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Research Article

Generalized Pythagorean fuzzy sets and new decision-making method

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ABSTRACT

The thought of the Pythagorean fuzzy soft set emerging from the Pythagorean fuzzy set, which is initiated by Yager, was generalized by including a parameter by Kirisci. In the present communication, a new decision-making method is defined. For this decision-making method, an algorithm is conceived. The reason for giving this algorithm is to facilitate the solution of multi-attribute decision-making difficulties. A numerical example concerning a medical diagnosis problem for the proposed algorithm has been successfully illustrated.

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INTRODUCTION

Uncertainty is a crucial concept for decision-making(DM) problems. It is not easy to make precise decisions in life since each piece of information contains vagueness, uncertainty, and imprecision. Fuzzy Set(FS) Theory, Zadeh's [1] pioneering work, proposed a membership function(MF) to solve problems such as vagueness, uncertainty, and imprecision, and this function took value in the range of [0,1]. FS Theory had solved many problems in practice, but there was no MF in real life, which only includes acceptances. Rejection is as important as acceptance in real life. Atanassov [2] clarified this problem and posed the intuitionistic fuzzy set(IFS) using the MF as well as the non-membership function(NF). In IFS, the sum of membership grade(MG) and non-membership grade(NG) is one. This condition is also a limitation for solutions of vagueness, uncertainty, and imprecision. Yager [3] has presented a solution to this situation and suggested Pythagorean Fuzzy Sets(PFS). PFS is more comprehensive than IFS because it uses the requirement that the summation of the squares of MG and NG is equal to or less than one. PFS is also a special status of the neutrosophic set [4]. There are many studies in the literature on FS, IFS, and PFS theories [5]-[31]. Despite all the possible solutions, these theories have limitations.

Molodtsov [32] proposed a new method called Soft Set in which preferences are given in different parameters for each alternative. First, the fuzzy soft set theory [21], then the intuitionistic fuzzy soft set theory [22], followed this development [33], [34]. Using the FSS and IFSS definitions, the definition of the Pythagorean fuzzy soft set(PFSS) has been given [23]. PFSS is a natural generalization of IFSS and is a parameterized family of PFSs. In [35]-[39], the

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main features of PFSS were examined and applied to various areas.

People may hesitate during DM. To avoid human hesitations from adversely affecting the DM process, hesitation value is also taken into account in PFS, just like IFS. Thus, experts may have hesitations about membership grades. If the expert participating in the decision process is only one person, this expert's error or bias will affect the process negatively. However, hesitation is subjective and the expert's hesitation can be directed by her/his perceptions. In this instance, enriching the decision continuum, assessing alternative decisions more meaningfully, and combining the subjective evaluations of more than one expert instead of the subjective evaluation of a single expert will provide a healthier DM process. With this in mind, a generalized intuitionistic fuzzy soft set(GIFSS) is characterized [5]. Feng et al. [7] identified some problems and difficulties in the definition of GIFSS and operations related to GIFSS in the manuscript of Agarwal et al [5]. Kirisci [19] defined GPFSS, considering the fixes in [7].

GPFSS ensures the frame for evaluating the reliability of the info in the PFSS to compensate for any distortion in the info given. It is very important to include the generalized parameter in the process to reduce the errors that may arise due to imprecise information, in consultation with the chairman. For instance, a patient may give wrong information to a physician about her/his symptoms. If the physician does not notice this wrong information, errors in diagnosis; and treatment will occur. In this case, an experienced physician can measure the reliability of the information given by the patient with a generalization parameter. Therefore, a generalization parameter is needed. This parameter indicates the level of confidence in the trustworthiness of the expert's knowledge of the subject and makes the approach very close to real-life cases. Thus, this parameter increases the reliability of the eventual decision. GPFSS has a generalization parameter to state the uncertainties.

