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Research Article

Wavelet estimation in nonparametric linear mixed-effects errors in variables model

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ABSTRACT

Nonparametric linear mixed effects models are preferred due to overcome the restrictions of linear models which need to satisfy distributional assumptions. In these models, smoothing approaches are needed to handle nonparametric part and chosen according to the type of data. When there is a measurement error in the nonparametric part, these smoothing techniques become more complicated. In this paper, we propose wavelet approach to smooth nonparametric function under known measurement error in nonparametric linear mixed effects model and then, we predict random effects parameter. Furthermore, a simulation study is done to demonstrate the theoretical findings by comparing with the case ignoring measurement error. The performances are much better for the proposed model than the no measurement error case.

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INTRODUCTION

Linear mixed-effects models (LMM) can be considered as an enlarged linear model that contains random effects together with the usual fixed effects [1]. They have acquired a lot of interest in statistical research because of their flexibility to handle many disciplines (e.g. longitudinal, clustered, multivariate, correlated, repeated measures, growth and dose–response curve data structures). However, in LMM information about the functional form of regression is needed and some distribution assumptions about random effects and random errors should be satisfied. Then, to overcome these restrictions which may not be satisfied by the parametric modeling, nonparametric linear mixed-effects models (NLMM) which provides flexibility to describe the functional association have become largely preferred instead of LMM [2-4].

For nonparametric features, the most famous smoothing approaches are kernel approaches, smoothing splines, penalized splines, regression polynomial splines and series-based smoothers, including wavelets [5-7]. The smoothing approaches are chosen according to the type of data.

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Longitudinal data which involve repeated observations of the same things at different points in time are used in NLMM.

Wavelets are not commonly handled for longitudinal data. Müller [8] investigated the nonparametric methods. Kernel-type smoothing methods, smoothing spline methods and regression (polynomial) spline methods for longitudinal data have been largely exposed [2, 9-17]. Rice and Wu [17] introduced NLMM for unequally sampled noisy curves. For longitudinal data, a NLMM was discussed on wavelet bases via a Bayesian structure by Lu and Huang [18]. Angelini et al. [19] examined wavelet regression estimation in NLMM. Ghoul et al. [20] proposed wavelet analysis in semiparametric mixed models for longitudinal data.

Although the wavelets have been considered as a very powerful mathematical tool and become an alternative approach to the Fourier transform, they are not commonly used in errors in variables case. Chichignoud et al. [21] introduced adaptive wavelet multivariate ($t \in [0,1]^d$, $d > 1$) nonparametric regression with errors in variables. They devised an adaptive estimator based on projection kernels on wavelets and a deconvolution operator in nonparametric model (NPM). Using theoretical arguments for nonparametric wavelet estimation Yalaz [22] represented a wavelet approach to estimate partially linear errors in variables model which is a semiparametric model (SPM) when explanatory variable of nonparametric part has measurement error. The author created the estimator for parametric part parameter of semiparametric regression using the idea described as wavelet approach for nonparametric function given in Chichignoud et al. [21]. Despite the improvements on wavelet approach in NPM and SPM with errors in variables, there is still a gap in NLMM uses wavelet approach considering the errors in variables. Although NPM, SPM and NLMM are totally different models and estimation procedures are also totally different, SPM and NLMM have nonparametric function which is handled in NPM. Hence, we are inspired from literature and created a unique solution for NLMM. The weight function given in Yalaz [22] helped us to generate the wavelet predictor in NLMM. In this study, we introduce wavelet approach to smooth nonparametric function under known measurement error in NLMM by expanding these previously studied topics.

We consider an experiment with n subjects and n_i observations over time for the ith subject. The response variable y_{ij} for the ith subject at time point t_{ij} satisfies.

$$y_{ij} = g\left(t_{ij}\right) + Z_{ij}^T b_i + \epsilon_{ij}, i = 1,\dots,n; j = 1,\dots,n_i \quad (1)$$

where $g(.)$ is a twice differentiable smooth function on some finite interval, $\in_{ij}$ are independent random noises with mean zero and the variance Σ_i, b_i are independent $q_i \times 1$ vectors of random effects associated with covariates Z_{ij} with mean zero and covariance matrix D_i.

