{Q}{\alpha x}=\frac{U_0 L}{x}\left(U_0>0\right), \tag{5}$$

where α is small angle of channel, Q is positive constant, U_0 is the characteristic velocity outside the boundary layer when $x = L$ and L is characteristic length along the direction of motion ($Q/\alpha = U_0 L$).

The velocity of suction/injection $v_w(x)$ through the porous channels is:

$$v_w(x)=\frac{S\sqrt{Q\upsilon}}{x\sqrt{\alpha}}, \tag{6}$$

where S denotes suction/injection parameter (with S>0 gives suction and S<0 provides injection, respectively). A graphical plot of flow is presented in Figure 1.

Since from (3), we get fluid pressure as function of x variable only, the gradient $\partial p/\partial x$ is acquired from central-line condition (4) as:

$$-\frac{1}{\rho}\frac{\partial p}{\partial x}=U\frac{dU}{dx}+\frac{\upsilon}{k}\varphi U+c\varphi U^2. \tag{7}$$

Using (7), we have from (2)

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{dU}{dx}+\upsilon\frac{\partial^2 u}{\partial y^2}+\frac{\upsilon}{k}\varphi(U-u)+c\varphi\left(U^2-u^2\right). \tag{8}$$

The above equations (1) and (8), admits following similarity solution [19]:

$$u=\frac{Q}{\alpha x}f(\eta) \text{ and } v=\sqrt{\frac{Q\upsilon}{\alpha}}\left[\eta f(\eta)-S\right]\Big/x, \tag{9}$$

where similarity variable η is defined as $\eta = y\sqrt{\frac{U}{\upsilon x}}$.

The equation of continuity (1) is identically satisfied and the equation (8) takes following form:

$$f''-(1-f^2)+K(1-f)+Sf'+\lambda(1-f^2)=0, \tag{10}$$

where $K=\upsilon\alpha\varphi/(k_0 Q)$ is the Darcy permeability parameter and $\lambda = c_0\varphi$ is Forchheimer parameter.