The DM method given in this work has been defined based on GPFSS. In Section 3, the GPFSS defined by Kirisci [19] is briefly described, and some properties are given. The solution to the problems that may occur in the comparison of two different GPFFSs is mentioned. In Section 4, we have framed the score, accuracy, and expectation functions to offer an algorithm to explain the DM problem by using GPFSS. We defined a PFWA operator similar to that defined by Yager [3]. We present the definitions of the reduced PFS(RPF), and reduced FS(RF) with reduced set operators. We proposed an algorithm by using the expectation function, RPF, and RF according to the GPFSS with a numerical example for medical DM.

PRELIMINARIES

Bellman and Zadeh [40] thought that the inherent vagueness that arises in people's daily life during DM also apply to objects. So there was also fuzziness and uncertainty in the objects. Starting from this idea, they introduced FSs in their decision-making method. They recommended that the decision-maker (DMr) could use $u_{C_j}(x_i)$ to state her/his predilection for the membership degree(MD) of an alternative (x_i) by a criterion C_j; and express the grade to which the alternative x_i fulfills the criterion C_j. Normally, the DMr in the convenient DM continuum may not only ensure the grade to which the alternative x_i fulfills criterion C_j but also offer the grade to which the alternative x_i does not meet criterion C_j. For this purpose, Atanassov [2] suggested the notion of IFS, which is portrayed by an MD and an ND fulfilling the case that the sum of its membership degree and non-membership degree is equal to or less than 1. The definitions of IFS are as follows:

Choose the set D as a universe. The set

$$\Gamma = \{< z, \Gamma(d_\Gamma(z), y_\Gamma(z)) > : z \in D\}$$

is called an IFS, where the function $d_\Gamma(z)$: $D \to [0,1]$ defined the MD and $y_\Gamma(z)$: $D \to [0,1]$ defines the ND of element $z \in D$ to Γ, respectively, and for every $z \in D$, it holds that $0 \leq d_\Gamma(z) + y_\Gamma(z) \leq 1$.

For $z \in D$ and any IFS Γ, $h_\Gamma(z) = 1 - d_\Gamma(z) - y_\Gamma(z)$ is called the degree of indeterminacy(ID) of z to Γ. For simpleness, Xu [27] denoted $A = \Gamma(d_A, y_A)$ as an intuitionistic fuzzy number(IFN), where d_A and y_A are MD and the ND of the element $z \in D$ to Γ, respectively.

At the same time, the DMr in many daily life DM problems can state their predilection about the grade of an alternative x_i by a criterion C_j fulfilling the situation that the sum of the grade to which the alternative x_i fulfills the criterion C_j ad the grade to which the alternative x_i does not meet the criterion C_j is bigger than 1. This case cannot be characterized by using IFS. Yager offered a new notion of PFS to model this condition, instead of requiring the DMr to modify their predilection info to comply with the limitations of IFSs.

Pythagorean MDs have three basic representations [3], [28], [29].

- $\alpha \in [0,1]$, $\beta \in [0,1]$, and $\alpha^2 + \beta^2 \leq 1$ for (α, β),
- $t \in [0,1]$ and $\gamma \in [0, \pi/2]$, for the polar coordinates (t,γ),
- $t \in [0,1]$, $\gamma \in [0, \pi/2]$, and $\eta = 1 - 2\gamma/\pi$ for (t,η).

The relationship between these items can be described as follows: $\alpha^2 + \beta^2 = t^2$, $\alpha = t.\cos\gamma$, $\beta = t.\sin\gamma$.

Choose the set D as a universe. The set

$$\Pi = \{< z, \Pi(d_\Pi(z), y_\Pi(z)) > : z \in D\}$$

is called a PFS, where the function $d_\Pi : D \to [0,1]$ defined the MD and $y_\Pi : D \to [0,1]$ defines the ND of $z \in D$ to the Π, respectively, and for every $z \in D$, it holds that $0 \leq d_\Pi^2(z) + y_\Pi^2(z) \leq 1$ [3], [28], [29]. For $z \in D$ and any PFS Π $h_\Pi = \sqrt{1 - d_\Pi^2(z) - y_\Pi^2(z)}$ is called the ID of z to Π.

