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Research Article

INFLUENCE OF IMPERFECT CONTACT CONDITIONS ON THE TORSIONAL WAVE DISPERSION IN PRE-STRAINED MANY LAYERED HOLLOW CYLINDER MADE OF HIGHLY ELASTIC MATERIALS

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ABSTRACT

In this paper, influence of the bonded imperfectness on torsional wave dispersion in the finitely pre-strained hollow compound circular cylinder made of highly-elastic material were investigated. The investigations are carried out within the scope of the piecewise homogeneous body model with the use of the Three-dimensional Linearized Theory of Elastic Waves in Initially Stresses Bodies (TLTEWISB). The mechanical relations of the materials of the cylinders are described through the harmonic potential. Numerical results on the effects of the imperfectness of the boundary condition on the influence of the initial stresses on the wave propagation velocity are presented and discussed.
Keywords: Torsional wave dispersion, imperfect contact condition, initial stress, compounded cylinder.

1. INTRODUCTION

The problem of wave propagation in pre-strained (pre-stressed) piecewise homogeneous elastic solid bodies is of interest in a number of physical and mechanical areas, such as geophysics, electrical devices, earthquake engineering, composite materials, ultrasonic non-destructive stress analysis of solids and others. Accordingly, a large number of investigations have been made in this field.

In many cases the control of the adhesion quality in the layered materials is made through measurement of the acoustic wave propagation velocity in these materials. Under these measurement procedures it is necessary to have information on the corresponding theoretical results related to the influence of the bonded imperfection on the dispersion of these waves. In connection with this, the investigation carried out in the present paper which relate to the study of the influence of the bonded imperfection on the torsional wave dispersion in the finitely pre-strained hollow circular cylinder has significant not only in a theoretical sense but also in a practical sense in the corresponding branches of modern engineering.

The subject of the papers [15] and [16] is the investigation of the dispersion relations of the torsional waves in a pre-stressed compounded cylinder.

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The torsional wave propagation in the compounded cylinder (without initial stresses) with an imperfect interface is studied in paper [7]. In [10] the investigations carried out in the papers [15] and [17] are developed for the case where the contact condition on the interface surface is imperfect. As in [7], the imperfectness of the contact condition is formulated according to the model used in [9]. Mo reover, in the present paper, as in [15] and [17], the mathematical formulations of the corresponding eigen-value problems are made within the scope of the piecewise homogenous body model with the use of the equations and relations of the TLTEWISB. It is assumed that the elasticity relations of the cylinders' materials are given through the Murnaghan Potential [14].

In these works it is assumed that the initial strains in the constituents are small and these strains are calculated within the scope of the classical linear theory of elasticity. The results of the investigations can be employed only for the compounded cylinders made from stiff materials. But these results are not suitable for the compounded cylinders fabricated from the high elastic materials such as elastomers, various type polymers and etc. Therefore in present paper attempt is made for the development of the investigations carried out in the papers [15] and [17] for the hollow compound cylinder made from high elastic materials, in other words for the case where the initial strains in the components of the cylinder are finite ones and the magnitude of those are not restricted. In this case, as in [17], it is assumed that in each component of the compounded cylinder there exists only the homogenous normal stress acting on the areas which are perpendicular to the lying direction of the cylinders. The mechanical relations of the materials of the cylinders are described through the harmonic potential in the papers [4] and [5].

The influence of the imperfectness of the interface conditions on the dispersion of the axisymmetric longitudinal waves in the pre-strained compound cylinder was investigated in [2] and [3].

The influence of the imperfectness of the contact condition on the torsional wave propagation in the initially stressed (stretched) hollow bi-material compound circular cylinder is investigated in [6] and [11] using with harmonic potential for the mechanical relations of the materials of the cylinders instead of the Murnaghan Potential as in papers [4] and [5].

The studies in this area have been collected in a book "Dynamics of Pre-Strained Bi-Material Elastic Systems, Linearized Three- Dimensional Approach" by Akbarov [1].

2. FORMULATION OF THE PROBLEM

We consider the sandwich compound (composite) circular cylinder shown in Fig. 1 and assume that in the initial state the radius of the internal circle of the inner hollow cylinder is R and the thickness of the inner and outer cylinders are $h^{(1)}$, $h^{(2)}$ and $h^{(3)}$, respectively. In the initial state we determine the position of the points of the cylinders by the Lagrangian coordinates in the cylindrical system of coordinates $Orqz$.

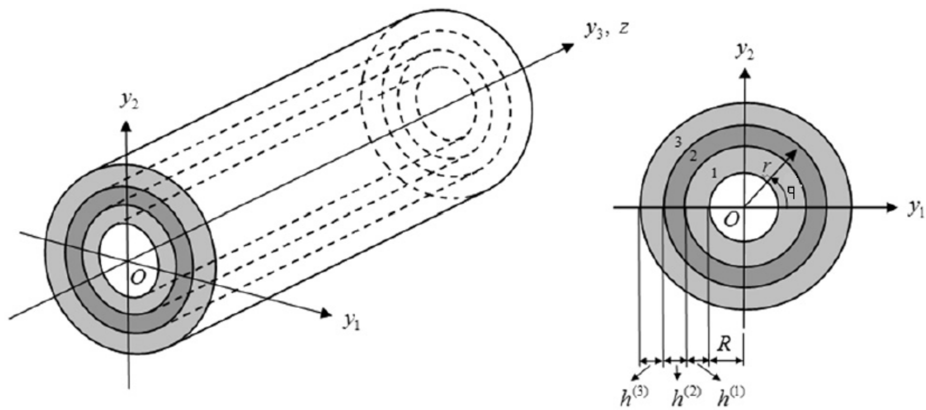


Figure 1. The geometry of the compound hollow cylinder.

Assume that the cylinders have infinite length in the direction of the Oz axis and the initial stress state in each component of the considered body is axisymmetric with respect to this axis and homogeneous. With the initial state of the cylinders we associate the Lagrangian cylindrical system of coordinates $O'r'q'z'$. The values related to the inner and external hollow cylinders will be denoted by the upper indices 1 and 2, respectively. Furthermore, we denote the values related to the initial state by an additional upper index, 0. Thus, the initial strain state in the inner and external hollow cylinders can be determined as follows.

$$u_r^{(k),0} = (\lambda_1^{(k)} - 1)r\,,\quad u_q^{(k),0} = 0\,,\quad u_z^{(k),0} = (\lambda_3^{(k)} - 1)z\,,\quad \lambda_1^{(k)} \neq \lambda_3^{(k)}\,,\quad k = 1,2,3 \tag{1}$$

where $u_{r,z}^{(k),0}$ is displacement and $\lambda_{1,3}^{(k)}$ is the elongation. We introduce the following notation:

$$\begin{aligned} &z' = \lambda_3^{(k)} z \quad r' = \lambda_1^{(k)} r \quad R' = \lambda_1^{(1)} R \\ &k = 1 \text{ for } R \le r \le R + h^{(1)} \\ &k = 2 \text{ for } R + h^{(1)} < r \le R + h^{(1)} + h^{(2)} \\ &k = 3 \text{ for } R + h^{(1)} + h^{(2)} < r \le R + h^{(1)} + h^{(2)} + h^{(3)} \end{aligned} \tag{2}$$

The values related to the system of the coordinates associated with the initial state below, i.e. with $O'r'q'z'$ will be denoted by upper prime.