The conditions in (4) become

$$f(0)=0,\ \ f(\infty)=1. \tag{11}$$

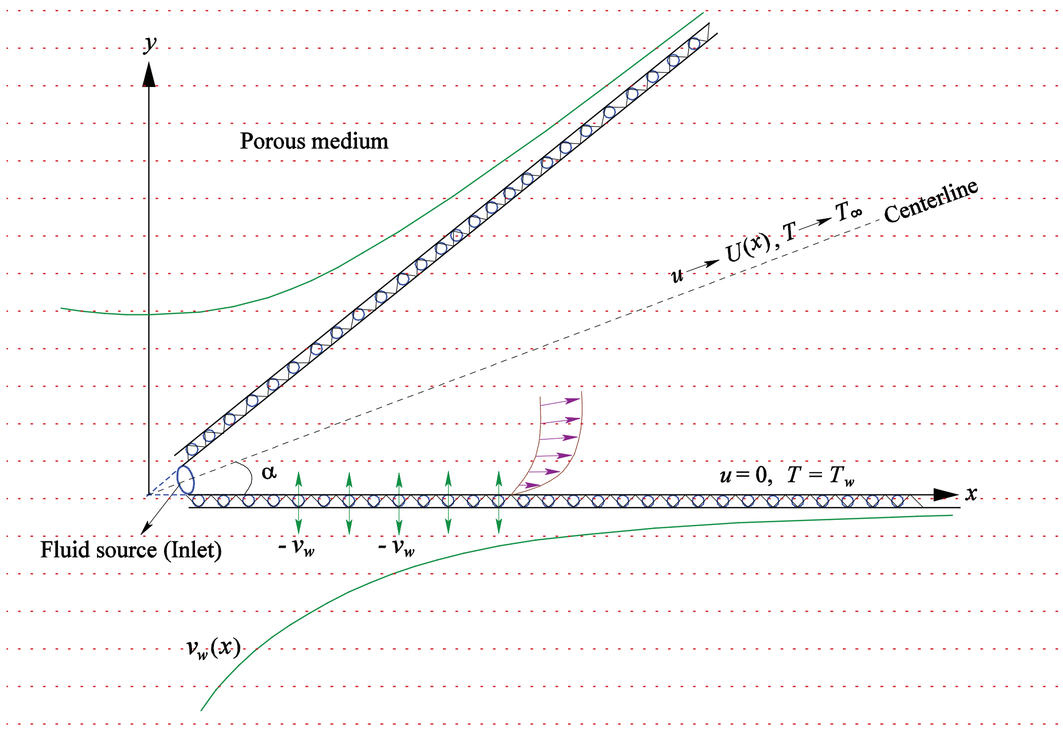

Figure 1. A sketch of divergent channel with flow region and other details.

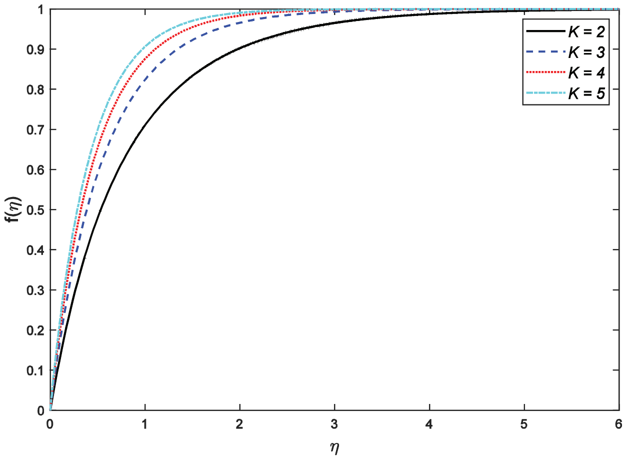

Figure 2. Velocity $f(\eta)$ for various values of K with $\lambda = 0$ and $S = 1$.

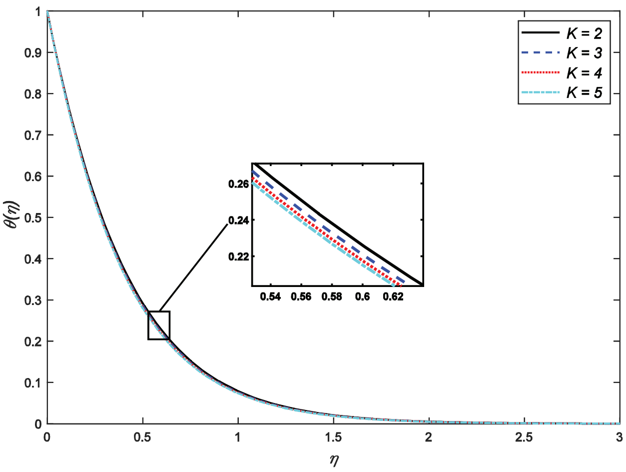

Figure 3. Temperature $\theta(\eta)$ for various values of K with $\lambda = 0$, $S = 1$, Pr = 2 and $m = 1$.

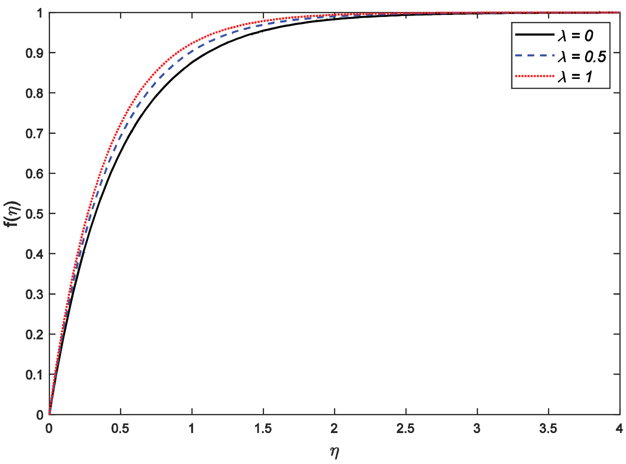

Figure 4. Velocity $f(\eta)$ for various values of λ with $K = 4$ and $S = 1$.

