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Research Article

Fixed point theorem for neutrosophic extended metric-like spaces and their application

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ABSTRACT

In this manuscript, our objective is to introduce the notion of neutrosophic extended metric-like spaces. We establish some fixed point theorems in this setting. Neutrosophic extended metric-like spaces metric space uses the idea of continuous triangular norms and continuous triangular conorms in an extended intuitionistic fuzzy metric-like space. Triangular norms are used to generalize with the probability distribution of triangle inequality in metric space conditions. Triangular conorms are known as dual operations of triangular norms. Triangular norms and triangular conorm are very significant for fuzzy operation. The obtained results boost the approaches of existing ones in the literature and are supported by some examples and an application.

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INTRODUCTION

After being given the notion of fuzzy sets (FS) by Zadeh [12], many researchers provide abundant generalizations. In this continuation, Kramosil and Michalek [16] originated the approach of fuzzy metric spaces. George and Veeramani [8] initiated the approach of fuzzy metric spaces. Garbiec [9] gave the fuzzy interpretation of the Banach contraction principle in fuzzy metric spaces. For basic concepts, see [7, 10, 11, 14, 23]. Harandi [21] is credited with coining the term metric like spaces (MLS) which elegantly generalizes the idea of metric spaces. Shukla and Abbas [22] reformulated definition (MLS) in this context, resulting in a fuzzy metric-like spaces (FMLS).

Mehmood [13] originated the approach of a fuzzy extended b-metric space (FEBMS) by replacing coefficient $b \geq 1$ with a function $\alpha: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$. The approach of intuitionistic fuzzy metric spaces was introduced by Park

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in [1] and Konwar [4] who initiated an intuitionistic fuzzy b-metric space (IFBMS). Kirişci and Simsek [17] generalized the approach of intuitionistic fuzzy metric space by presenting the approach of a neutrosophic metric space (NMS). Simsek and Kirişci [19] and Sowndrarajan et al. [18] proved some fixed point results in the setting of an NMS.

In this manuscript, we use the concepts of fuzzy extended b-metric space, NMS, and FMLS to introduce the notion of a neutrosophic extended metric-like space (NEMLS). So the notion of NEMLS is a generalization of fuzzy extended b-metric space, FMLS, IFBMS, NMS, and other concepts in the existing literature. Also, some fixed point (FP) results with non-trivial examples and an application are provided. For related works, see [1, 2, 3, 5, 6, 15, 20, 24-34].

In the end, some notations which are important for the understanding of this manuscript are accommodated in the appendix section to avoid obscurity and vagueness.

In this article, CTM is used for a continuous triangular norm and CTCM for a continuous triangular co-norm.

Preliminares

In this section, some basic definitions are given that are helpful to understand the main results.

Definition 2.1 [13] A 4-tuple $(\mathfrak{B}, \Delta_\alpha, \circ, \alpha)$ is called a FEBMS if $\mathfrak{B}$ is a non-empty set,$\alpha: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$, $\circ$ is a CTM and Δ_α is an FS on $\mathfrak{B} \times \mathfrak{B} \times (0, \infty)$, meeting the below circumstances for all $\vartheta, \delta, \beta \in \mathfrak{B}$ and $\varsigma, \sigma > 0$

$(F1)$ $\Delta_\alpha(\vartheta, \delta, 0) = 0$;

$(F2)$ $\Delta_\alpha(\vartheta, \delta, \varsigma) = 1 \Leftrightarrow \vartheta = \delta$; $(F3)$ $\Delta_\alpha(\vartheta, \delta, \varsigma) = \Delta_\alpha(\delta, \vartheta, \varsigma)$;

$(F4)$ $\Delta_\alpha(\vartheta, \beta, \alpha(\vartheta, \beta)(\varsigma + \sigma)) \geq \Delta_\alpha(\vartheta, \delta, \varsigma) \circ \Delta_\alpha(\delta, \beta, \sigma)$; $(F5)$ $\Delta_\alpha(\vartheta, \delta, \cdot): (0, \infty) \to [0,1$ is continuous.

Example 2.2 [13] Let $\mathfrak{B} = \{1,2,3\}$ and define Δ_α by

$$\Delta_\alpha(\vartheta, \delta, \varsigma) = \begin{cases} \dfrac{\varsigma}{\varsigma + d(\vartheta, \delta)} & \text{if } \varsigma > 0, \\ 0 & \text{if } \varsigma = 0 \end{cases}$$

Take $d(\vartheta, \delta) = (\vartheta - \delta)^2$ and given $\alpha: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$ as $\alpha(\vartheta, \delta) = 1 + \vartheta + \delta$.

Also take the CTM: $a \circ b = a \wedge b = \min\{a, b\}$. Then $(\mathfrak{B}, \Delta_\alpha, \circ, \wedge)$ is an EFBMS.

Park introduced the concept of intuitionistic fuzzy metric spaces (IFMSs) and utilized this idea to investigate fixed point results. Park defined the notion of IFMSs as follows:

Definition 2.3 [1] Suppose $E \neq \emptyset$ is an arbitrary set, assume a five-tuple $(E, R, S, *, \Delta)$ where $*$ is a CTM, Δ is a CTCM, and R, S are FSs on $E \times E \times (0, \infty)$. If $(E, R, S, *, \Delta)$ meet the following circumstances for all $\beta, \delta, \partial \in E$ and $\pi, \lambda > 0$:

(B1) $R(\beta, \delta, \lambda) + S(\beta, \delta, \lambda) \leq 1$,

(B2) $R(\beta, \delta, \lambda) > 0$,

(B3) $R(\beta, \delta, \lambda) = 1 \Leftrightarrow \beta = \delta$,

(B4) $R(\beta, \delta, \lambda) = R(\delta, \beta, \lambda)$,

(B5) $R(\beta, \partial, (\lambda + \pi)) \geq R(\beta, \delta, \lambda) * R(\delta, \partial, \pi)$,

(B6) $R(\beta, \delta, \cdot)$ is non decreasing (ND) function of R^+ and $\lim_{\lambda \to \infty} R(\beta, \delta, \lambda) = 1$,

(B7) $S(\beta, \delta, \lambda) > 0$,

(B8) $S(\beta, \delta, \lambda) = 0 \Leftrightarrow \beta = \delta$,

(B9) $S(\beta, \delta, \lambda) = S(\delta, \beta, \lambda)$,

(B10) $S(\beta, \partial, (\lambda + \pi)) \leq S(\beta, \delta, \lambda) \Delta S(\delta, \partial, \pi)$,

(B11) $S(\beta, \delta, \cdot)$ is non increasing (NI) function of R^+ and $\lim_{\lambda \to \infty} S(\beta, \delta, \lambda) = 0$.

Then $(E, R, S, *, \Delta)$ is an IFMS.