For convenience, (d_p(z), y_p(z)) was called the Pythagorean fuzzy number(PFN) by Zhang and Xu [31]. Yager suggested an alternative presentment of PFN which is $p = (r_p, k_p)$ is that $d_p = r_p \cos \theta_p$ ... $y_p = r_p \sin \theta_p$ where $k_p = 1 - 2\theta_p / \pi$.

The complement operator of PFN is indicated by $B^c = \Pi(y_B, d_B)$. This notation is distinct from IFN $A^c = \Gamma(y_A, d_A)$. The definition of A^c is suggested concerning the Sugeno [41] category of complements $c(z) = (1 - z)/(1 + \lambda z)$, $\lambda \in (-1,\infty)$ when $\lambda = 0$ (i.e. $c(x) = 1 - z$) while the definition of B^c is suggested concerning the Yager category of complements $c(z) = (1 - z^\sigma)^{1/\sigma}$, $(\sigma \in (0,\infty))$ when $\sigma = 2$, (i.e. $c(z) = \sqrt{1-z^2}$).

For PFNs $B_1 = \Pi(d_{B_1}, y_{B_1})$, $B_2 = \Pi(d_{B_2}, y_{B_2})$, $B = \Pi((d_B, y_B)$,

- $B_1 \cup B_2 = \Pi(\max (d_{B1}, d_{B2}), \min \cap (y_{B_1}, y_{B_2}))$,
- $B_1 \cap B_2 = \Pi(\min (d_{B1}, d_{B2})), \max (y_{B_1}, y_{B_2}))$.
- $B^c = \Pi(y_B, d_B)$,
- $B_1 \oplus B_2 = (\sqrt{d_2^1 + d_2^2 - d_2^1 \cdot d_2^2}), y_1 y_2)$,
- $B_1 \otimes B_2 = (d_1 d_2, \sqrt{y_2^1 + y_2^2 - y_2^1 \cdot y_2^2})$,
- $\alpha B = (\sqrt{1-(1-d^2)^a}\ y^\alpha)$,
- $B^\alpha = (d^\alpha, \sqrt{1-(1-y^2)^a})$.

Comparison of IFSs and PFSs:

IFS, offered by Atanassov [2] is an extension of FS Theory [1]. IFS is characterized by a membership degree and a non-membership degree and therefore can indicate the fuzzy character of data in more detail comprehensively. The prominent characteristic of IFS is that it assigns to each element a membership degree and a non-membership degree with their sum equal to or less than 1. However, in some practical DM processes, the sum of the membership degree and the non-membership degree to which an alternative satisfying a criterion provided by a decision maker may be bigger than 1, but their square sum is equal to or less than 1. Table 1 explains the difference between Pythagorean fuzzy sets and intuitionistic fuzzy sets.

Therefore, Yager [28] proposed PFS is characterized by a membership degree and a non-membership degree, which satisfies the condition that the square sum of its membership degree and non-membership degree is less than or equal to 1. Yager [29] gave an example to state this situation: a decision maker gives his support for membership of an alternative is $\frac{\sqrt{3}}{2}$ and his against membership is $\frac{1}{2}$. Owing to the sum of two values being bigger than 1, they are not available for IFS, but they are available for PFS since $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 \leq 1$. PFS is more capable than IFS to model the vagueness in the practical multi-criteria decision-making problems.

The main difference between PFNs and IFNs is their corresponding constraint conditions, which can be easily shown in Figure 1. Here, we observe that intuitionistic membership grades are all points under the line $x + y \leq 1$ and the Pythagorean membership grades are all points with $x^2 + y^2 \leq 1$.

