Journal of Advances in Manufacturing Engineering
Web page info: https://eds.yildiz.edu.tr/JAME
DOI: 10.14744/ytu.jame.2021.00002

Original Article

Investigation of the prediction capability of Yld89 yield criterion for highly anisotropic sheet materials

Bora ŞENER

Department of Mechanical Engineering, Yıldız Technical University, İstanbul, Turkey

ARTICLE INFO

Article history
Received: 26 January 2021
Accepted: 23 March 2021

Key words:
Plastic strain ratio, stress ratio, yield surface, Yld89 yield criterion.

ABSTRACT

In the present work, the prediction capability of Yld89 criterion from anisotropic yield functions was investigated in the view of the anisotropic behavior of the sheet metals. Investigation was conducted on two highly anisotropic sheet materials: an aluminum alloy (AA2090-T3) and an advanced high strength steel (TRIP 780). The in-plane variation of material anisotropy and normalized yield surface contours were considered in the evaluation of the prediction capability of the criterion. Firstly, the model coefficients were determined according to stress and strain based definitions. Then, the planar variations of the yield stress and plastic strain ratios and normalized yield surface contours of the materials were predicted according to both identification procedures. Finally, the computed results were compared with experiments to evaluate prediction capability of the model. It was observed from the comparisons that the planar variations of the yield stress ratio could successfully predicted by stress based definition, while the variations of the plastic strain ratios in the sheet plane could accurately predicted by strain based definition. Besides, it was determined that elastic region predicted from strain based definition was larger than stress based definition for AA2090-T3, while the predicted elastic region from stress based definition was slightly larger in than that of strain based definition for TRIP 780 material.

Cite this article as: Şener B. Investigation of the prediction capability of Yld89 yield criterion for highly anisotropic sheet materials. J. Adv. Manuf. Eng. 2021;2:1:7–13.

INTRODUCTION

Plastic behavior of the materials is described with yield criteria in phenomenological plasticity approach. The Tresca and von Mises yield criteria are two popular models which are used for isotropic cases. The former criterion considers the maximum shear stress, while the latter is derived from distortional energy. Apart from these two criteria, several isotropic yield criteria have been proposed by researchers in the literature. Hershey [1] and Hosford [2] proposed non-quadratic isotropic yield functions. Hershey and Hosford models contain an exponent which indicates the crystallographic structure of material. This exponent gives flexibility to the models and the computed yield surfaces from the criteria for body and face centered cubic materials (BCC and FCC) lie between Tresca and von Mises yield surfaces. However, these criteria don't contain shear stress and therefore they could be used only when

***Corresponding author.**
*E-mail address: borasen@yildiz.edu.tr

the stress components coincide with orthotropic directions of material. Then, Barlat and Richmond [3] modified Hosford yield function and add shear stress component by using stress invariants. Bassani [4] developed transversely isotropic yield functions for FCC and BCC materials and could successfully model the plastic behavior of these materials. Budiansky [5] developed a different yield criterion which is defined in polar coordinates and he could accurately define planar isotropy of BCC and FCC materials. However, these isotropic functions couldn't provide realistic results for sheet metal forming simulations. Because sheet metals exhibit planar anisotropy due to the preferred orientation of grains occurred in the rolling process and anisotropic yield functions are required to use to define plastic behavior of these materials. Hill [6] developed an anisotropic yield criterion by adding the coefficients to von Mises criterion in 1948. Hill48 yield criterion could be used for both three dimensional (3D) and plane stress state (2D) in finite element (FE) programs. It could give consistent results for steels, but this model has some limitations in strongly anisotropic materials such as aluminum alloys, titanium alloys and high strength steels. The limitations of Hill48 yield criterion in the modeling of anisotropic behavior of some aluminum alloys were firstly observed by Woodthorpe and Pearce [7] and this was referred as anomalous behavior in the literature. Thereafter, Hill suggested non-quadratic yield functions in order to define anomalous behavior of aluminum alloys in 1979, 1990 and 1993 [8–10]. Hill79 model could define the first anomalous behavior, but it couldn't describe the second anomalous behavior of aluminum alloys. Hill90 criterion describes both the first and second anomalous behavior, but this model requires long simulation time in FE analyses [11]. Hill93 criterion is an useful model for FE simulations, but this model hasn't contain shear stress component.