The concept of neutrosophic metric spaces was discussed by Kirişci and Simsek in his work and he defined the said concept as follows:

Definition 2.4 [17] Suppose $\mathfrak{B} \neq \emptyset$. Given a six tuple $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ where $*$ is a CTM, $\diamond$ is a CTCM, M_φ, N_φ and O_φ NS on $\mathfrak{B} \times \mathfrak{B} \times (0, \infty)$. If $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ meets the below circumstances for all $\vartheta, \delta, \beta \in \mathfrak{B}$ and $\varsigma, \sigma > 0$:

(I) $M_\varphi(\vartheta, \delta, \varsigma) + N_\varphi(\vartheta, \delta, \varsigma) + O_\varphi(\vartheta, \delta, \varsigma) \leq 3$,

(II) $0 \leq M_\varphi(\vartheta, \delta, \varsigma) \leq 1$,

(III) $M_\varphi(\vartheta, \delta, \varsigma) = 1 \Leftrightarrow \vartheta = \delta$,

(IV) $M_\varphi(\vartheta, \delta, \varsigma) = M_\varphi(\delta, \vartheta, \varsigma)$,

(V) $M_\varphi(\vartheta, \beta(\varsigma + \sigma)) \geq M_\varphi(\vartheta, \delta, \varsigma) * M_\varphi(\delta, \beta, \sigma)$,

(VI) $M_\varphi(\vartheta, \delta, \cdot): [0, \infty) \to [0,1]$ is continuous,

(VII) $\lim_{\varsigma \to \infty} M_\varphi(\vartheta, \delta, \varsigma) = 1$,

(VIII) $0 \leq N_\varphi(\vartheta, \delta, \varsigma) \leq 1$,

(IX) $N_\varphi(\vartheta, \delta, \varsigma) = 0 \Leftrightarrow \vartheta = \delta$,

(X) $N_\varphi(\vartheta, \delta, \varsigma) = N_\varphi(\delta, \vartheta, \varsigma)$,

(XI) $N_\varphi(\vartheta, \beta, (\varsigma + \sigma)) \leq N_\varphi(\vartheta, \delta, \varsigma) \diamond N_\varphi(\delta, \beta, \sigma)$,

(XII) $N_\varphi(\vartheta, \delta, \cdot): [0, \infty) \to [0,1]$ is continuous,

(XIII) $\lim_{\varsigma \to \infty} N_\varphi(\vartheta, \delta, \varsigma) = 0$,

(XIV) $0 \leq O_\varphi(\vartheta, \delta, \varsigma) \leq 1$,

(XV) $O_\varphi(\vartheta, \delta, \varsigma) = 0 \Leftrightarrow \vartheta = \delta$,

(XVI) $O_\varphi(\vartheta, \delta, \varsigma) = O_\varphi(\delta, \vartheta, \varsigma)$,

(XVII) $O_\varphi(\vartheta, \beta, (\varsigma + \sigma)) \leq O_\varphi(\vartheta, \delta, \varsigma) \diamond O_\varphi(\delta, \beta, \sigma)$,

(XVIII) $O_\varphi(\vartheta, \delta, \cdot): [0, \infty) \to [0,1]$ is a continuous,

(XIX) $\lim_{\varsigma \to \infty} O_\varphi(\vartheta, \delta, \varsigma) = 0$,

(XX) if $\varsigma \leq 0$ then $M_\varphi(\vartheta, \delta, \varsigma) = 0, N_\varphi(\vartheta, \delta, \varsigma) = 1, O_\varphi(\vartheta, \delta, \varsigma) = 1$,

Then $(M_\varphi, N_\varphi, O_\varphi)$ is a neutrosophic metric on and $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ is an NMS. The functions $M_\varphi(\vartheta, \delta, \varsigma)$, $N_\varphi(\vartheta, \delta, \varsigma)$ and $O_\varphi(\vartheta, \delta, \varsigma)$ represent the degree of nearness, naturalness, and non-nearness between ϑ and δ for ς, respectively.

RESULTS AND DISCUSSION

Now, we introduce the notion of NEMLS and utilize this concept to investigate some fixed point results.

Definition 3.1 Suppose $\mathfrak{B} \neq \emptyset$. Given a six tuple $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ where $*$ is a CTM, $\diamond$ is a CTCM,$\varphi: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$, M_φ, N_φ and O_φ are NSs on $\mathfrak{B} \times \mathfrak{B} \times (0, \infty)$. If $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, *, \diamond)$ meets the below circumstances for all $\vartheta, \delta, \beta \in \mathfrak{B}$ and $\varsigma, \sigma > 0$:

(i) $M_\varphi(\vartheta,\delta,\varsigma)+N_\varphi(\vartheta,\delta,\varsigma)+O_\varphi(\vartheta,\delta,\varsigma)\le 3$,
(ii) $0\le M_\varphi(\vartheta,\delta,\varsigma)\le 1$,
(iii) $M_\varphi(\vartheta,\delta,\varsigma)=1$ implies $\vartheta=\delta$,
(iv) $M_\varphi(\vartheta,\delta,\varsigma)=M_\varphi(\delta,\vartheta,\varsigma)$,
(v) $M_\varphi\big(\vartheta,\beta,\varphi(\vartheta,\beta)(\varsigma+\sigma)\big)\ge M_\varphi(\vartheta,\delta,\varsigma)*M_\varphi(\delta,\beta,\sigma)$,
(vi) $M_\varphi(\vartheta,\delta,\cdot):[0,\infty)\to[0,1]$ is continuous,
(vii) $\lim_{\varsigma\to\infty}M_\varphi(\vartheta,\delta,\varsigma)=1$,
(viii) $0\le N_\varphi(\vartheta,\delta,\varsigma)\le 1$,
(ix) $N_\varphi(\vartheta,\delta,\varsigma)=0$ implies $\vartheta=\delta$,
(x) $N_\varphi(\vartheta,\delta,\varsigma)=N_\varphi(\delta,\vartheta,\varsigma)$,
(xi) $N_\varphi\big(\vartheta,\beta,\varphi(\vartheta,\beta)(\varsigma+\sigma)\big)\le N_\varphi(\vartheta,\delta,\varsigma)\diamond N_\varphi(\delta,\beta,\sigma)$,
(xii) $N_\varphi(\vartheta,\delta,\cdot):[0,\infty)\to[0,1]$ is continuous,
(xiii) $\lim_{\varsigma\to\infty}N_\varphi(\vartheta,\delta,\varsigma)=0$,
(xiv) $0\le O_\varphi(\vartheta,\delta,\varsigma)\le 1$,
(xv) $O_\varphi(\vartheta,\delta,\varsigma)=0$ implies $\vartheta=\delta$,
(xvi) $O_\varphi(\vartheta,\delta,\varsigma)=O_\varphi(\delta,\vartheta,\varsigma)$,
(xvii) $O_\varphi\big(\vartheta,\beta,\varphi(\vartheta,\beta)(\varsigma+\sigma)\big)\le O_\varphi(\vartheta,\delta,\varsigma)\diamond O_\varphi(\delta,\beta,\sigma)$,
(xviii) $O_\varphi(\vartheta,\delta,\cdot):[0,\infty)\to[0,1]$ is a continuous,
(xix) $\lim_{\varsigma\to\infty}O_\varphi(\vartheta,\delta,\varsigma)=0$,
(xx) if $\varsigma\le 0$ then $M_\varphi(\vartheta,\delta,\varsigma)=0, N_\varphi(\vartheta,\delta,\varsigma)=1, O_\varphi(\vartheta,\delta,\varsigma)=1$,

Then (M_φ, N_φ, O_φ) is an extended neutrosophic metric-like (ENML) on and ($\mathfrak{B}$, M_φ, N_φ, O_φ, *, ◇) is an NEMLS.