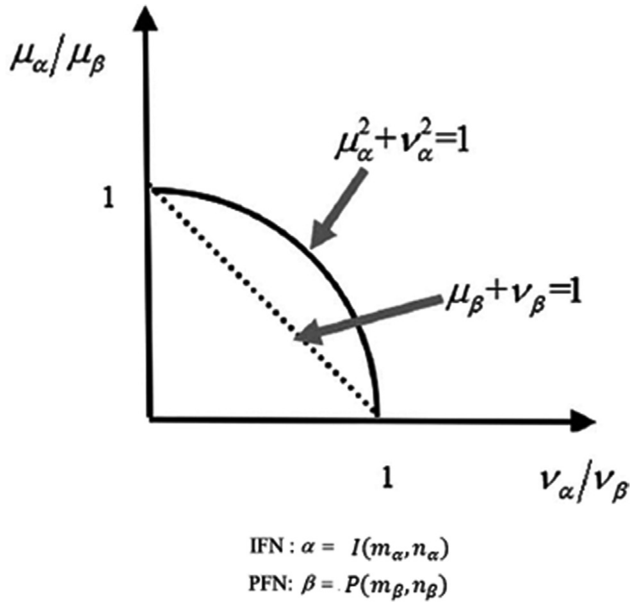

Figure 1. The PFNs and the IFNs.

Table 1: PFSs and IFSs

IFSs	PFSs
u + v □ 1	u+v □ 1 or u + v □ 1
0 □ u + v □ 1	0 □ $u^2 + v^2$ □ 1
w = 1 – [u + v]	$w = \sqrt{1-[u^2+v^2]}$
u + v + w = 1	$u^2 + v^2 + w^2 = 1$

One important implication of this is that it allows the use of the PFSs in situations in which we cannot use IFSs. An example of this would be a case in which a user indicates that their support for membership of x is $\frac{\sqrt{3}}{2}$ and their support against membership is $\frac{1}{2}$. As we noted these values are not allowable for intuitionistic membership grades but allowable as Pythagorean membership grades. Thus in this case, rather than requiring the user to change their information to satisfy the constraints of the IFS, we can use a PFS.

Comparison of GIFSSs and GPFSSs:

Agarwal et al. [5] defined Generalized Intuitionistic Fuzzy Soft Sets(GIFSS). Feng et al. [7] identified some problems and difficulties in the definition of GIFSS and operations related to GIFSS in the manuscript of Agarwal et al[5]. Kirisci [19] defined GPFSS, considering the fixes in

[7]. GPFSS ensures the frame for evaluating the reliability of the info in the PFSS to compensate for any distortion in the info given. The most important benefit of incorporating the generalized parameter into the analysis is to decrease the likelihood of errors induced by the imprecise info by taking the chairperson's view on the same. For example, a patient may give wrong information to a physician about her/his symptoms. If the physician does not notice this wrong information, errors in diagnosis; and treatment will occur. In this case, an experienced physician can measure the reliability of the information given by the patient with a generalization parameter. So, there is a requirement for a generalization parameter, demonstrating an expert's level of confidence in the reliability of the info, respectably making the approach quite close to real-world cases. This assists in extracting the singular bias from the input data and gets more credibility to the final decision. GPFSS has a generalization parameter to state the uncertainties.

□-SOFT SETS

Some definitions and properties in this section are taken from [19].

Take the set D to be a universe, the set E to be a parameter set, and let $\delta(D)$ denote the set of all PFSs. Consider the $\Pi \subseteq E$ for $\Pi = \{<z, \Pi(d_\Pi(z), y_\Pi(z))>: z \in D\}$. The pair (F,Π) is called PFSS on D, where $F: \Pi \to \delta(D)$ [23].

Consider the corresponding partial order $\leq_K$ as defined by $(m,n) \leq_K (p,r) \Leftrightarrow m \leq p$ and $n \geq r$, for all (m,n), $(p,r) \in K$. Pythagorean fuzzy value(PFV) is denoted by ordered pair $(m,n) \in K$. PFV is also called Pythagorean fuzzy number(PFN) [8].