In this study, the prediction capability of Yld89 yield criterion developed by Barlat and Lian [12] was investigated. An aluminum alloy (AA2090-T3) and an advanced high strength steel sheet (TRIP780) were selected in the study due to their high planar anisotropy coefficients. The planar variations of yield stress and plastic strain ratios and normalized yield surfaces were predicted with Yld89 criterion and the predicted results were compared with experiments.

Yld89 Yield Criterion

Barlat and Lian extended Hosford yield criterion and developed an anisotropic yield criterion in 1989 [12]. This yield criterion has been developed for 2D stress state and it can be expressed as follows:

$$2\sigma_e^{\ m} = a\left|K_1 + K_2\right|^m + a\left|K_1 - K_2\right|^m + c\left|2K_2\right|^m \tag{1}$$

K_1 and K_2 are the stress invariants and they are given in Eq. (2)

$$K_1 = \frac{\sigma_{xx} + h\sigma_{yy}}{2}, K_2 = \sqrt{\frac{\left(\sigma_{xx} - \sigma_{yy}\right)^2}{4} + p^2\sigma_{xy}^{\ 2}} \tag{2}$$

where a, c, h and p are material coefficients and the exponent m indicates the crystallographic structure of material. This parameter is taken as 6 and 8 for BCC and FCC metals, respectively. Effective stress of Yld89 criterion is determined from Eq. (1) and yield function can be written as follows:

$$f = \left(\frac{a\left|K_1 + K_2\right|^m + a\left|K_1 - K_2\right|^m + c\left|2K_2\right|^m}{2}\right)^{1/m} - \sigma_0 \tag{3}$$

If uniaxial yield stress at the angle θ with respect to rolling direction denotes with σ_θ, stress tensor components can be defined based on angle by using tensor transformations as follows: and

$$\sigma_{xx} = \sigma_\theta \cos^2\theta, \sigma_{yy} = \sigma_\theta \sin^2\theta \text{ and } \sigma_{xy} = \sigma_\theta \sin\theta\cos\theta \tag{4}$$

If stress components defined in Eq. (4) are substituted into Eq. (3) and angular variation of yield stress is determined as follows:

$$\sigma_\theta = \frac{\sigma_0}{\left\{\dfrac{a\left(F_1+F_2\right)^m + a\left(F_1-F_2\right)^m + 2^m cF_2^{\ m}}{2^{m+1}}\right\}^{1/m}} \tag{5}$$

If Eq. (5) is divided by σ_0, angular variation of yield stress ratio is determined as follows:

$$\overline{\sigma}_\theta = \frac{\sigma_\theta}{\sigma_0} = \frac{2^{m+1/m}}{\left\{a\left(F_1+F_2\right)^m + a\left(F_1-F_2\right)^m + 2^m cF_2^{\ m}\right\}^{1/m}} \tag{6}$$

where

$$F_1 = \cos^2\theta + h\sin^2\theta, F_2 = \sqrt{\left(cos^2\theta - hsin^2\theta\right)^2 + 4p^2sin^2\theta cos^2\theta} \tag{7}$$

Plastic strain ratio at the angle θ (r_θ) with respect to rolling direction is determined by using flow rule and volume constancy principle. The formula of (r_θ) can be written as follows:

$$r_\theta = -\frac{\left(\dfrac{\partial f}{\partial\sigma_{11}}\right)\sin^2\theta + \left(\dfrac{\partial f}{\partial\sigma_{22}}\right)\cos^2\theta - \left(\dfrac{\partial f}{\partial\sigma_{12}}\right)\sin\theta\cos\theta}{\dfrac{\partial f}{\partial\sigma_{11}} + \dfrac{\partial f}{\partial\sigma_{22}}} \tag{8}$$

The angular variation of plastic strain ratio is determined when stress transformation equations are substituted into Eq. (8). Yld89 coefficients could either be identified by using yield stress ratios (stress based definition) or by using plastic strain ratios (strain based definition).