Denote Y, b and $\in$ be the vectors obtained from stacking up the n subject-specific vectors of the same symbol, for instance, $Y = (Y_1^T,\dots,Y_n^T)^T$, $Z = \text{diag}(Z_1,\dots,Z_n)$, $\in = (\in_1^T,\dots,\in_n^T)^T$, $g(t) = (g(t_1)^T,\dots,g(t_n)^T)^T$, where $Y_i = (y_{i1},\dots,y_{in_i})^T$ and similarly for Z_i, $\in_i$ and $g(t_i)$, then the model (1) can be written as

$$Y = g(t) + Zb + \epsilon, \quad (2)$$

where the covariance matrix of Y is $V = ZDZ^T + \Sigma$, which is also equal to $\text{diag}(V_1,\dots,V_n)$, where $N = \sum_{i=1}^n n_i$ and $D = \text{diag}(D_1,\dots,D_n)$. The assumptions about the covariance matrix of random effects and the error variance are similar to Lindstrom and Bates [23] and Zaixing [24].

By following Yalaz [22, 25], we introduce wavelet predictor in NLMM when the variable of nonparametric part has measurement error. Here,

$$\tau = t + \Delta t,$$

where Δt are iid measurement errors. It is assumed that Δt has a known distribution which is proposed by Fan and Truong [26] for NPM.

This study is configured as follows. We define NLMM errors in variables method for one dimensional wavelets ($d = 1$) in Section 2, and in Section 3 we propose a predictor of random effects parameter. To analyze finite sample properties some Monte Carlo simulation studies are done in Section 4. In Section 5, conclusions are given.

THEORY

Approximation Kernels and Family of Estimators for Nonparametric Function in NLMM

We get NPM, if random effects part is embedded into the response variable in a NLMM:

$$\underbrace{Y - Zb}_{Y^*} = g(t) + \epsilon,$$

for $E[\in|t]=0$. Hence, if b is known, nonparametric function can be estimated using nonparametric methods.

Let $g(t) = \dfrac{\int y^* g_{t,y^*}\left(t, y^*\right) dy^*}{f(t)} = \dfrac{(gf)(t)}{f(t)}$ where $f(t)$ is defined as a classical deconvolution problem. Our mainly purpose is to estimate $(gf)(t)$. We denote $p(t) := g(t) \times f(t)$ and consider a father wavelet φ on the real line satisfying the following conditions described by Chichignoud et al. [21].

- φ is compactly supported on $[-A, A]$; A is a positive integer.

- Denote $\varphi_{0k}(t) = \varphi(t-k)$. There exists a positive integer N, such that for any t and k.

$$\int \sum_{k\in\mathbb{Z}} \underbrace{\varphi(t-k)}_{\varphi_{0k}(t)} \underbrace{\varphi(y^*-k)}_{\varphi_{0k}(y^*)} (y^*-t)^l dy^* = \delta_{0l},$$

$$l = 0,\ldots,\mathcal{N},$$

where δ_{0l} is the Kronecker delta which is defined as Hardle et al. [27]

$$\delta_{0l} = \begin{cases} 1 & , \quad 0l = 0, \\ 0 & , \quad otherwise. \end{cases}$$

- φ is of class the space of functions having all continuous derivatives C^r, where $r \geq 2$.

The associated projection kernel on the space

$$V_j := span\left\{ \varphi_{jk}, k \in \mathbb{Z} \right\}, \quad j \in \mathbb{N}$$

is given for any t and y^* by

$$K_j\left(t, y^*\right) = \sum_k \varphi_{jk}(t)\, \varphi_{jk}\left(y^*\right),$$

where

$$\varphi_{jk}(t) = 2^{\frac{j}{2}} \varphi\left(2^j t - k\right), \quad j \in \mathbb{N}, k \in \mathbb{Z}.$$