Definition 1. The set

$$(F,\Lambda(f)) = \left\{\left(\frac{z}{(d_F(z), y_F(z))}, f_F(z)\right): z \in E,\ d_F(z) \in [0,1],\ y_F(z) \in [0,1]\right\}$$

is called Pythagorean fuzzy parametrized Pythagorean fuzzy soft set (□-soft set), where Λ is a PFS, (F,Λ) is a PFSS and $f: \Lambda \to K$ is a PFS on Λ.

$\Omega(D)$ will be used as the representation of the set of all □-soft sets on D.

Example 1. Consider the set $D = \{u_1, u_2, u_3, u_4\}$ as objects. Let $E = \{e_1, e_2, e_3, e_4, e_5, e_6\}$ denote the parameter set. For u_1 and $\sigma \subseteq E$

$$(F,\sigma) = \left\{\frac{e_1}{(0.7,0.7)}, \frac{e_3}{(0.6,0.6)}, \frac{e_5}{(0.5,0.8)}, \frac{e_6}{(0.4,0.7)}\right\}$$

and

$f_\sigma(e_1) = (0.8,0.2)$, $f_\sigma(e_3) = (0.9,0.3)$, $f_\sigma(e_5) = (0.5,0.8)$, $f_\sigma(e_6) = (0.6,0.5)$ are state the expert opinion. This information can be observed in Table 2.

Let $\sigma,\varphi \subseteq E$. Choose the PFSSs (F,σ),(F,φ) and the □-soft sets $(F,\sigma(f))$,$(F,\varphi(g))$. If the following cases hold, for all $z \in E$,

Table 2: $(F, \sigma(f))$

D/σ	e_1	e_3	e_5	e_6
u_1	(0.7,0.7)	(0.6,0.6)	(0.8,0.2)	(0.4,0.7)
u_2	(0.5,0.6)	(0.4,0.9)	(0.8,0.6)	(0.5,0.6)
u_3	(0.9,0.4)	(0.8,0.4)	(0.6,0.7)	(0.7,0.4)
u_4	(0.7,0.5)	(0.6,0.5)	(0.5,0.8)	(0.8,0.3)
$f_\sigma(e_i)$	(0.8,0.2)	(0.9,0.3)	(0.5,0.8)	(0.6,0.5)

i. $(F,\sigma), \subseteq (F,\varphi)$, and $f_\sigma \leq_K g_\varphi$

ii. $d_f(z) \leq d_g(z)$, $y_f(z) \geq y_g(z)$

then $(F,\varphi(g))$ is called to be the □-soft subset of $(F,\sigma(f))$.

Choose the PFSSs (F,σ),(F,φ) and the □-soft sets $(F,\sigma(f))$. If $\sigma = \varphi$, $(F,\sigma), = (F,\varphi)$, and $f_\sigma = g_\varphi$, $F_{\sigma(f)} = F_{\varphi(g)}$.

For the PFSSs (F,σ),(F,φ) and the □-soft sets $(F,\sigma(f))$, $(F,\varphi(g))$ and $\sigma,\varphi \subseteq E$, $\tau = \sigma \cup \varphi$, $(F,\tau_\varepsilon(f)) = (F,\sigma(f)) \cup_\varepsilon (F,\varphi(g))$ is called the extended union, for $z \in \tau$, if

$(F,\sigma) \cup (F,\varphi) = (F,\tau)$; $f_\sigma \cup g_\varphi = h_\tau$

$$d_h(z) = \begin{cases} d_f(z), & z \in \sigma - \varphi, \\ d_g(z), & z \in \sigma - \varphi, \\ \max\{d_f(z), d_g(z)\} & z \in \sigma \cap \varphi, \end{cases}$$

$$y_h(z) = \begin{cases} y_f(z), & z \in \sigma - \varphi, \\ y_g(z), & z \in \sigma - \varphi, \\ \max\{y_f(z), y_g(z)\} & z \in \sigma \cap \varphi, \end{cases}$$

