Sigma J Eng & Nat Sci 38 (3), 2020, 1381-1398

Sigma Journal of Engineering and Natural Sciences
Sigma Mühendislik ve Fen Bilimleri Dergisi

Research Article

DYNAMICS ANALYSIS OF A TRUSS SYSTEM MODELLED BY THE FINITE ELEMENT METHOD IN THE FREQUENCY DOMAIN

Mehmet BOZCA*[1]

[1] *Yıldız Technical University, Machine Design Division, ISTANBUL;* ORCID: 0000-0002-2620-6053

Received: 27.03.2020 Revised: 25.05.2020 Accepted: 20.08.2020

ABSTRACT

The dynamic analysis of a truss system modelled by the finite element method in the frequency domain is studied. The truss system is modelled by 22 elements and has 44 degrees of freedom. The stiffness matrix and mass matrix of the truss system are obtained by using the finite element method. Differential equations of the truss system are obtained by using the obtained stiffness and mass matrix. By applying the Laplace transformation, the displacements of each node are calculated, and the equation is arranged in the frequency domain. The obtained differential equations are solved by using MATLAB. Eigen values are calculated and represented depending on the frequencies. Thus, static displacements, dynamic displacements, static reaction forces and dynamic reaction forces for each frequency are graphically obtained. Additionally, dynamic amplification factors are calculated and simulated depending on the frequencies. Dynamic displacements increased near the eigenvalues, and the dynamic amplification factors also increased dramatically depending on the related eigenvalues. By avoiding the natural frequency, it is possible to design a better structure to reduce vibration.
Keywords: Truss system, finite elements, eigenvalues, natural frequency.

1. INTRODUCTION

Truss systems are widely used in machine industries for tools such as cranes. Vibration characteristics and vibration dynamic factors are crucial for these truss systems. It is necessary to avoid the resonance frequency for these kinds of machine industries to prevent working accidents. Therefore, the vibration dynamic factors of the truss system are calculated depending on the frequency of each node of the truss system and obtained for the worst-case loading situations.

Furthermore, public or service structures should be designed so that their dynamic behaviour, expressed by their natural frequency and acceleration, will not affect the comfort of an occupant. For the serviceability limit state of a structure not to be exceeded when subjected to vibration, the natural frequency of the vibrations of the structure should be kept above appropriate values [1]. The following studies are presented in the literature:

A decoupled approach to the integrated optimum design of structures and robust control systems by using H_2 and H_∞ is presented. A truss system and beam system were modelled by

* Corresponding Author: e-mail: mbozca@yildiz.edu.tr, tel: (212) 383 27 93

using the finite el ement method. It is determined that the conventional simultaneous optimization approach can be approximated by a decoupled optimization approach [2].

The concurrent optimization of controlled structures for robustness was presented. A truss system was modelled by using a finite element model, and the controller was modelled by H_2 and H_∞ Thus, the structure was optimized by minimizing the singular value system [3].

The integrated optimization of beam structures and the LQR controller was studied. A beam system was modelled by using a finite element model. The structures and controllers are optimized simultaneously and successively. It is shown that simultaneous optimization of structures and controllers can be achieved by an equivalent decoupled optimization problem [4].

The structural optimization of a beam system modelled by the finite element method under stress constraints is realized. Dynamic amplification factors in terms of the frequency at each mode are obtained. It is shown that the dynamic amplification factor increases at the natural frequency [5].

An integrated optimization of structures and LQR control systems for reduced order models are studied. The structures and controllers are optimized simultaneously and successively. Since the degree-of-freedom (DOF) of a structure is very large in practice, model order reduction techniques must be employed at every controller design iteration during optimization, which increases the CPU time and can introduce modelling errors [6].

Equivalence and dynamic analyses for jointed trusses based on improved finite elements are presented. Two improved finite elements are presented for a link and a bending beam to calculate the dynamic characteristics of non-jointed and jointed trusses. The results indicate that the natural frequencies of the jointed structure increase with the excitation force and the stiffness of the joints [7].

2. STUDIES

Structural analysis of dynamic systems can be realised by using analytical methods or numerical methods. Analytical methods provide the exact behaviour of a system at any point within the system, while numerical solutions approximate exact solutions only at discrete point, called nodes. There are two common classes of numerical methods such as *finite difference method* and *finite elements method.*

With finite difference method, the differential equation is written for each node, and the derivatives are replaced by difference equations. Finite difference methods result in a set of simultaneous linear equations. Although finite difference methods are easy to understand and employ in simple problems. Finite difference methods become difficult to apply to problems with complex geometries or complex boundary conditions.

Finite element method uses integral formulation rather than difference equations to create a system of algebraic equations. Moreover, a continuous function is assumed to represent the approximate solution for each element. Then, the complete solution is generated by connecting or assembling the individual solutions, allowing for continuity at the inter-elemental boundaries.

2.1. Structural Finite Element Equations

Structural finite element analysis approaches are used to obtain the desired node displacement solution and dynamic analysis of the truss system.

2.1.1. Element Analysis

Structural finite element analysis requires knowledge of the behaviour of each element in a structure. Once each element is described, the governing equations of the entire structure may be derived for all structural systems. Energy methods are used to obtain governing equations. To apply energy theorems to the struc tural analysis of a system, it is necessary to define the strain energy, kinetic energy and change in the external dimensions due to bending. The main aim is to select element displacement functions that are uniquely specified when displacements at the nodes of the element are known [8].

