



Research Article

ASYMPTOTICALLY \mathcal{J} -STATISTICAL EQUIVALENT FUNCTIONS DEFINED ON AMENABLE SEMIGROUPS

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ABSTRACT

In this study, we introduce the notions of asymptotically \mathcal{J} -equivalence, asymptotically \mathcal{J}^* -equivalence, asymptotically strongly \mathcal{J} -equivalence and asymptotically \mathcal{J} -statistical equivalence for functions defined on discrete countable amenable semigroups. Also, we examine some properties of these notions and relationships between them.

Keywords: Statistical convergence, ideal convergence, asymptotically equivalence, folner sequence, amenable semigroups.

1. INTRODUCTION

In [1], Fast introduced the notion of statistical convergence for real sequences. Also this notion was studied in [2], [3] and [4], too. The idea of \mathcal{J} -convergence was introduced by Kostyrko et al. [5] which is based on the structure of the ideal \mathcal{J} of subset of the set \mathbb{N} (natural numbers). Then, by using ideal, Das et al. [6] introduced a new notion, namely \mathcal{J} -statistical convergence.

In [7], Day studied on amenable semigroups. Then, the notions of summability in amenable semigroups were examined in [8], [9], [10] and [11]. Recently, Nuray and Rhoades [12] introduced the notions of convergence, strongly summability and statistical convergence for functions defined on amenable semigroups. Also, the notions of \mathcal{J} -summable and \mathcal{J} -statistical convergence for functions defined on amenable semigroups were studied by Ulusu et al. [13].

In [14], Marouf presented definitions for asymptotically equivalent sequences and asymptotic regular matrices. Then, the notion of asymptotically equivalence has been developed by many researchers (see, [15, 16, 17]). Recently, the notions of asymptotically equivalence, strongly asymptotically equivalence and asymptotically statistical equivalence for function defined on amenable semigroups were introduced by Nuray and Rhoades [18].

Now, we recall the basic definitions and concepts that need for a good understanding of our study (see, [5, 6, 12, 13, 14, 18]).

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Let G be a discrete countable amenable semigroup with identity in which both right and left cancellation laws hold, and

$$w(G) = \{f \mid f: G \rightarrow \mathbb{R}\} \text{ and } m(G) = \{f \in w(G) : f \text{ is bounded}\}.$$

$m(G)$ is a Banach space with the supremum norm $\|f\|_\infty = \sup\{|f(g)| : g \in G\}$.

Namioka [19] showed that, if G is a countable amenable group, there exists a sequence $\{S_n\}$ of finite subsets of G such that

- i. $G = \bigcup_{n=1}^\infty S_n$,
- ii. $S_n \subset S_{n+1}$ ($n = 1, 2, \dots$),
- iii. $\lim_{n \rightarrow \infty} \frac{|S_n g \cap S_n|}{|S_n|} = 1, \lim_{n \rightarrow \infty} \frac{|g S_n \cap S_n|}{|S_n|} = 1,$

for all $g \in G$, where $|A|$ denotes the number of elements inside set A .

Any sequence of finite subsets of G satisfying (i), (ii) and (iii) is called a Folner sequence for G .

The sequence $S_n = \{0, 1, 2, \dots, n - 1\}$ is a familiar Folner sequence giving rise to the classical Cesàro method of summability.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancellation laws hold. A function $f \in w(G)$ is said to be convergent to s for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$, there exists a $n_0 \in \mathbb{N}$ such that $|f(g) - s| < \varepsilon$ holds, for all $n > n_0$ and $g \in G \setminus S_n$.

A function $f \in w(G)$ is said to be strongly Cesàro summable to s for any Folner sequence $\{S_n\}$ of G if

$$\lim_{n \rightarrow \infty} \frac{1}{|S_n|} \sum_{g \in S_n} |f(g) - s| = 0$$

holds.

A function $f \in w(G)$ is said to be statistically convergent to s for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$,

$$\lim_{n \rightarrow \infty} \frac{1}{|S_n|} |\{g \in S_n : |f(g) - s| \geq \varepsilon\}| = 0.$$

Let X is a non-empty set. A family of sets $\mathcal{J} \subset 2^X$ is called an ideal on X if

- i. $\emptyset \in \mathcal{J}$,
- ii. For each $A, B \in \mathcal{J}$, $A \cup B \in \mathcal{J}$,
- iii. For each $A \in \mathcal{J}$ and each $B \subset A$, $B \in \mathcal{J}$.