Example 3.2 Let $\mathfrak{B}$ = N. Define M_φ, N_φ, O_φ: $\mathfrak{B}\times\mathfrak{B}\times(0,\infty)\to[0,1]$ by

$$M_\varphi(\vartheta,\delta,\varsigma)=\frac{\varsigma}{\varsigma+\max\{\vartheta,\ \delta\}^2},\ N_\varphi(\vartheta,\delta,\varsigma)=\frac{\max\{\vartheta,\ \delta\}^2}{\varsigma+\max\{\vartheta,\ \delta\}^2}$$
$$O_\varphi(\vartheta,\delta,\varsigma)=\frac{\max\{\vartheta,\ \delta\}^2}{\varsigma}.$$

for all ϑ, $\delta\in\mathfrak{B}$ and $\varsigma>0$. Define the CTN by $a\circ b=a\cdot b$ and CTCN "◇" by $a\diamond b=\max\{a,b\}$ and define "φ" by

$$\varphi(\vartheta,\delta)=\begin{cases}1 & \text{if}\quad \vartheta=\delta,\\ \max\{\vartheta,\delta\} & if\ \vartheta\ne\delta\end{cases}$$

Then ($\mathfrak{B}$, M_φ, N_φ, O_φ,○,◇) be a NEMLS.

Proof: (i)–(iv), (vi) – (x), (xii) – (xvi) and (xviii) – (xx) are obvious. We shall prove (v), (xi) and (xix). We have $\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)[\max\{\vartheta,\delta\}^2+\max\{\delta,\beta\}^2]$.

Then
$$\varsigma\sigma\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)[(\varsigma\sigma+\sigma^2)\max\{\vartheta,\delta\}^2+(\varsigma\sigma+\varsigma^2)\max\{\delta,\beta\}^2].$$
That is
$$\varsigma\sigma\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)[(\varsigma+\sigma)\sigma\max\{\vartheta,\delta\}^2+(\varsigma+\sigma)\varsigma\max\{\delta,\beta\}^2].$$
Then
$$\varsigma\sigma\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)[(\varsigma+\sigma)\sigma\max\{\vartheta,\delta\}^2+(\varsigma+\sigma)\varsigma\max\{\delta,\beta\}^2+(\varsigma+\sigma)\max\{\vartheta,\delta\}^2\max\{\delta,\beta\}^2].$$
That is
$$\varsigma\sigma\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)(\varsigma+\sigma)[\sigma\max\{\vartheta,\delta\}^2+\varsigma\max\{\delta,\beta\}^2+\max\{\vartheta,\delta\}^2\max\{\delta,\beta\}^2].$$
Then
$$\varphi(\vartheta,\beta)(\varsigma+\sigma)\varsigma\sigma+\varsigma\sigma\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)(\varsigma+\sigma)\varsigma\sigma+\varphi(\vartheta,\beta)(\varsigma+\sigma)[\sigma\max\{\vartheta,\delta\}^2+\varsigma\max\{\delta,\beta\}^2+\max\{\vartheta,\delta\}^2\max\{\delta,\beta\}^2].$$
Then
$$\varsigma\sigma[\varphi(\vartheta,\beta)(\varsigma+\sigma)+\max\{\vartheta,\beta\}^2]\le\varphi(\vartheta,\beta)(\varsigma+\sigma)[\varsigma\sigma+\sigma\max\{\vartheta,\delta\}^2+\varsigma\max\{\delta,\beta\}^2+\max\{\vartheta,\delta\}^2\max\{\delta,\beta\}^2].$$
That is
$$\varsigma\sigma[\varphi(\vartheta,\beta)(\varsigma+\sigma)+\max\{\vartheta,\beta\}^2]\le\varphi(\vartheta,\beta)(\varsigma+\sigma)[\varsigma+\max\{\vartheta,\delta\}^2][\sigma+\max\{\delta,\beta\}^2].$$
Then
$$\frac{\varphi(\vartheta,\beta)(\varsigma+\sigma)}{\varphi(\vartheta,\beta)(\varsigma+\sigma)+\max\{\vartheta,\beta\}^2}\ge\frac{\varsigma\sigma}{[\varsigma+\max\{\vartheta,\delta\}^2][\sigma+\max\{\delta,\beta\}^2]}.$$
That is
$$\frac{\varphi(\vartheta,\beta)(\varsigma+\sigma)}{\varphi(\vartheta,\beta)(\varsigma+\sigma)+\max\{\vartheta,\beta\}^2}\ge\frac{\varsigma}{\varsigma+\max\{\vartheta,\delta\}^2}\cdot\frac{\sigma}{\sigma+\max\{\delta,\beta\}^2}.$$
Hence
$$\Rightarrow M_\varphi\big(\vartheta,\beta,\varphi(\vartheta,\beta)(\varsigma+\sigma)\big)\ge M_\varphi(\vartheta,\delta,\varsigma)\circ M_\varphi(\delta,\beta,\sigma).$$
(v) is satisfied.
$$\max\{\vartheta,\beta\}^2=\max\{\vartheta,\beta\}^2\max\{1,1\}$$
Then
$$\max\{\vartheta,\beta\}^2=\max\{\vartheta,\beta\}^2\max\left\{\frac{\max\{\vartheta,\delta\}^2}{\max\{\vartheta,\delta\}^2},\frac{\max\{\delta,\beta\}^2}{\max\{\delta,\beta\}^2}\right\}.$$
That is
$$\max\{\vartheta,\beta\}^2\le[\varphi(\vartheta,\beta)(\varsigma+\sigma)+\max\{\vartheta,\beta\}^2]\max\left\{\frac{\max\{\vartheta,\delta\}^2}{\varsigma+\max\{\vartheta,\delta\}^2},\frac{\max\{\delta,\beta\}^2}{\sigma+\max\{\delta,\beta\}^2}\right\}.$$
Then
$$\frac{\max\{\vartheta,\beta\}^2}{\varphi(\vartheta,\beta)(\varsigma+\sigma)+\max\{\vartheta,\beta\}^2}\le\max\left\{\frac{\max\{\vartheta,\delta\}^2}{\varsigma+\max\{\vartheta,\delta\}^2},\frac{\max\{\delta,\beta\}^2}{\sigma+\max\{\delta,\beta\}^2}\right\}.$$
Hence
$$N_\varphi\big(\vartheta,\beta,\varphi(\vartheta,\beta)(\varsigma+\sigma)\big)\le N_\varphi(\vartheta,\delta,\varsigma).$$
(xi) is satisfied.
$$\max\{\vartheta,\beta\}^2\le\varphi(\vartheta,\beta)\max\{\max\{\vartheta,\delta\}^2,\max\{\delta,\beta\}^2\}$$
Then
$$\frac{\max\{\vartheta,\beta\}^2}{\varsigma+\sigma}\le\varphi(\vartheta,\beta)\max\left\{\frac{\max\{\vartheta,\delta\}^2}{\varsigma},\frac{\max\{\delta,\beta\}^2}{\sigma}\right\}.$$
That is
$$\frac{\max\{\vartheta,\beta\}^2}{\varphi(\vartheta,\beta)(\varsigma+\sigma)}\le\max\left\{\frac{\max\{\vartheta,\delta\}^2}{\varsigma},\frac{\max\{\delta,\beta\}^2}{\sigma}\right\}.$$
Hence
$$\Rightarrow O_\varphi\big(\vartheta,\beta,\varphi(\vartheta,\beta)(\varsigma+\sigma)\big)\le O_\varphi(\vartheta,\delta,\varsigma)\diamond O_\varphi(\delta,\beta,\sigma).$$
(xvii) is satisfied.

Remark 3.3 The above example is also satisfied for $a\circ b=\min\{a,b\}$ and $a\diamond b=\max\{a,b\}$.

