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Research Article

ON EV-DEGREE AND VE-DEGREE BASED TOPOLOGICAL PROPERTIES OF THE SIERPIŃSKI GASKET FRACTAL

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ABSTRACT

In chemistry, pharmacology, medicine and physics molecular graphs have been used to model molecular substances, networks and fractals. Topological indices have been derived from the molecular graphs of chemical compounds, networks and fractals. Topological indices are important tools to analyze the underlying topology of fractals. Many topological indices have been used to understand and to investigate mathematical properties of fractal models. The Sierpiński gasket fractal is important for the study of fractals. Some physical properties of these type fractals were investigated by some researchers. Also certain topological indices of the Sierpiński gasket fractal have been calculated recently. Ve-degree and Ev-degree concepts have been defined recently in graph theory. Ev-degree and Ve-degree topological indices have been defined by using their corresponding classical degree based topological indices. In this study we calculate ev-degree and ve-degree topological indices for the Sierpiński gasket fractal.

Keywords: Ev-degree topological indices, Ve-degree topological indices, Sierspinski gasket fractal.
Mathematics Subject Classification: 05C07.

1. INTRODUCTION

Mandelbrot introduced fractals as the geometry of nature. Fractals are studied in various fields including chemistry, social science, mathematics, artificial engineering, computer science, economics, physics, and biology [1]. The Sierpiński gasket is probably the most classical example of a self-similar fractal lattice: in contrast to the translational invariance of lattices such as the square lattice or the honeycomb, one of the main properties of self-similar lattices is scaling-invariance. Construction of the Sierpiński gasket can be defined in multiple ways. One of them is first consider the equilateral triangle denoted by T_0. Subdivide T_0 into four smaller congruent equilateral triangles and remove the central one, except for external points. We thus obtain three smaller equilateral subtriangles and denote this by T_1. After one more step, we get nine smaller equilateral sub triangles and denote this by T_2. Continuing this process gives us the Sierpiński gasket in Figure 1.

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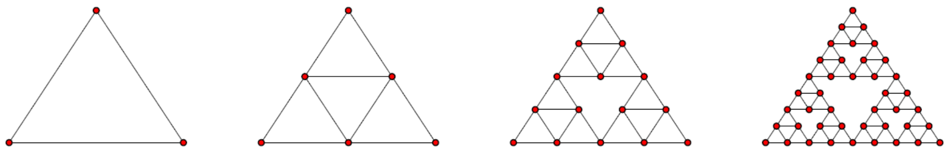

Figure 1. The growing Sierpiński networks G1, G2, G3 and G4.

For physical and electronic properties of the Sierpiński networks, we referred the interested reader to the references [2,3] and the references in these studies. For the mathematical properties of the the Sierpiński networks we re ferred the interested reader to the references [4,-6] and the references therein.

Graph theory has many applications for science, technology and social sciences. Graph theory enables suitable toys to researches to model real world problems. Chemical graph theory is one of the most using branches of graph theory. Chemical graph theory is considered the intersection of graph theory, chemistry and information science. In chemistry, pharmacology, medicine and physics molecular graphs has been used to model atomic and molecular substances. Topological indices have been derived from the molecular graphs of chemical compounds. Topological indices are important tools to analyze the underlying topology of networks. Many topological indices have been used to understand and to investigate mathematical properties of real world network models. Ve-degree and Ev-degree concepts have been defined recently in graph theory [7]. Ev-degree and Ve-degree topological indices have been defined by using their corresponding classical degree based topological indices [8-11].

The first study of the topological index of the Sierpiński gasket have been made by (Chen et al., 2019) in [12]. The authors investigated the eccentric distance sum of Sierpiński gasket and Sierpiński networks. As a continuation of this last study, we calculate ev-degree and ve-degree topological indices for the Sierpiński gasket in this study.

2. Preliminaries

In this section we give some basic and preliminary concepts which we shall use later. A graph G=(V,E) consists of two nonempty sets V and 2-element subsets of V namely E. The elements of V are called vertices and the elements of E are called edges. For a vertex v, deg(v) show the number of edges that incident to v. The set of all vertices which adjacent to v is called the open neighborhood of v and denoted by N(v). If we add the vertex v to N(v), then we get the closed neighborhood of v, N[v].

