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Research Article

NEW EXACT SOLUTIONS FOR ABLOWITZ-KAUP-NEWELL-SEGUR WATER WAVE EQUATION

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ABSTRACT

In this study, application of the improved Bernoulli sub-equation function method to Ablowitz-Kaup-Newell-Segur water wave equation is presented. Some new solutions have been successfully created. All the obtained solutions in this study satisfy the Ablowitz-Kaup-Newell-Segur Equation. In this paper, we have done all the calculations and graphs by Wolfram Mathematica 9.
Keywords: Ablowitz-Kaup-Newell-Segur water wave equation, improved Bernoulli sub-equation function method, exact solutions, analytical solutions.

1. INTRODUCTION

Nonlinear evolution equations (NLEEs) are widely used to model various nonlinear complex phenomena that arise in different field of nonlinear sciences. Since NLEEs define various aspects of our real life situations, it is important to look for exact and solitary wave solutions. Some methods used in the solution of these equations can be listed as follows. Such as the improved Bernoulli sub-equation function method [1-4], the extended sinh-Gordon equation expansion method[5-8] the exponential function method[9-10], the modified exp $(-\phi(\eta))$-expansion function[11] the implicit finite difference scheme and the Dufort–Frankel finite difference scheme methods[12] and the difference schemes method[13].

The Ablowitz-Kaup-Newell-Segur water wave equation[14-15] is used as a reduction for some nonlinear evolution equations. In this study, Ablowitz-Kaup-Newell-Segur water wave equation will be discussed and new solutions will be examined.
The AKNS equation is given by

$$4u_{xt} + u_{xxxt} + 8u_x u_{xy} + 4u_{xx}u_y - \alpha u_{xx} = 0, \quad (1)$$

where α is real constant non-zero value.

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2. The IBSEFM

Improved Bernoulli sub-equation function method (IBSEFM) formed by modifying the Bernoulli sub-equation function method will be given in this part.

Step 1. Let's consider the following fractional differential equation;

$$P\left(u, u_x, u_t, u_{xt}, \cdots\right) = 0, \tag{2}$$

and take the wave transformation,

$$u(x,t) = U(\gamma),\ \gamma = x - kt \tag{3}$$

where k are constants and will be determined later. Substituting Eq.(3) into Eq.(2), we obtain the following nonlinear ordinary differential equation;

$$N\left(U, U', U'', U''', \cdots\right) = 0. \tag{4}$$

Step 2. Considering trial equation of solution in Eq.(4), it can be written as follows,

$$U(\eta) = \frac{\sum_{i=0}^{n} a_i F^i(\eta)}{\sum_{j=0}^{m} b_j F^j(\eta)} = \frac{a_0 + a_1 F(\eta) + a_2 F^2(\eta) + \cdots + a_n F^n(\eta)}{b_0 + b_1 F(\eta) + b_2 F^2(\eta) + \cdots + b_m F^m(\eta)}. \tag{5}$$

According to the Bernoulli theory, we can consider the general form of Bernoulli differential equation for F' as follows,

$$F' = wF + dF^M, w \neq 0, d \neq 0, M \in R - \{0,1,2\}, \tag{6}$$

where $F = F(\eta)$ is Bernoulli differential polynomial. Substituting above relations in Eq.(4), it yields an equations of polynomial $\Omega(F)$ of F as follows,

$$\Omega(F) = \rho_S F^S + \cdots + \rho_1 F + \rho_0 = 0. \tag{7}$$

According to the balance principle, we can determine the relationship between n, m and M .

Step 3. The coefficients of $\Omega(F)$ all be zero will yield us an algebraic system of equations,

$$\rho_i = 0, i = 0, \cdots, s. \tag{8}$$

Solving this system, we will specify the values of $a_0, \cdots, a_n$ and $b_0, \cdots, b_m$.