Definition 3.4 Let ($\mathfrak{B}$, M_φ, N_φ, O_φ,○,◇) be a NEMLS and $\{\vartheta_n\}$ be a sequence in $\mathfrak{B}$.then $\{\vartheta_n\}$ is named to be:

(i) a convergent, if there exists $\vartheta\in\mathfrak{B}$ such that
$$\lim_{n\to\infty}M_\varphi(\vartheta_n,\vartheta,\varsigma)=M_\varphi(\vartheta,\vartheta,\varsigma),\ \lim_{n\to\infty}N_\varphi(\vartheta_n,\vartheta,\varsigma)=N_\varphi(\vartheta,\vartheta,\varsigma)$$
and $\lim_{n\to\infty}O_\varphi(\vartheta_n,\vartheta,\varsigma)=O_\varphi(\vartheta,\vartheta,\varsigma)$, for all $\varsigma>0$

(ii) a Cauchy sequence (CS), if and only if for each $\varsigma>0$, there exists $n_0\in$ N such that
$$\lim_{n\to\infty}M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma),\ \lim_{n\to\infty}N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma)\ \text{and}\ \lim_{n\to\infty}O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma)$$
exists and finite.

(iii) If every Cauchy sequence convergent in $\mathfrak{B}$, then ($\mathfrak{B}$, M_φ, N_φ, O_φ,○,◇) is called complete NEMLS.
$$\lim_{n\to\infty}M_\varphi\big(\vartheta_n,\vartheta_{n+q},\varsigma\big)=\lim_{n\to\infty}M_\varphi(\vartheta_n,\vartheta,\varsigma)=M_\varphi(\vartheta,\vartheta,\varsigma),$$
$$\lim_{n\to\infty}N_\varphi\big(\vartheta_n,\vartheta_{n+q},\varsigma\big)=\lim_{n\to\infty}N_\varphi(\vartheta_n,\vartheta,\varsigma)=N_\varphi(\vartheta,\vartheta,\varsigma),$$
$$\lim_{n\to\infty}O_\varphi\big(\vartheta_n,\vartheta_{n+q},\varsigma\big)=\lim_{n\to\infty}O_\varphi(\vartheta_n,\vartheta,\varsigma)=O_\varphi(\vartheta,\vartheta,\varsigma).$$

At this time, we shall prove extended Neutrosophic like Banach contraction results.

Theorem 3.5 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamondsuit)$ be a complete NEMLS in the company of $\varphi: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$ and suppose that

$$\lim_{\varsigma\to\infty} M_\varphi(\vartheta,\delta,\varsigma) = 1\,, \lim_{\varsigma\to\infty} N_\varphi(\vartheta,\delta,\varsigma) = 0 \text{ and } \lim_{\varsigma\to\infty} O_\varphi(\vartheta,\delta,\varsigma) = 0 \qquad (1)$$

for all ϑ, $\delta \in \mathfrak{B}$ and $\varsigma > 0$. Let $f: \mathfrak{B} \to \mathfrak{B}$ be a mapping satisfying

$$M_\varphi(f\vartheta, f\delta, k\varsigma) \geq M_\varphi(\vartheta,\delta,\varsigma),\ \ N_\varphi(f\vartheta, f\delta, k\varsigma) \leq N_\varphi(\vartheta,\delta,\varsigma), \\ O_\varphi(f\vartheta, f\delta, k\varsigma) \leq O_\varphi(\vartheta,\delta,\varsigma) \qquad (2)$$

for all $\vartheta, \delta \in \mathfrak{B}$, $0 < k < 1$ and $\varsigma > 0$. Further, suppose that for an arbitrary $\vartheta 0 \in \mathfrak{B}$, and $n, q \in$ N, we have

$$\varphi(\vartheta_n, \vartheta_{n+q}) < \frac{1}{k},$$

where $\vartheta_n = f^n\vartheta_0 = f\vartheta_{n-1}$. Then f has a unique FP.

Proof: Let ϑ_0 be a random element of $\mathfrak{B}$ and take $\vartheta_n = f^n\vartheta_0 = f\mathcal{a}_{n-1}$, $n \in$ N. By using (2) for all $\varsigma > 0$, we have

$$M_\varphi(\vartheta_n,\vartheta_{n+1},k\varsigma) = M_\varphi(f\vartheta_{n-1}, f\vartheta_n, k\varsigma) \geq M_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma) \geq M_\varphi\left(\vartheta_{n-2},\vartheta_{n-1},\frac{\varsigma}{k}\right) \\ \geq M_\varphi\left(\vartheta_{n-3},\vartheta_{n-2},\frac{\varsigma}{k^2}\right) \geq \cdots \geq M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{k^n}\right), \\ N_\varphi(\vartheta_n,\vartheta_{n+1},k\varsigma) = N_\varphi(f\vartheta_{n-1}, f\vartheta_n, k\varsigma) \leq N_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma) \leq N_\varphi\left(\vartheta_{n-2},\vartheta_{n-1},\frac{\varsigma}{k}\right) \\ \leq N_\varphi\left(\vartheta_{n-3},\vartheta_{n-2},\frac{\varsigma}{k^2}\right) \leq \cdots \leq N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{k^n}\right)$$

and

$$O_\varphi(\vartheta_n,\vartheta_{n+1},k\varsigma) = O_\varphi(f\vartheta_{n-1}, f\vartheta_n, k\varsigma) \leq O_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma) \leq O_\varphi\left(\vartheta_{n-2},\vartheta_{n-1},\frac{\varsigma}{k}\right) \\ \leq O_\varphi\left(\vartheta_{n-3},\vartheta_{n-2},\frac{\varsigma}{k^2}\right) \leq \cdots \leq O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{k^n}\right).$$

We obtain

$$M_\varphi(\vartheta_n,\vartheta_{n+1},k\varsigma) \geq M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{k^n}\right),\ N_\varphi(\vartheta_n,\vartheta_{n+1},k\varsigma) \leq N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{k^n}\right) \text{ and} \\ O_\varphi(\vartheta_n,\vartheta_{n+1},k\varsigma) \leq O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{k^n}\right). \qquad (3)$$

For any $q \in \mathbb{N}$, $\varsigma = \frac{q\varsigma}{\varsigma} = \frac{\varsigma}{q} + \frac{\varsigma}{q} + \cdots + \frac{\varsigma}{q}$ and using definition of a NEMLS, we deduce

$$M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \geq M_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \circ M_\varphi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right) \\ \circ M_\varphi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \circ \cdots \circ \\ M_\varphi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right).$$

Also,

$$N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \leq N_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \diamondsuit N_\varphi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right) \\ \diamondsuit N_\varphi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \diamondsuit \cdots \diamondsuit \\ N_\varphi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right)$$

and

$$O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \leq O_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \diamondsuit O_\varphi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right) \\ \diamondsuit O_\varphi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \diamondsuit \cdots \diamondsuit \\ O_\varphi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right).$$

Using (3) and the definition of an NEMLS, we deduce

$$M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \geq M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)k^n}\right) \circ M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)k^{n+1}}\right) \\ \circ M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)k^{n+2}}\right) \circ \cdots \circ \\ M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)k^{n+q-1}}\right).$$

Also,

$$N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \leq N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)k^n}\right) \diamondsuit N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)k^{n+2}}\right) \\ \diamondsuit N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)k^{n+3}}\right) \diamondsuit \cdots \diamondsuit \\ N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)k^{n+q-1}}\right)$$

and

$$O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \leq O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)k^n}\right) \diamondsuit O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)k^{n+2}}\right) \\ \diamondsuit O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)k^{n+3}}\right) \diamondsuit \cdots \diamondsuit \\ O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)k^{n+q-1}}\right).$$