2.1.1.1. Beam Element Stiffness Matrix

A typical planar beam element, with its displacement sign convention, is shown in Figure 1. The displacement coordinates q_1, q_2, q_4 and q_5 are components of the endpoint displacements, and q_3 and q_6 are endpoint rotations [8].

The longitudinal displacement of a point x on a beam $(0 \le x \le l)$ due to longitudinal strain is approximated by the following Equation [8]:

$$s(x) = -q_1 \frac{(x-l)}{l} + q_4 \frac{x}{l} \tag{1}$$

which is the exact solution for a beam element with a constant section and no axial distribution load. It should be emphasized that the longitudinal displacement $s(x)$ is due to only the longitudinal strain in the beam and not to the change in the length caused by the lateral displacement *w(x).*

The lateral displacement of the beam at point x is approximated by the following Equation [8]:

$$w(x) = \frac{q_2}{l^3}(2x^3 - 3lx^2 + l^3) - \frac{q_5}{l^3}(2x^3 - 3lx^2) + \frac{q_3}{l^2}(x^3 - 2lx^2 + l^2 x) + \frac{q_6}{l^2}(x^3 - lx^2) \tag{2}$$

which is the exact solution for beam elements with a constant cross section and no laterally distributed load.

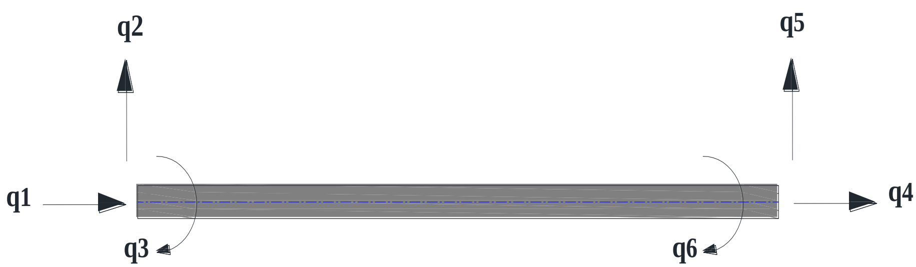

Figure 1. Planar beam element.

The strain energy SE due to the deformation of the beam is written as follows [8]:

$$SE = \frac{1}{2}\int_0^l hE\left(\frac{ds}{dx}\right)^2 dx + \frac{1}{2}\int_0^l EI\left(\frac{d^2w}{dx^2}\right)^2 dx \tag{3}$$

$$= \frac{1}{2}\int_0^l hE\left(\frac{q_1}{l} - \frac{q_4}{l}\right)^2 dx + \frac{1}{2}\int_0^l EI\left[\frac{q_2}{l^3}(12x-6l) - \frac{q_5}{l^3}(12x-6l) + \frac{q_3}{l^2}(6x-4l) + \frac{q_6}{l^2}(6x-2l9\right]^2 dx$$

where h is the cross-sectional area of the beam, I is the second moment of the cross-sectional area about its centroidal axis, and E is the Young's modulus of the material. Carrying out the integrations in Equation (3), the following quadratic form is obtained for $q = [q_1, q_2 ... q_6]^T$ [8]:

$$SE = \frac{1}{2} q^T k_B q \tag{4}$$

where k_B is the beam *element stiffness matrix* and is given as follows [8-10]:

$$k_B = \frac{E}{l^3}\begin{bmatrix} hl^2 & 0 & 0 & hl^2 & 0 & 0 \\ & 12I & 6lI & 0 & -12I & 6lI \\ & & 4l^2I & 0 & -6lI & 2l^2I \\ & & & hl^2 & 0 & 0 \\ & & & & 12I & -6lI \\ symmetric & & & & & 4l^2I \end{bmatrix} \tag{5}$$

2.1.1.2. Truss-Element Stiffness Matrix

If bending effects are neglected, a truss element is obtained for which only coordinates q_1 and q_4 of Figure 1 influence the strain energy. In this case, the strain energy is as given in Equation (4) but with the truss-element stiffness matrix k_T [8-10].

$$k_T = \frac{Eh}{l}\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 1 & 0 & 0 \\ & & & & 0 & 0 \\ symmetric & & & & & 0 \end{bmatrix} \tag{6}$$

Note that only q_1 and q_4 have an effect on the truss element strain energy. While q_2 and q_5 need to be retained in the analysis, since the truss element does not bend, $q_3 = q_6 = (q_5 - q_2)/l$, and these rotation variables may be suppressed [8].

2.1.1.3. Beam and Truss-Element Mass Matrices

The kinetic energy of a beam element, neglecting the rotary inertia of the beam cross section, is

$$KE = \frac{1}{2}\int_0^l \rho h\left[\left(\frac{ds}{dt}\right)^2 + \left(\frac{dw}{dt}\right)^2\right]dx \tag{7}$$

$$= \frac{1}{2}\int_0^l \rho h\left\{\begin{aligned}&\left[-\dot{q}_1\left(\frac{x-l}{l}\right)+\dot{q}_4\frac{x}{l}\right]^2\\ &+\left[\frac{\dot{q}_2}{l^3}(2x^3-3lx^2+l^3)+\frac{\dot{q}_5}{l^3}(2x^3-3lx^2)+\frac{\dot{q}_3}{l^2}(x^3-2lx^2+lx)+\frac{\dot{q}_6}{l^2}(x^3-lx^2)\right]^2\end{aligned}\right\}dx$$

where ρ is the mass density of the beam material and the dot over the variable ($\dot{}$) denotes the time derivative. The integration *KE* is obtained as follows [8]:

$$KE = \frac{1}{2}\dot{q}^T m_B \dot{q} \tag{8}$$

where m_B is *the beam-element mass matrix* and is given as follows [8-12]:

$$m_B = \frac{\rho h l}{420}\begin{bmatrix} 140 & 0 & 0 & 70 & 0 & 0 \\ & 156 & 22l & 0 & 54 & -13l \\ & & 4l^2 & 0 & 13l & -3l^2 \\ & & & 140 & 0 & 0 \\ & & & & 156 & -22l \\ symmetric & & & & & 4l^2 \end{bmatrix} \tag{9}$$

Since the kinetic energy of the beam element is positive for any $\dot{q}_i \neq 0$, it is expected that m_B is positive definite and is hence non-singular [8].