Step 4. When we solve nonlinear Bernoulli differential equation Eq.(6), we obtain the following two situations according to w and d ,

$$F(\eta) = \left[\frac{-d}{w} + \frac{E}{e^{w(M-1)\eta}}\right]^{\frac{1}{1-M}},\ \ w \neq d, \tag{9}$$

$$F(\eta) = \left[\frac{(E-1) + (E+1)\tanh\left(w(1-M)\eta/2\right)}{1 - \tanh\left(w(1-M)\eta/2\right)}\right]^{\frac{1}{1-M}},\ \ w = d, \tag{10}$$

3. APPLICATION

In this section, application of improved Bernoulli sub-equation function method to the AKNS is presented. Using the wave transformation on Eq. (1)

$$\emptyset(x,y,t) = U(\gamma),\ \gamma = x + y + ct\,. \tag{11}$$

Substituting Eq. (11) into Eq. (1), yields the following NODE,

$$(4c-d)U' + cU''' + 6(U')^2 = 0. \tag{12}$$

If we consider $V = U'$, we can obtain the following NODE,

$$(4c-d)V + cV'' + 6V^2 = 0. \tag{13}$$

Balancing Eq. (13) by considering the highest derivative V'' and the highest power V^2, we obtain

$$n + 2 = 2M + m.$$

Choosing $M = 3,\ m = 1,$ gives $n = 5$. Thus, the trial solution to Eq. (1) takes the following form,

$$U(\gamma) = \frac{a_0 + a_1F(\gamma) + a_2F^2(\gamma) + a_3F^3(\gamma) + a_4F^4(\gamma) + a_5F^5(\gamma)}{b_0 + b_1F(\gamma)}, \tag{14}$$

where $F' = wF + dF^3, w \neq 0, d \neq 0$. Substituting Eq. (14), its second derivative along with $F' = wF + dF^3, w \neq 0, d \neq 0$ into Eq. (13), yields a polynomial in F. Solving the system of the algebraic equations, yields the values of the parameter involved. Substituting the obtained values of the parameters into Eq. (13), yields the solutions to Eq. (1).

For $w \neq d$, we can find following coefficients:

Case 1.

$$a_1 = -\frac{\sqrt{a_0}a_3}{\sqrt{6}\sqrt{a_4}};\ a_2 = -\sqrt{6}\sqrt{a_0}\sqrt{a_4};\ b_1 = -\frac{a_3b_0}{\sqrt{6}\sqrt{a_0}\sqrt{a_4}};\ a_5 = -\frac{a_3\sqrt{a_4}}{\sqrt{6}\sqrt{a_0}};\ d = 4c + \frac{6a_0}{b_0};\ w = \frac{i\sqrt{a_4}}{2\sqrt{c}\sqrt{b_0}};\ \sigma = -\frac{i\sqrt{\frac{3}{2}}\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}; \tag{15}$$

Case 2.

$$a_3 = \frac{a_1a_2}{a_0};\ a_4 = \frac{a_2^2}{6a_0};\ b_1 = \frac{a_1b_0}{a_0};\ a_5 = \frac{a_1a_2^2}{6a_0^2};\ d = 4c + \frac{6a_0}{b_0};\ w = -\frac{ia_2}{2\sqrt{6}\sqrt{c}\sqrt{a_0}\sqrt{b_0}};\ \sigma = -\frac{i\sqrt{\frac{3}{2}}\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}; \tag{16}$$

Case 3.

$$a_1 = \frac{a_0b_1}{b_0};\ a_3 = \frac{a_2b_1}{b_0};\ a_4 = \frac{a_2^2}{6a_0};\ a_5 = \frac{a_2^2b_1}{6a_0b_0};\ c = -\frac{3a_0}{2\sigma^2b_0};\ d = \frac{6(-1+\sigma^2)a_0}{\sigma^2b_0};\ w = \frac{\sigma a_2}{6a_0}; \tag{17}$$

Case 4.

$$a_2 = \frac{6wa_0}{\sigma};\ a_3 = \frac{6wa_1}{\sigma};\ a_4 = \frac{6w^2a_0}{\sigma^2};\ b_1 = \frac{a_1b_0}{a_0};\ a_5 = \frac{6w^2a_1}{\sigma^2};\ c = -\frac{3a_0}{2\sigma^2b_0};\ d = \frac{6(-1+\sigma^2)a_0}{\sigma^2b_0}; \tag{18}$$