Table 1. Mechanical properties of AA2090-T3 [13]

Angle	Yield stress ratio	Plastic strain ratio
0	1.000	0.212
15	0.960	0.327
30	0.910	0.692
45	0.811	1.577
60	0.809	1.038
75	0.881	0.538
90	0.910	0.692
Biaxial	1.035	0.670

Table 2. Mechanical properties of TRIP780 [14]

Angle	Yield stress ratio	Plastic strain ratio
0	1.000	0.720
15	1.001	0.750
30	0.997	0.830
45	0.994	0.920
60	0.996	0.920
75	1.017	0.860
90	1.008	0.830
Biaxial	1.053	0.817

In this article, the coefficients obtained with stress based definition are denoted with $a_\sigma, c_\sigma, h_\sigma$ and p_σ while the coefficients obtained with strain based definition are denoted with and a_r, c_r, h_r and p_r. The coefficients are identified with uniaxial yield stress ratios along rolling, diagonal and transverse directions (RD, DD and TD) and biaxial yield stress ratio ($\bar{\sigma}_0, \bar{\sigma}_{45}, \bar{\sigma}_{90}$ and $\bar{\sigma}_b$) in stress based definition, while they are determined with plastic strain ratios along RD, DD and TD (r_0, r_{45}, r_{90}) and yield stress ratio along RD ($\bar{\sigma}_0$) in strain based definition. The equations with respect to stress and strain based definitions are given below:

$$h_\sigma = \frac{1}{\bar{\sigma}_{90}}, a_\sigma = 2\frac{\left(\frac{1}{\bar{\sigma}_b}\right)^m - (1-h)^m}{1+h^m-(1-h)^m} \tag{9}$$

$$a_\sigma + c_\sigma = 2 \tag{10}$$

$$h_r = 2\sqrt{\frac{r_0}{r_{90}}\left(\frac{1+r_{90}}{1+r_0}\right)}, c_r = 2\sqrt{\frac{r_0 r_{90}}{(1/r_0)(1/r_{90})}} \tag{11}$$

$$a_r + c_r = 2 \tag{12}$$

The coefficient p could be determined by using numerical methods. The difference between predicted and experimental stress ratios along diagonal direction is minimized in stress based definition, while the difference between plastic strain ratios along same direction is minimized in strain based definition. Interior point algorithm was used to minimize objective function in this study. Two objective functions are created for stress and strain based definitions and they are given in Eq. (13).

Table 3. Yld89 coefficients of AA2090-T3

Identification type	a	c	h	p
Stress based	0.4811	1.5188	1.0986	1.3258
Strain based	1.4655	0.5345	0.6532	1.1500

Table 4. Yld89 coefficients of TRIP780

Identification type	a	c	h	p
Stress based	1.4671	0.5329	0.9920	1.0117
Strain based	1.1285	0.8715	0.9607	1.0200

$$Obj_1 = \left(\sigma_{45_{pr}} - \sigma_{45_{\exp}}\right)^2, \quad Obj_2 = \left(r_{45_{pr}} - r_{45_{\exp}}\right)^2 \tag{13}$$

MATERIAL AND METHODS

Materials

Two strongly anisotropic materials AA2090-T3 (t = 1.6 mm) and TRIP 780 (t = 1.05 mm) were selected in this study. The mechanical properties of these materials were taken from the literature and they are given in Table 1 and Table 2, respectively.

Interior-Point Algorithm

Interior point algorithm is a numerical optimization method used to solve a sequence of approximate minimization problems. In the method inequalities constraints are transformed into equalities by adding slack variables. The optimization problem is defined as follows:

$$\min_x f(x), \text{subject to } h(x)=0 \text{ and } g(x)\le 0 \tag{14}$$

$$\min_{x,s} f_\mu(x,s) = \min_{x,s} f(x) - \mu\sum_i \ln(s_i), \text{subject to } h(x)=0 \text{ and } g(x)+s=0 \tag{15}$$

The added logarithmic term is called as barrier function. Method uses a direct step to solve Karush-Kuhn-Tucker (KKT) conditions [15].

RESULTS AND DISCUSSION

Yld89 coefficients of the materials were determined according to stress and strain based identifications. Then planar variations of the yield stress and plastic strain ratios and normalized yield surfaces were predicted by using the obtained coefficients for both of the materials and the numerical results were compared with experiments. Comparisons between theoretical and experimental results for stress and strain based definitions are given below.