An ideal $\mathcal{J} \subset 2^X$ is called non-trivial if $X \notin \mathcal{J}$ and a non-trivial ideal $\mathcal{J} \subset 2^X$ is called admissible if $\{x\} \in \mathcal{J}$ for each $x \in X$.

An admissible ideal $\mathcal{J} \subset 2^X$ is said to satisfy the condition (AP) if for every countable family of mutually disjoint sets $\{A_1, A_2, \dots\}$ belonging to \mathcal{J} there exists a countable family of sets $\{B_1, B_2, \dots\}$ such that $A_j \Delta B_j$ is a finite set for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^\infty B_j \in \mathcal{J}$.

A non-empty family of sets $\mathcal{F} \subset 2^X$ is called a filter on X if

- i. $\emptyset \notin \mathcal{F}$,
- ii. For each $A, B \in \mathcal{F}$, $A \cap B \in \mathcal{F}$,
- iii. For each $A \in \mathcal{F}$ and each $B \supset A$, $B \in \mathcal{F}$.

$\mathcal{J} \subset 2^X$ is a non-trivial ideal if and only if $\mathcal{F}(\mathcal{J}) = \{M \subset X : (\exists A \in \mathcal{J})(M = X \setminus A)\}$ is a filter on X , called the filter associated with \mathcal{J} .

Let G be a discrete countable amenable semigroup with identity in which both right and left cancellation laws hold, and $\mathcal{J} \subset 2^G$ be an admissible ideal. A function $f \in w(G)$ is said to be \mathcal{J} -convergent to s for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$,

$$\{g \in G : |f(g) - s| \geq \varepsilon\} \in \mathcal{J}$$

holds.

Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{J} \subset 2^{\mathbb{N}}$ be an admissible ideal. $f \in w(G)$ is said to be \mathcal{J} -statistical convergent to s , for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon, \delta > 0$

$$\left\{n \in \mathbb{N} : \frac{1}{|S_n|} |\{g \in S_n : |f(g) - s| \geq \varepsilon\}| \geq \delta\right\} \in \mathcal{J}$$

holds.

Two nonnegative sequences (x_k) and (y_k) are said to be asymptotically equivalent if

$$\lim_k \frac{x_k}{y_k} = 1$$

and it is denoted by $x \sim y$.

Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically equivalent for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$ there exists an $n_0 \in \mathbb{N}$ such that

$$\left| \frac{f(g)}{h(g)} - 1 \right| < \varepsilon$$

holds, for all $n > n_0$ and $g \in G \setminus S_n$. It is denoted by $f \sim h$.

Two nonnegative functions $f, h \in w(G)$ are said to be strongly Cesàro asymptotically equivalent for any Folner sequence $\{S_n\}$ of G if

$$\lim_{n \rightarrow \infty} \frac{1}{|S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - 1 \right| = 0$$

and it is denoted by $f \overset{w}{\sim} h$.

Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically statistical equivalent for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$

$$\lim_{n \rightarrow \infty} \frac{1}{|S_n|} \left| \left\{g \in S_n : \left| \frac{f(g)}{h(g)} - 1 \right| \geq \varepsilon \right\} \right| = 0,$$

and it is denoted by $f \overset{s}{\sim} h$.

2. MAIN RESULTS

In this section, we introduce the notions of asymptotically \mathcal{J} -equivalence, asymptotically \mathcal{J}^* -equivalence, asymptotically strongly \mathcal{J} -equivalence and asymptotically \mathcal{J} -statistical equivalence for functions defined on discrete countable amenable semigroups. Also, we examine some properties of these notions and relationships between of them. For the particular case; when the amenable semigroup is the additive positive integers, our definitions and theorems yield the results of [15, 17].

Definition 2.1 Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{J} \subset 2^G$ be an admissible ideal. Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically \mathcal{J} -equivalent of multiple L , for any Folner sequence $\{S_n\}$ of G if for every $\varepsilon > 0$

$$\left\{g \in G : \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon\right\} \in \mathcal{J}.$$

In this case, we write $f \overset{\mathcal{J}_L}{\sim} h$ and simply asymptotically \mathcal{J} -equivalent, if $L = 1$.