And now we give the definitions of ev-degree and ve-degree concepts which were given by (Chellali et al., 2017) in [7].

Definition 1 (**ve-degree**) Let G be a connected simple graph and $v\in V(G)$. The ve-degree of the vertex v, degve(v), equals the number of different edges that incident to any vertex from the closed neighborhood of v.

We also can restate the Definition 1 as follows: Let G be a connected simple graph and $v\in V(G)$. The ve-degree of the vertex v is the number of different edges between the other vertices with a maximum distance of two from the vertex v.

Definition2 (**ev-degree**) Let G be a connected graph and $e=uv\in E(G)$. The ev-degree of the edge e, degev(e), equals the number of vertices of the union of the closed neighborhoods of u and v.

The authors in (Chellali et al., 2017) also can give the Definition 2 as follows: Let G be a connected graph and $e=uv\in E(G)$. The ev-degree of the edge e, deguv(e)=degu+degv-ne, where ne means the number of triangles in which the edge e lies in.

Definition 3 (**ev-degree Zagreb index**) Let G be a connected graph and $e=uv\in E(G)$. The ev-degree Zagreb index of the graph G defined as;

$$M^{ev}(G) = \sum_{e\in E(G)} \deg_{ev} e^2 \qquad (1)$$

Definition 4 (**the first ve-degree Zagreb alpha index**) Let G be a connected graph and v∈V(G). The first ve-degree Zagreb alpha index of the graph G defined as;

$$M_1^{\alpha ve}(G) = \sum_{v\in V(G)} \deg_{ve} v^2 \tag{2}$$

Definition 5 (**the first ve-degree Zagreb beta index**) Let G be a connected graph and uv∈E(G). The first ve-degree Zagreb beta index of the graph G defined as;

$$M_1^{\beta ve}(G) = \sum_{uv\in E(G)} (\deg_{ve} u + \deg_{ve} v) \tag{3}$$

Definition 6 (**the second ve-degree Zagreb index**) Let G be a connected graph and uv∈E(G). The second ve-degree Zagreb index of the graph G defined as;

$$M_2^{ve}(G) = \sum_{uv\in E(G)} \deg_{ve} u \deg_{ve} v \tag{4}$$

Definition 7 (**ve-degree Randic index**) Let G be a connected graph and uv∈E(G). The ve-degree Randic index of the graph G defined as;

$$R^{ve}(G) = \sum_{uv\in E(G)} (\deg_{ve} u \deg_{ve} v)^{-1/2} \tag{5}$$

Definition 8 (**ev-degree Randic index**) Let G be a connected graph and e=uv∈E(G). The ev-degree Randic index of the graph G defined as;

$$R^{ev}(G) = \sum_{e\in E(G)} \deg_{ev} e^{-1/2} \tag{6}$$

Definition 9 (**ve-degree atom-bond connectivity index**) The ve-degree atom-bond connectivity (ve-ABC) index for a connected graph G defined as;

$$ABC^{ve}(G) = \sum_{uv\in E(G)} \sqrt{\frac{\deg_{ve}(u)+\deg_{ve}(v)-2}{\deg_{ve}(u)\times\deg_{ve}(v)}} \tag{7}$$

Definition 10 (**ve-degree geometric-arithmetic index**) The ve-degree geometric-arithmetic (ve-GA index) for a connected graph G defined as;

$$GA^{ve}(G) = \sum_{uv\in E(G)} \frac{2\sqrt{\deg_{ve}(u)\times\deg_{ve}(v)}}{\deg_{ve}(u)+\deg_{ve}(v)} \tag{8}$$

Definition 11 (**ve-degree harmonic index**) The ve-degree harmonic (ve-H) index for a connected graph G defined as;

$$H^{ve}(G) = \sum_{uv\in E(G)} \frac{2}{\deg_{ve}(u)+\deg_{ve}(v)} \tag{9}$$

Definition 12 (**ve-degree sum-connectivity index**) The ve-degree sum-connectivity (ve-χ) index for a connected graph G defined as;

$$\chi^{ve}(G) = \sum_{uv\in E(G)} (\deg_{ve}(u) + \deg_{ve}(v))^{-1/2} \tag{10}$$

3. RESULTS

We know from the reference 1 that, the Sierpiński gasket fractal S_n has $\frac{3}{2}(3^{n-1}+1)$ vertices and 3^n edges.