To obtain a truss element, bending is neglected and $w(x) = q_2 + (q_5 - q_2)x/l$. Equation (7) yields a quadratic form, as in Equation (8), but with the truss-element mass matrix [8-12].

$$m_T = \frac{\rho h l}{6}\begin{bmatrix} 2 & 0 & 0 & 1 & 0 & 0 \\ & 2 & 0 & 0 & 1 & 0 \\ & & 0 & 0 & 0 & 0 \\ & & & 2 & 0 & 0 \\ & & & & 2 & 0 \\ symmetric & & & & & 0 \end{bmatrix} \tag{10}$$

Note that $\dot{q}_3$ and $\dot{q}_6$ are suppressed in the kinetic energy expression and are not needed in a truss analysis. The velocities $\dot{q}_2$ and $\dot{q}_5$, however, play an important role. Since the third and sixth columns of m_T are zero, the matrix is singular. In fact, m_T is only of rank 4. It is also worth noting that, as in the case of the strain energy, the area *h* and length *l* may depend on the design variables [8].

2.1.2. Global Stiffness and Mass Matrices

The total strain and kinetic energy of a structure may be obtained by summing the strain and kinetic energies of all the elements that make up the structure. Before a meaningful expression for the total system strain and kinetic energies may be written, it is first necessary to define a system for the global displacements of all the nodes in the structure relative to a global coordinate system. Let $z_g \in R^n$ denote this global displacement vector [8].

2.1.2.1. Transformation from Local to Global Coordinates

Since the individual elements of the structure have their own inherent displacement coordinates relative to a body-fixed coordinate system, as in Figure 1 and Figure 2, the displacements must first be transformed from the element's body-fixed coordinate system to a coordinate system parallel to the global coordinates. Let q^i denote the vector of the nodal displacement coordinates of the i^{th} element in its body-fixed system. A rotation matrix S^i may be defined as follows [8]:

$$q^i = S^i \hat{q}^i \tag{11}$$

The transformed element displacements now coincide with the components of the global displacement vector z_g. Therefore, a *Boolean transformation matrix* β^i may be defined, consisting of only zeros and ones, that gives the following relation [8]:

$$\hat{q}^i = \beta^i z_g \tag{12}$$

Note that if $\hat{q}^i$ is an *r*-vector and z_g is an *n*-vector (*n*>*r*), then β^i is an *rxn* matrix that consists only of r unit components, with zeros as the remaining entries [8].

2.1.2.2. Generalized Global Stiffness Matrix

Denoting the i^{th} element stiffness matrix as k^i, the strain energy in the i^{th} elementary may be written as follows [8]:

$$SE^i = \frac{1}{2} q^{iT} k^i q^i \tag{13}$$

Substituting from Equations (11) and (12), SE^i is rewritten as follows [8]:

$$SE^i = \frac{1}{2} \hat{q}^{iT} S^{iT} k^i S^i \hat{q}^i = \frac{1}{2} z_g^T \beta^{iT} S^{iT} k^i S^i \beta^i z_g \tag{14}$$

The strain energy of the entire structure is now obtained by summing the strain energy over all NE elements in the structure to obtain

$$SE = \frac{1}{2} z_g^T \left[\sum_{i=0}^{NE} \beta^{iT} S^{iT} k^i S^i \beta^i \right] z_g \equiv \frac{1}{2} z_g^T K_g z_g \tag{15}$$

where K_g is *the generalized global stiffness matrix* and is given as follows [8-12]:

$$K_g = \sum_{i=1}^{NE} \beta^{iT} S^{iT} k^i S^i \beta^i \tag{16}$$

2.1.2.3. Reduced Global Stiffness Matrix

If all the boundary conditions associated with the structure have been imposed so that no rigid-body degrees-of-freedom exist, then the generalized global stiffness matrix K_g is positive definite, denoted simply by *K*, and it is called the *reduced global stiffness matrix*. However, if the generalized global stiffness matrix is assembled without considering the boundary conditions, it will generally not be positive definite [8].

2.1.2.4. Generalized Global Mass Matrix

As in the case of the strain energy, the kinetic energy of the i^{th} element may be written in terms of generalized velocities. Since the matrices S^i and β^i do not depend on generalized coordinates,

$$\dot{q} = S^i \dot{\hat{q}}^i \tag{17}$$

$$\dot{\hat{q}} = \beta^i \dot{z}_g \tag{18}$$

Using these relationships, the kinetic energy of the ith element may be written as follows [8]:

$$KE^i = \frac{1}{2} \dot{q}^{iT} m^i \dot{q}^i = \frac{1}{2} \dot{\hat{q}}^{iT} S^{iT} m^i S^i \dot{\hat{q}}^i = \frac{1}{2} \dot{z}_g^T \beta^{iT} S^{iT} m^i S^i \beta^i \dot{z}_g \tag{19}$$

Summing the kinetic energy over all elements, the total kinetic energy *KE* for the system is as follows [6]:

$$KE = \frac{1}{2}\dot{z}_g^T \left[\sum_{i=1}^{NE} \beta^{iT} S^{iT} m^i S^i \beta^i \right] \dot{z}_g \equiv \frac{1}{2}\dot{z}_g^T M_g \dot{z}_g \tag{20}$$

where M_g is the *generalized global mass matrix*, which is written as follows [7]:

$$M_g = \sum_{i=1}^{NE} \beta^{iT} S^{iT} m^i S^i \beta^i \tag{21}$$

Presuming that all structural elements have mass, it is impossible to obtain a nonzero velocity without investing a finite amount of kinetic energy. Therefore, a global system mass matrix will always be positive definite [8].