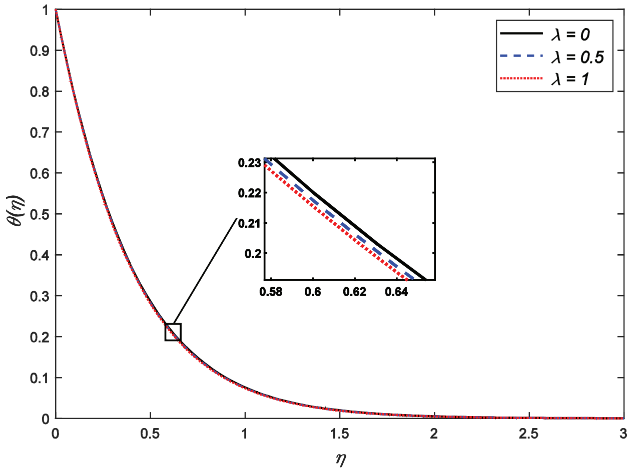

Figure 5. Temperature $\theta(\eta)$ for various values of λ with $K = 4$, $S = 1$, Pr = 2 and $m = 1$.

CONDITIONS OF SEPARATION CONTROL AND EXISTENCE OF BOUNDARY LAYER

Now, the situations under which the separation can be controlled by preventing back flow and boundary layer is maintained inside the divergent channel fixed in Darcy-Forchheimer porous material, will be explored. In above flow region, the quantity $f'(\eta)$ is linearly related to gradient of velocity, $\partial u/\partial y$. Across boundary layer region, $\partial u/\partial y$ slowly declines and goes to zero value at boundary layer termination-point, hence it corresponds that $\Phi[= f'(\eta)]$ is a decreasing type function of the variable η, so $\Phi'(\eta)<0$ in region of boundary layer and $\Phi \to 0$ as $\eta \to \infty$. Now, as η goes from 0 value to ∞, $f(\eta)$ rises from 0 (at wall) to 1 (at boundary layer termination-point) so that $f'(\eta)>0$. So, we may say that $\partial\Phi/\partial f = (\partial\Phi/\partial\eta)/(\partial f/\partial\eta)$ has negative value in region of boundary layer and $\Phi \to 0$ as $f \to 1$.

Equation (10) may be stated using the notation $f'(\eta) = \Phi$ and $f''(\eta) = \Phi(\partial\Phi/\partial f)$ as:

$$\frac{d\Phi}{df} = \frac{(1-\lambda)(1-f^2) - S\Phi - K(1-f)}{\Phi} \tag{12}$$

Taking limit as $f \to 1$ and also using $\Phi \to 0$ as $f \to 1$, the above equation (12) converts to

$$\lim_{f \to 1} \frac{d\Phi}{df} = \lim_{f \to 1} \frac{(1-\lambda)(1-f^2) - S\Phi - K(1-f)}{\Phi}.$$

As the limit in right hand side is taking indeterminate form, we can write

$$\lim_{f\to 1}\frac{d\Phi}{df}=\lim_{f\to 1}\frac{-2f(1-\lambda)-S\frac{d\Phi}{df}+K}{\frac{d\Phi}{df}}\Rightarrow$$

$$\lim_{f\to 1}\frac{d\Phi}{df}=\frac{-2(1-\lambda)-S\lim_{f\to 1}\frac{d\Phi}{df}+K}{\lim_{f\to 1}\frac{d\Phi}{df}}.$$