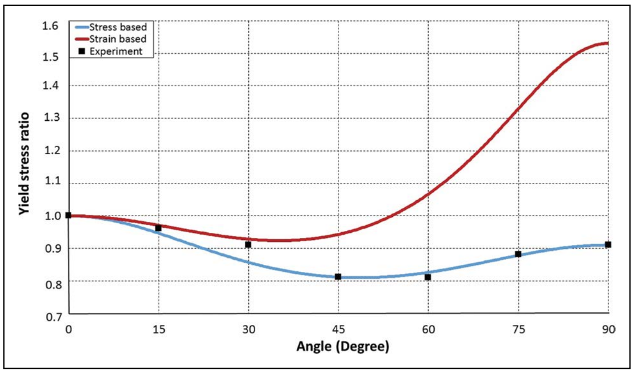

Figure 1. Comparison of yield stress ratios for AA2090-T3.

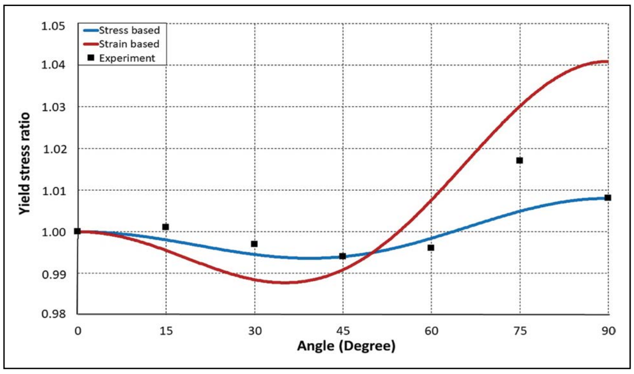

Figure 3. Comparison of yield stress ratios for TRIP780.

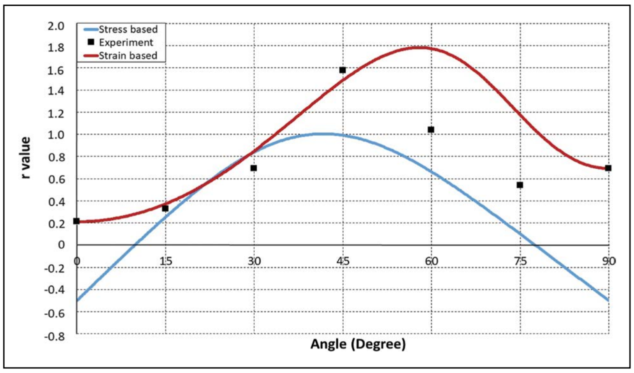

Figure 2. Comparison of plastic strain ratios for AA2090-T3.

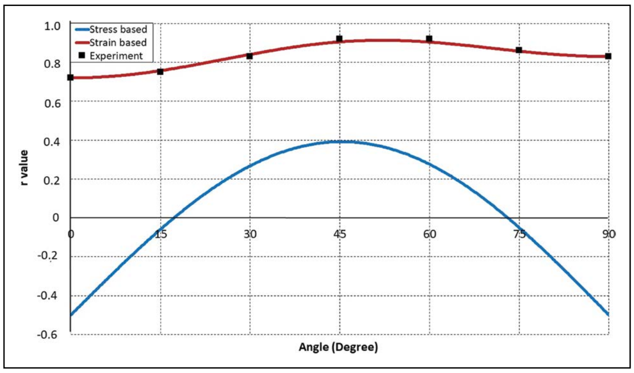

Figure 4. Comparison of plastic strain ratios for TRIP780.