2.1.2.5. Reduced Global Mass Matrix

If boundary conditions have been taken into account before the global displacement vector is defined, the reduced global mass matrix will be denoted by *M*, as in the case of the corresponding reduced global stiffness matrix *K*.

Note that in the case of member size design variables and on geometrical design variables that appear in the rotation matrices S^i, the global stiffness and mass matrices depend on design variables that appear in the element stiffness and mass matrices.

2.2. Dynamic Response of a Structure

Consider the case of the dynamic response of a structure with no boundary or interface conditions, that is, with independent generalized coordinates. Lagrange's equations apply in this case and may be written in matrix form, using $F = F_g, M = M_g$, and $K = K_g$, as

$$M\ddot{z} + Kz - F = 0 \tag{22}$$

The initial conditions of motion for such a system consist of specifying the position and velocity of the system at some initial time, e.g., t=0; that is, $z(0) = z^0$ and $\dot{z}(0) = \dot{z}^0$ [8].

2.3. Natural Vibration of a Structure

The natural vibration of a structure is defined as the harmonic motion of the structural system, with no applied load and *F=0.* In this case, the equation of the natural vibration of the structure is written as follows:

$$M\ddot{z} + Kz = 0 \tag{23}$$

2.3.1. Transfer Function

Assuming zero initial conditions, one obtains the following harmonic response of a structure by taking the Laplace transform of the transfer matrix (22) as follows [8]:

$$Ms^2 Z(s) + KZ(s) = F(s) \tag{24}$$

$$(Ms^2 + K)Z(s) = F(s) \tag{25}$$

Then, the transfer function of the system is written as follows:

$$TF = \frac{Z(s)}{F(s)} = \frac{1}{Ms^2 + K} \tag{26}$$

The complex Laplace transform variable s is substituted by s=jω, where ω is the excitation frequency, and j is an imaginary unit. Then, Equation (24) is written in the frequency domain as follows:

$$z = \left(-M\omega^2 + K\right)^{-1} F \tag{27}$$

2.4. Eigenvalues and Eigenvectors

A linear equation can be written in the following form [9-12]:

$$[A]\{Z\} = \{b\} \tag{28}$$

For a set of linear equations, the values of the elements of the $\{b\}$ matrix are typically nonzero. These types of problems render a set of linear equations of the following form [9-12]:

$$[A]\{Z\} - \lambda\{Z\} = 0 \tag{29}$$

In practice, Equation (29) can be written as follows [9-12]:

$$[[A] - \lambda[I]]\{Z\} = 0 \tag{30}$$

where **[I]** is the identity matrix having the same dimension as the **[A]** matrix. In Equation (30), the unknowns of matrix **{Z}** are called the *eigenvectors.*

2.4.1. Eigenvalues and eigenvectors of a dynamic system

By considering Equation (27), $z = \left(-M\omega^2 + K\right)^{-1} F$, the above equations can be applied for a dynamic system as follows:

If the matrices are the stiffness matrix **[K]** and the mass matrix **[M]**, then the eigenvectors **{Z}** will be the modes of vibration and the eigenvalues will be the square roots of the *natural frequencies* of the system [9-12].

The eigenvalue problem can be solved by considering Equation (27) as follows:

$$\left(-M\omega^2 + K\right) = 0 \tag{31}$$

We note that $\omega^2 = \lambda$ and based on the degree of freedom of the systems, each *natural frequency* is written as follows:

$$\omega_i^2 = \lambda_i \quad i = 1,\ldots,n$$

The roots of Equation (30) or Equation (31), the characteristic equation, are the natural frequencies of the dynamic system [9-12].

The relationship between the amplitudes $\{z_1,\ldots z_n\}$of a mass oscillating at its natural frequency is called the *natural mode* [9-12].

2.4.2. Reaction Forces R

The determination of the difference between the reaction forces and applied load is important. The reaction forces are written as follows [9-12]:

$$\{R\} = [K]\{u\} - \{F\} \tag{32}$$

where $\{R\}$ denotes the reaction force matrix, $[K]$ denotes the stiffness matrix, $\{u\}$ denotes the displacement matrix and $\{F\}$ denotes the applied load matrix [9-12].

2.4.3. Dynamic amplification factor ψ

The dynamic amplification factor ψ_D is described in terms of displacements as follows [9-12].

$$\psi_D = \frac{d_{ST} + d_{DN}}{d_{ST}} = 1 + \frac{d_{DN}}{d_{ST}} \tag{33}$$

where d_{ST} is the displacement in the static loading case and d_{DN} is the displacement in the dynamic loading case.