Denoting $(\partial\Phi/\partial f)_{f\to 1}=\Upsilon$, we have

$$\Upsilon^2+S\Upsilon+2(1-\lambda)-K=0, \tag{13}$$

which gives the value of Υ as

$$\Upsilon=\frac{1}{2}\left\{-S\pm\sqrt{S^2+4(K+2\lambda-2)}\right\} \tag{14}$$

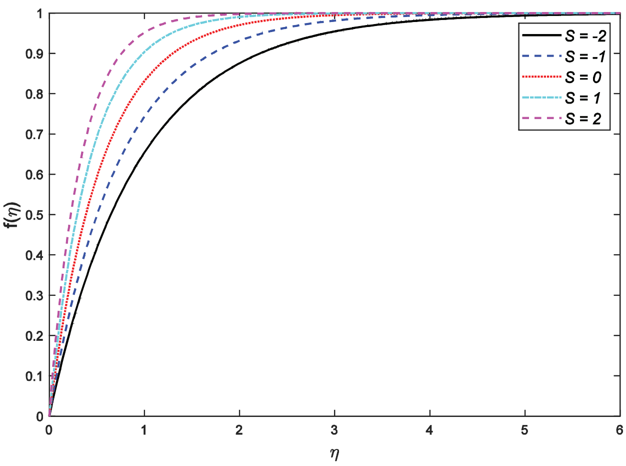

Figure 6. Velocity $f(\eta)$ for various values of S with K = 4 and λ = 0.5.

For existence of self-similar flow in boundary layer, the value of Υ must be negative in the region of boundary layer. Now it is clear that if $K + 2\Upsilon > 2$, then one value of Υ is definitely negative for any value of S (i.e., for suction, S>0 or injection, S<0 or without suction/injection, S=0), therefore boundary layer will exist and separation will be controlled. Next if $K + 2\Upsilon < 2$, then there will be two possibilities (i) $S\geq 2\sqrt{2}$ and (ii) $0<S<2\sqrt{2}$. For first possibility, the sum $K + 2\Upsilon$ can be attended any non-negative value less than 2 and in this occasion both values of Υ are negative; it confirms boundary layer structure. So, without any porous medium (K = 0 and λ = 0) boundary layer flow is possible only when $S\geq 2\sqrt{2}$, which is exactly same as study of Holstein[8]. For second possibility, the sum $K + 2\Upsilon$ can take any value less than 2 and greater than or equal to $2-(S^2/4)$and this case also two values of Υ are negative; so, boundary layer separation not occur. Lastly, $K + 2\Upsilon = 2$, then from (14) it is obvious that the values of Υ are 0 and $-2S$ and now if S>0 (only mass suction) then there exist one negative value of Υ; it guaranteed boundary layer existence.

HEAT TRANSFER

The energy analysis with variable wall temperature for the above diverging flow in porous material may be represented by the following boundary layer energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{15}$$

where T is temperature, c_p is specific heat and k is thermal conductivity.

Corresponding boundary conditions are

$$T=T_w=T_\infty+T_0x^m \text{ at } y=0;\ T\to T_\infty \text{ as } y\to\infty, \tag{16}$$

where T_w is variable channel wall temperature with T_0 being a constant, T_∞ is static temperature in free stream and m is power-law exponent.

Following dimensionless temperature function is introduced:

$$T=T_\infty+(T_w-T_\infty)\theta(\eta). \tag{17}$$

Using relations in (9) and (17), the equation (15) reduces to

$$\theta''+\Pr(S\theta'-mf\theta)=0, \tag{18}$$

where $\Pr=\dfrac{\upsilon\rho c_p}{\kappa}$ is well-known Prandtl number.

The boundary conditions in (16) become

$$\theta(0)=1,\ \ \theta(\infty)=0. \tag{19}$$

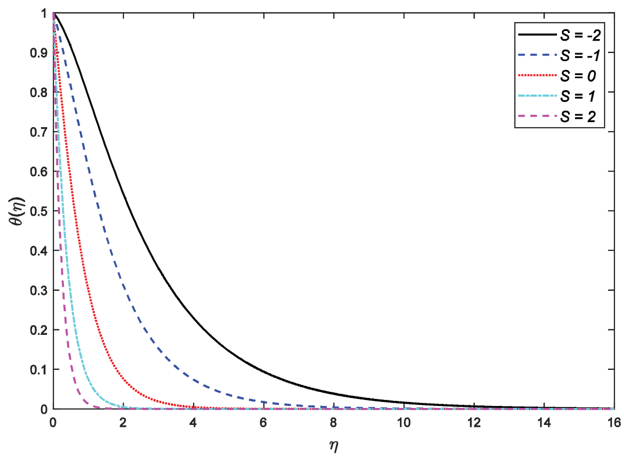

Figure 7. Temperature $\theta(\eta)$ for various values of S with K = 4, λ = 0.5, Pr = 2 and m = 1.

