Sigma J Eng & Nat Sci 10 (2), 2019, 201-211

Publications Prepared for the Sigma Journal of Engineering and Natural Sciences
Publications Prepared for the ICOMAA 2019 - International Conference on Mathematical Advances and Applications
Special Issue was published by reviewing extended papers

Research Article

ON INFINITE MATRICES AND SEQUENCE SPACES

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Received: 29.07.2019 Revised: 01.10.2019 Accepted: 11.11.2019

ABSTRACT

The purpose of this paper is to define the spaces $V_{\lambda_0}^{\sigma}(p)$ and $V_{\lambda}^{\sigma}(p)$ by using de la Valée poussin and invariant mean. Furthermore we characterize certain matrices in $(V_{\lambda}^{\sigma})_{\infty}$ which will up a gap in the existing literature.
Keywords: Infinite matrices, de la Vallée poussin, invariant mean, matrix transformations.
2000 Mathematics Subject Classification: 40C05, 40H05.

1. INTRODUCTION

Let w denote the set of all real and complex sequences $x = (x_k)$. By l_∞ and c, we denote the Banach spaces of bounded and convergent sequences $x = (x_k)$ normed by $\| x \| = \sup_k |x_k|$, respectively. A linear functional L on l_∞ is said to be a Banach limit [1] if it has the following properties:

1. $L(x) \geq 0$ if $n \geq 0$ (i.e. $x_n \geq 0$ for all n),
2. $L(e) = 1$ where $e = (1,1,\ldots)$,
3. $L(Dx) = L(x)$, where the shift operator D is defined by $D(x_n) = \{x_{n+1}\}$.

Let B be the set of all Banach limits on ℓ_∞. A sequence $x \in \ell_\infty$ is said to be almost convergent if all Banach limits of x coincide. Let $\hat{c}$ denote the space of almost convergent sequences. Lorentz [8] has shown that

$$\hat{c} = \left\{ x \in l_\infty : \lim_m d_{m,n}(x) \text{ exists uniformly in } n \right\}$$

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where

$$d_{m,n}(x) = \frac{x_n + x_{n+1} + x_{n+2} + \cdots + x_{n+m}}{m+1}.$$

If p_k is real and $p_k > 0$, we define (see, Maddox [9])

$$c_0(p) = \left\{ x : \lim_{k\to\infty} \left| x_k \right|^{p_k} = 0 \right\}$$

and

$$c(p) = \left\{ x : \lim_{k\to\infty} \left| x_k - l \right|^{p_k} = 0, \text{for some } l \right\}$$

If p_m is real such that $p_m > 0$ and $\sup p_m < \infty$, we define (see, Nanda [13])

$$\hat{c}_0(p) = \left\{ x : \lim_{m\to\infty} \left| d_{m,n}(x) \right|^{p_m} = 0, \text{uniformly in } n \right\}$$

and

$$\hat{c}(p) = \left\{ x : \lim_{m\to\infty} \left| d_{m,n}(x-le) \right|^{p_m} = 0, \text{for some } l, \text{uniformly in } n \right\}.$$

Shaefer [23] defined the σ-convergence as follows: Let σ be a one-to-one mapping from the set of natural numbers into itself. A continuous linear functional ϕ on l_∞ is said to be an invariant mean or a σ-mean provided that

(i) $\phi(x) \geq 0$ when the sequence $x = (x_k)$ is such that $x_k \geq 0$ for all k,

(ii) $\phi(e) = 1$ where $e = (1,1,1,\ldots)$, and

(iii) $\phi(x) = \phi(x_{\sigma(k)})$ for all $x \in l_\infty$.

We denote by V_σ the space of σ-convergent sequences. It is known that $x \in V_\sigma$ if and only if

$$\frac{1}{m}\sum_{k=1}^{m} x_{\sigma^k(n)} \to a \ limit$$

as $m \to \infty$, uniformly in n. Here $\sigma^k(n)$ denotes the k th iterate of the mapping σ at n.

A σ- mean extends the limit functional on c in the sense that $\phi(x) = \lim x$ for all $x \in c$ if and only if σ has no finite orbits, that is to say, if and only if , for all $n > 0, k \geq 1$ $\sigma^k(n) \neq n$.