Then the projection of $p(t)$ on V_j can be written as,

$$p_j(t) = K_j(p)(t) := \int K_j\left(t, y^*\right) p\left(y^*\right) dy^* = \sum_k p_{jk}\, \varphi_{jk}(t),$$

where

$$p_{jk} = \int p\left(y^*\right) \varphi_{jk}\left(y^*\right) dy^*.$$

In Chichignoud et al. [21], the authors adapted the kernel approach proposed by Fan and Truong [26] in their wavelet context and they introduced

$$\hat{p}_{jk} := \frac{1}{n}\sum_{i=1}^{n} Y_i^* \times (\mathcal{D}_j\varphi)_{j,k}\left(\tau_i\right) = 2^{\frac{j}{2}} \frac{1}{n}\sum_{i=1}^{n} Y_i^* \int \exp\left(-i2^j \tau_i - k\right) \frac{\phi_\varphi(s)}{\phi_\epsilon\left(2^j s\right)} ds,$$

$$\hat{p}_j(t) = \frac{1}{n}\sum_k \sum_{i=1}^{n} Y_i^* \times (\mathcal{D}_j\varphi)_{j,k}\left(\tau_i\right) \varphi_{jk}(t),$$

where $\phi_\varphi(s)$ is the Fourier transform of wavelet φ_{jk} and D_j is the deconvolution operator which is demonstrated by K_n in Fan and Truong [26] and defined as follows

$$\left(\mathcal{D}_j\varphi\right)\left(\tau\right) = \int \exp\left(-is\tau\right) \frac{\phi_\varphi(s)}{\phi_\epsilon\left(2^j s\right)} ds. \tag{3}$$

The authors also proposed a resolution level j selecting rule depending Goldenshluger and Lepskis' [28] methodology.

Constructions of Nonparametric Functions Estimator and Random Effects Predictor

For the model (2), lets denote the densities of τ and t by $f_\tau(.)$ and $f_t(.)$ respectively. Then, we describe the estimator of $f_t(.)$ as,

$$\hat{f}_n(t) = \frac{1}{n}\sum_k \sum_{i=1}^{n} (\mathcal{D}_j\varphi)_{j,k}\left(\tau_i\right) \varphi_{jk}(t),$$

where $(D_j\varphi)_{j,k}$ is given in (3).

Using the idea given in Yalaz [22], we demonstrate the weight function as

$$\omega_{ni}(.) = \frac{(\mathcal{D}_j\varphi)_{j,k}(.)}{\sum_i (\mathcal{D}_j\varphi)_{j,k}(.)} = \frac{1}{n}\frac{(\mathcal{D}_j\varphi)_{j,k}(.)}{\hat{f}_n(.)}.$$

As we mentioned before, if b is known, then the estimation of $g(.)$ can be found using the weight function given above as follows

$$g_n(t) = \sum_k \sum_{i=1}^{n} \omega_{ni}(t)\left(Y_i - Z_i b\right).$$

Hence, we need to predict random effects parameter, before to find the estimation of nonparametric function.

We show the variables as $\tilde{Y}_i = Y_i - \sum_k \sum_{i=1}^{n} \omega_{ni}\left(\tau_i\right) Y_i$ for $\tilde{Y} = (\tilde{Y}_1^T \ldots, \tilde{Y}_n^T)^T$ and $\tilde{Z}_i = Z_i - \sum_k \sum_{i=1}^{n} \omega_{ni}\left(\tau_i\right) Z_i$ for $\tilde{Z} = \text{diag}\left(\tilde{Z}_1,\ldots,\tilde{Z}_n\right)$. Then, we represent the model (2) as

$$\tilde{Y} = \tilde{Z}b + \epsilon, \tag{4}$$

where $\epsilon = \left(\epsilon_1^T,\ldots,\epsilon_n^T\right)^T$ and $b = \left(b_1^T,\ldots,b_n^T\right)^T$ which is a $(n \times q) \times 1$ vector of random effects with mean zero and covariance matrix $D = \text{diag}(D_1,\ldots,D_n)$. Because we translate the