By hypothesis, for all $n, q \in$ N, we obtain $\varphi(\vartheta_n, \vartheta_{n+q})k < 1$, with $0 < k < 1$. Therefore, by (1) and for $n \to \infty$,

$$\lim_{n\to\infty} M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = 1 \circ 1 \circ \cdots \circ = 1\,, \\ \lim_{n\to\infty} N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = 0 \diamondsuit 0 \diamondsuit \cdots \diamondsuit 0 = 0$$

and

$$\lim_{n\to\infty} O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = 0 \diamondsuit 0 \diamondsuit \cdots \diamondsuit 0 = 0.$$

That is, $\{\vartheta_n\}$ is a CS. Since $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamondsuit)$ is a complete NEMLS, let

$$\lim_{n\to\infty} M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = \lim_{n\to\infty} M_\varphi(\vartheta_n,\vartheta,\varsigma) = M_\varphi(\vartheta,\vartheta,\varsigma), \\ \lim_{n\to\infty} N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = \lim_{n\to\infty} N_\varphi(\vartheta_n,\vartheta,\varsigma) = N_\varphi(\vartheta,\vartheta,\varsigma), \\ \lim_{n\to\infty} O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = \lim_{n\to\infty} O_\varphi(\vartheta_n,\vartheta,\varsigma) = O_\varphi(\vartheta,\vartheta,\varsigma).$$

Now, we claim that ϑ is a FP of f. Using definition of the NEMLS and (1), we obtain

$$M_\varphi(f\vartheta,\vartheta,\varsigma) \geq M_\varphi\left(f\mathcal{a}, f\vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \circ M_\varphi\left(f\vartheta_n, \mathcal{a}, \frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \\ \geq M_\varphi\left(\mathcal{a},\vartheta_n,\frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))k}\right) \circ M_\varphi\left(\vartheta_{n+1},\vartheta_n,\frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \to 1 \circ 1 = 1 \text{ as } n \to \infty.$$

Also,

$$N_\varphi(f\vartheta,\vartheta,\varsigma) \leq N_\varphi\left(f\mathcal{a}, f\vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \diamondsuit N_\varphi\left(f\vartheta_n, \mathcal{a}, \frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \\ \leq N_\varphi\left(\mathcal{a},\vartheta_n,\frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))k}\right) \diamondsuit N_\varphi\left(\vartheta_{n+1},\vartheta_n,\frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \to 0 \diamondsuit 0 = 0 \text{ as } n \to \infty,$$

and

$$O_\varphi(f\vartheta,\vartheta,\varsigma) \leq O_\varphi\left(f\mathcal{a}, f\vartheta_n, \frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \diamondsuit O_\varphi\left(f\vartheta_n, \mathcal{a}, \frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \\ \leq O_\varphi\left(\mathcal{a},\vartheta_n,\frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))k}\right) \diamondsuit O_\varphi\left(\vartheta_{n+1},\vartheta_n,\frac{\varsigma}{2(\varphi(f\vartheta,\vartheta))}\right) \to 0 \diamondsuit 0 = 0 \text{ as } n \to \infty.$$

This implies that $f\vartheta = \vartheta$. To show its uniqueness, suppose that $fc = c$ for some $c \in \mathfrak{B}$, then

$$1 \geq M_\varphi(c,\vartheta,\varsigma) = M_\varphi(fc, f\vartheta,\varsigma) \geq M_\varphi\left(c,\vartheta,\frac{\varsigma}{k}\right) = M_\varphi\left(fc, f\vartheta,\frac{\varsigma}{k}\right) \\ \geq M_\varphi\left(c,\vartheta,\frac{\varsigma}{k^2}\right) \geq \cdots \geq M_\varphi\left(c,\vartheta,\frac{\varsigma}{k^n}\right) \to 1 \text{ as } n \to \infty.$$

Also,

$$0 \le N_\varphi(c,\vartheta,\varsigma) = N_\varphi(fc, f\vartheta,\varsigma) \le N_\varphi\left(c,\vartheta,\frac{\varsigma}{k}\right) = N_\varphi\left(fc, f\vartheta,\frac{\varsigma}{k}\right)$$
$$\le N_\varphi\left(c,\vartheta,\frac{\varsigma}{k^2}\right) \le \cdots \le N_\varphi\left(c,\vartheta,\frac{\varsigma}{k^n}\right) \to 0 \text{ as } n \to \infty,$$

and

$$0 \le O_\varphi(c,\vartheta,\varsigma) = O_\varphi(fc, f\vartheta,\varsigma) \le O_\varphi\left(c,\vartheta,\frac{\varsigma}{k}\right) = O_\varphi\left(fc, f\vartheta,\frac{\varsigma}{k}\right)$$
$$\le O_\varphi\left(c,\vartheta,\frac{\varsigma}{k^2}\right) \le \cdots \le O_\varphi\left(c,\vartheta,\frac{\varsigma}{k^n}\right) \to 0 \text{ as } n \to \infty.$$

By using the definition of a NEMLS, $\vartheta = c$.

Definition 3.6 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a NEMLS. A map $f: \mathfrak{B} \to \mathfrak{B}$ is an extended neutrosophic like contraction if there exists $0 < k < 1$ so that

$$\frac{1}{M_\varphi(f\vartheta, f\delta,\varsigma)} - 1 \le k\left[\frac{1}{M_\varphi(\vartheta,\delta,\varsigma)} - 1\right] \tag{4}$$

$$N_\varphi(f\vartheta, f\delta,\varsigma) \le k N_\varphi(\vartheta,\delta,\varsigma), \tag{5}$$

and

$$O_\varphi(f\vartheta, f\delta,\varsigma) \le k O_\varphi(\vartheta,\delta,\varsigma), \tag{6}$$

for all ϑ, $\delta \in \mathfrak{B}$ and $\varsigma > 0$.

Now, we prove a result for the above extended neutrosophic-like contraction.

Theorem 3.7 Let $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ be a complete NEMLS with $\varphi: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$ and suppose that

$$\lim_{\varsigma\to\infty} M_\varphi(\vartheta,\delta,\varsigma) = 1, \lim_{\varsigma\to\infty} N_\varphi(\vartheta,\delta,\varsigma) = 0 \text{ and } \lim_{\varsigma\to\infty} O_\varphi(\vartheta,\delta,\varsigma) = 0 \tag{7}$$

for all ϑ, $\delta \in \mathfrak{B}$ and $\varsigma > 0$. Let $f: \mathfrak{B} \to \mathfrak{B}$ be an extended Neutrosophic like contraction mapping in definition 3.3. Further, suppose that for an arbitrary $\vartheta_0 \in \mathfrak{B}$, and n, $q \in$ N, we have

$$\varphi(\vartheta_n, \vartheta_{n+q}) < \frac{1}{k},$$

where $\vartheta_n = f^n\vartheta_0 = f a_{n-1}$. Then f has a unique FP.

