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Research Article

BLOW UP SOLUTIONS FOR A CLASS OF NONLINEAR HIGHER-ORDER WAVE EQUATION WITH VARIABLE EXPONENTS

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ABSTRACT

In this paper, we consider a class of nonlinear higher-order wave equation with variable exponents $u_{tt}+(-\Delta)^m u+|u_t|^{p(x)-2}u_t=|u|^{q(x)-2}u$ in a bounded domain $\Omega\subset R^n$. We prove a finite time blow up result for the solutions with negative initial energy. This improves earlier results in the literature [18].

Keywords: Blow up, higher-order wave equation, variable exponent.

1. INTRODUCTION

In this paper, we consider the initial-boundary value problem for a class of nonlinear higher-order wave equation

$$u_{tt}+(-\Delta)^m u+|u_t|^{p(x)-2}u_t=|u|^{q(x)-2}u,\quad (x,t)\in\Omega\times(0,T), \tag{1}$$

with the inital-boundary conditions

$$u(x,0)=u_0(x),\quad u_t(x,0)=u_1(x),\quad x\in\Omega, \tag{2}$$

and

$$D^\alpha u(x,t)=0,\quad |\alpha|\le m-1,\ x\in\partial\Omega, \tag{3}$$

where $A=(-\Delta)^m$, $m\ge 1$ is a natural number, Ω is a bounded domain with smooth boundary $\partial\Omega$ in R^n $(n\ge 1)$ and $\alpha=(\alpha_1,\alpha_2,\dots,\alpha_n)$, $|\alpha|=\sum_{i=1}^n|\alpha_i|$, $D^\alpha=\prod_{i=1}^n\frac{\partial^{\alpha_i}}{\partial x_i^{\alpha_i}}$.

The variable exponents $p(.)$ and $q(.)$ are given as measurable functions on Ω satisfying

$$2\le p^-\le p(x)\le p^+<q^-\le q(x)\le q^+\le q^* \tag{4}$$

where

$$p^-=\underset{x\in\Omega}{essinf}\,p(x),\quad p^+=\underset{x\in\Omega}{esssup}\,p(x),$$
$$q^-=\underset{x\in\Omega}{essinf}\,q(x),\quad q^+=\underset{x\in\Omega}{esssup}\,q(x),$$

and

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$$q^* = \begin{cases} \infty, & if\ n = 1,2, \\ \frac{2n}{n-2}, & if\ n \geq 3. \end{cases}$$

When $m = 1$ and $p(x),\ q(x)$ are constants, (1) become the following famous wave equation

$$u_{tt} - \Delta u + |u_t|^{p-2}u_t = |u|^{q-2}u. \tag{5}$$

Firstly, Levine [8, 9] considered the interaction between the linear damping ($p = 2$), and source term by using the Concavity method. He showed that solutions blow up in finite time with negative initial energy. Later, Georgiev and Todorova in [6] extended the result to the nonlinear damping case ($p > 2$). See also [11, 16] for the related works of the blow up of the solution (5).

When $m = 2$ and $p(x),\ q(x)$ are constants, (1) become the following Petrovsky equation

$$u_{tt} + \Delta^2 u + |u_t|^{p-2}u_t = |u|^{q-2}u. \tag{6}$$

Messaoudi [12] studied the local existence and blow up of the solution to the equation (6). Wu and Tsai [17] obtained global existence and blow up of the solution of the problem (6). Later, Chen and Zhou [3] studied blow up of the solution of the problem (6) for positive initial energy.

When $m = 1$, Messaoudi et al. [13] considered the equation

$$u_{tt} - \Delta u + |u_t|^{p(x)-2}u_t = |u|^{q(x)-2}u. \tag{7}$$

They studied local existence and blow up of the solutions for the wave equation (7). For more results about the variable exponent spaces we refer the readers to [1, 2, 10, 14].

Motivated by the above results, in this paper, we prove the blow up result of the solution (1) under some conditions. Thus, we try to extend the previous results from constant-exponent nonlinearities to higher-order wave equation with variable-exponent nonlinearities.

The outline of this paper is as follows. In section 2, we state some results about the variable exponent Lebesgue and Sobolev spaces $L^{p(x)}(\Omega)$ and $W^{1,p(x)}(\Omega)$. In section 3, the blow up results will be proved.

