



Research Article

CRITICAL BUCKLING LOAD OF THE SIDE WALLS OF CABINET FURNITURE

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ABSTRACT

Loss of furniture stability may constitute a rapidly occurring, direct hazard to the user's life or health. Studies conducted to date have concerned the effect of the type of material and method of back wall attachment on its stability. In contrast, practically no analogous data are available on the stability of side walls. In the case of loading with a vertical force applied on a furniture item supported at three corners all the elements are subjected to torsional strain. However, the static pattern changes when the structure is supported at four corners. Then loading of a piece of furniture with a vertical force results in slight displacements in the direction of the acting force, whereas buckling and considerable deflections are perpendicular to the direction of that load. This has a significant effect on side wall stability. For this reason it was decided in this study to determine the effect of side wall structure on stability (buckling) and postbuckling behaviour of side walls in cabinet furniture. The analyses consisted in numerical calculations using the Finite Element method. Three types of side wall structures were used: particleboard, honeycomb panel and board-on-frame. Values of critical forces, lateral deflections and normal stresses in the direction of the acting load were calculated. Testing results confirmed that the honeycomb panel provides the most advantageous structure for side walls.

Keywords: Furniture, side wall, honeycomb, buckling, postbuckling, numerical analysis.

1. INTRODUCTION

Furniture durability, rigidity and stability of furniture are the most essential characteristics in terms of user safety. To date many studies have been conducted to describe rigidity and durability of furniture fastened in three corners (Ganowicz et al. 1978, Smardzewski et al. 2016). Tests have shown that torsional deformation of the structure depends solely on the torsional deformations of individual board elements in furniture (Fig.1a). In turn, rigidity of furniture depends on the rigidity and strength of joints, modulus of elasticity in shear, board thickness and their size dimensions (Smardzewski et al. 2014, Smardzewski 2015). A significant element addressed in furniture design is also connected with their stability. Loss of furniture stability may suddenly and directly threaten users' health or lives. This is particularly true in the case of furniture for children, since they are unable to promptly respond to direct hazard (Smardzewski 2015). Loss of stability may be a consequence of reaching critical load, resulting in structural failure due to considerable strains or due to bifurcation buckling, at which the structure starts to be deformed to a completely

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new form (Łodygowski and Kąkol, 2003). So far studies have been conducted on the effect of the type of material and manner of back wall attachment on its stability (Smardzewski and Dziegielewski 1993). In this case tests were conducted on a piece of furniture fastened in four corners with various types of back wall assembly. Analyses showed the smallest deflections for the back wall made of particleboard placed in a groove (Smardzewski and Dziegielewski 1993). In contrast, no such data are available on side wall stability. In the case of loading with a vertical force for a furniture item supported in three corners (Fig.1a) all the elements are subjected to torsional strain. Rigidity of the structure is then expressed as the quotient of force **F** and displacement **DF**. Nevertheless, the static model is changed when the structure is supported in four corners (Fig.1b). In this case loading of a furniture item with a vertical force **F** causes slight displacement **DF** in the direction of the force, while buckling and considerable deflections **w** are observed perpendicularly to the direction of this load.

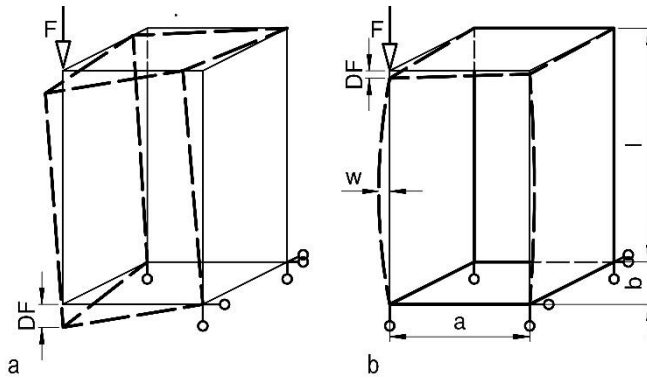


Figure 1. A scheme for the furniture strain under load:
a) element twisting, b) side wall buckling.