In case σ is the translation mapping $n \to n+1$, a σ-mean reduces to the unique Banach limit and V_σ reduces to $\hat{c}$.

In [23], Schaefer has defined the concept of $\sigma-$conservative , $\sigma-$regular and $\sigma-$coercive matrices and characterized matrix classes (c,V_σ), $(c,V_\sigma)_{reg}$ and (l_∞,V_σ) where V_σ denote the set of all bound sequences all of whose i nvariant means (or $\sigma-$ means) are equal. In [11], Mursaleen characterized the class $(c(p),V_\sigma)$, $(c(p),V_\sigma)_{reg}$ and $(l_\infty(p),V_\sigma)$ matrices which generalized the results due to Schaefer [23].

Matrix transformations between sequence spaces have been discussed by Savas and Mursaleen [21], Basarir and Savas [2], Vatan [4], Vatan and Simsek [5], Savas ([14],[15], [16], [17], [18], [19],[20]) and many others.

Recently, Khan and Rahman [3] studied the sequence space $ces\left[\left(p_r\right),\left(q_r\right)\right]$ and investigated some properties. Let $\left(q_r\right)$ be positive sequence of real numbers for $p=\left(p_j\right)$ with $\inf p_j>0$, we have

$$ces\left[\left(p_r\right),\left(q_r\right)\right]=\left\{x:\sum_{j=0}^{\infty}\left(\frac{1}{Q_{2^j}}\sum_j q_k\left|x_k\right|\right)^{p_j}<\infty\right\}$$

where

$$Q_{2^j}=q_{2^j}+q_{2^j+1}+q_{2^j+2}+\ldots+q_{2^{j+1}}$$

and $\sum_j$ denotes summation over the range $2^j\le k\le 2^{j+1}$. It is easy to see that this space is paranormed space under the paranorm

$$g(x)=\left(\sum_{j=0}^{\infty}\left(\frac{1}{Q_{2^j}}\sum_j q_k\left|x_k\right|\right)^{p_j}\right)^{\frac{1}{M}} \tag{1.1}$$

where

$$H=\sup p_j<\infty \text{ and } M=(1,H).$$

If we take $q_r=1$ for all r, then the space $ces\left[\left(p_r\right),\left(q_r\right)\right]$ reduces to the space $ces\left(p_r\right)$ studied by [6]. Also, if $p_r=p$ for all r, then the space $ces\left[\left(p_r\right),\left(q_r\right)\right]$ reduces to the space ces_p due to Lim [7].

It is easy to show that $ces\left[\left(p_r\right),\left(q_r\right)\right]$ is complete with paranorm (1.1) and it has Köthe-Toeplitz dual $ces^{+}\left[\left(p_r\right),\left(q_r\right)\right]$ defined by

$$ces^{+}\left[\left(p_r\right),\left(q_r\right)\right]=\left\{a=\left(a_k\right):\sum_{j=0}^{\infty}\left(Q_{2^j}\max_j\left(\frac{\left|a_k\right|}{q_k}\right)\right)^{t_j}B^{-t_j}<\infty \text{ for some } B>1\right\}.$$

It can be shown that $ces^{+}\left[\left(p_r\right),\left(q_r\right)\right]$ is isomorphic to $ces\left[\left(p_r\right),\left(q_r\right)\right]$ which is the dual space of $ces\left[\left(p_r\right),\left(q_r\right)\right]$, i.e., the space of all continuous linear functional on $ces\left[\left(p_r\right),\left(q_r\right)\right]$.

We write the following inequality which will be used later. For any $B > 0$ and any two complex numbers a and b, we have

$$|ab| \le B\left(|a|^t B^{-t} + |b|^p\right) \tag{1.2}$$

where $p > 1$ and $\frac{1}{p} + \frac{1}{t} = 1$ (see, Maddox [9]).