NLMM to LMM, we write the variance-covariance matrix of $\tilde{Y}$ as V where $V = \tilde{Z}D\tilde{Z}^T + \Sigma$ and the block diagonal covariance matrix of $\in$ is Σ. Under (4), we display the joint Gaussian distribution of b and $\tilde{Y}$ as

$$\begin{pmatrix} b \\ \tilde{Y} \end{pmatrix} \sim N\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} D & D\tilde{Z}^T \\ \tilde{Z}D & V \end{bmatrix}\right),$$

and the conditional distribution of $\tilde{Y}$ given b is $\tilde{Y} \mid b \sim N\left(\tilde{Z}b, \Sigma\right)$. Then, we maximize the joint density of $\tilde{Y}$ and b

$$\begin{aligned} f\left(\tilde{Y}, b\right) &= f\left(\tilde{Y}|b\right) f(b) \\ &= (2\pi)^{-\frac{q}{2}} \left|\Sigma\right|^{-\frac{1}{2}} |D|^{-\frac{1}{2}} \times \exp\left\{-\frac{1}{2}\right. \\ &\quad \left.\left[\left(\tilde{Y} - \tilde{Z}b\right)^T \Sigma^{-1}\left(\tilde{Y} - \tilde{Z}b\right) + b^T D^{-1} b\right]\right\}, \end{aligned} \tag{5}$$

where $|.|$ describes the determinant of a matrix. We derive $\log f(\tilde{Y}, b)$ by giving the log function of (5) as follows

$$\begin{aligned} \log f\left(\tilde{Y}, b\right) &= \log f\left(\tilde{Y}|b\right) + \log f(b) \\ &= -\frac{1}{2}\left\{\begin{matrix} q\log(2\pi) + \log\left|\Sigma\right| + \log|D| + \\ \left[\left(\tilde{Y} - \tilde{Z}b\right)^T \Sigma^{-1}\left(\tilde{Y} - \tilde{Z}b\right) + b^T D^{-1} b\right] \end{matrix}\right\}, \end{aligned} \tag{6}$$

By discarding the constant term and the log function, the partial derivative of (6) with respect to the element of b equal to zero and employing $\hat{b}$ to denote the solution find

$$\tilde{Z}^T \Sigma^{-1} \tilde{Y} - \left(\tilde{Z}^T \Sigma^{-1} \tilde{Z} + D^{-1}\right)\hat{b} = 0.$$

We compute $(\tilde{Z}^T \Sigma^{-1} \tilde{Z} + D^{-1})^{-1} = D - D\tilde{Z}^T V^{-1} \tilde{Z} D$ via the Sherman-Morrison-Woodbury Theorem ([29]) and after algebraic simplifications, we introduce the wavelet predictor in NLMM as

$$\hat{b} = D\tilde{Z}^T V^{-1} \tilde{Y}. \tag{7}$$

After prediction of random effects parameter, an estimation of nonparametric component $g(t)$ can be described as

$$\hat{g}_n(t) = \sum_k \sum_{i=1}^{n} \omega_{ni}(t)\left(Y_i - Z_i \hat{b}\right). \tag{8}$$

RESULTS AND DISCUSSION

Simulation Study

In this section, we benefit from Monte Carlo simulation approach to demonstrate the finite sample properties of the estimators by using MATLAB.

We consider the model with two random effects ($q = 2$) as follows;

$$y_{ij} = b_1 + b_2 time_{ij} + g\left(t_{ij}\right) + \varepsilon_{ij},$$

$$b_i \overset{iid}{\sim} N(0, D),\ \varepsilon_{ij} \overset{iid}{\sim} N\left(0, \sigma_\epsilon^2 I_{n_i}\right),$$

where $D = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$ is the AR(1) process with $\rho = 0.60$ and $time_{ij}$ shows time which was taken as the same set of occasions, $\{t_{ij} = j$ for $i = 1,\ldots,n, j = 1,\ldots,n_i\}$. We consider $n = 4, 8, 16, 32$ subjects and per subject $n_i = 8$ observations. Then, we calculate the simulation results with the sample sizes of $N = \Sigma_{i=1}^{n} n_i = 32,64,128,256$.

Nonparametric functions are considered as

$$g_1(t) = 4.26(\exp(-3.25t) - 4\exp(-6.5t) + 3\exp(-9.75t)),$$

$$g_2(t) = \begin{cases} 4t^2(3-4t), & 0 \le t \le 0.5, \\ \frac{4}{3}t(4t^2 - 10t + 7) - 15, & 0.5 < t \le 0.75, \\ \frac{16}{3}t(1-t)^2, & 0.75 < t \le 1. \end{cases}$$

In all examples, the density of both the true regressor t and the measurement error Δt are chosen as the most common combinations of ordinarily smooth distributions which are summarized in Table 1.