Within this framework, we investigate the axisymmetric torsional wave propagation along the $O'z'$ axis in the considered body by the use of the following field equations.

The equation of motion is:

$$\frac{\partial}{\partial r'} Q'^{(k)}_{r'q} + \frac{\partial}{\partial z'} Q'^{(k)}_{qz} + \frac{1}{r'}\left(Q'^{(k)}_{r'q} + Q'^{(k)}_{qr'}\right) = \rho'^{(k)} \frac{\partial^2}{\partial t^2} u'^{(k)}_q \tag{3}$$

The elasticity relations are:

$$Q'^{(k)}_{r'q} = \omega'^{(k)}_{1221} \frac{\partial u'^{(k)}_q}{\partial r'} - \omega'^{(k)}_{1212} \frac{u'^{(k)}_q}{r'}\,,\quad Q'^{(k)}_{z'q'} = \omega'^{(k)}_{1331} \frac{\partial u'^{(k)}_q}{\partial z'}\,, \tag{4}$$

$$Q'^{(k)}_{qr'} = \omega'^{(k)}_{2121} \frac{\partial u'^{(k)}_q}{\partial r'} - \omega'^{(k)}_{2112} \frac{u'^{(k)}_q}{r'}$$

In Eqs. (3)-(4) through the $Q'^{(k)}_{r'q}$, $Q'^{(k)}_{z'q'}$ and $Q'^{(k)}_{qr'}$ are the perturbation of the components of Kirchhoff stress tensors. $u'^{(k)}_{q}$ is the perturbation of the components of the displacement vector. ω's are the constants determined through the mechanical constants of the cylinders materials and through the initial stress state. $\rho'^{(k)}$ is the density.

Green's strain tensors with the displacement vector u in the cylindrical coordinates system are:

$$
\begin{aligned}
&\varepsilon_{rr}=\frac{\partial u_r}{\partial r}+\frac{1}{2}\left(\frac{\partial u_r}{\partial r}\right)^2+\frac{1}{2}\left(\frac{\partial u_q}{\partial r}\right)^2+\frac{1}{2}\left(\frac{\partial u_z}{\partial r}\right)^2\\
&\varepsilon_{qq}=\frac{1}{r}\frac{\partial u_q}{\partial q}+\frac{u_r}{r}+\frac{1}{2r^2}\left(\frac{\partial u_r}{\partial q}-u_q\right)^2+\frac{1}{2r^2}\left(\frac{\partial u_q}{\partial q}-u_r\right)^2+\frac{1}{2r^2}\left(\frac{\partial u_z}{\partial q}\right)^2\\
&\varepsilon_{zz}=\frac{\partial u_z}{\partial z}+\frac{1}{2}\left(\frac{\partial u_r}{\partial z}\right)^2+\frac{1}{2}\left(\frac{\partial u_q}{\partial z}\right)^2+\frac{1}{2}\left(\frac{\partial u_z}{\partial z}\right)^2\\
&\varepsilon_{rz}=\frac{1}{2}\left(\frac{\partial u_z}{\partial r}+\frac{\partial u_r}{\partial z}+\frac{\partial u_r}{\partial r}\frac{\partial u_r}{\partial z}+\frac{\partial u_q}{\partial r}\frac{\partial u_q}{\partial z}+\frac{\partial u_z}{\partial r}\frac{\partial u_z}{\partial z}\right)\\
&\varepsilon_{rq}=\frac{1}{2}\left(\frac{\partial u_q}{\partial r}+\frac{1}{r}\frac{\partial u_r}{\partial q}-\frac{1}{r}u_q+\frac{1}{r}\frac{\partial u_r}{\partial r}\left(\frac{\partial u_r}{\partial q}-u_r\right)+\frac{1}{r}\frac{\partial u_q}{\partial r}\left(\frac{\partial u_q}{\partial q}-u_q\right)+\frac{\partial u_z}{\partial r}\frac{\partial u_z}{\partial q}\right)\\
&\varepsilon_{qz}=\frac{1}{2}\left(\frac{1}{r}\frac{\partial u_z}{\partial q}+\frac{\partial u_q}{\partial z}+\frac{1}{r}\frac{\partial u_r}{\partial z}\left(\frac{\partial u_r}{\partial q}-u_r\right)+\frac{1}{r}\frac{\partial u_q}{\partial z}\left(\frac{\partial u_q}{\partial q}-u_q\right)+\frac{1}{r}\frac{\partial u_z}{\partial q}\frac{\partial u_z}{\partial z}\right)
\end{aligned}
\tag{5}
$$

Using the Eqs. (1)-(5) we obtain the following initial strains:

$$
\varepsilon_{rr}^{(k),0}=\varepsilon_{qq}^{(k),0}=\frac{1}{2}\left(\left(\lambda_1^{(k)}\right)^2-1\right),\ \varepsilon_{zz}^{(k),0}=\frac{1}{2}\left(\left(\lambda_3^{(k)}\right)^2-1\right),\ \varepsilon_{rq}^{(k),0}=\varepsilon_{rz}^{(k),0}=\varepsilon_{qz}^{(k),0}=0 \tag{6}
$$

According to the Eq. (6), the following relations can be written:

$$
\frac{\partial}{\partial\varepsilon_{rr}^{(k),0}}=\frac{\partial}{\partial\varepsilon_{qq}^{(k),0}}=\frac{1}{\lambda_1^{(k)}}\frac{\partial}{\partial\lambda_1^{(k)}},\quad \frac{\partial}{\partial\varepsilon_{zz}^{(k),0}}=\frac{1}{\lambda_3^{(k)}}\frac{\partial}{\partial\lambda_3^{(k)}} \tag{7}
$$

Consider the definition of the stress and strain tensors in the large elastic deformation theory. For this purpose we use the Lagrange coordinates r, q and z in the cylindrical system of coordinates Orqz.

Consider the determination of the Kirchhoff stress tensor. The use of various types of stress tensors in the large (finite) elastic deformation theory is connected with the reference of the components of these tensors to the unit area of the relevant surface elements in the deformed or un-deformed state. This is because, in contrast to the linear theory of elasticity, in the finite elastic deformation theory, the difference between the areas of the surface elements taken before and after deformation must be accounted for in the derivation of the equation of motion and under satisfaction of the boundary conditions. According to the aim of the present investigation, we here consider two types of stress tensors denoted by $\tilde{q}$ and $\tilde{s}$ the components of which refer to the unit area of the relevant surface elements in the un-deformed state, but which act on the surface elements in the deformed state. The physical components $S_{(ij)}$ of the stress tensor $\tilde{s}$ are determined through the strain energy potential $\Phi = \Phi(\varepsilon_{rr}, \varepsilon_{qq}, ..., \varepsilon_{qz})$ by the use of the following expression:

$$S_{(ij)} = \frac{1}{2}\left(\frac{\partial}{\partial \varepsilon_{(ij)}} + \frac{\partial}{\partial \varepsilon_{(ji)}}\right)\Phi \tag{8}$$

Elasticity relationship of cylinders is expressed by the harmonic potential as in [20].

$$\Phi = \frac{1}{2}\lambda\left(s_1\right)^2 + \mu s_2 \tag{9}$$

$$\begin{gathered} s_1 = \sqrt{1+2\varepsilon_1} + \sqrt{1+2\varepsilon_2} + \sqrt{1+2\varepsilon_3} - 3 \\ s_2 = \left(\sqrt{1+2\varepsilon_1} - 1\right)^2 + \left(\sqrt{1+2\varepsilon_2} - 1\right)^2 + \left(\sqrt{1+2\varepsilon_3} - 1\right)^2 \end{gathered} \tag{10}$$

In Eqs. (8)-(9)-(10), λ and μ are material constants and ε_i are the principal values of Green's strain tensor.