2. (σ, λ)-CONVERGENCE

We define the following:

Let $\lambda = (\lambda_m)$ be a non-decreasing sequence of positive numbers tending to ∞ such that

$$\lambda_{m+1} \le \lambda_m + 1, \ \lambda_1 = 1.$$

A sequence $x = (x_k)$ of real numbers is said to be (σ, λ) - convergent to a number L if and only if $x \in V_\sigma^\lambda$, where

$$V_\lambda^\sigma = \{x \in l_\infty : \lim_{m\to\infty} t_{mn}(x) = L \text{ uniformly in n}; L = (\sigma,\lambda) - lim x\},$$

$$t_{mn}(x) = \frac{1}{\lambda_m} \sum_{i \in I_n} x_{\sigma^i(n)},$$

and $I_n = [n - \lambda_n + 1, n]$ (see, [11]). Note that $c \subset V_\lambda^\sigma \subset l_\infty$. For $\sigma(n) = n+1, V_\lambda^\sigma$ is reduced to the space $\hat{V}_\lambda$ of almost λ-convergent sequences and if we take $\sigma(n) = n+1$ and $\lambda = n$, V_σ^λ reduce to $\hat{c}$ (see, [8]) .

It is quite natural to expect that the sequence V_λ^σ and $V_{\lambda_0}^\sigma$ can be extended to $V_\lambda^\sigma(p)$ and $V_{\lambda_0}^\sigma\left(p\right)$ just as $\hat{c}$ and $\hat{c}_0$ were extended to $\hat{c}(p)$ and $\hat{c}_0(p)$ respectively.

The main object of this paper is to characterize matrix transformations between some sequence spaces. We first define the sequence spaces $V_\lambda^\sigma(p)$ and $V_{\lambda_0}^\sigma(p)$ (the definitions are given below) and characterize certain matrices in $(V_\lambda^\sigma)_\infty$.

If p_m is real such that $p_m > 0$ and $\sup p_m < \infty$, we define

$$V_{\lambda_0}^{\sigma}(p) = \left\{ x : \lim_{m\to\infty} \left| t_{m,n}(x) \right|^{p_m} = 0, \text{uniformly in } n \right\}$$

$$V_{\lambda}^{\sigma}\left(p\right) = \left\{ x : \lim_{m\to\infty} \left| t_{m,n}(x-le) \right|^{p_m} = 0, \text{for some } l, \text{uniformly in } n \right\}.$$

and

$$(V_{\lambda}^{\sigma})_{\infty}\left(p\right) = \left\{ x : \sup_{m,n} \left| t_{m,n}(x) \right|^{p_m} < \infty \right\}.$$

In particular, if $p_m = p > 0$ *for all* m, we have $V_{\lambda_0}^{\sigma}(p) = V_{\lambda_0}^{\sigma}$, $V_{\lambda}^{\sigma}(p) = V_{\lambda}^{\sigma}$ and $(V_{\lambda}^{\sigma})_{\infty}(p) = (V_{\lambda}^{\sigma})_{\infty}$.

3. MAIN RESULTS

Let X and Y be two nonempty subsets of the space w of complex sequences. Let $A = \left(a_{nk}\right), \left(n, k = 1, 2, ...\right)$ be an infinite matrix of complex numbers. We write $Ax = \left(A_n(x)\right)$ if $A_n(x) = \sum_k a_{nk} x_k$ converges for each n .(Throughout $\sum_k$ denotes summation over k from $k = 1$ to $k = \infty$). If $x = (x_k) \in X \Rightarrow Ax = \left(A_n(x)\right) \in Y$ we say that A defines a (matrix) transformation from X to Y and we denote it by $A : X \to Y$. By (X, Y) we mean the class of matrices A such that $A : X \to Y$.