2. PRELIMINARIES

In this section, we state some results about the variable exponent Lebesgue and Sobolev spaces $L^{p(x)}(\Omega)$ and $W^{1,p(x)}(\Omega)$ (see [4, 5, 7, 15]).

Let $p: \Omega \to [1, \infty]$ be a measurable function, where Ω is a domain of R^n. We define the variable exponent Lebesgue space by

$$L^{p(x)}(\Omega) = \left\{u: \Omega \to R, u\ is\ measurable\ and\ \int_\Omega |u|^{p(x)}dx < \infty\right\},$$

endowed with the Luxemburg norm

$$\|u\|_{p(x)} = inf\left\{\lambda > 0: \int_\Omega \left|\frac{u}{\lambda}\right|^{p(x)} dx \leq 1\right\},$$

$L^{p(x)}(\Omega)$ is a Banach space.

The variable exponent Sobolev space $W^{1,p(x)}(\Omega)$ is defined by

$$W^{1,p(x)}(\Omega) = \{u \in L^{p(x)}(\Omega): \nabla u\ exists\ and\ |\nabla u| \in L^{p(x)}(\Omega)\}.$$

Variable exponent Sobolev space is a Banach space with respect to the norm

$$\|u\|_{1,p(x)} = \|u\|_{p(x)} + \|\nabla u\|_{p(x)}.$$

The space $W_0^{1,p(x)}(\Omega)$ is defined as the closure of $C_0^\infty(\Omega)$ in $W^{1,p(x)}(\Omega)$ with respect to the norm $\|u\|_{1,p(x)}$. For $u \in W_0^{1,p(x)}(\Omega)$, we can define an equivalent norm

$$\|u\|_{1,p(x)} = \|\nabla u\|_{p(x)}.$$

Let the variable exponent $p(.)$ satisfy th e log-Hölder continuity condition:

$$|p(x)-p(y)| \leq \frac{A}{\log\frac{1}{|x-y|}}, \quad for\ all\ x,y \in \Omega\ \ with\ |x-y|<\delta, \tag{8}$$

where $A>0$ and $0<\delta<1$.

Lemma 1 *[4]* (*Poincare inequality*) Let Ω be a bounded domain of R^n and $p(.)$ satisfies log-Hölder condition, then

$$\|u\|_{p(.)} \leq c\|\nabla u\|_{p(.)}, \ \ for\ all\ u \in W_0^{1,p(.)}(\Omega),$$

where $c=c(p^-,p^+,|\Omega|)>0$.

Lemma 2 *[4]* Let $p(.) \in C(\bar{\Omega})$ and $q:\Omega \to [1,\infty)$ be a measurable function and satisfy

$$\underset{x\in\bar{\Omega}}{essinf}\left(p^*(x)-q(x)\right)>0.$$

Then the Sobolev embedding $W_0^{1,p(.)}(\Omega) \hookrightarrow L^{q(.)}(\Omega)$ is continuous and compact. Where

$$p^*(x)=\begin{cases}\frac{np^-}{n-p^-},\ if\ p^-<n,\\ \infty,\ if\ p^- \geq n.\end{cases}$$

Next, we state the local existence theorem of problem (1), that can be obtained by combining arguments in [6, 12, 13].

Theorem 3 (*Local existence*). Assume that (4) and (8) and $(u_0,u_1) \in H_0^m(\Omega) \times L^2(\Omega)$ hold, then there exists a unique solution u of (1) satisfying

$$u \in C\left([0,T):H_0^m(\Omega)\right), \ \ u_t \in C\left([0,T):L^2(\Omega)\right) \cap L^{p(.)}\left(\Omega \times (0,T)\right).$$

3. BLOW UP OF SOLUTIONS

In this section, we are going to consider the blow up of the solution for problem (1). Firstly, we give following lemma.