With the help of Figure 1 and the following table, we give the ev-degree of the edges of the Sierpiński gasket fractal S_n.

Table 1. The ev-degree of the edges of the Sierpiński gasket fractal S_n.

Number of Edges	Ev-degree
6	5
3^{n-1}	6
$3^n - 3^{n-1} - 6$	7

And the following table we give the ve-degree of the vertices of the Sierpiński gasket fractal S_n

Table 2. The ve-degree of the vertices of the Sierpiński gasket network S_n

Number of Vertices	Ve-degree
3	7
6	11
$3+3^{n-2}$	14
$\mathbf{\frac{7x3^n - 189}{18}}$	**13**

Table 3. The ve-degree of the end vertex of edges of the Sierpiński gasket fractal S_n.

Number of Edges	Ve-degrees of its end vertices
6	(7,11)
6	(11,13)
6	(11,14)
$4n-4$	(13,13)
$3^n-4n-14$	(13,14)

And we begin to compute ev-degree and ve-degree topological indices.

Theorem 1 *The ev-degree and ve-degree topological indices of the Sierpiński gasket fractal* S_n *are given in the following table.*

Table 4. The topological indices of the Sierpiński gasket network S_n.

Topological index	Symbol of the topological index	Topological index value of the Sierpsinki gasket nfractal
ev-degree Zagreb index	$M^{ev}(S_n)$	$61x3^n - 49x3^{n-1} - 144$
the first ve-degree Zagreb alpha index	$M_1^{\alpha ve}(S_n)$	$\frac{1575x3^n - 6573}{18}$
the first ve-degree Zagreb beta index	$M_1^{\beta ve}(S_n)$	$3^{n+3} - 4n - 80$
the second ve-degree Zagreb index	$M_2^{ve}(S_n)$	$182x3^n - 52n - 990$
ve-degree Randic index	$R^{ve}(S_n)$	$\frac{6}{\sqrt{77}} + \frac{6}{\sqrt{143}} + \frac{6}{\sqrt{154}} + \frac{4n-4}{13} + \frac{3^n - 4n - 14}{\sqrt{182}}$

ev-degree Randic index	$R^{ev}(S_n)$	$\frac{6}{\sqrt{5}}-\frac{6}{\sqrt{7}}+3^{n-1}\left(\frac{1}{\sqrt{6}}-\frac{1}{\sqrt{7}}\right)+\frac{3^n}{\sqrt{7}}$
ve-degree atom-bond connectivity index	$ABC^{ve}(S_n)$	$6x\sqrt{\frac{16}{77}}++6x\sqrt{\frac{22}{143}}+6x\sqrt{\frac{23}{154}}+(4n-4)x\sqrt{\frac{24}{13}}+(3^n-4n-14)x\frac{5}{182}$
ve-degree geometric-arithmetic index	$GA^{ve}(S_n)$	$\frac{12\sqrt{77}}{18}+\frac{12\sqrt{143}}{24}+\frac{12\sqrt{154}}{25}+\frac{(8n-4)x13}{26}+\frac{2x(3^n-4n-14)\sqrt{182}}{27}$
ve-degree harmonic index	$H^{ve}(S_n)$	$\frac{247}{150}+\frac{4n-4}{13}+\frac{2x(3^n-4n-14)}{27}$
ve-degree sum-connectivity index	$\chi^{ve}(S_n)$	$6x(\frac{1}{3\sqrt{2}}+\frac{1}{2\sqrt{6}}+\frac{1}{5})+\frac{8n-8}{\sqrt{26}}+\frac{2x(3^n-4n-14)}{\sqrt{27}}$