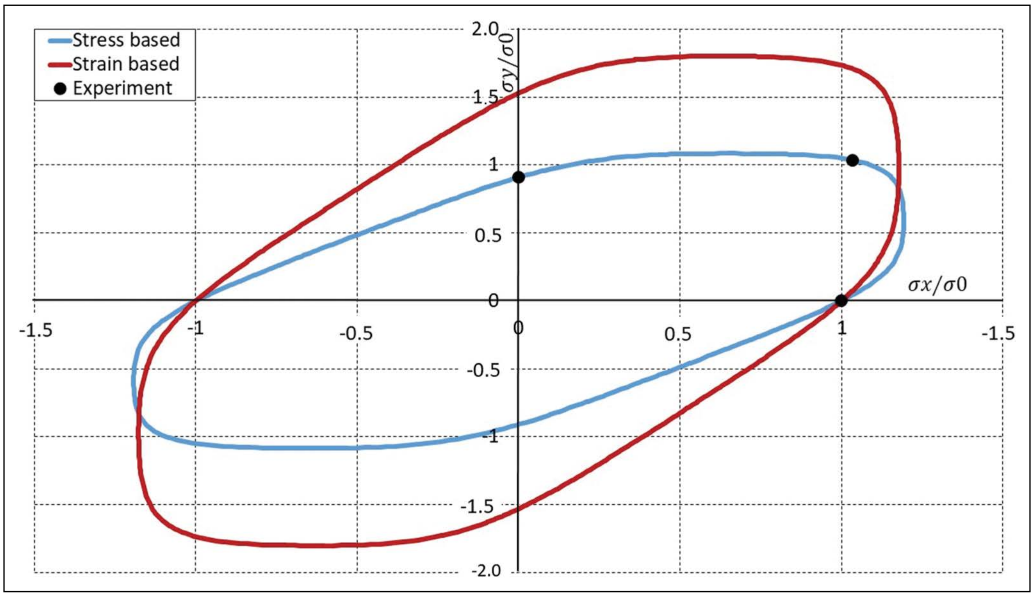

Figure 5. The computed yield surfaces for AA2090-T3.

Planar Variations of Yield Stress and Plastic Strain Ratios

Yld89 model parameters of both materials were determined according to stress and strain based identification procedures. The determined coefficients for AA2090-T3 and TRIP780 materials are given in Table 3 and Table 4, respectively.

The variations of the yield stress and plastic strain ratios in the sheet plane were predicted by using Eq. (6) and Eq. (8). The predicted results from both identification methods were compared with each other and experimental results. Comparison results for AA2090-T3 and TRIP 780 materials were given from Figure 1–4.

It is seen from Figure 1–4 that stress based identifica-

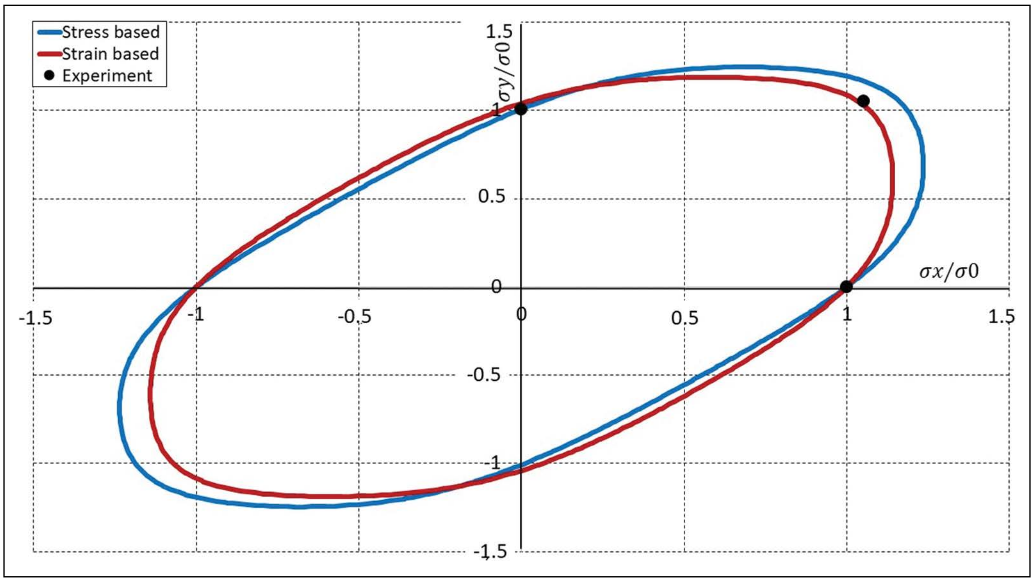

Figure 6. The computed yield surfaces for TRIP780.