We now characterize the matrices in the class $\left(ces\left[\left(p_r\right), \left(q_r\right)\right], \left(V_{\lambda}^{\sigma}\right)_{\infty}\right)$. We write

$$t_{m,n}(x) = t_{m,n}(Ax) = \sum_k a\left(m, n, k\right) x_k$$

where

$$a\left(m, n, k\right) = \frac{1}{\lambda_m} \sum_{i\in I_n} a_{\sigma^i(n),k} \ .$$

Theorem 3.1 Let $1 < p_j < \sup p_j < \infty$ and $\frac{1}{p_j} + \frac{1}{t_j} = 1$ for $j = 0, 1, 2, ... A \in \left(ces\left[\left(p_r\right), \left(q_r\right)\right], \left(V_{\lambda}^{\sigma}\right)_{\infty}\right)$ if and only if there exists an integer $B > 1$ such that

$$W\left(B\right) = \sup_{m,n} \sum_{j=0}^{\infty} \left(Q_{2^j} A_j\left(m, n\right)\right)^{t_j} B^{-t_j} < \infty \qquad (3.1)$$

where $A_j(r,n) = \max_j \left(\frac{a(m,n,k)}{q_k} \right)$ and for every m, $\max_j$ means maximum over $\left[2^j, 2^{j+1}\right]$.

Proof. Sufficiency: Suppose that there exists an integers $B > 1$ such that $W(B) < \infty$. Then by inequality (1.2), we have

$$\sum_{k=0}^{\infty} \left| a(m,n,k) x_k \right| = \sum_{j=0}^{\infty} \sum_j \left| a(m,n,k) x_k \right|$$

$$\leq \sum_{j=0}^{\infty} Q_{2^j} \max_j \left(\frac{a(m,n,k)}{q_k} \right) \frac{1}{Q_{2^j}} \sum_j q_k \left| x_k \right|$$

$$\leq B \left[\sup_{m,n} \sum_{j=0}^{\infty} \left(Q_{2^j} A_j(m,n) \right)^{t_j} B^{-t_j} + \sum_{j=0}^{\infty} \left(\frac{1}{Q_{2^j}} \sum_j q_k \left| x_k \right| \right)^{p_j} \right]$$

$$\leq B \left[\sup_{r,n} \sum_{j=0}^{\infty} \left(Q_{2^j} A_j(m,n) \right)^{t_j} B^{-t_j} + \sum_{j=0}^{\infty} \left(\frac{1}{Q_{2^j}} \sum_j q_k \left| x_k \right| \right)^{p_j} \right] < \infty$$

Therefore $A \in \left(ces\left[(p_r),(q_r)\right], \left(V_{\sigma}^{\lambda}\right)_{\infty} \right)$.

Necessity: Suppose that $A \in \left(ces\left[(p_r),(q_r)\right], \left(V_{\lambda}^{\sigma}\right)_{\infty} \right)$ but

$$\sup_{m,n} \sum_{j=0}^{\infty} \left(Q_{2^j} A_j(m,n) \right)^{t_j} B^{-t_j} = \infty$$

for all $B > 1$. Then $\sum_{k=1}^{\infty} a(m,n,k) x_k$ converges uniform in n for all m and $x \in ces\left[(p_r),(q_r)\right]$, hence $a(r,n,k)_{k=1,2,\ldots} \in ces^{+}\left[(p_r),(q_r)\right]$ for all m and n. It is easy to see that each $t_{m,n}$ defined by $t_{m,n}(x) = \sum_{k=1}^{\infty} a(m,n,k) x_k$ is an element of $ces^{+}\left[(p_r),(q_r)\right]$. Since $ces\left[(p_r),(q_r)\right]$ is complete and since $\sup_{m,n}\left| t_{m,n}(x) \right| < \infty$ on $ces\left[(p_r),(q_r)\right]$, by the uniform boundedness principle, there exists a number L independent m,n,x and a number $\delta > 1$ such that

$$\left| t_{m,n}(x) \right| < L \qquad (3.2)$$

for all n, m and $x \in S[0,\delta]$ where $S[0,\delta]$ is the closed sphere in $ces[(p_r),(q_r)]$ with center at the origin 0 and radius δ. We now choose integer $E > 1$ such that $E\delta^M > L$. Since

$$\sup_{m,n} \sum_{j=0}^{\infty} \left(Q_{2^j} A_j(m,n)\right)^{t_j} E^{-t_j} = \infty,$$