Proof: Let ϑ_0 be in $\mathfrak{B}$ and take $\vartheta_n = f^n\vartheta_0 = f a_{n-1}$, $n \in$ N. By using (4), (5) and (6) for all $\varsigma > 0$, $n > q$, we have

$$\frac{1}{M_\varphi(\vartheta_n,\vartheta_{n+1},\varsigma)} - 1 = \frac{1}{M_\varphi(f\vartheta_{n-1},\vartheta_n,\varsigma)} - 1$$
$$\le k\left[\frac{1}{M_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma)} - 1\right] = \frac{k}{M_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma)} - k.$$

Thus,

$$\frac{1}{M_\varphi(\vartheta_n,\vartheta_{n+1},\varsigma)} \le \frac{k}{M_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma)} + (1-k)$$
$$\le \frac{k^2}{M_\varphi(\vartheta_{n-2},\vartheta_{n-1},\varsigma)} + k(1-k) + (1-k).$$

Continuing in this way, we get

$$\frac{1}{M_\varphi(\vartheta_n,\vartheta_{n+1},\varsigma)} \le \frac{k^n}{M_\varphi(\vartheta_0,\vartheta_1,\varsigma)} + k^{n-1}(1-k) + k^{n-2}(1-k) + \cdots + k(1-k) + (1-k)$$
$$\le \frac{k^n}{M_\varphi(\vartheta_0,\vartheta_1,\varsigma)} + (k^{n-1} + k^{n-2} + \cdots + 1)(1-k) \le \frac{k^n}{M_\varphi(\vartheta_0,\vartheta_1,\varsigma)} + (1-k^n).$$

We obtain

$$\frac{1}{\frac{k^n}{M_\varphi(\vartheta_0,\vartheta_1,\varsigma)} + (1-k^n)} \le M_\varphi(\vartheta_n,\vartheta_{n+1},\varsigma). \tag{8}$$

Also,

$$N_\varphi(\vartheta_n,\vartheta_{n+1},\varsigma) = N_\varphi(f\vartheta_{n-1},\vartheta_n,\varsigma) \le kN_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma) = N_\varphi(f\vartheta_{n-2},\vartheta_{n-1},\varsigma)$$
$$\le k^2 N_\varphi(\vartheta_{n-2},\vartheta_{n-1},\varsigma) \le \cdots \le k^n N_\varphi(\vartheta_0,\vartheta_1,\varsigma) \tag{9}$$

and

$$O_\varphi(\vartheta_n,\vartheta_{n+1},\varsigma) = O_\varphi(f\vartheta_{n-1},\vartheta_n,\varsigma) \le kO_\varphi(\vartheta_{n-1},\vartheta_n,\varsigma) = O_\varphi(f\vartheta_{n-2},\vartheta_{n-1},\varsigma)$$
$$\le k^2 O_\varphi(\vartheta_{n-2},\vartheta_{n-1},\varsigma) \le \cdots \le k^n O_\varphi(\vartheta_0,\vartheta_1,\varsigma). \tag{10}$$

For any $q \in \mathbb{N}, \varsigma = \frac{q\varsigma}{\varsigma} = \frac{\varsigma}{q} + \frac{\varsigma}{q} + \cdots + \frac{\varsigma}{q}$ and using the definition of a NEMLS, we deduce

$$M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \ge M_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \circ M_\varphi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right)$$
$$\circ M_\varphi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \circ \cdots \circ$$
$$M_\varphi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right).$$

Also,

$$N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \le N_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \diamond N_\varphi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right)$$
$$\diamond N_\varphi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \diamond \cdots \diamond$$
$$N_\varphi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right)$$

and

$$O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \le O_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \diamond O_\varphi\left(\vartheta_{n+1},\vartheta_{n+2},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right)$$
$$\diamond O_\varphi\left(\vartheta_{n+2},\vartheta_{n+3},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \diamond \cdots \diamond$$
$$O_\varphi\left(\vartheta_{n+q-1},\vartheta_{n+q},\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right).$$

One writes

$$M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \ge \frac{1}{\frac{k^n}{M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right)} + (1-k^n)}$$
$$\circ \frac{1}{\frac{k^{n+1}}{M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right)} + (1-k^{n+1})}$$
$$\circ \frac{1}{\frac{k^{n+2}}{M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right)} + (1-k^{n+2})} \circ \cdots \circ$$
$$\frac{1}{\frac{k^{n+q-1}}{M_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right)} + (1-k^{n+q-1})}.$$

Also,

$$N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \le k^n N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \diamond k^{n+1} N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\right)}\right)$$
$$\diamond k^{n+2} N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \diamond \cdots \diamond$$
$$k^{n+q-1} N_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right)$$

and

$$O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) \le k^n O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\right)}\right) \diamond k^{n+1} O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta\right.}\right.$$
$$\diamond k^{n+2} O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\right)}\right) \diamond \cdots \diamond$$
$$k^{n+q-1} O_\varphi\left(\vartheta_0,\vartheta_1,\frac{\varsigma}{q\left(\varphi(\vartheta_n,\vartheta_{n+q})\varphi(\vartheta_{n+1},\vartheta_{n+q})\varphi(\vartheta_{n+2},\vartheta_{n+q})\cdots\varphi(\vartheta_{n+q-1},\vartheta_{n+q})\right)}\right).$$

By hypothesis for all $n, q \in$ N, we obtain $\varphi(\vartheta_n, \vartheta_{n+q})k < 1$. Therefore, from the definition of a NEMLS, (1) and for $n \to \infty$,

$$\lim_{n\to\infty} M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = 1 \circ 1 \circ \cdots \circ = 1$$
$$\lim_{n\to\infty} N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = 0 \diamond 0 \diamond \cdots \diamond 0 = 0$$

and

$$\lim_{n\to\infty} O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = 0 \diamond 0 \diamond \cdots \diamond 0 = 0.$$

That is, $\{\vartheta_n\}$ is a CS. Since $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi,\circ,\diamond)$ is complete, let

$$\lim_{n\to\infty} M_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = \lim_{n\to\infty} M_\varphi(\vartheta_n,\vartheta,\varsigma) = M_\varphi(\vartheta,\vartheta,\varsigma),$$
$$\lim_{n\to\infty} N_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = \lim_{n\to\infty} N_\varphi(\vartheta_n,\vartheta,\varsigma) = N_\varphi(\vartheta,\vartheta,\varsigma),$$
$$\lim_{n\to\infty} O_\varphi(\vartheta_n,\vartheta_{n+q},\varsigma) = \lim_{n\to\infty} O_\varphi(\vartheta_n,\vartheta,\varsigma) = O_\varphi(\vartheta,\vartheta,\varsigma).$$

Now, we investigate that ϑ is an FP of f. Using the definition of a NEMLS and (1), we obtain

$$\frac{1}{M_\varphi(\mathrm{f}\vartheta_n,\mathrm{f}\vartheta,\varsigma)} - 1 \le k\left[\frac{1}{M_\varphi(\vartheta_n,\vartheta,\varsigma)} - 1\right] = \frac{k}{M_\varphi(\vartheta_n,\vartheta,\varsigma)} - k$$
$$\Rightarrow \frac{1}{\frac{k}{M_\varphi(\vartheta_n,\vartheta,\varsigma)}+(1-k)} \le M_\varphi(\mathrm{f}\vartheta_n,\mathrm{f}\vartheta,\varsigma).$$

Using the above inequality, we obtain

$$M_\varphi(\vartheta,\mathrm{f}\vartheta,\varsigma) \ge M_\varphi\left(\vartheta,\vartheta_{n+1},\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \circ M_\varphi\left(\mathrm{f}\vartheta_n,\mathrm{f}\vartheta,\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right)$$
$$\ge M_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \circ \frac{1}{\frac{k}{M_\varphi\left(\vartheta_n,\vartheta,\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right)} + (1-k)} \to 1 \circ 1 = 1 \text{ as } n \to \infty.$$