For the PFSSs (F,σ),(F,φ) and the □-soft sets $(F,\sigma(f))$, $(F,\varphi(g))$ and σ, $\varphi \subseteq E$, $\rho = \sigma \cup \varphi$, $(F, \rho_\in(f)) = (F,\sigma(f)) \cap_\in (F,\varphi(g))$ is called the extended intersection, for $z \in \rho$, if

$$d_h(z) = \begin{cases} d_f(z), & z \in \sigma - \varphi, \\ d_g(z), & z \in \sigma - \varphi, \\ \max\{d_f(z), d_g(z)\} & z \in \sigma \cap \varphi, \end{cases}$$

$$y_h(z) = \begin{cases} y_f(z), & z \in \sigma - \varphi, \\ y_g(z), & z \in \sigma - \varphi, \\ \max\{y_f(z), y_g(z)\} & z \in \sigma \cap \varphi, \end{cases}$$

For the PFSSs (F, σ), (F, φ) and the □-soft sets $(F, \sigma(f))$, $(F, \varphi(g))$ and σ, $\varphi \subseteq E$, $\tau = \sigma \cup \varphi$, $(F, \tau_\varepsilon(f)) = (F, \sigma(f)) \cup_\varepsilon (F, \varphi(g))$ is called the restricted union, for $z \in \tau$, if

$(F,\tau) = (F,\sigma) \cup_\varepsilon (F,\varphi)$

and

$d_h(z) = max\{d_f(z), d_g(z)\}$; $y_h(z) = min\{y_f(z), y_g(z)\}$.

For the PFSSs (F,σ), (F,φ) and the □-soft sets $(F,\sigma(f))$, $(F,\varphi(g))$ and $\sigma,\varphi \subseteq E$, $\rho = \sigma \cup \varphi$, $(F,\rho_\varepsilon(f)) = (F,\sigma(f)) \cap_\varepsilon (F,\varphi(g))$ is called the restricted intersection, for $z \in \rho$, if

$(F,\rho) = (F,\sigma) \cap_\varepsilon (F,\varphi)$

and

$d_h(z) = min\{d_f(z), d_g(z)\}$; $y_h(z) = max\{y_f(z), y_g(z)\}$.

Take a PFSSs (F,σ) and let $m, n \in [0,1]$ such that $m^2 + n^2 \leq 1$. Therefore, (F,σ) is said to be a (m,n)-constant PFSS. If $d_{F(z)} = \tilde{m}$ and $y_{F(z)} = \tilde{n}$ for all $z \in \sigma$, then it is denoted by $Z_\sigma^{(m,n)}$.

The relative null PFSS and relative whole PFSS according to the set E are denoted by $Z_\sigma^{(0,1)}$ and $Z_\sigma^{(1,0)}$, respectively. Then, for $(F,\sigma(f)) \in D$,

i. If $(F,\sigma) = Z_\sigma^{(0,1)}$, $d_f(z) = 0$, $y_f(z) = 1$ for all $z \in \sigma$, then a relative null □-soft set according to σ is denoted by $(F,\sigma(f))$.
ii. If $(F,\sigma) = Z_\sigma^{(1,0)}$, $d_f(z) = 1$, $y_f(z) = 0$ for all $z \in \sigma$, then a relative whole □-soft set according to σ is denoted by $(F,\sigma(f))$.

NEW DECISION-MAKING METHOD

If condition $T(G) = d_G^2 - y_G^2$ holds, then, for all $G = d_G, y_G \in K$, $T:K \to [-1,1]$ is called score function[42]. Consider the PFNs G,H. If $T(G) < T(H)$, then $G > H$; if $T(G) > T(H)$, then $G>H$; $T(G) = T(H)$, then $G \sim H$.