there exists $m_0 > 1$ such that

$$R = \sum_{j=0}^{m_0} \left(Q_{2^j} A_j(m,n)\right)^{t_j} E^{-t_j} > 1$$

Define a sequence

$$x_k = 0 \text{ if } k \geq 2^{m_0+1}$$

and

$$x_{N(j)} = Q_{2^j}^{t_j} \delta^{\frac{M}{p_j}} \left[\left\{sgna\left(m,n,N(j)\right)\right\}\right] \left|a\left(m,n,N(j)\right)\right|^{t_j-1} R^{-1} E^{-\frac{t_j}{p_j}},$$

$$x_k = 0 \text{ if } 0 \leq j \leq m_0 \text{ and } k \neq N(j)$$

where $N(j)$ is the smallest integer such that

$$\left|a\left(m,n,N(j)\right)\right| = \max_j \left(\frac{\left|a(m,n,k)\right|}{q_k}\right).$$

So we get $g(x) < \delta$ but $\left|t_{mn}(x)\right| > L$, which contradicts by (3.2). This completes the proof. □

By specializing the sequences (p_r) and (q_r) of the spaces $ces[(p_r),(q_r)]$ in Theorem 1. We get the spaces $ces(p_r)$ and ces_p defined by [6] and Lim [7].

We have

Corollary 3.1 *Let* $1 < p_j < \sup p_j < \infty$. *Then* $A \in \left(ces(p), \left(V_\sigma^\lambda\right)_\infty\right)$ *if and only if there exists an integer* $B > 1$ *such that* $W(B) < \infty$, *where*

$$W(B) = \sup_{m,n} \sum_{j=0}^{\infty} \left(2^j A_j(m,n)\right)^{t_j} B^{-t_j} \text{ and } \frac{1}{p_j} + \frac{1}{t_j} = 1 \left(j = 0,1,2,\ldots\right).$$

Proof. If we take $q_r = 1$ for every r in Theorem 1, then we obtain the result. □

Corollary 3.2 *Let* $1 < p < \infty$ *and* $\frac{1}{p} + \frac{1}{t} = 1$. *Then* $A \in \left(ces_p, \left(V_\lambda^\sigma \right)_\infty \right)$ *if and only if*

$$\sup_{m,n} \left(\sum_{j=0}^{\infty} \left(2^j A_j(m,n) \right)^t \right)^{\frac{1}{t}} < \infty.$$

Proof. If we take $q_r = 1$ and $p_r = p$ for every r in Theorem 1, then we obtain the proof of Corollary. □

Theorem 3.2 *Let* $1 < p_j < \sup p_j < \infty$ *and*

$$\frac{1}{p_j} + \frac{1}{t_j} = 1, (j = 1, 2, ...). A \in \left(ces\left[(p_r), (q_r) \right], \left(V_\sigma^\lambda \right)_\infty \right)$$ *if and only if*

(i) $\lim_{m \to \infty} a(m,n,k) = a_k$ uniformly in n and for fixed k,

(ii) there exists $B > 1$ such that $W(B) < \infty$,

where

$$W(B) = \sup_{r,n} \sum_{j=0}^{\infty} \left(Q_{2^j} A_j(m,n) \right)^{t_j} B^{-t_j}.$$

Proof. Suppose that $A \in \left(ces\left[(p_r), (q_r) \right], \left(V_\lambda^\sigma \right)_\infty \right)$. Then $t_{m,n}(x) = \sum_{k=1}^{\infty} a(m,n,k) x_k$ exists for every $m \geq 1$ and $\lim_{m \to \infty} \left| t_{mn}(x) \right|$ uniformly in n exists for every $x \in ces\left[(p_r), (q_r) \right]$. Therefore by a similar argument to that in Theorem 1 we have the condition (i) is obtained by taking $x = e_k \in ces\left[(p_r), (q_r) \right]$, where e_k is a sequence with 1 at the k^{th} place and zero elsewhere.

