INVERTED DECOUPLING PID CONTROLLER DESIGN FOR A MIMO SYSTEM

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ABSTRACT
Eliminating loop interaction is a critical issue in the control of Multi Input Multi Output (MIMO) systems. In this study, an experimental setup, Festo Didactic MPS-PA, is used as MIMO system. Inverted decoupling PID method is applied to the system to eliminate the loop interactions of MIMO system. For this purpose, firstly, the system is modelled. Then, decoupler blocks are designed to eliminate the loop interactions. Finally, the modelled system is simulated, and experiments are performed on the real system. The effect of inverted decoupling PID control on the system performance and the effect of classical PID control on the system performance are compared.

Keywords: System modelling and identification, decoupling PID, MIMO systems, process control, process automation, festo didactic MPS-PA.

1. INTRODUCTION
In recent years, multivariable control techniques have gained increasing importance. MIMO systems are commonly used in industry. There are various problems in the control applications of the systems. One of the most important problem of the systems is the negative effect of inputs and outputs on each other [1].

Due to the loop interactions in MIMO systems, it is difficult to adapt the control methods that are used in Single Input Single Output (SISO) systems to MIMO systems. In order to control of MIMO systems, firstly, the mathematical model of the systems is obtained. Thanks to the model, the relationship between system variables can be analysed with ease. Eliminating the loop interaction in MIMO systems is called as decoupling control. In decoupling control methods, a decoupler is added to the modelled system to work with the main controller.

Since MIMO systems are used extensively in the industry, especially in the chemical industry, various studies have been carried out on the decoupling control [2]. Some of these studies; PID controller that controls multi-loop systems by decomposing into more than one-loop systems [3], nonlinear multivariable decoupling PID with ANN [4], intelligent decoupling PID control for
complex industrial systems [5], analytical decoupling control for MIMO systems [6], normalized decoupling control for high dimensional MIMO systems [7], MIMO control with inverted decoupling [8], an extended approach to inverted decoupling control [9], a new method for decoupling control of two input two output (TITO) systems [10], ideal compensator based PID controller for TITO systems [11]. In addition to these studies, there are also studies involving Fuzzy Logic, Artificial Neural Network (ANN) and auto-tuning decoupling PID control in the control of multivariable systems such as; Intelligent decoupling control method using ANN for systems with nonlinear and strong interaction [12], auto-tuning decoupling controller method using a combination of Cerebellar Model Articulation Controller Neural Networks and PID controller in multivariable systems [13], intelligent decoupling PID controller containing conventional PID [14], inverted decoupling PID controller application on the approximate linear model of a system [15].

In this study, inverted decoupling PID technique is applied to eliminate the interaction in the selected MIMO system. A data acquisition card, NI USB-6211, is used to acquire step response of output variables. Transfer functions of the fluid flow rate, pressure and loop interactions are obtained with the step responses. The decouplers are designed by using the transfer functions, and they are added to the system model. The loop interaction in the system is eliminated by the designed PID controllers and decouplers. Also, PID parameters are determined experimentally by the Ziegler-Nichols method. The modelled system is verified with experimental results.

The paper is organized as follows; the multivariable and decoupling control structures are mentioned in Section 2. In Section 3, the experimental set, system modelling and decoupler design are described. In Section 4, simulation and experimental results are given. Concluding remarks are given in Section 5.

2. MULTIVARIABLE DECOUPLING METHOD

2.1. General Structure of Multivariable Control

Due to the structure of MIMO systems, there is an interaction between inputs and outputs. Figure 1 shows the control structure of SISO systems. Unlike SISO systems, transfer functions are in matrix form in multivariate systems [16].