3. NUMERICAL EXAMPLE

Definition of truss

A *truss* is an engineering structure consisting of straight members connected at their ends by means of bolts, rivets, pins, or welding. Trusses offer practical solutions to many structural problems in engineering such as tower, bridges, and roofs of buildings. Two-dimensional trusses (or *plane trusses*) is defined as a truss whose members lie in a single plane.

A truss system modelled by the finite element method is shown in Figure 2. The applied load is applied at 21. The node and worst case loading conditions are investigated. There are 40 element numbers, 22 node numbers and 44 displacement numbers for the truss system.

Boundary conditions

The displacements are zero on node 1, node 5 and node 22. That is, z_1=0, z_2=0, z_5=0, z_6=0 and z_{43}=0, z_{44}=0.

During the dynamic analysis of the truss system in the frequency domain, static and dynamic reaction forces and the displacement on node 21 are considered.

The vibration analysis of the truss system is investigated by using the obtained stiffness and mass matrix in the equation of motion of the system.

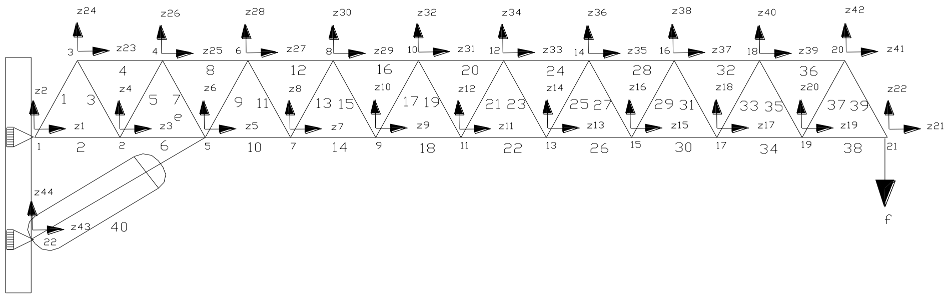

Figure 2. 44-degree-of-freedom truss structure

Generalized global stiffness matrix K_g

$$K_g = \sum_{i=1}^{NE} \beta^{iT} S^{iT} k^i S^i \beta^i \tag{16}$$

Element stiffness matrix k^i

By deleting the rows and columns for q_3 and q_6 of the stiffness matrix of the truss system obtained by Equation (6), the following equation is obtained:

$$k^i = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{6}$$

Generalized global mass matrix M_g

$$M_g = \sum_{i=1}^{NE} \beta^{iT} S^{iT} m^i S^i \beta^i \tag{21}$$

Element mass matrix m^i

By deleting the rows and columns for q_3 and q_6 of the mass matrix of the truss system obtained by Equation (6), the following equation is obtained.

$$m^i = \frac{\rho h l}{6} \begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \tag{10}$$

The matrix S^i is written as follows:

$$S^i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ -\sin\theta_i & -\cos\theta_i & 0 & 0 \\ 0 & 0 & \cos\theta_i & -\sin\theta_i \\ 0 & 0 & -\sin\theta_i & -\cos\theta_i \end{bmatrix} \tag{11}$$

β^i matrix is a [4x44] matrix. It is obtained by putting 1 in for the i^{th} element in the corresponding row and column of the global coordinate system.

Simulation results

The dynamic analysis of the truss system is investigated by using the following simulation parameters shown in Table 1. The simulation results are shown in Figure 3 to Figure 10.

Table 1. Simulation parameters

Parameters	**Unit**	**Value**
Elasticity modulus *E*	[N/m^2]	21.10^{10}
Diameter of truss d	[m]	10.10^{-3}
Cross-sectional area of the truss h	[m^2]	$282{,}74.10^{-6}$
Truss element length l	[m]	1
Material density ρ	[kg/m^3]	7850
Applied load *F*	[N]	3000

Eigenvalues of the system

The natural frequencies of the dynamic system are a characteristic of system vibration response related with eigenvalues. Therefore, eigenvalues are calculated to observe dynamic system vibration response.

The eigenvalues of the system are presented in Table 2. The truss system was modelled by 22 elements and has 44 degrees of freedom. Therefore, 44 eigenvalues are obtained during the simulation study since the eigenvalues of the system are equal to the degrees of freedom of the system.

For the 44 degrees of freedom of the system, we have 44 *natural frequencies* of the system.

$$\omega_i^2 = \lambda_i \quad i = 1,...,44$$

Additionally, for the 44 degree-of freedom of the system we have 44 *natural modes*.

Table 2. Eigenvalues of the truss system

Node i^{th}	**Eigenvalue i^{th}**	**Node i^{th}**	**Eigenvalue i^{th}**	**Node i^{th}**	**Eigenvalue i^{th}**	**Node i^{th}**	**Eigenvalue i^{th}**
1	$3.0709.10^9$	**12**	$1.7567.10^9$	**23**	$1.3109.10^8$	**34**	$1.0377.10^9$
2	$2.7519.10^9$	**13**	$1.7304.10^9$	**24**	$1.8523.10^8$	**35**	$9.9406.10^8$
3	$2.6404.10^9$	**14**	$1.6796.10^9$	**25**	$2.8425.10^8$	**36**	$9.4950.10^8$
4	$2.6054.10^9$	**15**	$1.5730.10^9$	**26**	$2.7880.10^8$	**37**	$8.3624.10^8$
5	$2.4216.10^9$	**16**	$1.3563.10^9$	**27**	$3.6923.10^8$	**38**	$8.5939.10^8$
6	$2.3663.10^9$	**17**	$3.5099.10^5$	**28**	$1.2424.10^9$	**39**	$7.4144.10^8$
7	$2.1851.10^9$	**18**	$3.4334.10^6$	**29**	$4.6927.10^8$	**40**	$7.3238.10^8$
8	$2.0589.10^9$	**19**	$6.2757.10^6$	**30**	$5.4677.10^8$	**41**	1.0000
9	$1.8284.10^9$	**20**	$1.9259.10^7$	**31**	$5.8621.10^8$	**42**	1.0000
10	$1.7810.10^9$	**21**	$5.2348.10^7$	**32**	$1.1229.10^9$	**43**	1.0000
11	$1.7746.10^9$	**22**	$7.9138.10^7$	**33**	$6.6329.10^8$	**44**	1.0000

Reaction force simulations

The static reaction force is shown in Figure 3 Although the frequency increases from 0,1 [1/s] to 10 [1/s], the static displacement is constant.