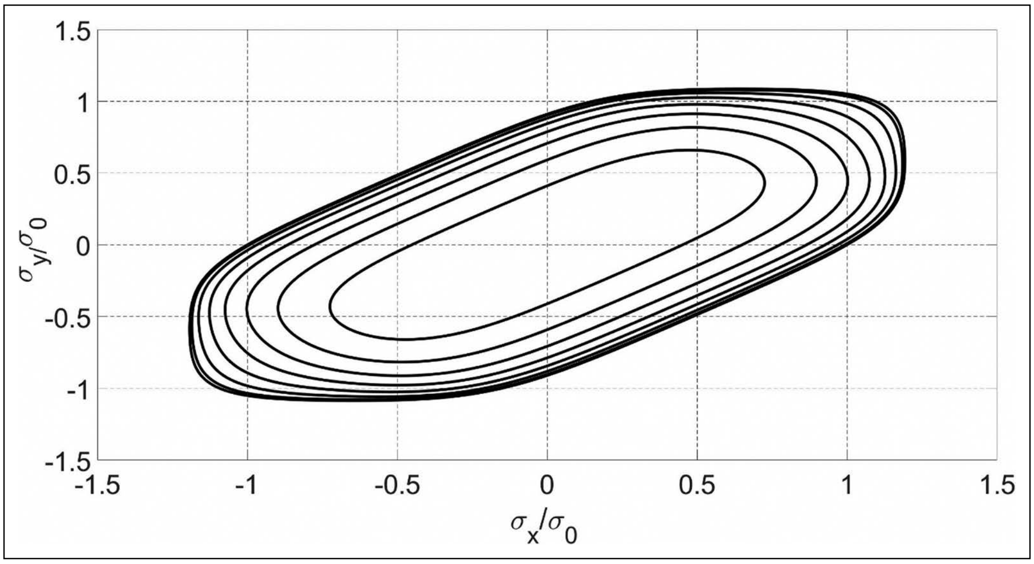

Figure 7. Yield surfaces computed from stress based identification in different shear stresses (AA2090-T3).

tion could accurately predict the planar variation of yield stress ratios, while strain based identification predict the planar variation of the plastic strain ratios for both of the materials. In addition to that the error percentages between the predicted and experimental plastic strain ratios along diagonal direction in strain based definition were determined as 5.4% and 1.58% for AA2090-T3 and TRIP 780, respectively. When the experimental and predicted yield stress ratios along the diagonal direction were investigated, it was observed that the predicted and experimental yield stress ratios at 45^0 were the same in stress based definition for both of the materials. These results indicate that interior point algorithm could successfully minimize the objective functions.

Normalized Yield Surfaces

The yield surfaces of the materials were predicted according to both identification methods and the predicted surfaces were compared with each other and experimental results. Comparison results for AA2090-T3 and TRIP780 are shown in Figure 5 and Figure 6, respectively. It is seen from Figure 5 and Figure 6 that the contours of the yield surfaces change according to coefficient identification procedures. It is observed from Figure 5 that the predicted elastic region by using strain based definition was larger than that of the stress based definition and also the biaxial yield stress ratio of AA2090-T3 alloy could be accurately predicted with stress based definition. On the other hand, a different result was observed for TRIP780 steel. As it is seen from Figure 6

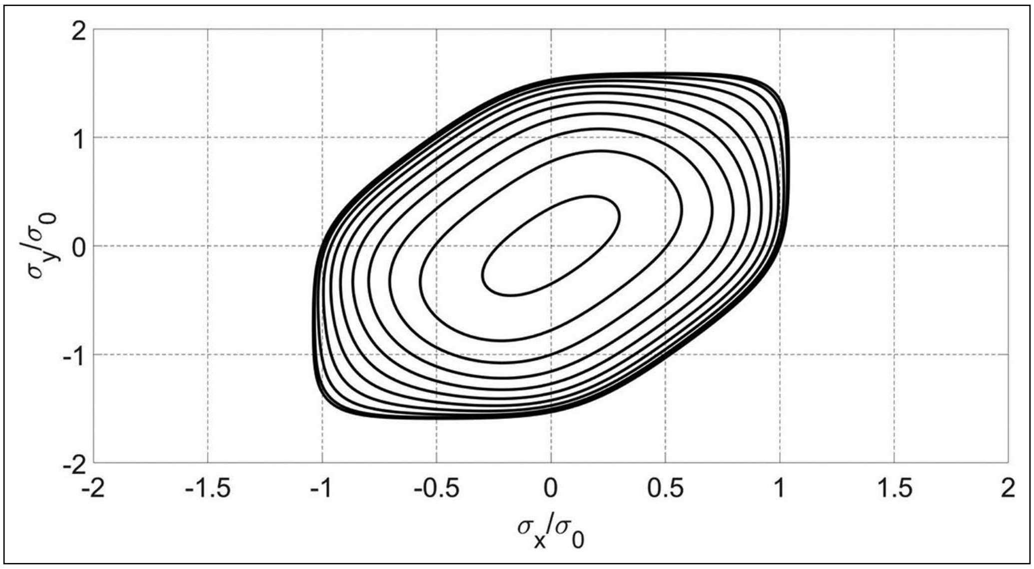

Figure 8. Yield surfaces computed from strain based identification in different shear stresses (AA2090-T3).