Example 2.1

- If we take $\mathcal{J} = \mathcal{J}_f$ be an ideal of all finite subsets of G , then we get asymptotically equivalent in [18] with respect to Folner sequence.

- Let $\mathcal{J}_d = \{H \subset G: \delta(H) = 0\}$. Then, \mathcal{J}_d is an admissible ideal and asymptotically \mathcal{J}_d -equivalence coincides with asymptotically statistical equivalent in [18] with respect to the Folner sequence.

Remark 2.1 The asymptotical \mathcal{J} -equivalence of $f, h \in w(G)$ depends on the particular choice of the Folner sequence.

By assuming $\mathcal{J} = \mathcal{J}_d$, let us show this by an example.

Example 2.2 Let $G = \mathbb{Z}^2$ and take two Folner sequences as follows:

$\{S_n^1\} = \{(i, j) \in \mathbb{Z}^2: |i| \leq n, |j| \leq n\}$ and $\{S_n^2\} = \{(i, j) \in \mathbb{Z}^2: |i| \leq n, |j| \leq n^2\}$, and define $f, h \in w(G)$ by

$$f(g) := \begin{cases} \frac{|ij|+3}{|ij|+2} & , \text{ if } (i, j) \in A, \\ 1 & , \text{ if } (i, j) \notin A \end{cases} \text{ and } h(g) := \begin{cases} \frac{|ij|+1}{|ij|+2} & , \text{ if } (i, j) \in A, \\ 1 & , \text{ if } (i, j) \notin A \end{cases}$$

where $A = \{(i, j) \in \mathbb{Z}^2: i \leq j \leq n, i = 0, 1, 2, \dots, n; n = 1, 2, \dots\}$.

Since for the Folner sequence $\{S_n^2\}$

$$\lim_{n \rightarrow \infty} \frac{1}{|S_n^2|} \left| \left\{ g \in S_n^2: \left| \frac{f(g)}{h(g)} - 1 \right| \geq \varepsilon \right\} \right| = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)(n+2)}{2}}{(2n+1)(2n^2+1)} = 0,$$

then $f(g), h(g)$ are asymptotically \mathcal{J}_d -equivalent.

But, since for the Folner sequence $\{S_n^1\}$

$$\lim_{n \rightarrow \infty} \frac{1}{|S_n^1|} \left| \left\{ g \in S_n^1: \left| \frac{f(g)}{h(g)} - 1 \right| \geq \varepsilon \right\} \right| = \lim_{n \rightarrow \infty} \frac{\frac{(n+1)(n+2)}{2}}{(2n+1)^2} = \frac{1}{4} \neq 0,$$

then $f(g), h(g)$ are not asymptotically \mathcal{J}_d -equivalent.

Definition 2.2 Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{J} \subset 2^G$ be an admissible ideal. Two nonnegative functions $f, g \in w(G)$ are said to be asymptotically \mathcal{J}^* -equivalent of multiple L , for any Folner sequence $\{S_n\}$ for G if there exists $M \subset G$ such that $M \in \mathcal{F}(\mathcal{J})$ (i.e., $G \setminus M \in \mathcal{J}$) and an $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $\varepsilon > 0$

$$\left| \frac{f(g)}{h(g)} - L \right| < \varepsilon,$$

for all $n > n_0$ and all $g \in M \setminus S_n$. In this case, we write $f \stackrel{\mathcal{J}^*}{\sim} L h$ and simply asymptotically \mathcal{J}^* -equivalent, if $L = 1$.

Theorem 2.1 Let $\mathcal{J} \subset 2^G$ be an admissible ideal. If two nonnegative functions $f, h \in w(G)$ are asymptotically \mathcal{J}^* -equivalent of multiple L , for Folner sequence $\{S_n\}$ for G , then f, h are asymptotically \mathcal{J} -equivalent of multiple L for same sequence.