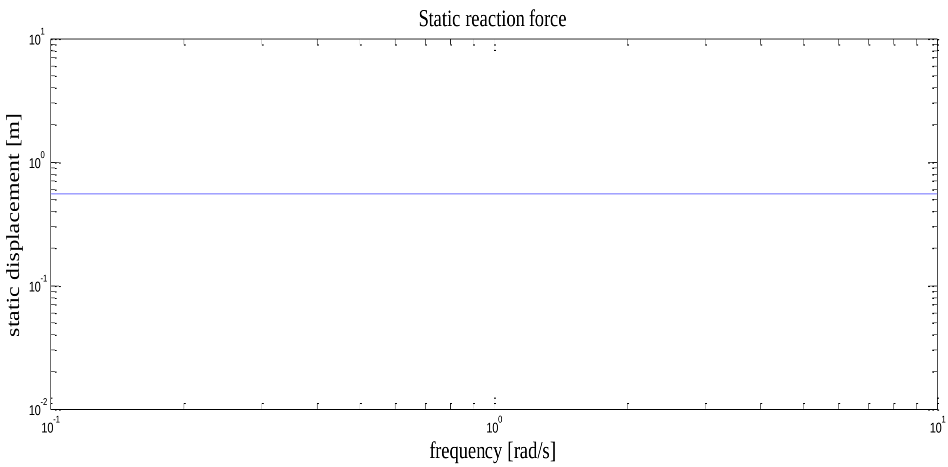

Figure 3. Static reaction force

The dynamic reaction force for the low frequency range is shown in Figure 4. When the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic reaction force dramatically increases near the natural frequency.

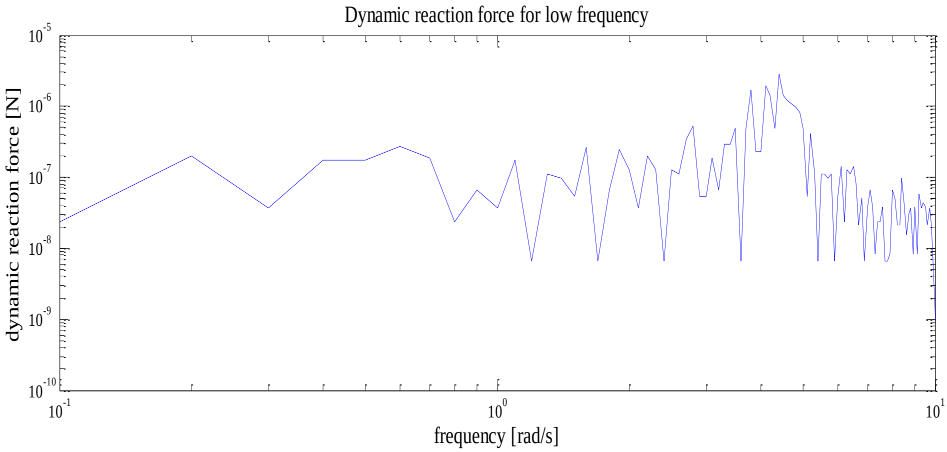

Figure 4. Dynamic reaction force for the low frequency range

The dynamic reaction force for the high frequency range is shown in Figure 5. When the frequency increases from 1 [1/s] to 100 [1/s], the dynamic reaction force dramatically increases near the natural frequency.

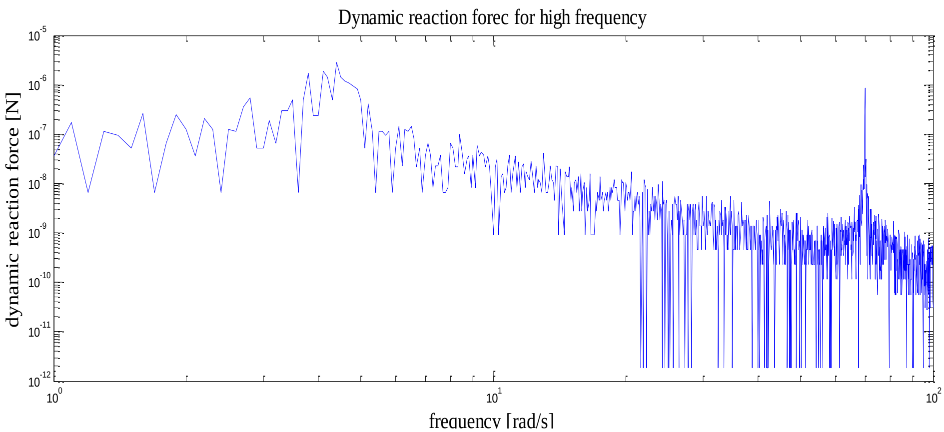

Figure 5. Dynamic reaction force for the high frequency range

Displacement simulations

The static displacement for the z_{22} direction for the low frequency range is shown in Figure 4. Although the frequency increases from 0,1 [1/s] to 10 [1/s], the static displacement is constant.