Substituting Eq. (15) into Eq. (13) gives

$$u_1(x,y,t)=\frac{\left(6\sqrt{a_0}\left(36e^{\frac{4i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k^4a_0^2-12\sqrt{6}e^{\frac{3i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k^3a_0^{3/2}\sqrt{a_4}-36e^{\frac{2i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k^2a_0a_4-2\sqrt{6}e^{\frac{i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k\sqrt{a_0}a_4^{3/2}+a_4^2\right)\left(6e^{\frac{2i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k^2\eta a_0^{3/2}+\sqrt{a_0}\left(-\eta a_4-6i\sqrt{c}e^{\frac{i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k\sqrt{a_4}\sqrt{b_0}\right)+i\sqrt{6}\sqrt{c}a_4\sqrt{b_0}\right)\right)}{((6e^{\frac{i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k\sqrt{a_0}+\sqrt{6}\sqrt{a_4})^2(6e^{\frac{2i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k^2a_0-a_4)(6e^{\frac{2i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k^2a_0-4\sqrt{6}e^{\frac{i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}k\sqrt{a_0}\sqrt{a_4}+a_4)b_0)} \tag{19}$$

Substituting Eq. (16) into Eq. (13) gives

$$u_2(x,y,t)=\frac{a_0\left(\eta+\frac{i\sqrt{6}\sqrt{c}a_2\sqrt{b_0}}{\sqrt{a_0}\left(6e^{\frac{i\sqrt{6}\eta\sqrt{a_0}}{\sqrt{c}\sqrt{b_0}}}ka_0-a_2\right)}\right)}{b_0} \tag{20}$$

Substituting Eq. (17) into Eq. (13) gives

$$u_3(x,y,t)=\frac{a_0\left(ct+x+y-\frac{18ka_0}{\sigma(-6ka_0+e^{2(ct+x+y)\sigma}a_2)}\right)}{b_0} \tag{21}$$

Substituting Eq. (18) into Eq. (13) gives

$$u_4(x,y,t)=\frac{\left(ct+x+y-\frac{3k}{e^{2(ct+x+y)\sigma}w-k\sigma}\right)a_0}{b_0} \tag{22}$$

Choosing the suitable values of parameters, we performed the numerical simulations of the obtained solutions for (19-21) equation by plotting their 2D and 3D.