RESULTS AND DISCUSSION

Equations (10) and (18) are non-linear coupled second order ODEs. To get solutions of (10) and (18) along with boundary conditions (11) and (19), MATLAB "bvp4c" package is used. During the solution by MATLAB "bvp4c" package, the equations are transformed to IVP (initial value problem) of a 1st order system since MATLAB bvp4c package solves only 1st order IVPs and the system of ODEs is given by:

$$f' = p, \;\; p' = (1-\lambda)(1-f^2) - K(1-f) - Sp \tag{20}$$

$$\theta' = \delta, \; \delta = \Pr(mf\theta - S\delta) \tag{21}$$

with

$$f(0) = 0 \text{ and } \theta(0) = 1. \tag{22}$$

Bvp4c based on finite-difference method is a scheme with 4th order accuracy and confer C^1-continuous solution with 10^{-5} is taken as tolerance level. For continuous solution, the error control is based on residual error. Adequate preliminary guess values of unspecified initial conditions are supposed to initiate the numerical integration. The guesses are chosen in such a manner that the obtained solutions should satisfy all the boundary conditions of the problem and convergent asymptotically to free stream boundary condition. Graphical presentations of computed solutions for various parameters are prepared.

The effects of 1st and 2nd order resistance for porous medium on velocity and temperature are important aspects in practical application purpose and those are depicted in Figures 2-5. Figures confirm that due to both Darcy and non-Darcy resistive forces thicknesses of velocity and thermal boundary layers reduce and these phenomena are as anticipated because porous medium produces resistance to the transport processes. Though these forces reduce boundary layer thicknesses, but flow separation and vortex formation with back flow are deferred due to these resistance forces of porous medium, which approves existence of boundary layer flow. In addition, fluid temperature also diminishes with both types of resistance due to porous medium. Physically, augmenting Darcy permeability parameter resistive frictional forces augmented which diminish the boundary layer thickness, while augmenting non-Darcy Forchheimer parameter inertia factor (which relates to 2nd resistive force) the boundary layer shrinks. Also, when frictional forces due to both Darcy and non-Darcy resistances are strong enough to confine the generated vorticity, then boundary layer flow exists without any condition on applied suction/injection, i.e., even without any suction/injection. Also, for the increment of mass suction the velocity of the fluid augments and hence corresponding boundary layer shrinks (Figure 6), whereas temperature and corresponding boundary layer both reduces (Figure 7). Contrary outcomes are witnessed for applied mass injection. Mass suction through porous channel controls the existence of boundary layer flow by delaying the occurrence of back flow. Practically, when suction applied then fluid mass is taken away from the surface, whereas opposite happens with injection. As usual, due to rises of Prandtl number and power law exponent produces reductions of thermal boundary layer and temperature of the flow field (Figures 8 & 9). Practically, thermal diffusion rate diminishes with higher Prandtl number, whereas increase in power law exponent the convection processes inside the fluid declines and it is leading to decrease in the temperature gradient.

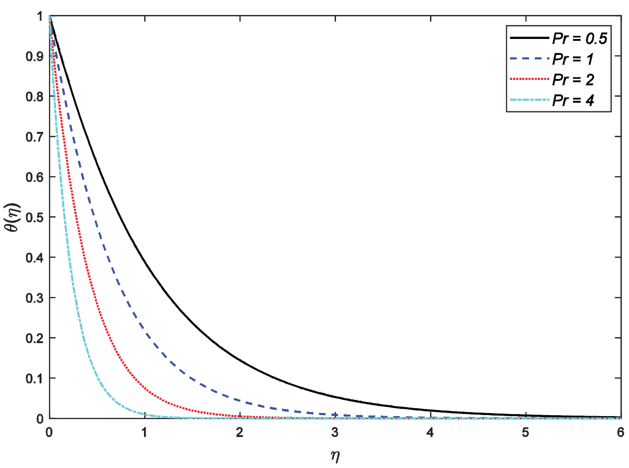

Figure 8. Temperature $\theta(\eta)$ for various values of Pr with $K = 4$, $\lambda = 0.5$, $S = 1$ and $m = 1$.