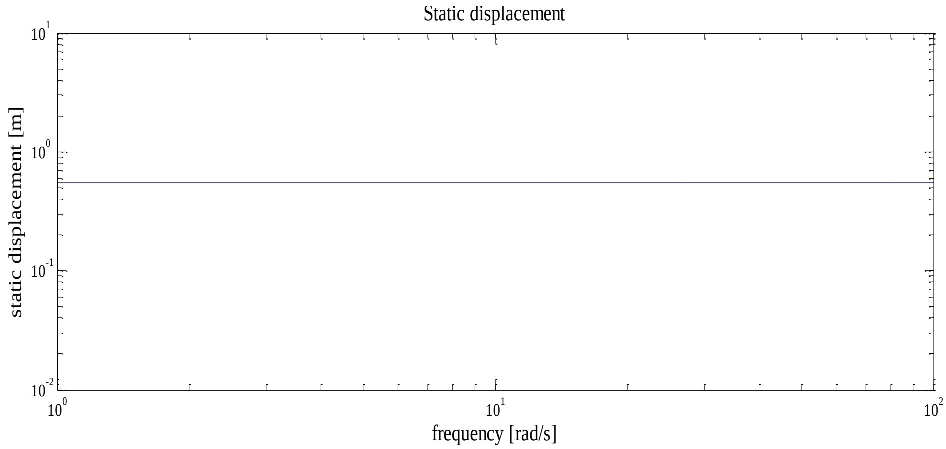

Figure 6. Static displacement for the z_{22} direction

The dynamic displacement for the z_{22} direction for the low frequency range is shown in Figure 7. While the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic displacement dramatically increases near the natural frequency.

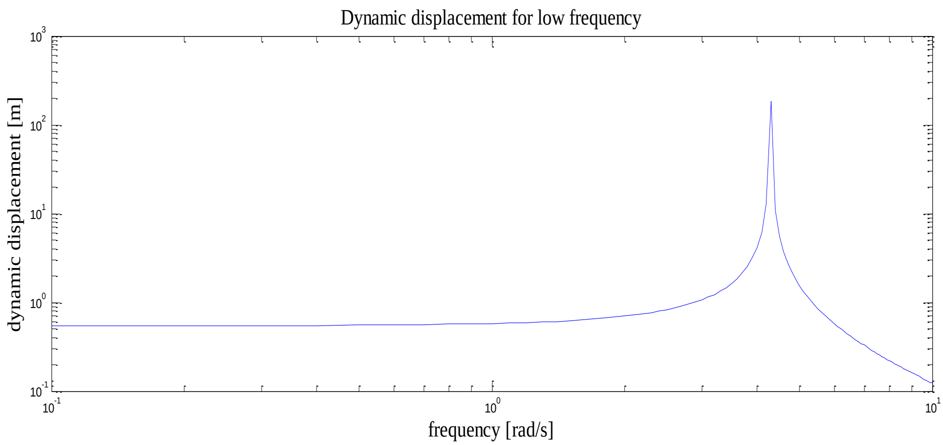

Figure 7. Dynamic displacement for the z_{22} direction for $\omega = 0.1:0.1:10$ (low frequency)

The dynamic displacement for the z_{22} direction for the high frequency range is shown in Figure 8. While the frequency increases from 1 [1/s] to 100 [1/s], the dynamic displacement dramatically increases near the natural frequency.

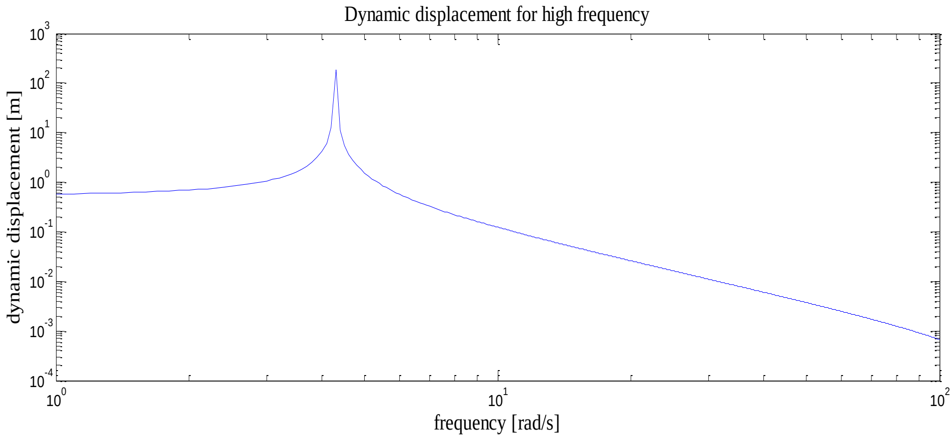

Figure 8. Dynamic displacement for the z_{22} direction for $\omega = 1:0.1:100$ (high frequency)

Dynamic factor simulations

The dynamic factor for the z22 direction for the low frequency range is shown in Figure 9. While the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic factor dramatically increases near the natural frequency.