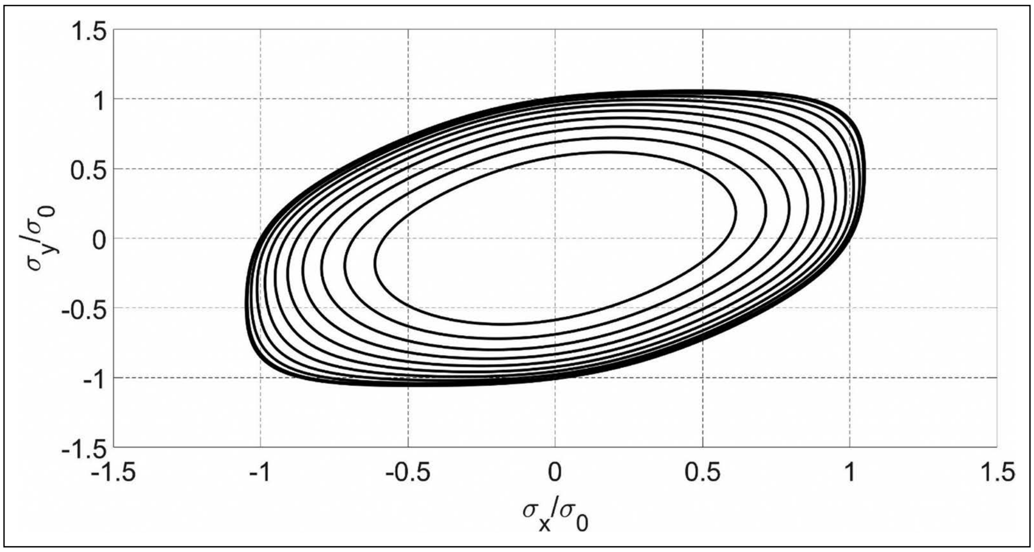

Figure 9. Yield surfaces computed from stress based identification in different shear stresses (TRIP780).

that the predicted elastic region by using strain based definition was slightly smaller than that of stress based definition.

Shear stress was assumed as zero ($\sigma_{xy} = 0$) in these comparisons. In order to conduct a comprehensively investigation, yield surfaces computed from both identification procedures were plotted in different shear stress levels and the results were given from Figure 7–10 for both of the materials.

CONCLUSION

In this study, the prediction capability of Yld89 yield criterion was investigated and AA2090-T3 aluminum alloy and TRIP780 advanced high strength steel sheets were selected as test materials. The coefficients of the yield criterion for both of the materials were determined by using stress and strain based identification procedures. Variations of the yield stress and plastic strain ratios of the materials in the sheet plane were predicted according to the both identification procedure and then normalized yield surfaces were computed in two dimensional stress space. The predicted results were compared with experimental results in order to investigate the prediction capability of the yield function. According to the obtained results from the study, the following conclusions could be drawn:

1. Yld89 yield criterion couldn't simultaneously predict the planar variation of the both yield stress and plastic strain ratios of the materials. Because the model has only four coefficients and these coefficients could be identified either yield stress or plastic strain ratios.
2. The contours of the normalized yield surfaces are changed according to the coefficient identification procedure. The elastic region predicted from strain based definition was larger than that of stress based definition