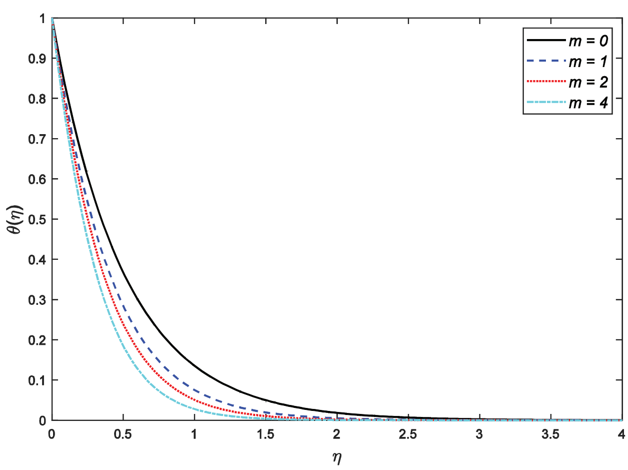

Figure 9. Temperature $\theta(\eta)$ for various values of m with $K = 4$, $\lambda = 0.5$, $S = 1$ and Pr = 2.

CONCLUSIONS

The effects of resistance for non-Darcy porous medium on the boundary layer existence in divergent channel flow and in addition the impacts on velocity and temperature are explored in this investigation. Important obtained features of the problem are summarized below:

(a) The self-similar flow in boundary layer controlling the separation is possible when $K + 2\Upsilon > 2$ for any value of S.
(b) If frictional forces due to both Darcy and non-Darcy resistances are stronger enough to confine the generated vorticity, then boundary layer flow exists without any condition on suction/injection and even for without any suction/injection.
(c) If $K + 2\Upsilon = 2$, then boundary layer exists if $S > 0$, i.e., only for suction cases.
(d) Fluid temperature weakens with both types of resistive forces due to non-Darcy porous medium.
(e) For Darcy and non-Darcy resistance, thicknesses of velocity and thermal boundary layers reduce.
(f) Velocity augments and temperature diminishes with suction increment and for injection outcomes are contrary.
(g) Growths of Prandtl number and power law exponent causes drop of thermal boundary layer thickness.

NOMENCLATURE

c	variable Forchheimer inertia coefficient
c_o	a constant
$f(\eta)$	dimensionless velocity
k	variable permeability
k_o	a constant
K	permeability parameter
L	characteristic length
m	power-law exponent
p	pressure
Pr	Prandtl number
Q	volume flow rate
S	suction/injection parameter
T	temperature
T_0	being a constant
T_w	variable channel wall temperature
T_∞	static temperature in free stream
u, v	velocity components along x- and y-axes
U	free stream velocity
U_o	characteristic velocity
v_w	suction/injection velocity
x, y	cartesian coordinate measured along the channel wall and normal to it, respectively

Greek symbols

α	angle of the channel
κ	thermal conductivity
μ	dynamic viscosity
υ	kinematic viscosity
λ	Forchheimer parameter
η	similarity variable
ρ	density
c_p	specific heat
φ	porosity
θ	dimensionless temperature

Subscripts

nf	nanofluid
f	base fluid
s	solid particle

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Jeffery GB. L. The two-dimensional steady motion of a viscous fluid. Lond Edinb Dublin Philos Mag J Sci 1915;29:455-465. [CrossRef]
[2] Hamal G. Spiralförmige Bewegungen zäher Flüssigkeiten. Jahresber Dtsch Math-Verein [Article in German] 1917;25:34-60.
[3] Dean WR. LXXII. Note on the divergent flow of fluid. Lond Edinb Dublin Philos Mag J Sci 1934;18:759-777. [CrossRef]
[4] Rosenhead L. The steady two-dimensional radial flow of viscous fluid between two inclined plane walls. Proc R Soc Lond A 1940;175:436-467. [CrossRef]
[5] Wang JW, Price GM. Laminar flow development and heat transfer in converging plane-walled channels. Appl Sci Res 1972;25:361-371. [CrossRef]
[6] James DF, Saringer JH. Flow of dilute polymer solutions through converging channels. J Non-Newton Fluid Mech 1982;11:317-339. [CrossRef]
[7] Barış S. Flow of a second-grade visco-elastic fluid in a porous converging channel. Turkish J Eng Env Sci 2003;27:73-81.