We consider the normal distribution as an example of a supersmooth distribution, and the Laplace (or double explonential) distribution, uniform distribution and beta distribution for the ordinarily smooth case. Because *Beta* (2,2) and *Beta* (0.5,2) distributions reflect two different behaviors on [0,1] we use them. Finally, following the asymptotic considerations given in Chichignoud et al. [21], we choose the primary resolution level j that we have used throughout our simulations as $j(n) = \log_2(\log(n)) + 1$.

The average values of 100 replicates of the mean squared error of response variable (MSE) and the mean squared

Table 1: Examples

Example 1	Example 2	Example 3
$t \to Beta(2,2)$	$t \to Beta(0.5,2)$	$t \to Uniform[0,1]$
$\Delta t \to L(0,0.001)$	$\Delta t \to L(0,0.001)$	$\Delta t \to L(0,0.001)$
$\in \to N(0,0.25)$	$\in \to N(0,0.25)$	$\in \to N(0,0.25)$
$\sigma_\epsilon^2 = 0.25$	$\sigma_\epsilon^2 = 0.25$	$\sigma_\epsilon^2 = 0.25$

Table 2: Simulation results g_1

	***n* = 32**		***n* = 64**		***n* = 128**		***n* = 256**	
Example 1								
	MSE	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**
Our	0.2388	0.1910	0.6923	0.2220	1.0313	0.3119	2.5672	0.2685
NoME	0.6536	0.5122	1.1017	0.4781	2.2137	0.4912	4.4953	0.4749
Example 2								
	MSE	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**
Our	0.3055	0.1960	0.5782	0.2430	0.8895	0.7299	1.2934	1.5225
NoME	0.6719	0.4866	1.3092	0.4689	2.5972	0.4104	5.1700	0.4113
Example 3								
	MSE	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**
Our	0.2707	0.1387	0.4807	0.2204	0.9833	0.2877	3.3393	0.3815
NoME	0.6415	0.5112	1.1410	0.4661	2.2870	0.4546	4.7235	0.4069

Table 3: Simulation results g_2

	***n* = 32**		***n* = 64**		***n* = 128**		***n* = 256**	
Example 1								
	MSE	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**
Our	0.2156	0.1944	0.6595	0.1661	0.8946	0.3563	2.2521	0.1975
NoME	0.5859	0.5228	0.9974	0.4768	2.0009	0.4841	4.0860	0.4816
Example 2								
	MSE	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**
Our	0.3779	0.3670	0.6439	0.2807	1.3524	0.3938	1.4073	1.7035
NoME	0.7777	0.4121	1.4087	0.4848	2.8928	0.4215	5.6872	0.4205
Example 3								
	MSE	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**	**MSE**	**MSEP**
Our	0.2720	0.1184	0.4633	0.1936	1.0232	0.2354	3.1527	0.4068
NoME	0.6076	0.5220	1.0576	0.4589	2.2507	0.4735	4.5764	0.4188

error of predictor (MSEP) for measurement error case (Our) and ignoring measurement error case (NoME) which are considered in four different sample sizes are given in Table 2 for g_1 and given in Table 3 for g_2. When we compare our results with ignoring measurement error case, it can be easily seen that the results are encouraging.

We also compared the finite sample and the asymptotic distributions of our estimator and the estimator ignoring measurement. In Figures (1-3), the abscissa is $Z = (Var(g(t)))^{-½}\ (g(t) - E(g(t)))$ and the ordinate is probability. The empirical cumulative distribution function (CDF) curve of the estimator (indicated by dashed line) fits very well with the normal CDF (indicated by solid line) curve and it is better than the NoME case (indicated by dotted line).