We need to observe here that the volume of deflection **w** and the value of critical force causing buckling of a side wall will depend on the type and thickness of used material. Moreover, in an attempt to minimise weight of furniture, popular particleboards are being replaced with honeycomb panels. This may have a significant effect on side wall stability. Thus in this study it was decided to determine the effect of side wall structure on stability (buckling) and postcritical (postbuckling) behaviour of side walls in cabinet furniture.

2. MATERIALS AND METHODS

2.1. Stability of thin panels

When investigating a freely supported rectangular board with dimensions **a** and **l** and thickness **t** (Fig.2) uniformly loaded with compressive force **s_x** along shorter sides **a**, the strain function **w** may be written in the form (Timoshenko and Woinowsky-Krieger, 1987; Yamaguchi, 1999):

$$w = A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{l}, \tag{1}$$

where **m** and **n** denote the number of sine half-waves in directions **x** and **y**. By introducing this equation to the general differential equation for the deflection of a rectangular board:

$$D \left(\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} \right) + t \left(\sigma_x \frac{\partial^2 w}{\partial x^2} + 2\tau_{xy} \frac{\partial^2 w}{\partial x \partial y} + \sigma_y \frac{\partial^2 w}{\partial y^2} \right) = 0, \tag{2}$$

where:

$D = \frac{Et^3}{12(1-\nu^2)}$, E - modulus of linear elasticity of the board, ν - Poisson's ratio of the board, and adopting the assumption on balance of a deflected board ($w \neq 0$) we obtain an expression describing critical stresses causing loss of its stability:

$$\sigma_x = \frac{\pi^2 D}{ta^2} K. \tag{3}$$

Thus critical force F_{cr} causing loss of stability may be described as:

$$F_{cr} = \frac{\pi^2 D}{a} K, \tag{4}$$

where: $\alpha=l/a$ and $K = \left(\frac{m}{\alpha} + n^2 \frac{\alpha}{m}\right)^2$. For a side wall of cabinet furniture subjected to uniform compression along edge a the value of coefficient K depends on the method of support and the l/a ratio (Fig. 2).

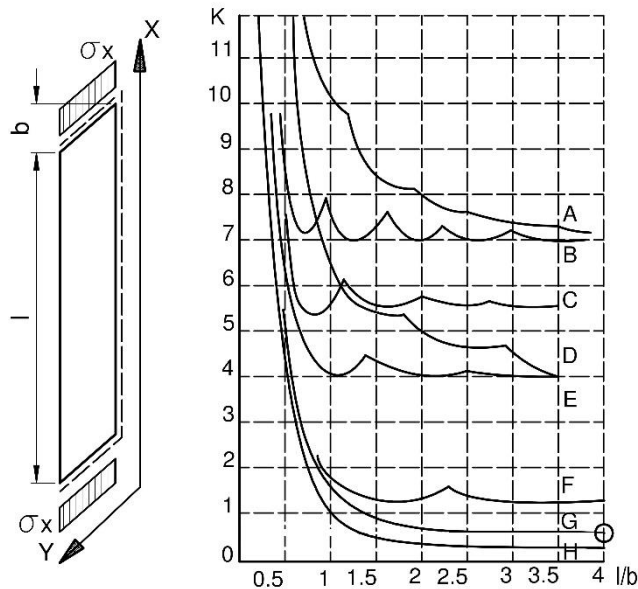


Figure 2. Variation in index K for a rectangular board uniformly compressed along edge a for various methods of margin support: A - four edges fastened, B - two long edges fastened, shorter edges with articulated supports, C - one longer edge fastened, three edges with articulated supports, D - two shorter edges fastened, longer edges with articulated supports, E - all four edges with articulated supports, F - one longer edge fastened, the other longer edge free, the other edges with articulated supports, G - one longer edge free, the other edges with articulated supports, H - two longer edges free, shorter edges with articulated supports.

2.2. Side wall structure

For the purpose of these analyses virtual models were prepared for side walls with dimensions $t=22$ mm, $a=400$ mm, $l=1800$ mm. Three design solutions were selected: an element

manufactured solely of particleboard PB, an element manufactured as honeycomb panel with a cardboard core with hexagonal cells, and a frame element manufactured as board-on-frame, but with no cardboard core (Fig. 3). In the case of honeycomb panels the frame on the circumference was manufactured from particleboard PB of 16 mm in thickness. Facing was made from HDF board of 3 mm in thickness. Faces were glued to the frame and the cardboard core using PVAc adhesive applied onto the HDF board at 120 g/m². Physic-mechanical properties of used boards were determined in accordance with respective standards (BS EN 310:1993, BS EN 322:1993, BS EN 323:1993) and presented in Table 1.