Using the Eqs. (9)-(10) we obtain the following expression for the strain energy potential in the initial state:

$$\Phi^{(k),0} = \frac{1}{2}\lambda^{(k)}\left(2\lambda_1^{(k)} + \lambda_3^{(k)} - 3\right)^2 + \mu^{(k)}\left(2\left(\lambda_1^{(k)} - 1\right)^2 + \left(\lambda_3^{(k)} - 1\right)^2\right) \tag{11}$$

Using Eqs. (10)-(11) we obtain the following expressions for the stresses in the initial state:

$$\begin{gathered} S_{zz}^{(k),0} = \left[\lambda^{(k)}\left(2\lambda_1^{(k)} + \lambda_3^{(k)} - 3\right) + 2\mu^{(k)}\left(\lambda_3^{(k)} - 1\right)\right]\left(\lambda_3^{(k)}\right)^{-1} \\ S_{rq}^{(k),0} = S_{rz}^{(k),0} = S_{zq}^{(k),0} = S_{rr}^{(k),0} = S_{qq}^{(k),0} = 0 \end{gathered} \tag{12}$$

The stress tensor $\tilde{q}$ is called the Kirchhoff stress tensor. Using kirchhoff stress tensor in the initial state we obtain:

$$\begin{gathered} \omega_{1221}'^{(k)} = \omega_{1212}'^{(k)} = \frac{\mu^{(k)}}{\lambda_3^{(k)}}, \\ \omega_{1331}'^{(k)} = \frac{\lambda_1^{(k)}}{\lambda_1^{(k)} + \lambda_3^{(k)}}\left(2\mu^{(k)} - \lambda^{(k)}\left(2\lambda_1^{(k)} + \lambda_3^{(k)} - 3\right)\right) + \frac{1}{\lambda_3^{(k)}} S_{33}^{(k),0} \end{gathered} \tag{13}$$

Torsional wave propagation in the compound hollow cylinder will be investigated by the use of Eqs. (3)-(4)-(13) as in [5].

The imperfectness of the contact conditions is identified by discontinuities of the displacements and forces across the mentioned interface. A review of the mathematical modeling of the various type incomplete contact conditions for elastodynamics problems has been detailed in a paper [13]. It follows from this paper that for most models the discontinuity of the displacement u^+ and force f^+ vectors on one side of the interface are assumed to be linearly related to the displacement u_- and force f_- vectors on the other side of the interface. This statement, as in the paper [19], can be presented as follows:

$$[f] = Cu^- + Df^-, \quad [u] = Gu^- + Ff^- \tag{14}$$

where C, D, G and F are three-dimensional (3 x 3) matrices and the square brackets indicate a jump in the corresponding quantity across the interface. Consequently, if the interface is at $r = R + h_1$, then

$$[u] = u\big|_{r=R+h_1+0} - u\big|_{r=R+h_1-0} \quad [f] = f\big|_{r=R+h_1+0} - f\big|_{r=R+h_1-0} \tag{15}$$

It follows from Eq. (14) that we can write incomplete contact conditions for various particular cases by the selection of the matrices C, D, G and F. One of such selections was made in the paper [9], according to which, it is assumed that C = D = G = 0. In this case it is obtained from Eq. (14) that;

$$[f] = 0, \ [u] = Ff^- \tag{16}$$

where F is a constant diagonal matrix. The model Eq. (16) simplifies significantly the solution procedure of the corresponding problems and is adequate sufficiently with many real cases. Therefore, this model (i.e. the model Eq. (16)) has been used in many investigations carried out [2], [7], [8], [9], [10], [12] and [18]. According to this statement, we also use the model Eq. (16) for the mathematical formulation of the incomplete contact conditions which can be written for the problem under consideration as follows:

$$\begin{gathered}
u_q^{(2)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)} - u_q^{(1)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)} = \frac{\mathrm{RF}t_{rq}'^{(1)}}{\mu_1} \\
u_q^{(3)}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)} - u_q^{(2)}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)} = \frac{\mathrm{RF}t_{rq}'^{(2)}}{\mu_2} \\
t_{rq}'^{(1)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)} = t_{rq}'^{(2)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)} \Rightarrow t_{rq}'^{(1)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)} - t_{rq}'^{(2)}\Big|_{r'=\lambda_2^{(1)}\kappa R\left(1+\frac{h_1}{R}\right)} = 0 \\
t_{rq}'^{(2)}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)} = t_{rq}'^{(3)}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)} \Rightarrow t_{rq}'^{(2)}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)} - t_{rq}'^{(3)}\Big|_{r'=\lambda_2^{(2)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}\right)} = 0 \\
t_{rq}'^{(1)}\Big|_{r'=\lambda_2^{(1)}\kappa R} = 0 \\
t_{rq}'^{(3)}\Big|_{r'=\lambda_2^{(3)}\kappa R\left(1+\frac{h_1}{R}+\frac{h_2}{R}+\frac{h_3}{R}\right)} = 0
\end{gathered} \tag{17}$$

The parameter F in Eq. (17) characterizes the shear-spring type imperfectness between the cylinders under consideration. $F = 0$ determines continuous contact condition and $F > 0$ determines imperfect contact condition. In this paper, the influence of this parameter F on the dispersion curves are investigated.

3. SOLUTION PROCEDURE AND OBTAINING THE DISPERSION RELATION

As we assume that the harmonic torsional wave propagates along the Oz axis, we can accordingly represent the displacement $u_q^{(m)}(r, z, t)$ as,

$$u_q'^{(m)}(r', z', t) = -\frac{\partial}{\partial r'}\psi^{(m)}(r', z', t) \tag{18}$$

where the function $\psi^{(m)}$ in Eq. (18) satisfies the equation written below.

$$\left[\Delta_1 + (\xi'^{(m)})^2 \frac{\partial^2}{\partial z'^2} - \frac{\rho'}{\omega'_{1221}} \frac{\partial^2}{\partial t^2}\right] \psi = 0 \tag{19}$$

Where

$$\Delta'_1 = \frac{\partial^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial}{\partial r'} \quad \xi'^{(m)2}_n = \frac{2\lambda_3^{(m)3}}{\lambda_2^{(m)2}\left(\lambda_2^{(m)} + \lambda_3^{(m)}\right)} \tag{20}$$