[8] Magyari E. Backward boundary layer heat transfer in a converging channel. Fluid Dyn Res 2007;39:493-504. [CrossRef]

[9] Maranzoni A, Pilotti M, Tomirotti M. Experimental and numerical analysis of side weir flows in a converging channel. J Hydraulic Eng 2017;143:04017009. [CrossRef]

[10] Holstein H. Ähnliche laminare Reibungsschichten an durchlässigen Wändern. ZWB-VM 1943.

[11] Gersten K, Körner H. Wärmeübergang unter Berücksichtigung der Reibungswärme bei laminaren Keilströmungen mit veränderlicher Temperatur und Normalgeschwindigkeit entlang der Wand. Int J Heat Mass Transf [Article in German] 1968;11:655-673. [CrossRef]

[12] Eagles PM. The stability of a family of Jeffery-Hamel solutions for divergent channel flow. J Fluid Mech 1966;24:191-207. [CrossRef]

[13] Kamel MT. Flow of a micropolar fluid in a diverging channel. Int J Eng Sci 1987;25:759-768. [CrossRef]

[14] Drazin PG. Flow through a diverging channel: instability and bifurcation. Fluid Dyn Res 1999;24:321-327. [CrossRef]

[15] Sadeghy K, Khabazi N, Taghavi SM. Magnetohydrodynamic (MHD) flows of viscoelastic fluids in converging/diverging channels. Int J Eng Sci 2007;45:923-938. [CrossRef]

[16] Esmaeilpour M, Ganji DD. Solution of the Jeffery-Hamel flow problem by optimal homotopy asymptotic method. Comput Math Appl 2010;59:3405-3411. [CrossRef]

[17] Bhattacharyya K, Layek GC. MHD boundary layer flow of dilatant fluid in a divergent channel with suction or blowing. Chin Phys Lett 2011;28:084705. [CrossRef]

[18] Sheikholeslami M, Ganji DD, Ashorynejad HR, Rokni HB. Analytical investigation of Jeffery-Hamel flow with high magnetic field and nanoparticle by Adomian decomposition method. Appl Math Mech 2012;33:25-36. [CrossRef]

[19] Layek GC, Kryzhevich SG, Gupta AS, Reza M. Steady magnetohydrodynamic flow in a diverging channel with suction or blowing. Z Angew Math Phys 2013;64:123-143. [CrossRef]

[20] Gerdroodbary MB, Takami MR, Ganji DD. Investigation of thermal radiation on traditional Jeffery-Hamel flow to stretchable convergent/divergent channels. Case Stud Ther Eng 2015;6:28-39. [CrossRef]

[21] Gepner SW, Floryan JM. Flow dynamics and enhanced mixing in a converging-diverging channel. J Fluid Mech 2016;807:167-204. [CrossRef]

[22] Khan U, Adnan, Ahmed N, Mohyud-Din ST. Soret and Dufour effects on Jeffery-Hamel flow of second-grade fluid between convergent/divergent channel with stretchable walls. Results Phys 2017;7:361-372. [CrossRef]

[23] Akinshilo AT, Ilegbusi A, Ali HM, Surajo A-J. Heat transfer analysis of nanofluid flow with porous medium through Jeffery Hamel diverging/converging channel. J Appl Comput Mech 2020;6:433-444.