Proof. Suppose that $f, g \in w(G)$ are asymptotically \mathcal{J}^* -equivalent of multiple L for Folner sequence $\{S_n\}$ for G . Then, there exists $M \subset G, M \in \mathcal{F}(\mathcal{J})$ (i.e., $H = G \setminus M \in \mathcal{J}$) and an $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $\varepsilon > 0$

$$\left| \frac{f(g)}{h(g)} - L \right| < \varepsilon,$$

for all $n > n_0$ and all $g \in M \setminus S_n$. Therefore, obviously

$$A_\varepsilon = \left\{ g \in G: \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \subset H \cup S_{n_0}.$$

Since \mathcal{J} is admissible, $H \cup S_{n_0} \in \mathcal{J}$ and so $A_\varepsilon \in \mathcal{J}$. Hence, $f, g \in w(G)$ are asymptotically \mathcal{J} -equivalent of multiple L .

Theorem 2.2 Let $\mathcal{J} \subset 2^G$ be an admissible ideal that satisfy the condition (AP). If two nonnegative functions $f, h \in w(G)$ are asymptotically \mathcal{J} -equivalent of multiple L , for Folner sequence $\{S_n\}$ for G , then f, g are asymptotically \mathcal{J}^* -equivalent of multiple L for same sequence.

Proof. Let \mathcal{J} satisfies the condition (AP) and suppose that $f(g), h(g) \in w(G)$ are asymptotically \mathcal{J} -equivalent of multiple L for Folner sequence $\{S_n\}$ for G . Then, for every $\varepsilon > 0$ we have

$$\left\{g \in G: \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \in \mathcal{J}.$$

Denote by

$$A_1 = \left\{g \in G: \left| \frac{f(g)}{h(g)} - L \right| \geq 1 \right\} \text{ and } A_n = \left\{g \in G: \frac{1}{n} \leq \left| \frac{f(g)}{h(g)} - L \right| < \frac{1}{n+1} \right\}$$

for $n \geq 2, n \in \mathbb{N}$. Obviously, $A_i \cap A_j = \emptyset$ for $i \neq j$. By the condition (AP), there exists a sequence of sets $(B_n)_{n \in \mathbb{N}}$ such that $A_j \triangle B_j$ are infinite sets for $j \in \mathbb{N}$ and $B = \bigcup_{j=1}^{\infty} B_j \in \mathcal{J}$. It is sufficient to prove that there exist $M \subset G, M \in \mathcal{F}(\mathcal{J})$ (i.e., $M = G \setminus B$) and an $n_0 = n_0(\varepsilon) \in \mathbb{N}$ such that for every $\varepsilon > 0$

$$\left| \frac{f(g)}{h(g)} - L \right| < \varepsilon,$$

for all $n > n_0$ and all $g \in M \setminus S_n$.

Let $\eta > 0$. Choose $k \in \mathbb{N}$ such that $\frac{1}{k+1} < \eta$. Then, for every $\eta > 0$ we have

$$\left\{g \in G: \left| \frac{f(g)}{h(g)} - L \right| \geq \eta \right\} \subset \bigcup_{j=1}^{k+1} A_j.$$

Since $A_j \triangle B_j$ ($j = 1, 2, \dots, k + 1$) are finite sets, there exists n_0 such that

$$\bigcup_{j=1}^{k+1} B_j \cap (M \setminus S_{n_0}) = \bigcup_{j=1}^{k+1} A_j \cap (M \setminus S_{n_0}). \tag{1}$$

If $g \in M \setminus S_{n_0}$ and $g \notin \bigcup_{j=1}^{k+1} B_j$, then $g \notin \bigcup_{j=1}^{k+1} A_j$ by (1). But we have

$$\left| \frac{f(g)}{h(g)} - L \right| < \frac{1}{k+1} < \eta.$$

Hence, $f, g \in w(G)$ are asymptotically \mathcal{J}^* -equivalent of multiple L .

Definition 2.3 Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{J} \subset 2^{\mathbb{N}}$ be an admissible ideal. Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically strongly \mathcal{J} -equivalent of multiple L , for any Folner sequence $\{S_n\}$ for G if for every $\varepsilon > 0$

$$\left\{n \in \mathbb{N}: \frac{1}{|S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \in \mathcal{J}.$$

In this case, we write $f \overset{[\mathcal{J}L]}{\sim} h$.