Proof of Theorem1. From the Figure 1, Table 1, Table 2 and Table 3;

$$M^{ev}(S_n)=\sum_{e\in E(S_n)} deg_{ev}e^2 = 6x5^2+3^{n-1}x6^2+(3^n-3^{n-1}-6)x7^2 = 61x3^n-49x3^{n-1}-144$$

$$M_1^{\alpha ve}(S_n)=\sum_{v\in V(S_n)} deg_{ve}v^2 = 3x7^2+6x11^2+(3+3^{n-2})x14^2+\left(\frac{7x3^n-189}{18}\right)x13^2=\frac{1575x3^n-6573}{18}$$

$$M_1^{\beta}(S_n)=\sum_{uv\in E(S_n)}(deg_{ve}u+deg_{ve}v)=6x18+6x24+6x25+(4n-4)x26+(3^n-4n-14)x27=3^{n+3}-4n-80$$

$$M_2^{ve}(S_n)=\sum_{uv\in E(S_n)} deg_{ve}u deg_{ve}v=6x77+6x143+6x154+(4n-4)x169+(3^n-4n-14)x182=182x3^n-52n-990$$

$$R^{ve}(S_n)=\sum_{uv\in E(S_n)}(deg_{ve}u deg_{ve}v)^{-1/2}=\frac{6}{\sqrt{77}}+\frac{6}{\sqrt{143}}+\frac{6}{\sqrt{154}}+\frac{4n-4}{13}+\frac{3^n-4n-14}{\sqrt{182}}$$

$$R^{ev}(S_n)=\sum_{e\in E(S_n)} deg_{ev}e^{-1/2}=\frac{6}{\sqrt{5}}+\frac{3^{n-1}}{\sqrt{6}}+\frac{3^n}{\sqrt{7}}-\frac{3^{n-1}}{\sqrt{7}}-\frac{6}{\sqrt{7}}=\frac{6}{\sqrt{5}}-\frac{6}{\sqrt{7}}+3^{n-1}\left(\frac{1}{\sqrt{6}}-\frac{1}{\sqrt{7}}\right)+\frac{3^n}{\sqrt{7}}$$

$$ABC^{ve}(S_n)=\sum_{uv\in E(S_n)}\sqrt{\frac{deg_{ve}(u)+deg_{ve}(v)-2}{deg_{ve}(u)\times deg_{ve}(v)}}=6x\sqrt{\frac{16}{77}}+6x\sqrt{\frac{22}{143}}+6x\sqrt{\frac{23}{154}}+(4n-4)x\sqrt{\frac{24}{13}}+(3^n-4n-14)x\frac{5}{182}$$

$$GA^{ve}(S_n)=\sum_{uv\in E(S_n)}\frac{2\sqrt{deg_{ve}(u)\times deg_{ve}(v)}}{deg_{ve}(u)+deg_{ve}(v)}=\frac{12\sqrt{77}}{18}+\frac{12\sqrt{143}}{24}+\frac{12\sqrt{154}}{25}+\frac{(8n-4)x13}{26}+\frac{2x(3^n-4n-14)\sqrt{182}}{27}$$

$$H^{ve}(S_n) = \sum_{uv\in E(S_n)} \frac{2}{deg_{ve}(u)+deg_{ve}(v)} = \frac{12}{18} + \frac{12}{24} + \frac{12}{25} + \frac{8n-8}{26} + \frac{2x(3^n-4n-14)}{27} = \frac{247}{150} + \frac{4n-4}{13} + \frac{2x(3^n-4n-14)}{27}$$

$$\chi^{ve}(S_n) = \sum_{uv\in E(S_n)} (deg_{ve}(u) + deg_{ve}(v))^{-1/2} = \frac{6}{\sqrt{18}} + \frac{6}{\sqrt{24}} + \frac{6}{\sqrt{25}} + \frac{8n-8}{\sqrt{26}} + \frac{2x(3^n-4n-14)}{\sqrt{27}} = 6x(\frac{1}{3\sqrt{2}} + \frac{1}{2\sqrt{6}} + \frac{1}{5}) + \frac{8n-8}{\sqrt{26}} + \frac{2x(3^n-4n-14)}{\sqrt{27}}$$

4. CONCLUSIONS

In this study we investigated the ev-degree and ve-degree topological properties of the Sierpiński gasket fractal. The other topological indices of the Sierpiński gasket fractal and other type fractals are worth to study for future researches. Also mathematical properties of ev-degree and ve-degree topological indices are interesting studies for further studies.

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