It follows from the problem statement that the presentation

$$\psi^{(m)}(r',z',t) = \varphi^{(m)}\left(r'\right)\mathrm{e}^{\mathrm{i}(\kappa z' - \omega t)} \tag{21}$$

holds. Thus, we obtain from Eqs. (19)-(21) the following equation for unknown function $\varphi^{(m)}(r')$

$$\left[\frac{d^2}{dr'^2} + \frac{1}{r'}\frac{d}{dr'} - (\xi'^{(m)})^2\kappa^2 + \frac{\lambda_3^{(m)}\rho'^{(m)}\omega^2}{\mu^{(m)}}\right]\varphi(r')\mathrm{e}^{\mathrm{i}(\kappa z' - \omega t)} = 0 \tag{22}$$

Introducing the notation

$$(s^{(m)})^2 = (\xi'^{(m)})^2\kappa^2 + \frac{\lambda_3^{(m)}\rho'^{(m)}\omega^2}{\mu^{(m)}} \tag{23}$$

The solution to the Eq. (22) can be written as follows.

$$\begin{aligned} s^{(1)2} > 0 \;\; &\Rightarrow \;\; \phi^{(1)}\left(r'\right) = A^{(1)}\,\mathrm{J}_0\left(s^{(1)}\kappa r'\right) + B^{(1)}\,\mathrm{Y}_0\left(s^{(1)}\kappa r'\right) \\ s^{(2)2} > 0 \;\; &\Rightarrow \;\; \phi^{(2)}\left(r'\right) = A^{(2)}\,\mathrm{J}_0\left(s^{(2)}\kappa r'\right) + B^{(2)}\,\mathrm{Y}_0\left(s^{(2)}\kappa r'\right) \end{aligned} \tag{24}$$

$$\begin{aligned} s^{(1)2} < 0 \;\; &\Rightarrow \;\; \phi^{(1)}\left(r'\right) = A^{(1)}\,\mathrm{I}_0\left(s^{(1)}\kappa r'\right) + B^{(1)}\,\mathrm{K}_0\left(s^{(1)}\kappa r'\right) \\ s^{(2)2} < 0 \;\; &\Rightarrow \;\; \phi^{(2)}\left(r'\right) = A^{(2)}\,\mathrm{I}_0\left(s^{(2)}\kappa r'\right) + B^{(2)}\,\mathrm{K}_0\left(s^{(2)}\kappa r'\right) \end{aligned} \tag{25}$$

Using the Eqs. (4)-(18)-(21)-(24)-(25) we obtain the following dispersion equation from the condition Eq. (17).

$$\det\left\|\alpha_{ij}(\mathrm{c}/\,\mathrm{c}_2^{(2)}, \mathrm{kR}, \mathrm{h}_1, \mathrm{h}_2, \mathrm{h}_3, \mathrm{F}, \lambda_m^{(k)}, \lambda^{(\mathrm{k})}, \mu^k, \rho^{(\mathrm{k})})\right\| = 0, \qquad \mathrm{i;j} = 1,2,3,4,5,6 \tag{26}$$

Where,

$$\alpha_{11}=\begin{cases}\underbrace{\left[\ \mathrm{J}_1\left(s^{(1)}\kappa r'\right)\right.}-\dfrac{RF}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)\mathrm{J}_2\left(s^{(1)}\kappa r'\right)\ \Big] & ,\mathrm{s}^{(1)}>0\\ \underbrace{\left[\ -\mathrm{I}_1\left(s^{(1)}\kappa r'\right)\right.}-\dfrac{RF}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)\mathrm{I}_2\left(s^{(1)}\kappa r'\right)\ \Big] & ,\mathrm{s}^{(1)}<0\end{cases}$$

$$\alpha_{12}=\begin{cases}\underbrace{\left[\mathrm{Y}_1\left(s^{(1)}\kappa r'\right)-\dfrac{RF}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)\mathrm{Y}_2\left(s^{(1)}\kappa r'\right)\right]}_{s^2>0\ \Rightarrow\ \alpha_{42}} & ,\mathrm{s}^{(1)}>0\\ \left[\underbrace{\underbrace{\mathrm{K}_1\left(s^{(1)}\kappa r'\right)}_{0}-\dfrac{RF}{\lambda_3^{(1)}}\left(\mathrm{s}^{(1)}\kappa\right)\mathrm{K}_2\left(s^{(1)}\kappa r'\right)}_{s^2<0\ \Rightarrow\ \alpha_{42}}\right] & ,\mathrm{s}^{(1)}<0\end{cases}$$

$$\alpha_{13}=\begin{cases}-\dfrac{\mathrm{s}^{(2)}}{\mathrm{s}^{(1)}}\mathrm{J}_1\left(s^{(2)}\kappa r'\right) & ,s^{(2)}>0\\ \dfrac{\mathrm{s}^{(2)}}{\mathrm{s}^{(1)}}\mathrm{I}_1\left(s^{(2)}\kappa r'\right) & ,\mathrm{s}^{(2)}<0\end{cases}\qquad \alpha_{14}=\begin{cases}-\dfrac{\mathrm{s}^{(2)}}{\mathrm{s}^{(1)}}\mathrm{Y}_1\left(s^{(2)}\kappa r'\right) & ,\mathrm{s}^{(2)}>0\\ -\dfrac{\mathrm{s}^{(2)}}{\mathrm{s}^{(1)}}\mathrm{K}_1\left(s^{(2)}\kappa r'\right) & ,\mathrm{s}^{(2)}<0\end{cases}$$

$$\alpha_{15}=0\qquad \alpha_{16}=0\qquad \alpha_{21}=0\qquad \alpha_{22}=0$$

$$\alpha_{23}=\begin{cases}\underbrace{\left[\ \mathrm{J}_1\left(s^{(2)}\kappa r'\right)\right.}-\dfrac{RF}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)\mathrm{J}_2\left(s^{(2)}\kappa r'\right)\ \Big] & ,\mathrm{s}^{(2)}>0\\ \underbrace{\left[\ -\mathrm{I}_1\left(s^{(2)}\kappa r'\right)\right.}-\dfrac{RF}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)\mathrm{I}_2\left(s^{(2)}\kappa r'\right)\ \Big] & ,\mathrm{s}^{(2)}<0\end{cases}$$

$$\alpha_{24}=\begin{cases}\underbrace{\left[\mathrm{Y}_1\left(s^{(2)}\kappa r'\right)-\dfrac{RF}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)\mathrm{Y}_2\left(s^{(2)}\kappa r'\right)\right]}_{s^2>0\ \Rightarrow\ \alpha_{42}} & ,\mathrm{s}^{(2)}>0\\ \left[\underbrace{\underbrace{\mathrm{K}_1\left(s^{(2)}\kappa r'\right)}_{0}-\dfrac{RF}{\lambda_3^{(2)}}\left(\mathrm{s}^{(2)}\kappa\right)\mathrm{K}_2\left(s^{(2)}\kappa r'\right)}_{s^2<0\ \Rightarrow\ \alpha_{42}}\right] & ,\mathrm{s}^{(2)}<0\end{cases}$$