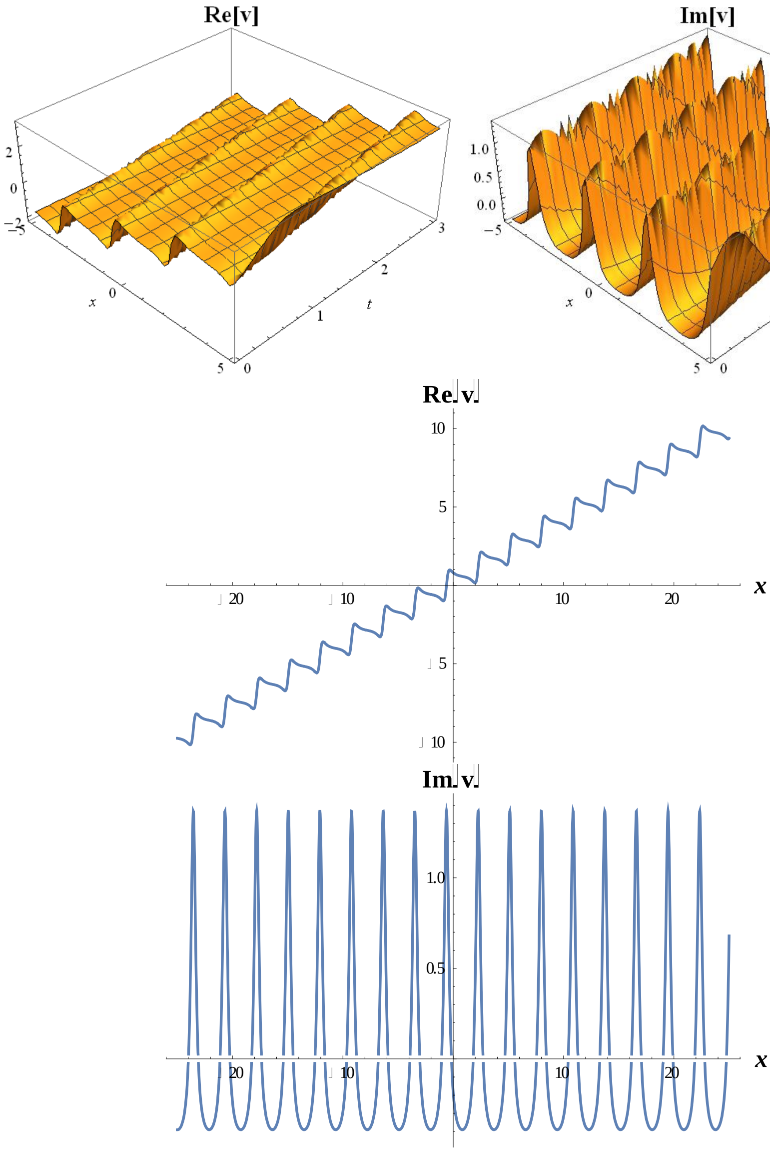

Figure 1. The 3D and 2Dsurfaces of the solution Eq.(19) for suitable values

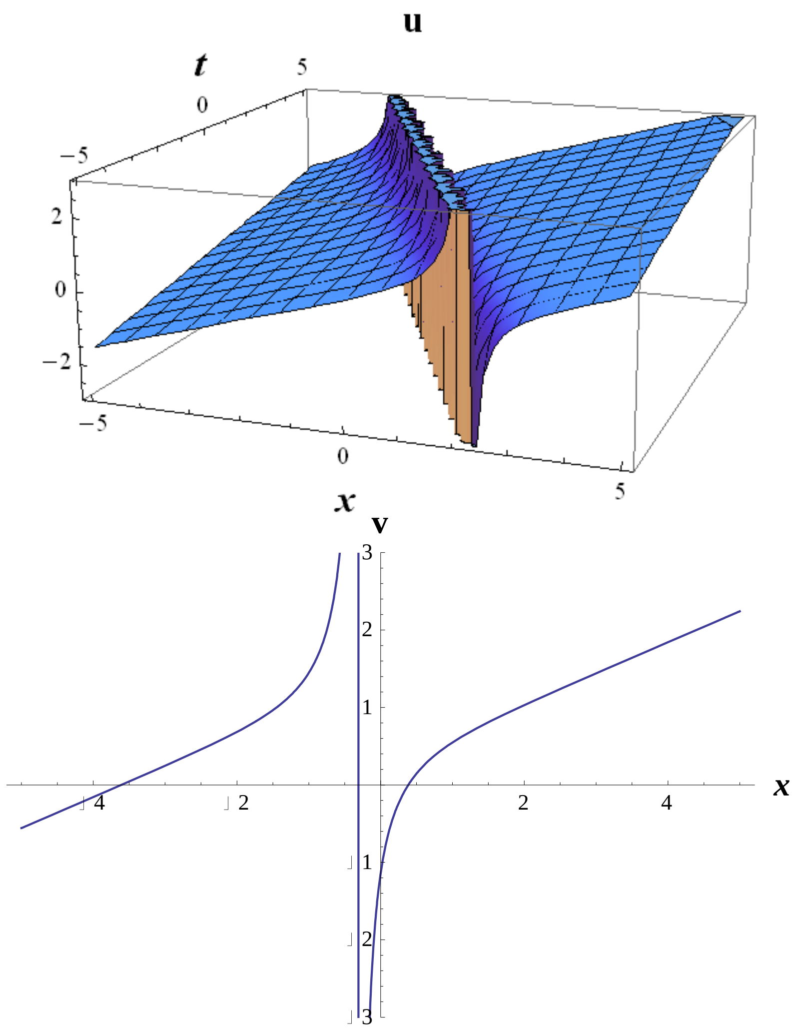

Figure 2. The 3D and 2D surfaces of the solution Eq.(21) for suitable values

4. CONCLUSIONS

In this article, new solutions are obtained for the Ablowitz-Kaup-Newell-Segur Equation using the IBSEFM method. We have seen that the results we obtained are new solutions when we compare them with previous ones. Our results might be useful in explaining the physical meaning of various nonlinear models arising in the field of nonlinear sciences. IBSEFM is powerful and efficient mathematical tool that can be used to handle various nonlinear mathematical models.

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