If condition $U(N) = d_G^2 - y_G^2$ holds, then, for all $G = d_G, y_G \in K$, $U:K \to [-1,1]$ is called score function[30]. Consider the PFNs G,H. If $U(G) < U(H)$, then $G < H$; if $U(G) > U(H)$, then $G > H$; $U(G) = U(H)$, then $G \sim H$.

The binary process $\leq_{(T,U)} \in K$ can be defined as follows[21]:

$G \leq_{(T,U)} H \Leftrightarrow (T(G) < T(H) \vee T(G) = T(H) \wedge U(G) \leq U(H))$.

for $G, H \in K$.

Definition 2. The mapping $V:K \to [0,1]$ is called the expectation score function such that for all $G = d_G, y_G \in K$,

$$V(G) = \frac{d_G^2 - y_G^2 + 1}{2}$$

In this function, if we take $(d_G^2)^* = 1 - y_G^2$, then [19]

$$V(G) = \frac{d_G^2 + (1 - y_G^2)}{2} = \frac{d_G^2 + (d_G^2)^*}{2}$$

For V, the following cases are hold:

i. $V(0,1) = 0$ and $V(1,0) =$ held
ii. $V(d_H, y_H)$ is increasing according to d_G,
iii. $V(d_H, y_H)$ is decreasing according to y_G.

Definition 3. Take the PFVs $Y_j = (d_j, y_j) \in K$ and the weighted vector $\xi = (\xi_1, \xi_2, \ldots \xi_k)^T$ for j=1,2,…,k, $\xi_j \in [0,1]$ with $\sum_{j=1}^k \xi_j = 1$. If the condition

$$\Omega_W(Y_j) = \left(\sum_{j=1}^k \xi_j d_j, \sum_{j=1}^k \xi_j y_j\right)$$

is holds, then the mapping $\Omega_{it}:K^n \to K$ is called the Pythagorean fuzzy weighted averaging (Ω_W) operator.

The weight vector ξ is calculated with $\frac{V_{f(z)}}{\sum_{z\in 1} V_{f(z)}}$. Now, we will obtain a PFS, which is called reduced PFS of an Ω-soft set.

Let $|D|$ represent the cardinality of PFSS D and $\xi_j \in [0,1]$ with $\sum_{j=1}^k \xi_j = 1$. The set RPF=$\{(u,d_\rho, y_\rho):u\in D\}$ is called Reduced Pythagorean Fuzzy Set(RPF) of Ω-soft set, where

$$d_p(u) = \frac{1}{|D|}\sum_{e\in E, u\in D} \xi_j d_j, \quad y_\rho(u) = \frac{1}{|D|}\sum_{e\in E, u\in D} \xi_j y_j$$

for $d_\rho:D \to [0,1]$, $d_\rho:D \to [0,1]$. d_ρ and y_ρ are called Reduced Set Operators of RPF. It is easily seen that RPF is a PFS on D.

In a similar idea, we can give the following definition:

The set RF = $\{(u,t_\varepsilon):u\in D\}$ is called Reduced Fuzzy Set of RPF on D, such that

$$t_\beta(u) = d_\beta(u)\left[\sqrt{1 - y_p^2(u)}\right]$$

for $t_\varepsilon:D \to [0,1]$.

Algorithm:

Step 1: Using the P parameter set obtained from expert opinions, establishes the PFSS X.

Step 2: Thanks to expert knowledge, establishes Ω – soft set using the options set obtained.

Step 3: Computes the set RPF of the Ω – soft set.

Step 4: Calculates the set RF of the set RPF.

Step 5: For making a decision, selects the results of RF which has the maximum membership degree.