Table 1. Physical and mechanical properties of tested materials
(laboratory conditions T=26 °C; H=40%)

Type of material	Thicknes [mm]	MC	Density	MOE	MOR
		[%]	[kg/m ³]	[MPa]	[MPa]
HDF PB	3.0	6.14/0.20	846/12	4071/416	41.0/4.0
	16	5.76/0.95	664/11	1672/100	6.6/0.9

/Standard deviation

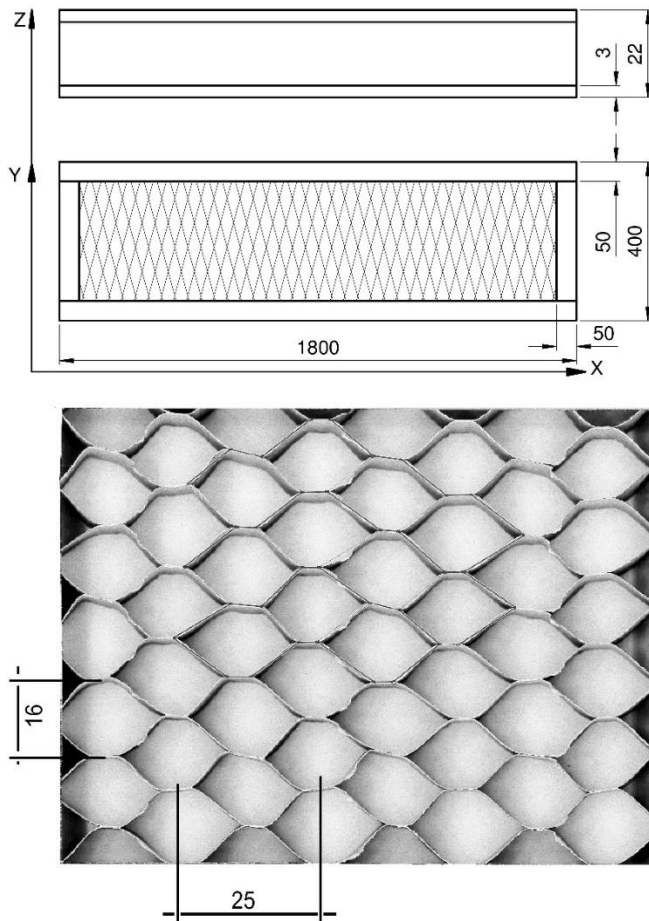


Figure 3. Dimensions of tested panels and the shape of core cells.

In the case of hexagonal core cells (Fig. 4) it was decided to determine their relative density and elastic properties.

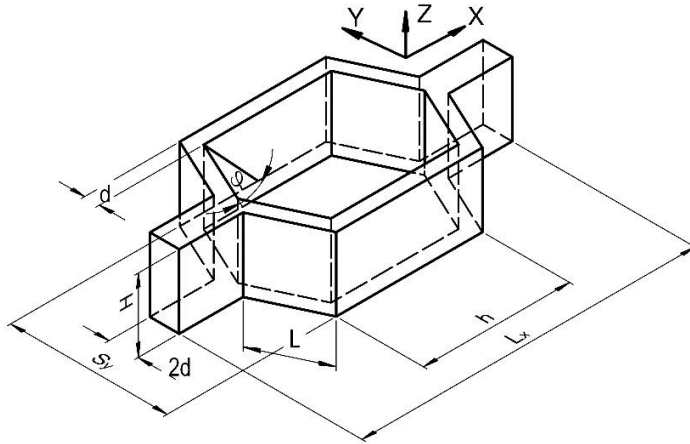


Figure 4. A hexagonal core cell.