$$\alpha_{25}=\begin{cases}-\dfrac{\mathrm{s}^{(3)}}{\mathrm{s}^{(2)}}\mathrm{J}_1\left(s^{(3)}\kappa r'\right) & ,\mathrm{s}^{(3)}>0\\ \dfrac{\mathrm{s}^{(3)}}{\mathrm{s}^{(2)}}\mathrm{I}_1\left(s^{(3)}\kappa r'\right) & ,\mathrm{s}^{(3)}<0\end{cases}\qquad \alpha_{26}=\begin{cases}-\dfrac{\mathrm{s}^{(3)}}{\mathrm{s}^{(2)}}\mathrm{Y}_1\left(s^{(3)}\kappa r'\right) & ,\mathrm{s}^{(3)}>0\\ -\dfrac{\mathrm{s}^{(3)}}{\mathrm{s}^{(2)}}\mathrm{K}_1\left(s^{(3)}\kappa r'\right) & ,\mathrm{s}^{(3)}<0\end{cases}$$

$$\alpha_{31}=\begin{cases}-\dfrac{\mu^{(1)}}{\mu^{(2)}\lambda_3^{(1)}}\left\{J_2(s^{(1)}\kappa r')\right\} & ,\ s^{(1)}>0\\ -\dfrac{\mu^{(1)}}{\mu^{(2)}\lambda_3^{(1)}}\left\{I_2(s^{(1)}\kappa r')\right\} & ,\ s^{(1)}<0\end{cases}\qquad \alpha_{32}=\begin{cases}-\dfrac{\mu^{(1)}}{\mu^{(2)}\lambda_3^{(1)}}\left\{Y_2(s^{(1)}\kappa r')\right\} & ,s^{(1)}>0\\ -\dfrac{\mu^{(1)}}{\mu^{(2)}\lambda_3^{(1)}}\left\{K_2(s^{(1)}\kappa r')\right\} & ,s^{(1)}<0\end{cases}$$

$$\alpha_{33}=\begin{cases}\left(\dfrac{s^{(2)}}{s^{(1)}}\right)^2\dfrac{1}{\lambda_3^{(2)}}J_2\left(s^{(2)}\kappa r'\right) & ,\ s^{(2)}>0\\ \left(\dfrac{s^{(2)}}{s^{(1)}}\right)^2\dfrac{1}{\lambda_3^{(2)}}I_2\left(s^{(2)}\kappa r'\right) & ,\ s^{(2)}<0\end{cases}\qquad \alpha_{34}=\begin{cases}\left(\dfrac{s^{(2)}}{s^{(1)}}\right)^2\dfrac{1}{\lambda_3^{(2)}}Y_2\left(s^{(2)}\kappa r'\right) & ,\ s^{(2)}>0\\ \left(\dfrac{s^{(2)}}{s^{(1)}}\right)^2\dfrac{1}{\lambda_3^{(2)}}K_2\left(s^{(2)}\kappa r'\right) & ,\ s^{(2)}<0\end{cases}$$

$$\alpha_{35}=0\quad \alpha_{36}=0\quad \alpha_{41}=0\quad \alpha_{42}=0$$

$$\alpha_{43}=\begin{cases}-\dfrac{\mu^{(2)}}{\mu^{(3)}\lambda_3^{(2)}}\left\{J_2(s^{(2)}\kappa r')\right\} & ,s^{(2)}>0\\ -\dfrac{\mu^{(2)}}{\mu^{(3)}\lambda_3^{(2)}}\left\{I_2(s^{(2)}\kappa r')\right\} & ,s^{(2)}<0\end{cases}\qquad \alpha_{44}=\begin{cases}-\dfrac{\mu^{(2)}}{\mu^{(3)}\lambda_3^{(2)}}\left\{Y_2(s^{(2)}\kappa r')\right\} & ,s^{(2)}>0\\ -\dfrac{\mu^{(2)}}{\mu^{(3)}\lambda_3^{(2)}}\left\{K_2(s^{(2)}\kappa r')\right\} & ,s^{(2)}<0\end{cases}$$

$$\alpha_{45}=\begin{cases}\left(\dfrac{s^{(3)}}{s^{(2)}}\right)^2\dfrac{1}{\lambda_3^{(3)}}J_2\left(s^{(3)}\kappa r'\right) & ,\ s^{(3)}>0\\ \left(\dfrac{s^{(3)}}{s^{(2)}}\right)^2\dfrac{1}{\lambda_3^{(3)}}I_2\left(s^{(3)}\kappa r'\right) & ,\ s^{(3)}<0\end{cases}\qquad \alpha_{46}=\begin{cases}\left(\dfrac{s^{(3)}}{s^{(2)}}\right)^2\dfrac{1}{\lambda_3^{(3)}}Y_2\left(s^{(3)}\kappa r'\right) & ,\ s^{(3)}>0\\ \left(\dfrac{s^{(3)}}{s^{(2)}}\right)^2\dfrac{1}{\lambda_3^{(3)}}K_2\left(s^{(3)}\kappa r'\right) & ,\ s^{(3)}<0\end{cases}$$

$$\alpha_{51}=\begin{cases}-\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}J_2\left(s^{(1)}\kappa r'\right) & ,\ s^{(1)}>0\\ -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}I_2\left(s^{(1)}\kappa r'\right) & ,\ s^{(1)}<0\end{cases}\qquad \alpha_{52}=\begin{cases}-\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}Y_2\left(s^{(1)}\kappa r'\right) & ,\ s^{(1)}>0\\ -\dfrac{\mu^{(1)}}{\lambda_3^{(1)}}K_2\left(s^{(1)}\kappa r'\right) & ,\ s^{(1)}<0\end{cases}$$

$$\alpha_{53}=0\quad \alpha_{54}=0\quad \alpha_{55}=0\quad \alpha_{56}=0\quad \alpha_{61}=0\quad \alpha_{62}=0\quad \alpha_{63}=0\quad \alpha_{64}=0$$

$$\alpha_{65}=\begin{cases}-\dfrac{\mu^{(3)}}{\lambda_3^{(3)}}J_2\left(s^{(3)}\kappa r'\right) & ,\ s^{(3)}>0\\ -\dfrac{\mu^{(3)}}{\lambda_3^{(3)}}I_2\left(s^{(3)}\kappa r'\right) & ,\ s^{(3)}<0\end{cases}\qquad \alpha_{66}=\begin{cases}-\dfrac{1}{\lambda_3^{(3)}}\left\{Y_2\left(s^{(3)}\kappa r'\right)\right\} & ,s^{(3)}>0\\ -\dfrac{1}{\lambda_3^{(3)}}\left\{K_2\left(s^{(3)}\kappa r'\right)\right\} & ,s^{(3)}<0\end{cases}\qquad (27)$$

Thus, the dispersion equation for the considered torsional wave propagation problem has been derived in the form presented in Eq. (27).