APPLICATION

Consider Table 2 in Example 1 values. As in Table 3, let's get the table values of $(F,\sigma(g))$. According to Table 2,3, calculate extended intersection as in Table 4:

The values of Tables 2,3,4 are considered. The expectation values V are calculated and,

$$\xi = \{0.29143898, \ 0.13843352, \ 0.30054645, \ 0.1111111, \ 0.15846995\}^T$$

is obtained. The ξ values are used to calculate the RF.

$$RPF = \{(u_1,0.12525,0.17675), \ (u_2,0.1095,0.18725), \ (u_3,0.182\ ,0.138), \ (u_4,0.152,0.134)\}$$

and

$$RF = \{(u_1,0.123246), \ (u_2,0.107529), \ (u_3,0.18018), \ (u_4,0.15048)\}.$$

Table 3: $(F,\sigma(g))$

E/φ	e_2	e_3	e_5	e_6
u_1	(0.6,0.6)	(0.4,0.7)	(0.7,0.6)	(0.1,0.9)
u_2	(0.3,0.8)	(0.8,0.4)	(0.6,0.5)	(0.4,0.8)
u_3	(0.6,0.7)	(0.8,0.6)	(0.3,0.7)	(0.7,0.5)
u_4	(0.6,0.6)	(0.7,0.4)	(0.8,0.5)	(0.5,0.6)
$g_\varphi(e_i)$	(0.5,0.7)	(0.9,0.4)	(0.6,0.5)	(0.6,0.7)

Table 4: Extended Intersection

$P/\rho_\in$	e_1	e_2	e_3	e_5	e_6
u_1	(0.7,0.7)	(0.6,0.6)	(0.4,0.7)	(0.7,0.6)	(0.1,0.9)
u_2	(0.5,0.6)	(0.3,0.8)	(0.4,0.9)	(0.6,0.6)	(0.4,0.8)
u_3	(0.9,0.4)	(0.6,0.7)	(0.8,0.6)	(0.3,0.7)	(0.7,0.5)
u_4	(0.7,0.5)	(0.6,0.6)	(0.6,0.5)	(0.5,0.5)	(0.5,0.6)
$h_{\rho_\varphi}(e_i)$	(0.8,0.2)	(0.5,0.7)	(0.9,0.4)	(0.5,0.8)	(0.6,0.7)

Table 5: Results

	e_1	e_2	e_3	e_5	e_6
$h_{\rho_\mathcal{E}}$	(0.8,0.2)	(0.5,0.7)	(0.9,0.4)	(0.5,0.8)	(0.6,0.7)
$V_{f_{\rho_\mathcal{E}}}$	0.8	0.38	0.825	0.305	0.435
ξ	0.29143898	0.13843352	0.30054645	0.1111111	0.15846995

Among the results obtained, the maximum value is 0.18018 and since it belongs to u_3 at this value, the choice of DM will be u_3.

CONCLUSION

The new FS, identified as PFS, was presented to the literature by Yager [28]. PFS charmed the care of numerous scientists in a little while [8], [19], [23], [24], [31], [37], [42]. Peng et al. [23] have described PFSSs. Kirisci [19] has extended PFSS to Ω – soft set and investigated some characteristics. In this study, new a DM algorithm with Ω – soft set is proposed. Kirisci [19] defined score, accuracy, expectation functions, and Ω_W operator, for the DM process. Belonging to the DM mechanism with the working principle of the algorithm, a numerical example was solved.

NOMENCLATURE

DMr	Decision maker
DM	Decision-making
FS	Fuzzy Set
IFS	Intuitionistic Fuzzy Set
PFS	Pythagorean Fuzzy Set
PFSS	Pythagorean Fuzzy Soft Set
GIFSS	Generalized Intuitionistic Fuzzy Soft Set
GPFSS	Generalized Pythagorean Fuzzy Soft Set
Ω-soft set	Pythagorean Fuzzy Parameterized Pythagorean Fuzzy Soft Set
IFN	Intuitionistic Fuzzy Number
PFN	Pythagorean Fuzzy Number
MF	Membership Function
NF	Non-membership Function
RPF	Reduced Pythagorean Fuzzy Set
RF	Reduced Fuzzy Set

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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