Relative density ρ is the ratio of cell density ρ^* to density of cell matter ρ_s , and it is described in the form:

$$\rho = \frac{\rho^*}{\rho_s}, \quad (5)$$

where: cell density ρ^* is a ratio of cell mass to its volume, while density of cell matter ρ_s is the ratio of cell matter mass to the volume of cell matter. Moreover, considering that the cell wall height H is constant, the notation may take the final form as

$$\rho_p = \frac{F_s}{F^*}, \quad (6)$$

where: F_s and F^* , respectively, is the surface area of the matter and the cell surface area. Finally the surface area of the rectangle, in which the honeycomb cell is inscribed, may be presented as:

$$F^* = 4(L \cos(\varphi) + d)(h + L \sin(\varphi) - d \cot(\varepsilon)). \quad (7)$$

Using trigonometric relations seen in Fig. 4 the following were obtained:

$$F_1 = 2L \cos(\varphi)(h - 2d \cot(\varepsilon) + L \sin(\varphi)), \quad (8)$$

$$F_2 = 2((L \cos(\varphi) + d) - 2d)(h - 2d \cot(\varepsilon)), \quad (9)$$

$$F_3 = 2L \sin(\varphi) \cos(\varphi) (L - d \cot(\varepsilon)). \quad (10)$$

Relative density ρ_p is thus the ratio of surface area of core matter F_s to the surface area of the rectangle, in which it is inscribed F^* ,

$$\rho = 1 - \frac{F_1 + F_2 + F_3}{F^*}. \quad (11)$$

For a single cell two longitudinal moduli of elasticity E_x , E_y and two Poisson's ratios ν_{xy} , ν_{yx} were established:

$$E_x = \frac{E_s d^3 \left(\frac{h}{L} + \sin(\varphi)\right)}{L^3 \cos^3(\varphi)}, \quad (12)$$

where: E_s - modulus of linear elasticity of the matter.

$$E_y = \frac{E_s d^3 \cos(\varphi)}{L^3 \left(\frac{h}{L} + \sin(\varphi)\right) \sin^2(\varphi)}, \quad (13)$$

$$u_{xy} = \frac{\sin(\varphi)\left(\frac{h}{L} + \sin(\varphi)\right)}{\cos^2(\varphi)}, \tag{14}$$

$$u_{yx} = \frac{\cos^2(\varphi)}{\left(\frac{h}{L} + \sin(\varphi)\right)\sin(\varphi)}. \tag{15}$$

Table 2 presents calculated values of elastic properties of the used cell.

Table 2. Characteristic properties of the hexagonal cell

E_s	d	h	L	f	L_x	S_v	r	E_x	E_v	u_{xy}	u_{yx}
MPa	mm			°	mm			MPa			
6000	0.15	3.9	11.5	45	23.99	16.62	0.02493	0.0389	0.0178	0.1017	1.48

In view of the above-mentioned material data, for the investigated side wall with one longer edge being free and three other edges with articulated supports, the critical force causing buckling of the side wall of thickness $t=22$ mm is equal to:

$$F_{cr} = \frac{3.14^2(1672 \cdot 22^3)}{400 \cdot 12(1-0.3^2)} \cdot 0.4 = 16074 \text{ N}. \tag{16}$$

2.3. Numerical calculations

Numerical calculations were performed applying the Finite Element Method (FEM) with the use of the Abaqus v. 6.13-1 software. Selected side wall structures were modelled using rectangular, 8-node solid elements type C3D8R. On average 5500 elements and 12100 nodes were used. The grid model, the method of support and loading of tested structures are presented in Fig. 5. Elastic properties of the materials given in Tables 1 and 2 were applied. First values of critical forces F_{cr} , were calculated, then deflections U_3 and normal stresses S_{22} were calculated in the direction of loading in the postcritical range.

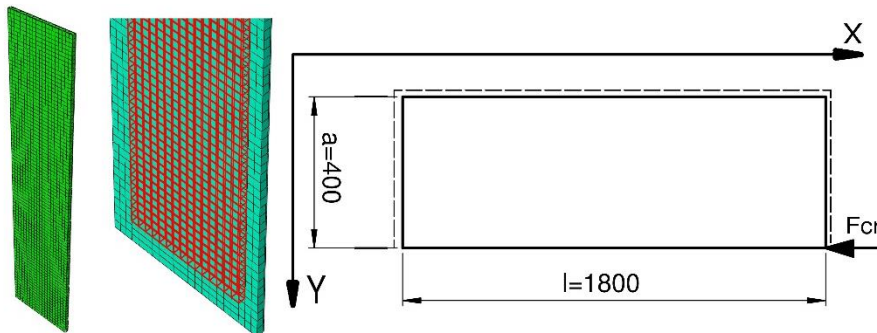


Figure 5. Grid model and loading method for the side wall.