4. NUMERICAL RESULTS AND DISCUSSIONS

We have found that the first lowest mode which is non-dispersive homogenous hollow cylinder, becomes dispersive for a compound one. The limiting value of the torsional wave speed for the case considered is determined from dispersion Eqs. (26)-(27) by using power series expansions of Bessel functions, retaining only the dominant term as $k\mathrm{R} \to 0$:

$$\alpha = \frac{\left(\left(\lambda_2^{(2)}\right)^4 \zeta_2{}^4 - \left(\lambda_2^{(1)}\right)^4 \zeta_1{}^4\right)}{\left(\lambda_2^{(1)}\right)^4 \left(\zeta_1{}^4 - 1\right)}, \qquad \beta = \frac{\left(\left(\lambda_2^{(3)}\right)^4 \zeta_3{}^4 - \left(\lambda_2^{(2)}\right)^4 \zeta_2{}^4\right)}{\left(\lambda_2^{(1)}\right)^4 \left(\zeta_1{}^4 - 1\right)} \tag{28}$$

$$\frac{c}{c_2{}^{(1)}} = \left[\frac{\frac{\mu^{(1)}}{\lambda_3^{(1)}}\left(\xi_1{}^{,(1)}\right)^2 + \frac{\mu^{(2)}}{\lambda_3^{(2)}}\alpha\left(\xi_1{}^{,(2)}\right)^2 + \frac{\mu^{(3)}}{\lambda_3^{(3)}}\beta\left(\xi_1{}^{,(3)}\right)^2}{\mu^{(1)} + \mu^{(2)}\frac{c_2{}^{(1)2}}{c_2{}^{(2)2}}\alpha + \mu^{(3)}\frac{c_2{}^{(1)2}}{c_2{}^{(3)2}}\beta}\right]^{1/2} \tag{29}$$

Where,

$$\zeta_1 = 1 + \frac{h1}{R} \qquad \zeta_2 = 1 + \frac{h1}{R} + \frac{h2}{R} \qquad \zeta_3 = 1 + \frac{h1}{R} + \frac{h2}{R} + \frac{h3}{R} \tag{30}$$

In the case where $\lambda_3^{(m)} = \lambda_2^{(m)} = 1.0$, the Eq. (28)-(29) transforms to the following one.

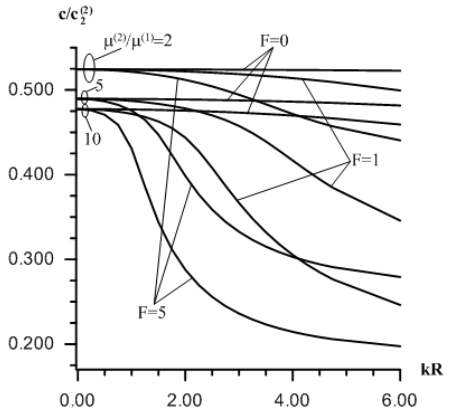
$$\left(\frac{c}{c_2{}^{(1)}}\right)^2 = \frac{\mu^{(1)} + \mu^{(2)}\alpha}{\mu^{(1)} + \mu^{(2)}\alpha\frac{c_2{}^{(1)2}}{c_2{}^{(2)2}}} \tag{31}$$

Moreover, the Eq. (29) is a generalization of the corresponding one attained in the paper [15] for the finite initial strain state. Note that in the paper [15] this type expression was obtained for the small initial strain state.

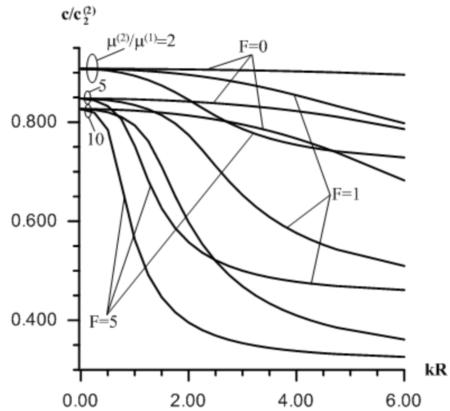
It follows from the Eqs. (20)-(29) that the limit values of $c/c_2^{(1)}$, where $c_2^{(1)} = \sqrt{\mu^{(1)}/\rho^{(1)}}$ decrease with $\mu^{(1)}/\mu^{(2)}$ and increase with $\lambda = (\lambda_3^{(1)} = \lambda_3^{(2)})$ Consequently, the initial stretching (compression) of the compound cylinder along the torsional wave propagation direction causes to increase (to decrease) of the limit velocity of this wave as $kR \to 0$. According to the known physical-mechanical consideration, the other limit value of the velocity of the considered wave, i.e. the limit velocity as $kR \to \infty$ must be equal to min $\left\{c_2^{(1)}(\lambda_3^{(1)}), c_2^{(2)}(\lambda_3^{(2)})\right\}$,i.e. the following relation must be hold.

$$c \to \min\left\{c_2^{(1)}(\lambda_3^{(1)}), c_2^{(2)}(\lambda_3^{(2)}), c_2^{(3)}(\lambda_3^{(3)})\right\} \text{ as } kR \to \infty \tag{32}$$

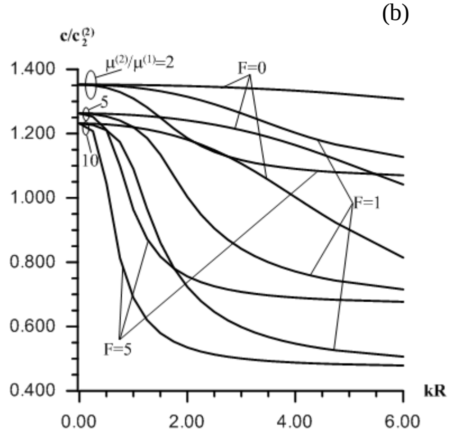
Accuracy of the algorithms used in the considered problem for a similar situation in [5] paper had proven in paper [6] and [11]. Thereafter, $c/c_2^{(2)}$ and kR have been studied.



(a)



(b)



(c)

Figure 2. The influence of the parameter F on dispersion curves under varios values of stiffness. $h^{(1)}/R = h^{(3)}/R = 0.1$ and $h^{(2)}/R = 0.4$. a) $\lambda_3 = 0.6$, b) $\lambda_3 = 1.0$, c) $\lambda_3 = 1.4$.

The parameter F_1 describes the imperfectness between the inner and middle layers. The parameter F_2 describes the imperfectness between the middle and outher layers. The notation $F(= F_1 = F_2)$ is introduced and the influence of the parameter F which characterizes the imperfectness in the layers of the cylinders on the dispersion curves obtained for various values of the problem parameters is studied. Also, the notation $\lambda_3 (= \lambda_3^{(1)} = \lambda_3^{(2)} = \lambda_3^{(3)})$ is introduced and the influence of the elongation parameter λ_3 which characterizes the initial strains in the cylinder on the dispersion curves is considered.

First, we consider the case where $F_1 = F_2$, $\mu^{(1)} = \mu^{(3)}$ and $\rho^{(1)} = \rho^{(2)} = \rho^{(3)}$. Fig. 2 shows the influence of the parameter F on dispersion curves obtained for the first lowest mode in the cases where $\lambda_3 = 0.6$ (Fig. 2a), $\lambda_3 = 1.0$ (Fig. 2b) and $\lambda_3 = 1.4$ (Fig. 2c) for various values of the parameter $\mu^{(2)} / \mu^{(1)}$ under $h^{(1)} / R = h^{(3)} / R = 0.1$ and $h^{(2)} / R = 0.4$. It follows from Fig. 2 the increase in middle cylinder's material stiffness, i.e. an increase in the values of $\mu^{(2)} / \mu^{(1)}$ causes to decrease in the value of $c/c_2^{(2)}$.