![Figure 1. SISO System Block Diagram](image)

According to the given block diagram there is the output obtained in Eq.4.
\[ U(s) = K(s)[R(s) - Y(s) - N(s)] \]  
(1)

\[ Y(s) = G(s)K(s)[R(s) - Y(s) - N(s)] + G_d(s)D(s) \]  
(2)

\[ (1 + G(s)K(s))Y(s) = G(s)K(s)R(s) + G_d(s)D(s) - G(s)K(s)N(s) \]  
(3)

\[ Y(s) = (1 + G(s)K(s))^{-1}G(s)K(s)R(s) + (1 + G(s)K(s))^{-1}G_d(s)D(s) \]
\[ - (1 + G(s)K(s))^{-1}G(s)K(s)N(s) \]  
(4)

Decouplers give the system a new structure and eliminate interactions. The decouplers are widely used, and indicated by \( W_1(s) \).

\[ G_s(s) = G(s)W_1(s) \]  
(5)

\[ K(s) = W_1(s)K_s(s) \]  
(6)

In Eq. 5 and 6, \( G_s(s) \) is transfer function of the new system, and \( K_s(s) \) is transfer function of the new controller.

Multivariable systems are examined as discrete systems due to their symmetrical structure. Hence, they are designed as a single-loop symmetrical control system.

2.2. Decoupling Control

In general, there are three different decoupling method applied to the MIMO systems: (i) Dynamic Decoupling, (ii) Static Decoupling and (iii) Approximate Decoupling. Dynamic Decoupling is eliminates the interactions in all loops. In this method, the \( G_s(s) \) transfer function defined in Eq. 5 is a diagonal matrix. Static Decoupling is eliminates the steady state interactions in all loops. In this method, the \( G_s(0) \) transfer function is a diagonal matrix. The decoupler is defined as \( W_1(s) = G_0^{-1}(s) \). In Approximate Decoupling method, the transfer function \( G(jw_0) \) is a diagonal matrix for \( w_0 \). The decoupler is defined as \( W_1(s) = G_0^{-1}(s) \). \( G_0(s) \) is approximately equal to \( G(jw_0) \). Bandwidth frequency is a good choice for \( w_0 \) because the greatest interaction in the system is around this frequency.

The aim of these methods is completely eliminate the effects of loop interactions. The decoupler blocks separate the MIMO system into independent SISO subsystems.

2.3. Mathematical Modelling of Multivariable Systems and Decoupling Control

Multivariable systems can be represented in different structures such as P canonical form and V canonical form. These forms are shown in Fig. 2. The P form is more commonly used than the V form [17].

There are many methods and block structures in the literature for multivariable decoupling control algorithms. Fig. 3 shows the most commonly used decoupling control diagram in the literature [18].
In Fig.3, $U_1(s)$, $U_2(s)$ are the controller outputs and $U_1^*(s)$, $U_2^*(s)$ are the decoupling controller outputs. The system output is given in Eq.8.

$$Y(s) = G(s)U^*(s), \quad U^*(s) = G_c^*(s)U(s), \quad U(s) = G_c(s)[W(s) - Y(s)]$$

$$Y(s) = G(s)G_c^*(s)U(s) = G(s)G_c(s)G_c(s) [W(s) - Y(s)]$$

where $G_c(s)$ is diagonal matrix.

If

$$X = G(s)G_c^*(s) = diag[x_1, x_2]$$

then the decoupling controllers can be artificially created as,

$$G_c^*(s) = G^{-1}(s)X, \quad G^{-1}(s) = \frac{adj(G(s))}{\det(G(s))}$$
\[
\det(G(s)) = G_{11}(s)G_{22}(s) - G_{12}(s)G_{21}(s) \tag{11}
\]
\[
\text{adj} = \begin{bmatrix}
G_{22}(s) & -G_{12}(s) \\
-G_{21}(s) & G_{11}(s)
\end{bmatrix} \tag{12}
\]
\[
X = \text{diag}[x_1 - x_2] \tag{13}
\]
\[
G_c^*(s) = G^{-1}(s)X = \begin{bmatrix}
G_{22}(s)x_1 & -G_{12}(s)x_2 \\
-G_{21}(s)x_1 & G_{11}(s)x_2
\end{bmatrix} \frac{1}{\det(G(s))} \tag{14}
\]

The simplest form of a decoupling matrix is a unit matrix.

\[
G_{c,11}^*(s) = G_{c,22}^*(s) = 1 \tag{15}
\]
\[
G_{c,12}^*(s) = -G_{12}(s)/G_{11}(s) \quad \text{and} \quad G_{c,21}^*(s) = -G_{21}(s)/G_{22}(s) \tag{16}
\]

The decoupling elements are independent of the forward path controllers as shown in Eq. 15 and 16, respectively. It means that controller modes can be changed without the need for a new decoupler design.