Definition 2.4 Let G be a discrete countable amenable semigroup with identity in which both right and left cancelation laws hold, and $\mathcal{J} \subset 2^{\mathbb{N}}$ be an admissible ideal. Two nonnegative functions $f, h \in w(G)$ are said to be asymptotically \mathcal{J} -statistical equivalent of multiple L , for any Folner sequence $\{S_n\}$ for G if for every $\varepsilon, \delta > 0$

$$\left\{n \in \mathbb{N}: \frac{1}{|S_n|} \left| \left\{g \in S_n: \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \right| \geq \delta \right\} \in \mathcal{J}.$$

In this case, we write $f \overset{S(\mathcal{J}L)}{\sim} h$.

Remark 2.2 The asymptotically \mathcal{J} -statistical equivalence of $f, h \in w(G)$ depend on the particular choice of the Folner sequence.

Theorem 2.3 Let $\mathcal{J} \subset 2^{\mathbb{N}}$ be an admissible ideal. If two nonnegative function $f, h \in w(G)$ are asymptotically strongly \mathcal{J} -equivalent of multiple L , for Folner sequence $\{S_n\}$ of G , then f and h are asymptotically \mathcal{J} -statistical equivalent to multiple L for same sequence.

Proof. Suppose that $f, h \in w(G)$ are asymptotically strongly \mathcal{J} -equivalent of multiple L , for Folner sequence $\{S_n\}$ for G . For any fixed $\varepsilon > 0$, we have

$$\sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| = \sum_{\substack{g \in S_n \\ \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon}} \left| \frac{f(g)}{h(g)} - L \right| + \sum_{\substack{g \in S_n \\ \left| \frac{f(g)}{h(g)} - L \right| < \varepsilon}} \left| \frac{f(g)}{h(g)} - L \right| \geq \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \right| \cdot \varepsilon$$

and this inequality gives that

$$\frac{1}{\varepsilon |S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| \geq \frac{1}{|S_n|} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \right|.$$

Hence, for any $\delta > 0$,

$$\left\{ n \in \mathbb{N} : \frac{1}{|S_n|} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \right| \geq \delta \right\} \subseteq \left\{ n \in \mathbb{N} : \frac{1}{|S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| \geq \delta \cdot \varepsilon \right\}$$

holds. Therefore, due to our acceptance, the set in the right of above inclusion belongs to \mathcal{J} , so we get

$$\left\{ n \in \mathbb{N} : \frac{1}{|S_n|} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \right| \geq \delta \right\} \in \mathcal{J}.$$

This completes the proof.

Theorem 2.4 Let $\mathcal{J} \subset 2^{\mathbb{N}}$ be an admissible ideal. If $f, h \in m(G)$ are asymptotically \mathcal{J} -statistical equivalent of multiple L for Folner sequence $\{S_n\}$ for G , then f, h are asymptotically strongly \mathcal{J} -equivalent of multiple L for same sequence.

Proof. Suppose that $f, h \in m(G)$ are asymptotically \mathcal{J} -statistical equivalent of multiple L for Folner sequence $\{S_n\}$ of G . Since $f, g \in m(G)$, set $\| \frac{f}{g} \|_{\infty} + L = M$. Then, for given $\varepsilon > 0$ we have

$$\frac{1}{|S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| = \frac{1}{|S_n|} \sum_{\substack{g \in S_n \\ \left| \frac{f(g)}{h(g)} - L \right| \geq \frac{\varepsilon}{2}} \left| \frac{f(g)}{h(g)} - L \right| + \frac{1}{|S_n|} \sum_{\substack{g \in S_n \\ \left| \frac{f(g)}{h(g)} - L \right| < \frac{\varepsilon}{2}} \left| \frac{f(g)}{h(g)} - L \right| \leq \frac{M}{|S_n|} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - L \right| \geq \frac{\varepsilon}{2} \right\} \right| + \frac{\varepsilon}{2},$$

and so

$$\left\{ n \in \mathbb{N} : \frac{1}{|S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \subseteq \left\{ n \in \mathbb{N} : \frac{1}{|S_n|} \left| \left\{ g \in S_n : \left| \frac{f(g)}{h(g)} - L \right| \geq \frac{\varepsilon}{2} \right\} \right| \geq \frac{\varepsilon}{2M} \right\}.$$

Therefore, due to our acceptance, the right set belongs to \mathcal{J} , so we get

$$\left\{ n \in \mathbb{N} : \frac{1}{|S_n|} \sum_{g \in S_n} \left| \frac{f(g)}{h(g)} - L \right| \geq \varepsilon \right\} \in \mathcal{J}.$$

This completes the proof.

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