[24] Liu P, Ho JY, Wong TN, Toh KC. Laminar film condensation inside and outside vertical diverging/converging small channels: A theoretical study. Int J Heat Mass Transf 2020;149:119193. [CrossRef]

[25] Hoseinzadeh S, Moafi A, Shirkhani A, Chamkha AJ. Numerical validation heat transfer of rectangular cross-section porous fins. J Thermophy Heat Transf 2019;33:698-704. [CrossRef]

[26] Hoseinzadeh S, Heyns PS, Chamkha AJ, Shirkhani A. Thermal analysis of porous fins enclosure with the comparison of analytical and numerical methods. J Therm Anal Calorim 2019;138:727-735. [CrossRef]

[27] Hoseinzadeh S, Sohani A, Ashrafi TG. An artificial intelligence-based prediction way to describe flowing a Newtonian liquid/gas on a permeable flat surface. J Therm Anal Calorim 2022;147:4403-4409. [CrossRef]

[28] Hoseinzadeh S, Sohani A, Shahverdian MH, Shirkhani A, Heyns S. Acquiring an analytical solution and performing a comparative sensitivity analysis for flowing Maxwell upper-convected fluid on a horizontal surface. Therm Sci Eng Prog 2021;23:100901. [CrossRef]

[29] Ashrafi TG, Hoseinzadeh S, Sohani A, Shahverdian MH. Applying homotopy perturbation method to provide an analytical solution for Newtonian fluid flow on a porous flat plate. Math Meth Appl Sci 2021;44:7017-7030. [CrossRef]

[30] Ahmadi N, Rezazadeh S, Dadvand A, Mirzaee I. Numerical investigation of the effect of gas diffusion layer with semicircular prominences on polymer exchange membrane fuel cell performance and species distribution. J Renew Energy Envior 2015;2:36-46. [CrossRef]

[31] Xu J, Qin H, Li H, Lei Z. Numerical simulation for hydrocarbon production analysis considering Pre-Darcy flow in fractured porous media. Eng Analy Bound Elem 2022;134:360-376. [CrossRef]

[32] Jabbary A, Arnesa SR, Samanipour H, Ahmadi N. Numerical investigation of 3D rhombus designed PEMFC on the cell performance. Int J Green Energy 2021;18:425-442. [CrossRef]

[33] Hoseinzadeh S, Ghasemiasl R, Havaei D, Chamkha AJ. Numerical investigation of rectangular thermal energy storage units with multiple phase change materials. J Mol Liq 2018;271:655-660. [CrossRef]

[34] Ghasemia MH, Hoseinzadeh S, Memon S. A dual-phase-lag (DPL) transient non-Fourier heat transfer analysis of functional graded cylindrical material under axial heat flux. Int Commun Heat Mass Transf 2022;131:105858. [CrossRef]

[35] Mahmoud MAA, Mahmoud MA-E, Waheed SE. Hydromagnetic boundary layer micropolar fluid flow over a stretching surface embedded in a non-Darcian porous medium with radiation. Math Prob Eng 2006;2006:39392. [CrossRef]
[36] Hatami M, Mosayebidorcheh S, Vatani M, Mosayebidorcheh T, Ganji DD. Differential transformation method for analysis of nonlinear flow and mass transfer through a channel filled with a porous medium. J Ther Eng 2020;6:24-40. [CrossRef]
[37] Paul A, Nath JM, Das TK. An investigation of the MHD Cu-Al2O3/H2O hybrid-nanofluid in a porous medium across a vertically stretching cylinder incorporating thermal stratification impact. J Ther Eng 2023;9:799-810. [CrossRef]
[38] Ebrahimi A, Roohi E. Flow and thermal fields investigation in divergent micro/nano channels. J Ther Eng 2016;2:709-714. [CrossRef]
[39] Xuyi Z, Fuqiang W, Xuhang S, Ziming C, Xiangtao G. Analysis of heat transfer performance of the absorber tube with convergent-divergent structure for parabolic trough collector. J Ther Eng 2021;7:1843-1856. [CrossRef]
[40] Verma AK, Bhattacharyya K, Rajput S, Mandal MS, Chamkha AJ, Yadav D. Buoyancy driven non-Newtonian Prandtl-Eyring nanofluid flow in Darcy-Forchheimer porous medium over inclined non-linear expanding sheet with double stratification. Waves Random Complex Media 2022. [CrossRef]
[41] Verma AK, Rajput S, Bhattacharyya K, Chamkha AJ. Nanoparticle's radius effect on unsteady mixed convective copper-water nanofluid flow over an expanding sheet in porous medium with boundary slip. Chem Eng J Adv 2022;12:100366. [CrossRef]