Sufficiency: The conditions (i)-(ii) hold. From (i), we have

$$\sum_{j=0}^{\infty} \left(Q_{2^j} A_j(m,n) \right)^{t_j} B^{-t_j} \leq W(B) < \infty \tag{3.3}$$

By using (3.3) it is easy to check that $\sum_{k=1}^{\infty} a_k x_k$ is absolutely convergent for each $x \in ces\left[(p_r), (q_r) \right]$. Moreover for each $x \in ces\left[(p_r), (q_r) \right]$ and $\epsilon > 0$, we choose integer number $m_0 > 1$ such that

$$g_{m_0}\left(x\right) = \sum_{j=0}^{\infty}\left(\frac{1}{Q_{2^j}}\sum_j q_k\left|x_k\right|\right)^{p_j} < \epsilon^M .$$

Define the matrix $\left(b\left(m,n,k\right)\right)_{r=1}^{\infty}$ where $\left(b\left(m,n,k\right)\right) = \left(a\left(m,n,k\right) - a_k\right)$ for all n. By the condition (ii) and inequality (1.2), we have, for all n

$$\sum_{k=m_0+1}^{\infty}\left|b\left(m,n,k\right)x_k\right| \le B\left[\sum_{j=m_0}^{\infty}\left(\left(Q_{2^j}V_j\left(m,n\right)\right)^{t_j} B^{-t_j} + 1\right)\right]\left(g_{m_0}\left(x\right)\right)^{\frac{1}{M}}$$

where

$$W_j\left(m,n\right) = \max_j\left(\frac{\left|a\left(m,n,k\right)\right| - a_k}{q_k}\right)$$

By inequality above, we get

$$\sum_{j=m_0}^{\infty}\left(\left(Q_{2^j}W_j\left(m,n\right)\right)^{t_j} B^{-t_j}\right) \le 2W\left(B\right) < \infty.$$

Therefore

$$\lim_{m\to\infty}\sum_{k=1}^{\infty}a\left(m,n,k\right)x_k = \sum_{k=1}^{\infty}a_k x_k \text{uniformly in } n.$$

This shows that $A \in \left(ces\left[\left(p_r\right),\left(q_r\right)\right],\left(V_\lambda^\sigma\right)_\infty\right)$ which proves the Theorem. □

Corollary 3.3 *Let* $1 < p_j < \sup p_j < \infty$. *Then* $A \in \left(ces(p),(V_\lambda^\sigma)_\infty\right)$ *if and only if*

(i) $\lim_{r\to\infty} a\left(m,n,k\right) = a_k$ uniformly in n and for fixed k,

(ii) there exists $B > 1$ such that $W(B) < \infty$,

where

$$W\left(B\right) = \sup_{r,n}\sum_{j=0}^{\infty}\left(2^j A_j\left(m,n\right)\right)^{t_j} B^{-t_j}$$

Proof. If $q_r = 1$ for every r in Theorem 2, then we get the conditions (i)-(ii). □

Corollary 3.4 *Let* $1 < p < \infty$ *and* $\frac{1}{p_j} + \frac{1}{t_j} = 1.$ *Then* $A \in \left(ces_p, V_\lambda^\sigma\right)$ *if and only if*

(i) $\lim_{r\to\infty} a\left(m,n,k\right) = a_k$ uniformly in n and for fixed k,

(ii) $\sup_{r,n}\left(\sum_{j=0}^{\infty}\left(2^{j}A_{j}\left(m,n\right)\right)^{t}\right)^{\frac{1}{t}}<\infty.$

Proof. If $q_r = 1$ and $p_r = p$ for all r in Theorem 2 , then we get the proof of the corollary. □

Theorem 3.3 *Let* $1 < p_j < \sup p_j < \infty$ *and* $\frac{1}{p_j}+\frac{1}{t_j}=1,\left(j=1,2,...\right)$. *Then*

$$A \in \left(ces\left[\left(p_r\right),\left(q_r\right)\right],\left(V_{\lambda_0}^{\sigma}\right)\right) \quad \textit{if and only if}$$

(i) $\lim_{r\to\infty} a\left(m,n,k\right)=0$ uniformly in n and for fixed k,

(ii) there exists $B>1$ such that $W(E)<\infty$,

where

$$W\left(B\right)=\sup_{r,n}\sum_{j=0}^{\infty}\left(Q_{2^j}A_j\left(m,n\right)\right)^{t_j}B^{-t_j}.$$

Proof. Theorem 3 can be provided by using an argument similar to that in Theorem 2. □

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