Also,

$$N_\varphi(\vartheta,\mathrm{f}\vartheta,\varsigma) \le N_\varphi\left(\vartheta,\vartheta_{n+1},\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \diamond N_\varphi\left(\mathrm{f}\vartheta_n,\mathrm{f}\vartheta,\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right)$$
$$\le N_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \diamond kN_\varphi\left(\vartheta_n,\vartheta,\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \to 0 \diamond 0 = 0 \text{ as } n \to \infty$$

and

$$O_\varphi(\vartheta,\mathrm{f}\vartheta,\varsigma) \le O_\varphi\left(\vartheta,\vartheta_{n+1},\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \diamond O_\varphi\left(\mathrm{f}\vartheta_n,\mathrm{f}\vartheta,\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right)$$
$$\le O_\varphi\left(\vartheta_n,\vartheta_{n+1},\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \diamond kO_\varphi\left(\vartheta_n,\vartheta,\frac{\varsigma}{2\varphi(\vartheta,\mathrm{f}\vartheta)}\right) \to 0 \diamond 0 = 0 \text{ as } n \to \infty.$$

This implies that $f\vartheta = \vartheta$. Now, we show uniqueness. Suppose $fc = c$ for some $c \in \mathfrak{B}$, then

$$\frac{1}{M_\varphi(\vartheta,c,\varsigma)} - 1 = \frac{1}{M_\varphi(\mathrm{f}\vartheta,\mathrm{f}c,\varsigma)} - 1$$
$$\le k\left[\frac{1}{M_\varphi(\vartheta,c,\varsigma)} - 1\right] < \frac{1}{M_\varphi(\vartheta,c,\varsigma)} - 1,$$

which is a contradiction. Also,

$$N_\varphi(\vartheta,c,\varsigma) = N_\varphi(\mathrm{f}\vartheta,\mathrm{f}c,\varsigma) \le kN_\varphi(\vartheta,c,\varsigma) < N_\varphi(\vartheta,c,\varsigma)$$

a contradiction, and

$$O_\varphi(\vartheta,c,\varsigma) = O_\varphi(\mathrm{f}\vartheta,\mathrm{f}c,\varsigma) \le kO_\varphi(\vartheta,c,\varsigma) < O_\varphi(\vartheta,c,\varsigma).$$

It is also a contradiction. Therefore, we must have $M_\varphi(\vartheta, c, \varsigma) = 1$, $N_\varphi(\vartheta, c, \varsigma) = 0$ and $O_\varphi(\vartheta, c, \varsigma) = 0$, hence $\vartheta = c$.

Example 3.8 Let $\mathfrak{B}$ = N. Define

$$\varphi(\vartheta,\delta) = \begin{cases} 1 & \text{if } \quad \vartheta = \delta, \\ \max\{\vartheta,\delta\} & if\ \vartheta \ne \delta \end{cases}$$

Also, take

$$M_\varphi(\vartheta,\delta,\varsigma) = \frac{\varsigma}{\varsigma + \max\{\vartheta,\delta\}^2},\ N_\varphi(\vartheta,\delta,\varsigma) = \frac{\max\{\vartheta,\delta\}^2}{\varsigma + \max\{\vartheta,\delta\}^2}$$
$$\text{and } O_\varphi(\vartheta,\delta,\varsigma) = \frac{\max\{\vartheta,\delta\}^2}{\varsigma}$$

with $a \circ b = a.b$ and $a \diamond b = \max\{a, b\}$. Then $(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi,\circ,\diamond)$ is a complete NEMLS.

Define $f: \mathfrak{B} \to \mathfrak{B}$ by

$$\mathrm{f}(\vartheta) = \begin{cases} 1 & \text{if } \vartheta \in \{1,2\}, \\ \dfrac{\vartheta}{7} & \text{if otherwise} \end{cases}$$

Then we have four cases:

(a) If $\vartheta, \delta \in \{1,2\}$, then $\mathrm{f}\vartheta = \mathrm{f}\delta = 0$;
(b) If $\vartheta \in \{1,2\}$ and $\delta \notin \{1,2\}$, then $\mathrm{f}\vartheta = 0$ and $\mathrm{f}\delta = \frac{\delta}{7}$;
(c) If $\delta \in \{1,2\}$ and $\vartheta \notin \{1,2\}$, then $\mathrm{f}\delta = 0$ and $\mathrm{f}\vartheta = \frac{\delta}{7}$;
(d) If $\vartheta, \delta \notin \{1,2\}$ then $\mathrm{f}\vartheta = \frac{\delta}{7}$ and $\mathrm{f}\delta = \frac{\delta}{7}$.

In all above cases, one gets

$$M_\varphi(\mathrm{f}\vartheta,\mathrm{f}\delta,k\varsigma) \ge M_\varphi(\vartheta,\delta,\varsigma),\ N_\varphi(\mathrm{f}\vartheta,\mathrm{f}\delta,k\varsigma) \le N_\varphi(\vartheta,\delta,\varsigma)$$
$$\text{and } O_\varphi(\mathrm{f}\vartheta,\mathrm{f}\delta,k\varsigma) \le O_\varphi(\vartheta,\delta,\varsigma)$$

are satisfied for $k \in \left[\frac{1}{2}, 1\right)$. Then

$$\frac{1}{M_\varphi(\mathrm{f}\vartheta,\mathrm{f}\delta,\varsigma)} - 1 \le k\left[\frac{1}{M_\varphi(\vartheta,\delta,\varsigma)} - 1\right],\ N_\varphi(\mathrm{f}\vartheta,\mathrm{f}\delta,\varsigma) \le kN_\varphi(\vartheta,\delta,\varsigma)$$
$$\text{and } O_\varphi(\mathrm{f}\vartheta,\mathrm{f}\delta,k\varsigma) \le O_\varphi(\vartheta,\delta,\varsigma)$$

are satisfied for $k \in \left[\frac{1}{2}, 1\right)$.

Observe that all circumstances of Theorem 3.5 and Theorem 3.7 are fulfilled, and 1 is the unique FP of f.

AN APPLICATION TO INTEGRAL EQUATIONS

Let $\mathfrak{B} = C([e, g], \mathbb{R})$ be the set of continuous functions defined on $[e, g]$. Consider the integral equation:

$$\vartheta(l) = f(j) + \beta \int_e^g F(l,j)\vartheta(l)dj \text{ for } l,j \in [e,g] \qquad (11)$$

where $\beta > 0$, $f(j)$ is a fuzzy function of $j \in [e, g]$ and $F \in \mathfrak{B}$. Define M_φ, N_φ and O_φ by

$$M_\varphi(\vartheta(l),\delta(l),\varsigma) = \sup_{l\in[e,g]} \frac{\varsigma}{\varsigma+\max\{\vartheta(l),\ \delta(l)\}^2} \text{ for all } \vartheta,\delta \in \mathfrak{B} \text{ and } \varsigma > 0$$
$$N_\varphi(\vartheta(l),\delta(l),\varsigma) = \sup_{l\in[e,g]} \frac{\max\{\vartheta(l),\ \delta(l)\}^2}{\varsigma+\max\{\vartheta(l),\ \delta(l)\}^2} \text{ for all } \vartheta,\delta \in \mathfrak{B} \text{ and } \varsigma > 0$$

and

$$O_\varphi(\vartheta(l),\delta(l),\varsigma) = \sup_{l\in[e,g]} \frac{\max\{\vartheta(l),\ \delta(l)\}^2}{\varsigma}$$

for all $\vartheta, \delta \in \mathfrak{B}$ and $\varsigma > 0$

where the CTN and CTCN are defined by $a \circ b = a.b$ and $a \diamond b = \max\{a, b\}$.

