Table 1. The absolute errors of sine-cosine wavelet method for different k values for Example 1

t	(k=6, L=1)	(k=7, L=1)	(k=8, L=1)
0	1.6248E-06	2.0329E-07	2.5428E-08
0.2	1.2640E-04	1.1938E-05	3.0823E-05
0.4	9.5463E-04	2.4670E-05	2.3622E-05
0.6	2.1195E-04	5.5888E-04	5.3325E-05
0.8	1.9748E-04	1.8901E-04	4.9591E-05

Table 2. The absolute errors of sine-cosine wavelet method for different k values for Example 2

t	(k=6, L=1)	(k=7, L=1)	(k=8, L=1)
0	1.2881E-05	2.2776E-06	4.0270E-07
0.2	2.3397E-04	2.2190E-05	5.7513E-06
0.4	1.2548E-04	3.2547E-04	3.1143E-05
0.6	2.2836E-04	6.0073E-04	5.7348E-04
0.8	1.8421E-04	1.7620E-04	4.6190E-04

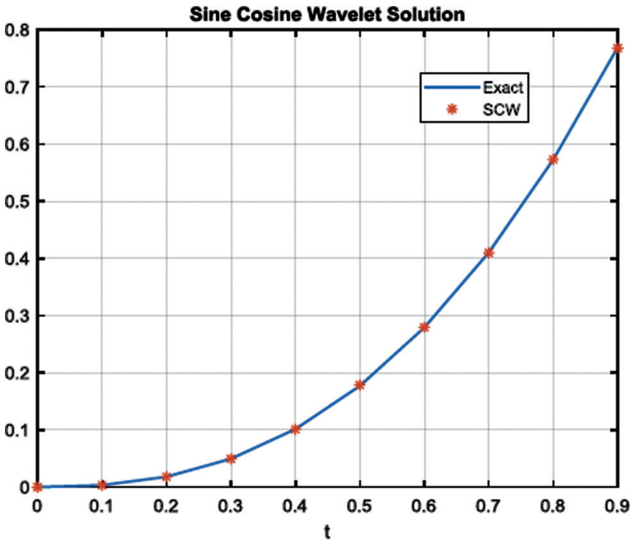

Figure 2. Sine-cosine wavelet and the exact solution for Example 2.

ILLUSTRATIVE EXAMPLES

In this section, we apply the method to two examples and we have carried out all of the numerical calculations using Matlab R2020a.

Example 1: We first consider following B-T equation

$$D^{3/2}y(t)+D^2y(t)+y(t)=t^3+6t+\frac{8}{\Gamma(0.5)t^{0.5}},$$
$$y(0)=y'(0)=0,\qquad t\in[0,1),$$

with exact solution is $y(t) = t^3$.

By applying the method described in section IV, the values of the unknown matrix C^T are obtained. Figure 1 represents the sine -cosine wavelet solution for k = 6, L = 1 and the exact solution. From Figure 1, it is clear that sine-cosine wavelet solution provides good approximation with exact solution.

The absolute errors for different k values of sine-cosine wavelet solution (L = 1) are given in Table 1. As results suggest, when the values of k increase, the absolute error decreases and our solution converges to the exact solution which proves the validity of the method.

Example 2: In this example, we consider the following B-T equation:

$$D^{3/2}y(t)+D^2y(t)+y(t)=\frac{15}{4}\sqrt{t}+\frac{15}{8}\sqrt{\pi t}+t^2\sqrt{t},$$
$$y(0)=y'(0)=0,\qquad t\in[0,1),$$

with exact solution is $y(t) = t^{2.5}$.

By using the proposed technique presented in section IV, we obtain the approximate solution of B-T equation. Table 2 represents absolute errors for k = 6, 7, 8 and L=1. In addition, the comparison of the numerical solutions and the exact solutions for k = 6 and L=1 is given in Figure 2. As can be seen, numerical results show the efficiency of our solution. Numerical results also demonstrate that the method is fast and can be applied to real-time problems.

CONCLUSIONS

In this study, we propose sine-cosine wavelet method with block pulse functions to solve fractional Bagley–Torvik differential equations. This method converts this fractional differential equation to system of linear algebraic equations. We obtain the numerical solutions of the Bagley-Torvik equations by solving this system. Since the basis functions of sine–cosine wavelets are orthogonal, the implementation of present approach is simple and easy. Numerical examples are included to illustrate the validity and the accuracy of the proposed numerical method.

DATA AVAILABILITY STATEMENT

The authors confirm that the data that supports the findings of this study are available within the article. Raw data that support the finding of this study are available from the corresponding author, upon reasonable request.

CONFLICT OF INTEREST

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

ETHICS

There are no ethical issues with the publication of this manuscript.

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