If we look at the issue we are mainly interested in, with increasing of F, value of $c/c_2^{(2)}$ decreases. Moreover, we see that in Fig. 2b(there is no initial strain $\lambda_3 = 1.0$), with the increasing stiffness of middle cylinder $\mu^{(2)} / \mu^{(1)}$, the influence of increasing F became more obvious. Looking at the whole Fig. 2, increasing of λ_3 the influence of the parameter F considerably increases and propagation velocity significantly decreases along with kR.

According to the mechanical consideration, the values of $c / c_2^{(2)}$ must increase with the thickness of the middle layer of the cylinder ($h^{(2)} / R$). This prediction is confirmed by the graphs given in Fig. 3, which are constructed in the cases where $\lambda_3 = 0.6$ (Fig. 3a), $\lambda_3 = 1.0$ (Fig. 3b) and $\lambda_3 = 1.4$ (Fig. 3c) for various values of $h^{(2)} / R$ under $h^{(1)} / R = h^{(3)} / R = 0.1$ and $\mu^{(2)} / \mu^{(1)} = 2.0$.

As seen in Fig. 3 wave propagation velocity increases with the $h^{(2)} / R$ and in parallel with the increase in the value of the contacting simulating F, wave propagation speed is relatively decreases. Moreover, we see that in Fig. 3, with the increasing of λ_3 the influence of the parameter F considerably increases and propagation velocity decreases along with kR.

In all the considered cases, the limit velocity of the wave propagation as $kR \to 0$ is the same and independent of the parameter F. Effects of parameter F decreases with kR. Finally, we note that the wave propagation velocity approch to the $\min\left\{c_R^{(m)}\right\}$ as $kR \to \infty$, where $c_R^{(m)} (m = 1,2,3)$ is a Rayleigh wave velocity of the m-th material. Consequently, the high wavenumber limit value of the torsional wave propagation velocity also does not depend on the parameter F.

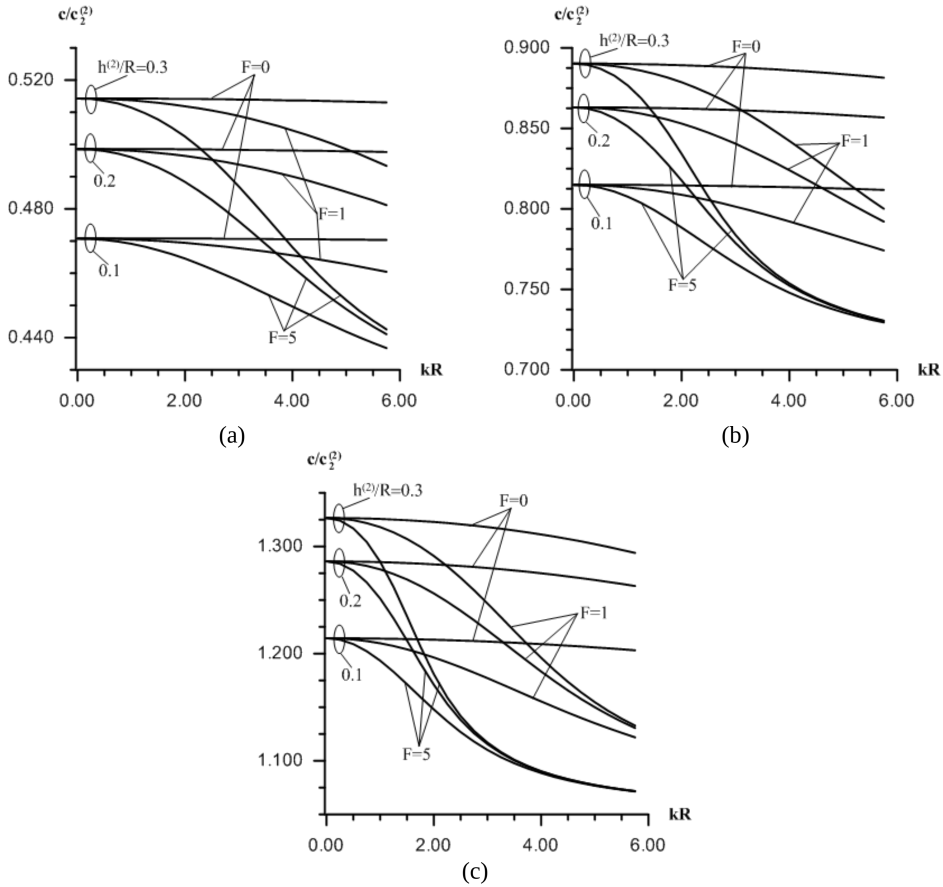


Figure 3. The influence of the parameter F on dispersion curves under varios values of $h^{(2)}/R$. $h^{(1)}/R = h^{(3)}/R = 0.1$ and $\mu^{(2)}/\left(\mu^{(1)} = \mu^{(3)}\right) = 2.0$. a) $\lambda_3 = 0.6$, b) $\lambda_3 = 1.0$, c) $\lambda_3 = 1.4$.

Assume that the imperfectness of the contact condition of cylinder layers are different. Now we case where $F_1 \neq F_2$.

The influence of the parameters F_1 and F_2 on dispersion curves under varios values of the initial strains are presented in Fig. 4 in the cases where $\lambda_3 = 0.6$ (Fig. 4a), $\lambda_3 = 1.0$ (Fig. 4b), $\lambda_3 = 1.4$ (Fig. 4c) under $\mu^{(2)}/\left(\mu^{(1)} = \mu^{(3)}\right) = 2.0$, $h^{(1)}/R = h^{(3)}/R = 0.1$ and $h^{(2)}/R = 0.4$. We see that with the growing of the initial strains value, the influence of the parameters F_1 and F_2 becomes more prominent. Meanwhile increasing of F_2 is more efficient on dispersion curves than that of the parameter F_1.