3. RESULTS AND DISCUSSION

Figure 6 presents the form of buckling of side walls under critical loading. It may be concluded from the figure that in the critical state the side walls manufactured from particleboard and from honeycomb panel exhibit a similar form of buckling by deflection of the face edge of the panel. In the case of the board-on-frame the central part of the facing suffered considerable buckling. For individual types of structures the value of critical force was as follows:

- particleboard: 16428 N,
- honeycomb panel with cardboard core: 29205 N,
- board-on-frame with not cardboard core: 5838 N.

It results from the given values that the side wall manufactured from honeycomb panel shows the highest value of critical force. This is a direct consequence of the high value of the modulus of linear elasticity of HDF facings (4071 MPa) as well as cardboard core binding these facings. Despite the low value of the modulus of linear elasticity of the honeycomb core (0.0389 MPa), its presence has a significant effect on the maintenance of facing stability. The structure with no core loses stability under a 5-fold lower critical load (5838 N). In the case of the side wall manufactured entirely from particleboard the value of critical force is almost 50% lower in relation to honeycomb panel and is equal to the value calculated analytically.

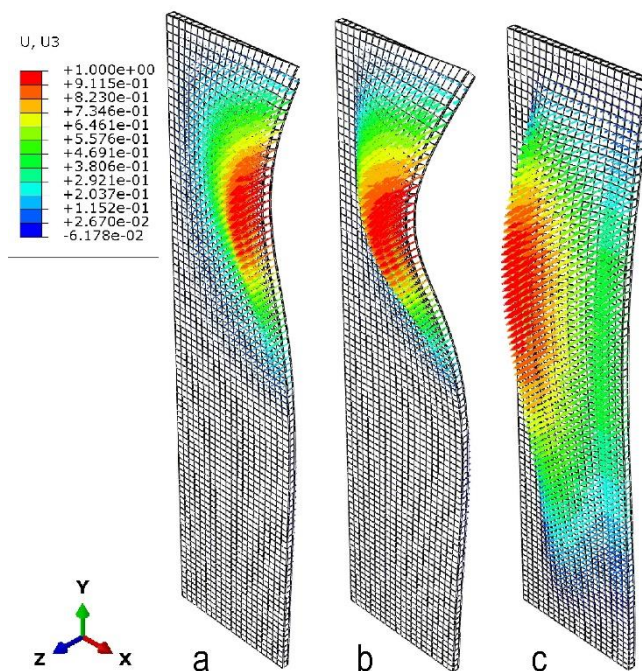


Figure 6. The form of buckling in the side wall under critical load:
 [1] particleboard, b) honeycomb panel, c) board-on-frame.