3. EXPERIMENTAL SETUP

In this study, Festo Didactic MPS-PA Compact Workstation experimental setup is used as MIMO system shown in Fig. 4. The components numbered in the experimental setup are: (1) Pressure sensor, (2) Proportional valve, (3) Flow sensor, (4) Pump motor.

The experimental setup is designed as 4 closed loop SISO systems with an industrial controller. It is possible to use these closed loops separately or cascade with PLC or any controller [19]. These are:

- Level control system
- Flow rate control system
- Pressure control system
- Temperature control system

![Figure 4. Festo MPS-PA Compact Workstation experimental setup](image)
In Fig. 5, block diagram of the designed measurement and control system is given.

3.1. Modelling of Flow Rate-Pressure System

Flow rate-Pressure system has two inputs and two outputs. Experimentally, data are received from the system, all transfer functions in the system are found, and the system is modelled. Fig. 6 shows the system model.

In Fig. 6, \( U_1(s) \) is control signal applied to the pump, \( U_2(s) \) is control signal applied to the proportional valve, \( Y_1(s) \) is output signal of flow rate and \( Y_2(s) \) is output signal of pressure. General transfer function of 2x2 system is as follows:

\[
Y_1(s) = G_{11}(s)U_1(s) + G_{12}(s)U_2(s) \tag{17}
\]

\[
Y_2(s) = G_{21}(s)U_1(s) + G_{22}(s)U_2(s) \tag{18}
\]
There is a pump motor or proportional valve in every cycle in the system. Therefore, transfer functions are designed in second order.

\[
G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}
\]

\[G_{11}(s) = \frac{0.038576s + 0.00337}{s^2 + 0.14223s + 0.00704}
\]

\[G_{12}(s) = \frac{0.2426s + 0.00351}{s^2 + 0.71824s + 0.00991}
\]

\[G_{21}(s) = \frac{0.06655s + 0.00151}{s^2 + 0.51484s + 0.006264}
\]

\[G_{22}(s) = \frac{0.42814s^2 + 0.385s + 0.0182}{s^2 + 1.10336s + 0.05312}
\]

Fig. 7 shows simulation results of the system transfer functions with the unit step responses in the experimental setup.

**Figure 7.** Simulation results of the Flow rate-Pressure system.
3.2. Design of Decoupler Blocks

Designed multivariable control block diagram for the system is given in Fig.8. \( G_{c1}(s) \) and \( G_{c2}(s) \) are controllers, \( D_1(s) \) and \( D_2(s) \) are decoupler blocks, and \( G_{11}(s) \), \( G_{12}(s) \), \( G_{21}(s) \), \( G_{22}(s) \) are the transfer functions of the overall system.

Inverted decoupling method is used due to simpler in application and design than other methods [20]. Transfer functions of the decoupler blocks are given in Eq. 24 and 25.

\[
D_1(s) = -\frac{G_{12}(s)}{G_{11}(s)}
\]

\[
D_2(s) = -\frac{G_{21}(s)}{G_{22}(s)}
\]

Thus, the transfer functions of the decoupler blocks designed using the transfer functions of the flow rate-pressure system:

\[
D_1(s) = \frac{-0.0666s^3 - 0.011s^2 - 0.0007s - 1.065e - 05}{0.0386s^3 + 0.0232s^2 + 0.002s + 2.111e - 05}
\]

\[
D_2(s) = \frac{-0.2426s^3 - 0.2712s^2 - 0.0168s - 1.864e - 04}{0.4281s^4 + 0.6925s^3 + 0.299s^2 + 0.0169s + 1.8e - 04}
\]

In the experiments and Simulink® program, PID parameters are performed by the Ziegler-Nichols method [21]. The Ziegler-Nichols coefficient adjustment formulations are shown in Table 1.