CONCLUSION

This paper presents wavelet estimation of the NLMM when nonparametric part has measurement error. If the measurement error is known, prediction of random effects parameter using wavelet approach is possible. And so, we introduce the predictor of b based on projection kernels on wavelets and borrowing the ideas of deconvolution technique. We implemented some simulations to illustrate the theoretical results. Because in literature NLMM is not

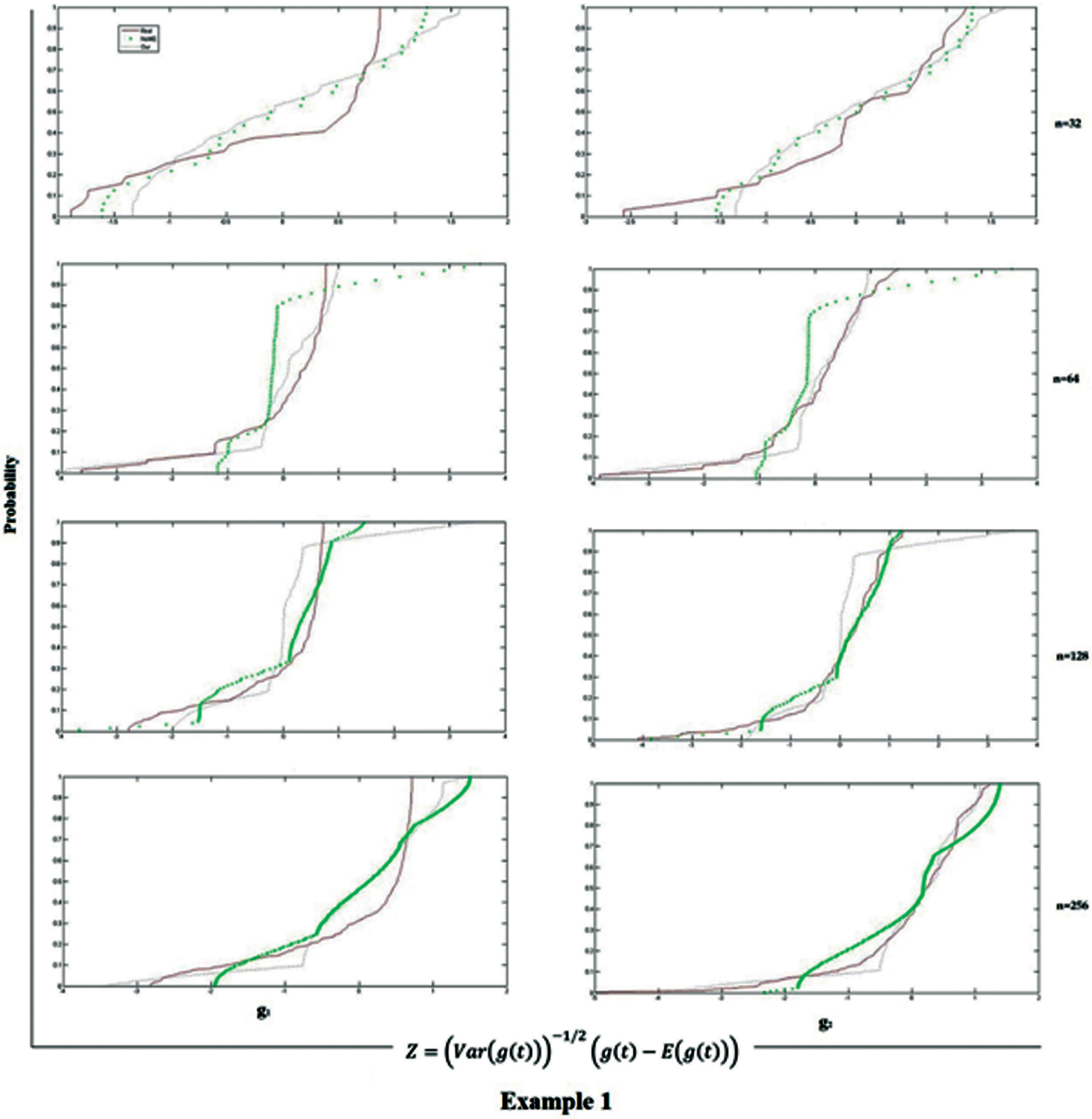

Figure 1. Comparison of the finite sample and asymptotic distributions of the estimator in Example 1.