Given $\varphi: \mathfrak{B} \times \mathfrak{B} \to [1, \infty)$ as

$$\varphi(\vartheta, \delta) = \begin{cases} 1 & \text{if } \vartheta = \delta; \\ \max\{\vartheta, \delta\} & \text{if otherwise.} \end{cases}$$

$(\mathfrak{B}, M_\varphi, N_\varphi, O_\varphi, \circ, \diamond)$ is a complete NEMLS.

Theorem 4.1 Suppose the below conditions hold:

I. $\max\{F(l, j)\vartheta(l), F(l, j)\delta(l)\} \leq \max\{\vartheta(l), \delta(l)\}$ for $\vartheta, \delta \in \mathfrak{B}$, $k \in (0, 1)$ and $\forall\ l, j \in [e, g]$;

II. $\beta \int_e^g dj \leq k < 1$.

Then integral equation (11) has a unique solution.
Proof: Define $f: \mathfrak{B} \to \mathfrak{B}$ by

For all $\vartheta, \delta \in \mathfrak{B}$, we have

$f\vartheta(l) = f(j) + \beta \int_e^g F(l, j)e(l)dj$ for all $l, j \in [e, g]$.

$$M_\varphi(f\vartheta(l), f\delta(l), k\varsigma) = \sup_{l\in[e,g]} \frac{k\varsigma}{k\varsigma + \max\{f\vartheta(l), f\delta(l)\}^2}$$
$$= \sup_{l\in[e,g]} \frac{k\varsigma}{k\varsigma + \max\{f(j) + \beta \int_e^g F(l,j)e(l)dj, f(j) + \beta \int_e^g F(l,j)e(l)dj\}^2}$$
$$= \sup_{l\in[e,g]} \frac{k\varsigma}{k\varsigma + \max\{\beta \int_e^g F(l,j)e(l)dj, \beta \int_e^g F(l,j)e(l)dj\}^2}$$
$$= \sup_{l\in[e,g]} \frac{k\varsigma}{k\varsigma + \max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2 \left(\beta \int_e^g dj\right)^2}$$
$$\geq \sup_{l\in[e,g]} \frac{\varsigma}{\varsigma + \max\{\vartheta(l), \delta(l)\}^2}$$
$$\geq M_\varphi(\vartheta(l), \delta(l), \varsigma).$$

Also,

$$N_\varphi(f\vartheta(l), f\delta(l), k\varsigma) = \sup_{l\in[e,g]} \frac{\max\{f\vartheta(l), f\delta(l)\}^2}{k\varsigma + \max\{f\vartheta(l), f\delta(l)\}^2}$$
$$= \sup_{l\in[e,g]} \frac{\max\{f(j) + \beta \int_e^g F(l,j)e(l)dj, f(j) + \beta \int_e^g F(l,j)e(l)dj\}^2}{k\varsigma + \max\{f(j) + \beta \int_e^g F(l,j)e(l)dj, f(j) + \beta \int_e^g F(l,j)e(l)dj\}^2}$$
$$= \sup_{l\in[e,g]} \frac{\max\{\beta \int_e^g F(l,j)e(l)dj, \beta \int_e^g F(l,j)e(l)dj\}^2}{k\varsigma + \max\{\beta \int_e^g F(l,j)e(l)dj, \beta \int_e^g F(l,j)e(l)dj\}^2}$$
$$= \sup_{l\in[e,g]} \frac{\max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2 \left(\beta \int_e^g dj\right)^2}{k\varsigma + \max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2 \left(\beta \int_e^g dj\right)^2}$$
$$\leq \sup_{l\in[e,g]} \frac{\max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2}{\varsigma + \max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2}$$
$$\leq \sup_{l\in[e,g]} \frac{\max\{\vartheta(l), \delta(l)\}^2}{\varsigma + \max\{\vartheta(l), \delta(l)\}^2}$$
$$\leq N_\varphi(\vartheta(l), \delta(l), \varsigma).$$

Moreover,

$$O_\varphi(f\vartheta(l), f\delta(l), k\varsigma) = \sup_{l\in[e,g]} \frac{\max\{f\vartheta(l), f\delta(l)\}^2}{k\varsigma}$$
$$= \sup_{l\in[e,g]} \frac{\max\{f(j) + \beta \int_e^g F(l,j)e(l)dj, f(j) + \beta \int_e^g F(l,j)e(l)dj\}^2}{k\varsigma}$$
$$= \sup_{l\in[e,g]} \frac{\max\{\beta \int_e^g F(l,j)e(l)dj, \beta \int_e^g F(l,j)e(l)dj\}^2}{k\varsigma}$$
$$= \sup_{l\in[e,g]} \frac{\max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2 \left(\beta \int_e^g dj\right)^2}{k\varsigma}$$
$$\leq \sup_{l\in[e,g]} \frac{\max\{F(l,j)\vartheta(l), F(l,j)\delta(l)\}^2}{\varsigma}$$
$$\leq \sup_{l\in[e,g]} \frac{\max\{\vartheta(l), \delta(l)\}^2}{\varsigma}$$
$$= O_\varphi(\vartheta(l), \delta(l), \varsigma).$$

Therefore, all circumstances of Theorem 3.5 are fulfilled. Hence, operator f has a single FP. This implies that integral equation (11) has a single solution.

CONCLUSION

This study aims to define a neutrosophic extended metric-like space and examine some properties. This work is the extended form of a fuzzy metric like space see [21, 22]. This new concept can also be studied to the fixed point theory, as in metric fixed metric theory and so it can construct the NEMLS fixed point theory. As is well known, in recent years, the study of metric fixed point theory has been widely researched because this theory has a fundamental role in various areas of mathematics, science, and economic studies. This work can easily extend in the structure of neutrosophic controlled metric like spaces, neutrosophic triple partial g-metric like space, and many others.

APPENDIX

Definition 6.1 [1] A binary operation (BO) $\circ : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous triangle norm if it meets the below assertions:

1. $a \circ b = b \circ a$, $(\forall)\ a, b \in [0, 1]$;
2. $\circ$ is continuous;
3. $a \circ 1 = a$, $(\forall)\ a \in [0, 1]$;
4. $(a \circ b) \circ \varkappa = a \circ (b \circ \varkappa)$, $(\forall)\ a, b, \varkappa \in [0, 1]$;
5. If $a \leq \varkappa$ and $b \leq d$, with $a, b, \varkappa, d \in [0, 1]$, then $a \circ b \leq \varkappa \circ d$.

Definition 6.2 [1] A BO $\diamond : [0, 1] \times [0, 1] \to [0, 1]$ is called a continuous triangle conorm, if it meets the below assertions:

1. $a \diamond b = b \diamond a$, $(\forall)\ a, b \in [0, 1]$;
2. $\diamond$ is continuous;
3. $a \diamond 0 = 0$, $(\forall)\ a \in [0, 1]$;
4. $(a \diamond b) \diamond \varkappa = a \diamond (b \diamond \varkappa)$, $(\forall)\ a, b, \varkappa \in [0, 1]$;
5. If $a \leq \varkappa$ and $b \leq d$, with $a, b, \varkappa, d \in [0, 1]$, then $a \diamond b \leq \varkappa \diamond d$.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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