PID parameters are obtained by the Ziegler-Nichols method. They are experimentally verified by using Table 2 [22]. The PID parameters used in the system are determined as \( K_p = 2.47 \), \( T_i = 1.2 \), \( T_d = 0.325 \).
Table 1. The Ziegler-Nichols coefficient adjustment formulas

<table>
<thead>
<tr>
<th>Controller</th>
<th>$K_p$</th>
<th>$T_i$</th>
<th>$T_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>P Control</td>
<td>$K_u/2$</td>
<td>$P_u/1.2$</td>
<td></td>
</tr>
<tr>
<td>PI Control</td>
<td>$K_u/2.2$</td>
<td>$P_u/2$</td>
<td></td>
</tr>
<tr>
<td>PID Control</td>
<td>$K_u/1.7$</td>
<td>$P_u/8$</td>
<td></td>
</tr>
</tbody>
</table>

Table 2. Effect of PID parameters changes on the system

<table>
<thead>
<tr>
<th>Controller</th>
<th>Rise Time</th>
<th>Overshoot</th>
<th>Settling Time</th>
<th>Steady State Error</th>
<th>Stability</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Small change</td>
<td>Decrease</td>
<td>Decrease</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Decrease</td>
<td>Increase</td>
<td>Increase</td>
<td>Decrease significantly</td>
<td>Decrease</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Minor Decrease</td>
<td>Minor Decrease</td>
<td>Minor Decrease</td>
<td>No effect</td>
<td>Improves</td>
</tr>
</tbody>
</table>

The Simulink® model of the system is shown in Fig. 9.

Figure 9. The Simulink® model of the system

4. EXPERIMENTAL AND SIMULATION RESULTS

Experimental and simulation results show that inverted decoupling PID method gives better results. Fig. 10 and Fig. 11 show the experimental and simulation results of flow rate and pressure changes in the system. Fig. 12 shows the pressure change when the flow rate is increased. With the designed controller, it was observed that the effect on pressure decreased experimentally from 24.55% to 11.55%, and in simulation it decreased from 11.65% to 5.3%. The settling time of the pressure is 7 s with approximately 3% error in the actual system and 3 s in the simulation. With the designed controller, the settling time of the flow rate reference value is changed as 10 s in the real system and 8 s in the simulation.

Fig. 13 shows the flow rate change when the pressure is increased. With the designed controller, the effect on the flow rate does not change experimentally remarkably. However, in simulation, it decreased from 11.2% to 4.9%. Experimentally, 2% steady-state error is observed at the flow rate in the normal system, while the flow rate reaches the reference value with 0.6% error.
after 10 s in the designed system. With the designed controller, the settling time of the pressure reference value was decreased as 4 s in real system and 5 s in simulation.

Figure 10. Real system results

Figure 11. Simulation results
5. CONCLUSION

In this study, an experimental study is performed on the control of input-output effects in MIMO systems. Festo Didactic MPS-PA Compact Workstation is used as a MIMO system. In this system, the flow rate and pressure are controlled simultaneously. The system and input-output effects are modelled in MATLAB by acquiring data from the system. The inverted decoupling PID controller is designed to eliminate the input-output effects of the system. Using the Ziegler-Nichols method, the PID parameters are determined approximately and manual adjustments are performed to obtain appropriate parameters. Designed inverted decoupling PID controller gives better results for this system. When simulation results and experimental data are considered, the
modelled MIMO system can be used for different control methods without need for long-term experimental studies such systems.

Due to the increasing use of MIMO systems in industry, conventional control methods may inadequate. Decoupling PID control method eliminates the interaction in MIMO systems and gives more successful results than classical PID control. In subsequent studies, better results can be obtained with self-tuning PID parameters by using techniques such as fuzzy logic or artificial neural networks. This study results can be used such as the petrochemical industry or artificial heart.

REFERENCES


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