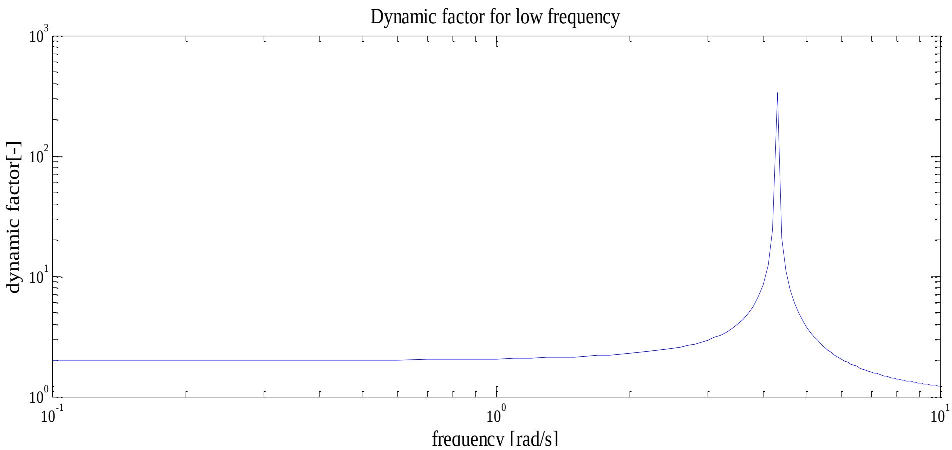

Figure 9. Dynamic factor for the z_{22} direction for $\omega = 0.1:0.1:10$ (low frequency)

The dynamic factor for the z22 direction for the high frequency range is shown in Figure 10. While the frequency increases from 1 [1/s] to 100 [1/s], the dynamic factor dramatically increases near the natural frequency.

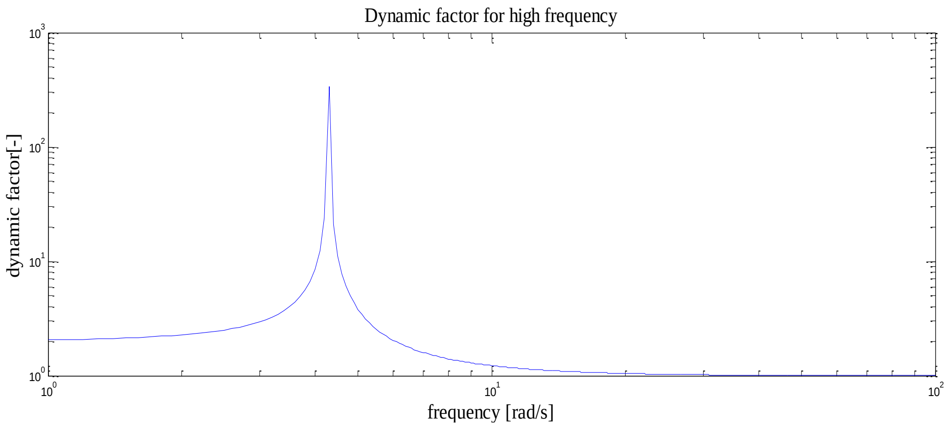

Figure 10. Dynamic factor for the z_{22} direction for $\omega = 1:0.1:100$ (high frequency)

Results and discussion

During the simulation, the following results are obtained. There are 44 eigenvalues of the system and the eigenvalues are equal to the degrees of freedom of the system. For the 44 degrees of freedom of the system, we have 44 *natural frequencies* of the system. Additionally, for the 44 degrees of freedom of the system, we have 44 *natural modes*.

In the low frequency case**,** while the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic reaction force dramatically increases. When the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic displacement dramatically increases. Similarly, as the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic factor dramatically increases.

In the high frequency case, while the frequency increases from 1 [1/s] to 100 [1/s], the dynamic reaction force dramatically increases. When the frequency increases from 0,1 [1/s] to 10 [1/s], the dynamic displacement dramatically increases. Similarly, as the frequency increases from 1 [1/s] to 100 [1/s], the dynamic factor dramatically increases.

4. CONCLUSIONS

Finite element method is an effective numerical method in structural analysis of a dynamic system. Therefore, the dynamic analysis of the truss system is modelled by the finite element method in the frequency domain. Since trusses offer practical solutions to many structural problems in engineering structures.The truss system is modelled by 22 elements and has 44 degrees of freedom. Furthermore, the natural frequencies of the dynamic system are a characteristic of system vibration response related with eigenvalues. Therefore, 44 eigenvalues are obtained during the simulation study since the eigenvalues of the system are equal to the degrees of freedom of the system. The static reaction forces and dynamic reaction forces are calculated and simulated depending on the frequency. The static displacement and dynamic displacement are calculated and simulated depending on the frequency. The dynamic amplification factors are calculated in terms of the frequency at each node. The following conclusions can be summarized based on the numerical simulation results:

- The modes of vibration and the eigenvalues will be the square roots of the *natural frequencies* of the system.
- The relationship between the amplitudes of a mass oscillating at its natural frequency is called a *natural mode*.

- In the low-frequency case, the static reaction force is constant, while the dynamic reaction force increases. Moreover, static displacement is constant, although the frequency increases. However, the dynamic displacement dramatically increases near the natural frequency, while the frequency increases. The dynamic amplification factor increases at the natural frequency since the dynamic displacements increase.
- In the high-frequency case, the static reaction force is constant, while the dynamic reaction force increases. Moreover, the static displacement is constant although the frequency increases. However, the dynamic displacement dramatically increases near the natural frequency, while the frequency increases. The dynamic amplification factor increases at the natural frequency since the dynamic displacements increase.
- The eigenvalues of the system are equal to the degrees of freedom of the system. By avoiding the natural frequency, it is possible to design a better structure to reduce vibration.

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