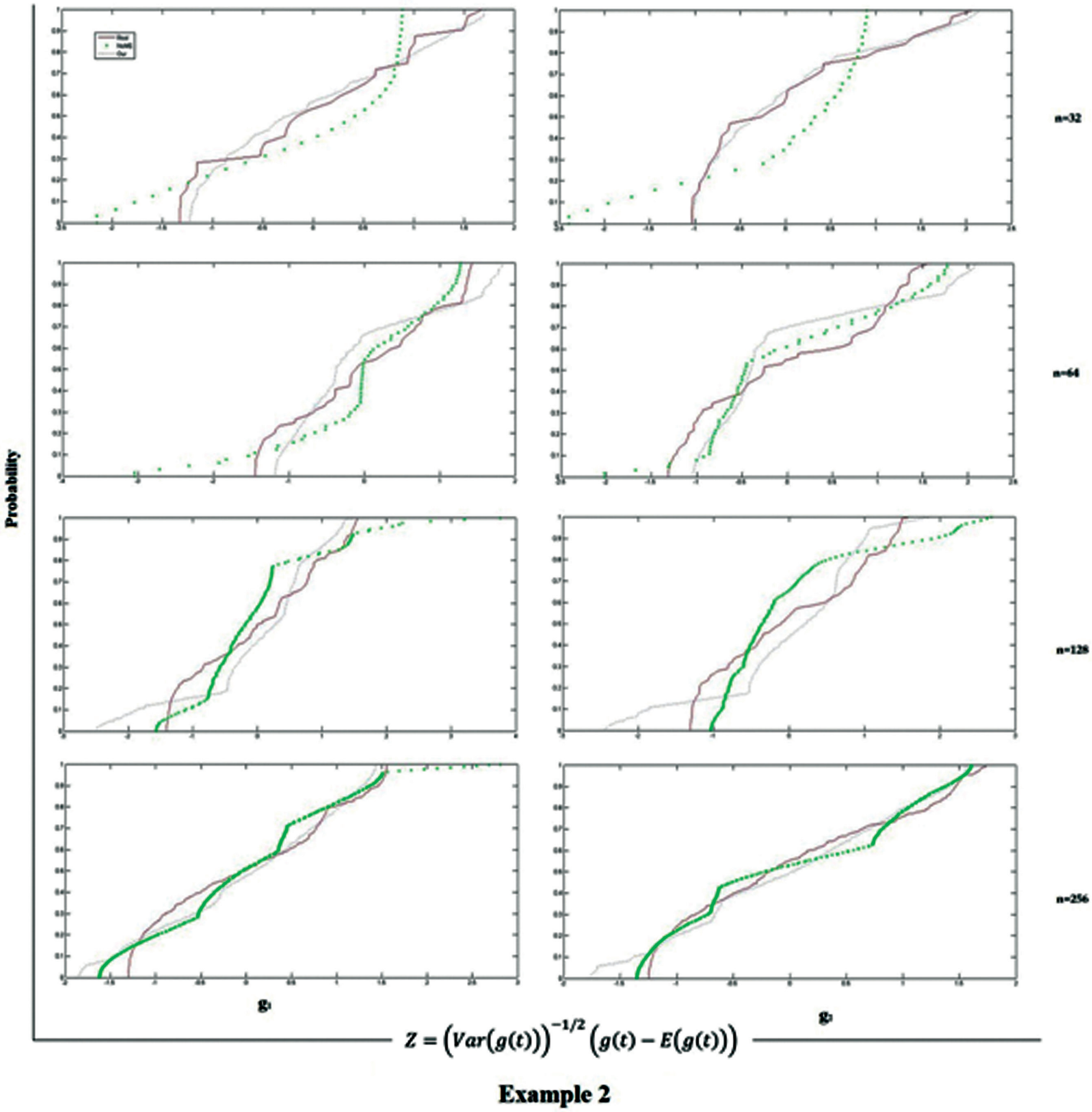

Figure 2: Comparison of the finite sample and asymptotic distributions of the estimator in Example 2.