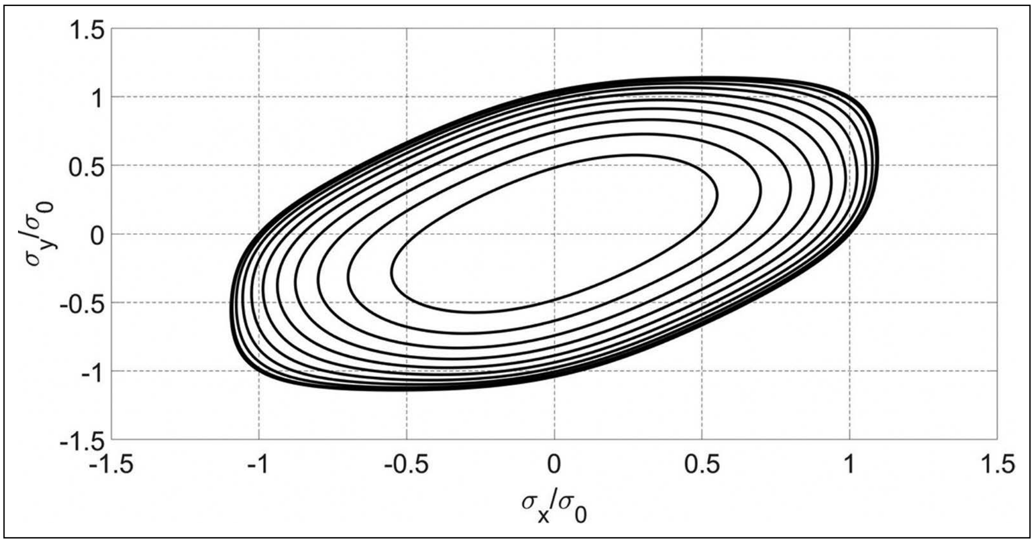

Figure 10. Yield surfaces computed from strain based identification in different shear stresses (TRIP780).

for AA2090-T3 material, while the opposite result was observed for TRIP780 steel.

3. Biaxial yield stress ratio could successfully predicted by stress based definition for AA2090-T3 material, while the slight difference between was observed for TRIP 780 steel.

Data Availability Statement

All graphs and data obtained or generated during the investigation appear in the published article.

Confilict of Interest

The author declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Ethics

There are no ethical issues with the publication of this manuscript.

REFERENCES

[1] Hershey, A.V. (1954). The plasticity of an isotropic aggregate of anisotropic face centered cubic crystals, Journal of Applied Mechanics, 21, 241-249.

[2] Hosford, W.F. (1972). A generalized isotropic yield criterion, Journal of Applied Mechanics, 39, 607-609.

[3] Barlat, F., Richmond, O. (1987). Prediction of tri-component plane stress yield surfaces and associated flow and failure behavior of strongly textured F.C.C polycrystalline sheets, Materials Science and Engineering, 95, 15-29.

[4] Bassani, J.L. (1977). Yield characterization of metals with transversely isotropic plastic properties, International Journal of Mechanical Sciences, 19, 651-660.

[5] Budiansky, B. (1984). Anisotropic plasticity of plane-isotropic sheets, Studies in Applied Mechanics, 6, 15-29.

[6] Hill, R. (1948). A theory of the yielding and plastic flow of anisotropic metals, Proceedings of the Royal Society of London Series A Mathematical and Physical Sciences, 193(A), 281-297.

[7] Woodthorpe, J., Pearce, R. (1970). The anomalous behavior of aluminum sheet under balanced biaxial tension, International Journal of Mechanical Sciences, 12, 341-347.

[8] Hill, R. (1979). Theoretical plasticity of textured aggregates, Mathematical Proceedings of the Cambridge Philosophical Society, 85, 179-191.

[9] Hill, R. (1990). Constitutive modeling of orthotropic plasticity in sheet metals, Journal of the Mechanics and Physics of Solids, 38, 405-417.

[10] Hill, R. (1993). A user-friendly theory of orthotropic plasticity in sheet metals, International Journal of Mechanical Sciences, 35, 19-25.

[11] Lin, S.B., Ding, J.L. (1996). A modified from of Hill's orientationdashdependent yield criterion for orthotropic sheet metals, Journal of the Mechanics and Physics of Solids, 44, 1739-1764.

[12] Barlat, F., Lian, J. (1989). Plastic behavior and stretchability of sheet metals. Part I: a yield function for orthotropic sheets under plane stress conditions, International Journal of Plasticity, 5, 51-66.

[13] Barlat, F., Lege, D.J., Brem, J.C., Warren, C.J. (1991b). Constitutive behavior for anisotropic materials and application to a 2090-T3 Al-Li alloy, Modeling the Deformation of Crystalline Solids, Warrendale, pages 189-203.

[14] Stoughton, T.B., Shi, M.F., Huang, G., Yoon, J.W. (2013). Material characterizations for benchmark 1 and benchmark 2, AIP Conference Proceedings, 1567(1), 9-14.

[15] Nocedal, J., Wright, S.J. (2006). Numerical Optimization, Springer, 2nd Ed., pages 563-592, United States of America.