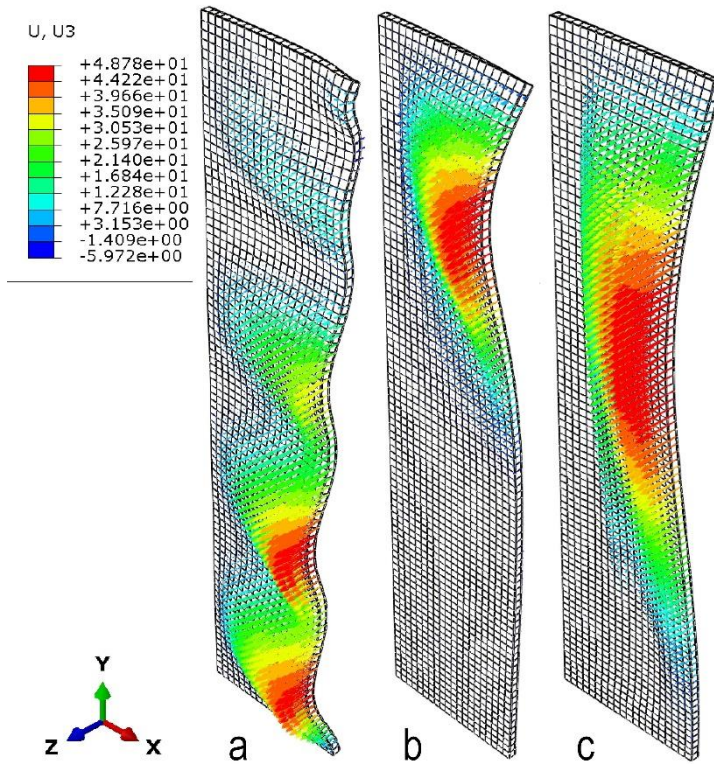


Figure 7. Form of buckling in the side wall at postcritical loading:
 [1] a) particleboard, b) honeycomb panel, c) board-on-frame.

Figure 7 presents the form of buckling for analogous side walls in the postcritical state, i.e. when load exceeded the critical value. It may be concluded from this figure that in the postcritical state the side wall manufactured from particleboard suffers multiwave deflection. The side wall made from honeycomb panel retains the original form of buckling, while the side wall made from the board-on-frame suffers double buckling in the central part of facings. For this board respective transverse deflections U_3 are 48 mm, 36 mm and 89 mm. Thus the most advantageous rigidity following bifurcation is found for honeycomb panels.

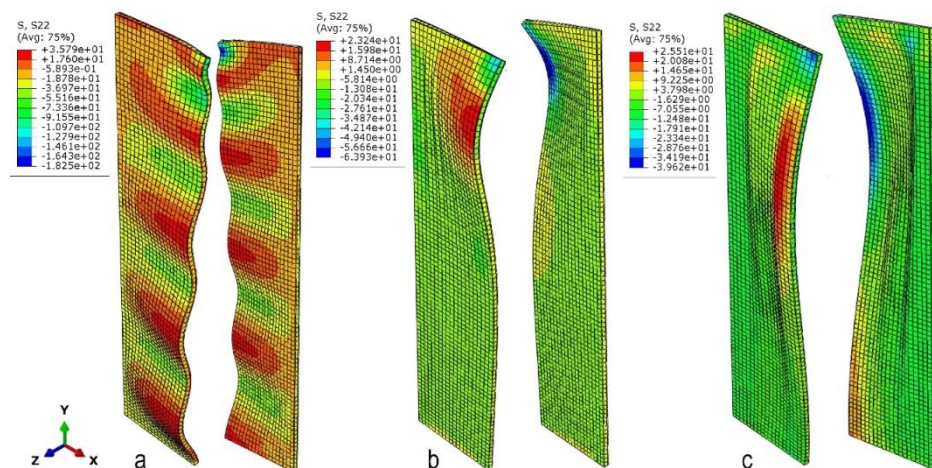


Figure 8. Distribution of normal stresses s_{22} in panels after exceeding critical loads: [1] particleboard, b) honeycomb panel, c) board-on-frame.

In turn, Figure 8 presents the distribution of normal stresses in panels after critical loads are exceeded. This figure shows that for individual structures of side walls tensile stresses are 35.7 MPa, 23.2 MPa and 25.5 MPa, respectively. Compressive stresses were 182 MPa, 63.9 MPa and 39.6 MPa. It may be concluded from these data that a side wall manufactured from honeycomb panel exhibits the best properties for initial stability as well as very good mechanical strength parameters after critical loads are exceeded.

4. CONCLUSION

Conducted numerical calculations for the stability of side walls in cabinet furniture and the obtained results made it possible to formulate the following conclusions and observations:

1. The side wall manufactured from honeycomb panel loses stability at the highest value of critical load, 2-fold greater in comparison to buckling of the same element made from particleboard,
2. In the postcritical state the smallest deflections are found for elements made from honeycomb panel, while they are greatest for that of board-on-frame,
3. Normal stresses have the most advantageous distributions in facings of the honeycomb panel and they are manifested in the sites of the greatest buckling amplitude,
4. From the practical point of view it is recommended to use honeycomb panels to design side walls of cabinet furniture.

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