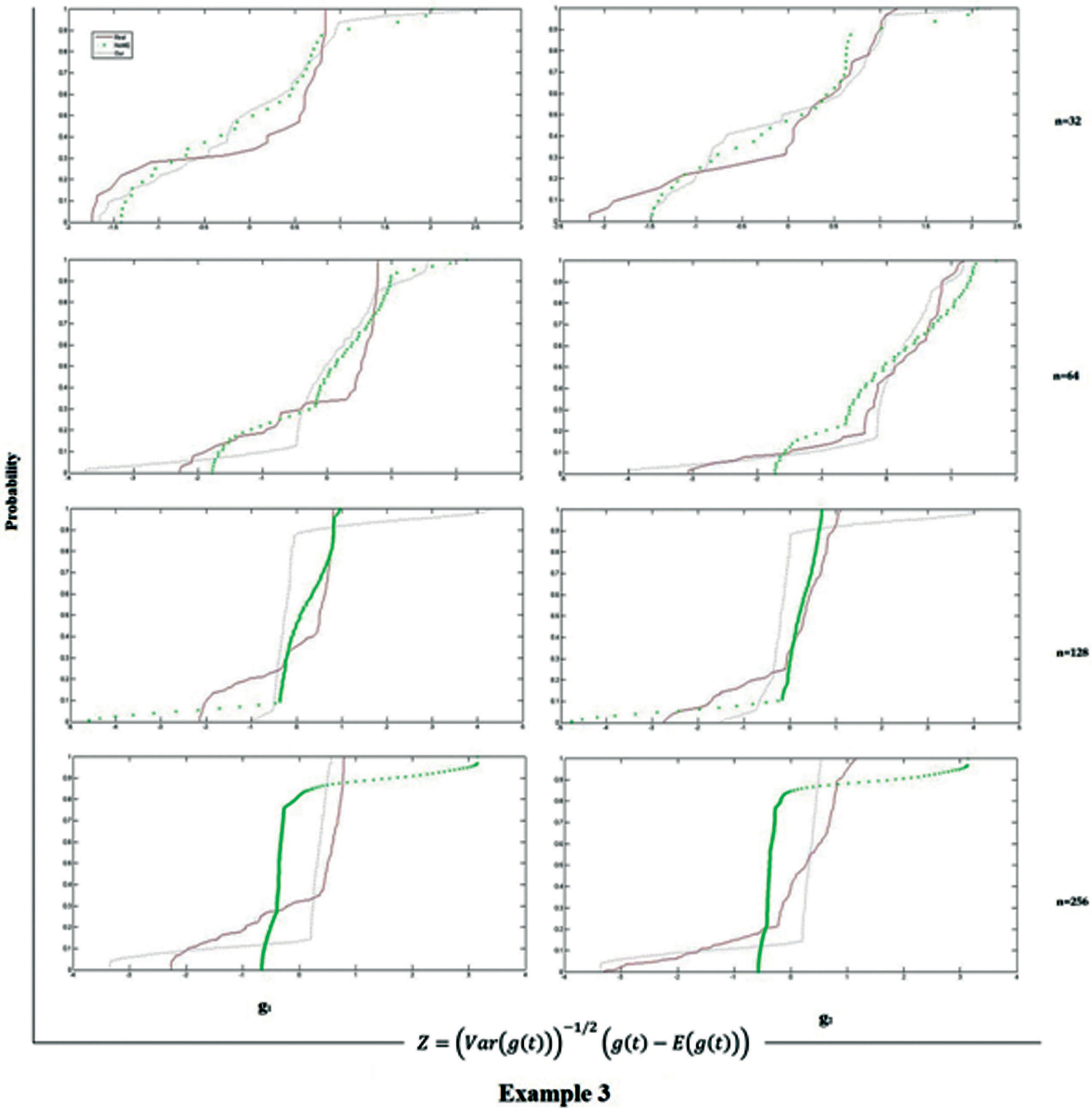

Figure 3: Comparison of the finite sample and asymptotic distributions of the estimator in Example 3.

considered using wavelet approach, we compared our work with the circumstances ignoring measurement error. We aimed at estimating the two different regression functions at different three points with three different densities and finally compared these results. It is discussed in the simulation that the resulting rates are comparable to no measurement error case. The performances are much better for the proposed model then the model ignoring measurement errors. Asymptotic normality of proposed predictor is still open one and should be investigated.

AUTHORSHIP CONTRIBUTIONS

Authors equally contributed to this work.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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