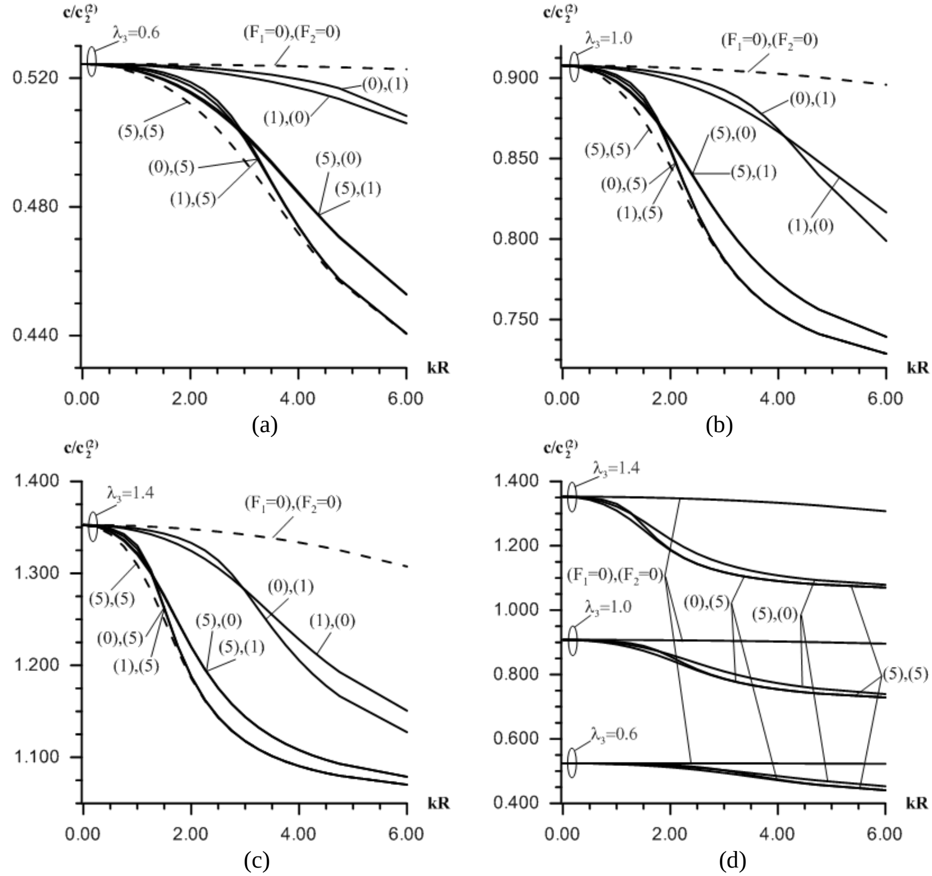


Figure 4. The influence of the parameter $F(F_1, F_2)$ on dispersion curves under varios values of λ_3. $\mu^{(2)}/\mu^{(1)} = 2.0$, $h^{(1)}/R = h^{(3)}/R = 0.1$ and $h^{(2)}/R = 0.4$. a) $\lambda_3 = 0.6$, b) $\lambda_3 = 1.0$, c) $\lambda_3 = 1.4$.

Now we consider how the parameters F_1 and F_2 effect on the dispersion curves under various values of $\mu^{(2)}/\mu^{(1)}$. Corresponding dispersion curves are given in Fig. 5. from which again follows that the influence of the parameter F_2 on the wave propagation is more than that of the parameter F_1. When F_2 increases, the wave propagation speed further decreases. Considering Fig. 5(b)-(c), for a high value of F_2, it is observed that dispersion curves overlapping conflicts.

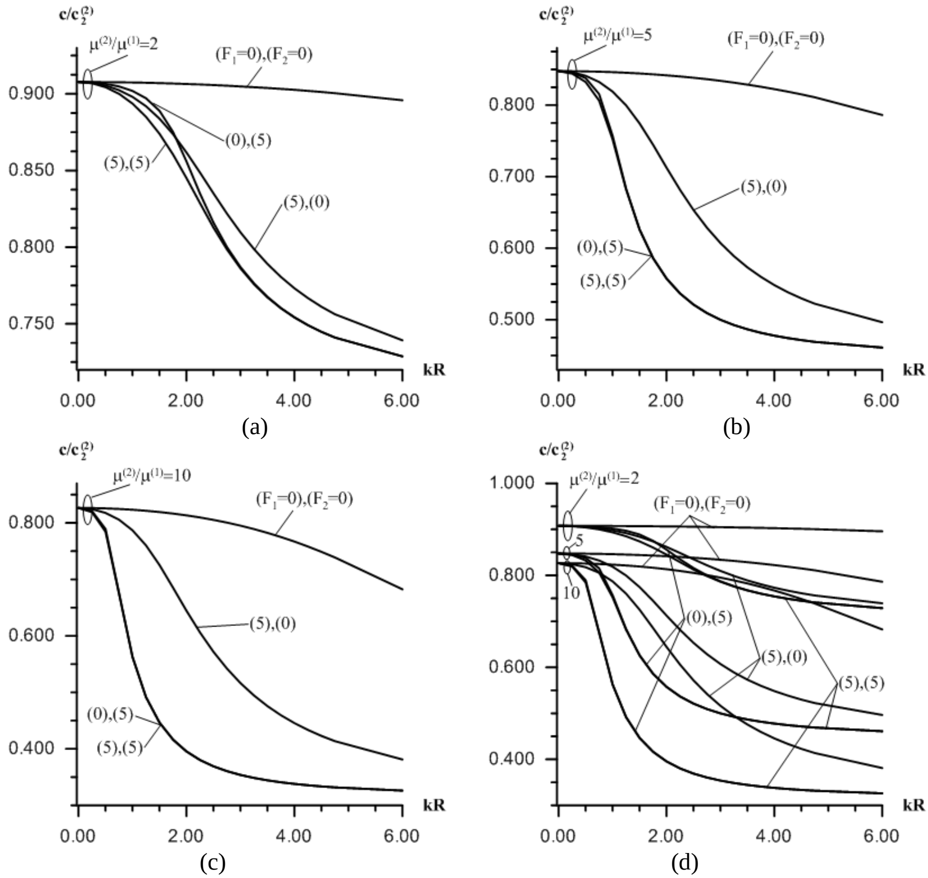


Figure 5. The influence of the parameter $F(\mathrm{F}_1, \mathrm{F}_2)$ on dispersion curves under various values of stiffness. $h^{(1)}/R = h^{(3)}/R = 0.1$, $h^{(2)}/R = 0.4$ and $\lambda_3 = 1$. a) $\mu^{(2)}/\left(\mu^{(1)} = \mu^{(3)}\right) = 2$, b) $\mu^{(2)}/\left(\mu^{(1)} = \mu^{(3)}\right) = 5$, c) $\mu^{(2)}/\left(\mu^{(1)} = \mu^{(3)}\right) = 10$

5. CONCLUSION

In the present paper, the influence of the bonded imperfectness on torsional wave dispersion in a three-layered (sandwich) hollow circular cylinder with finite homogeneous axisymmetric initial strains has been studied for the first lowest mode. The basic numerical investigations were made for the case where the materials of the inner and external hollow cylinders were the same and the materials of the middle hollow cylinder is more stiffer than that of the external hollow cylinders. Concrete numerical results are presented basically for the case where the initial strains in the cylinders are equal to each other. According to these results the following concrete conclusions are indicated.

The case where $F_1 = F_2 (= F)$;

- The increase in middle cylinder's material stiffness, i.e. an increase in the values of $\mu^{(2)}/\mu^{(1)}$ causes to decrease in the value of $c/c_2^{(2)}$ and with the increasing of F, value of $c/c_2^{(2)}$ further decreases.
- With the increasing of λ_3 the influence of the parameter F considerably increases and propagation velocity decreases along with kR.
- With the increase in the value of the contacting simulating F, wave propagation speed is relatively decreases.

The case where $F_1 \neq F_2$;

- With the growing of the initial strains and the middle cylinder stiffness value, the influence of the parameters F_1 and F_2 becomes more prominent. Meanwhile increasing of F_2 is more efficient on dispersion curves than that of the parameter F_1.
- Low and high wavenumber limit values of the torsional wave propagation velocity do not depend on the parameter $F(F_1, F_2)$ and are determined through the Eqs. (29)-(32).

According to the foregoing results, it can be concluded that the considered type imperfection causes to decrease of the wave propagation velocity and the initial stretching acts significantly not only the wave propagation velocity, but also the magnitude of the mentioned influence of the imperfection of the contact on this velocity.

The results and the method of the present investigation can be used for determination and control/minimizing of the noise of threelayered polymer pipes which are used to transfer various types of liquids. At the same time, the results of these investigations can be used for determination and control of the adhesion quality in the layered materials.

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