Lemma 4 [*13*] If $q:\Omega \to [1,\infty)$ is a measurable function and

$$2 \leq q^- \leq q(x) \leq q^+ < \frac{2n}{n-2};\ \ n \geq 3 \tag{9}$$

holds. Then, we have following inequalities:

(i) $\rho_{q(.)}^{\frac{s}{q^-}}(u) \leq c\left(\|\nabla u\|^2+\rho_{q(.)}(u)\right)$, (10)

(ii) $\|u\|_{q^-}^s \leq c\left(\|\nabla u\|^2+\|u\|_{q^-}^{q^-}\right)$, (11)

(iii) $\rho_{q(.)}^{\frac{s}{q^-}}(u) \leq c\left(|H(t)|+\|u_t\|^2+\rho_{q(.)}(u)\right)$, (12)

(iv) $\|u\|_{q^-}^s \leq c\left(|H(t)|+\|u_t\|^2+\|u\|_{q^-}^{q^-}\right)$, (13)

(v) $c\|u\|_{q^-}^{q^-} \leq \rho_{q(.)}(u)$ (14)

for any $u \in H_0^1(\Omega)$ and $2 \leq s \leq q^-$. Where $c>1$ a positive constant and $H(t)=-E(t)$ will be specified later. Also where $\rho_{q(.)}(u)=\int_\Omega |u|^{q(.)}dx$.

Now, we state and prove our blow up result.

Theorem 5 Under the assumptions of Theorem 3, and the initial energy $E(0)<0$. Then the solution (1) blows up in finite time T^*, and

$$T^* \leq \frac{1-\sigma}{\xi\sigma\Psi^{\frac{\sigma}{1-\sigma}}(0)},$$

where $\Psi(t)$ and σ are given in (18) and (19) respectively.

Proof. Multiplying u_t on two sides of the problem (1) and integrate over the domain Ω, we have

$$\frac{d}{dt}\left[\frac{1}{2}\|u_t\|^2+\frac{1}{2}\left\|A^{1/2}u\right\|^2-\int_\Omega \frac{1}{q(x)}|u|^{q(x)}dx\right]=-\int_\Omega \frac{1}{p(x)}|u_t|^{p(x)}dx,$$

$$E'(t)=-\int_\Omega |u_t|^{p(x)}dx, \tag{15}$$

where

$$E(t)=\frac{1}{2}\|u_t\|^2+\frac{1}{2}\left\|A^{1/2}u\right\|^2-\int_\Omega \frac{1}{q(x)}|u|^{q(x)}dx. \tag{16}$$

Set

$$H(t)=-E(t)$$

then $E(0)<0$ and (15) gives $H(t)\geq H(0)>0$. Also, by the definition $H(t)$, we have

$$\begin{aligned}H(t)&=-\frac{1}{2}\|u_t\|^2-\frac{1}{2}\left\|A^{1/2}u\right\|^2+\int_\Omega \frac{1}{q(x)}|u|^{q(x)}dx\\&\leq \int_\Omega \frac{1}{q(x)}|u|^{q(x)}dx\\&\leq \frac{1}{q^-}\rho_{q(.)}(u).\end{aligned} \tag{17}$$

Define

$$\Psi(t)=H^{1-\sigma}(t)+\varepsilon\int_\Omega uu_t dx, \tag{18}$$

where ε small to be chosen later and

$$0<\sigma\leq min\left\{\frac{q^- -p^+}{(p^+-1)q^-},\frac{q^- -2}{2q^-}\right\}. \tag{19}$$

Our goal is to show that $\Psi(t)$ satisfies a differential inequality of the form

$$\Psi'(t)\geq \xi\Psi^{ב}(t),\qquad ב>1.$$

This, of course, will lead to a blow up in finite time.

By taking a derivative of (18) and using Eq. (1), we obtain

$$\begin{aligned}\Psi'(t)&=(1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon\int_\Omega (u_t^2+uu_{tt})dx\\&=(1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon\|u_t\|^2+\left\|A^{1/2}u\right\|^2\\&\quad+\varepsilon\int_\Omega |u|^{q(.)}dx-\varepsilon\int_\Omega uu_t|u_t|^{p(.)-2}dx.\end{aligned} \tag{20}$$

By using the definition of the $H(t)$, it follows that

$$\begin{aligned}-\varepsilon q^-(1-\xi)H(t)&=\frac{\varepsilon q^-(1-\xi)}{2}\|u_t\|^2+\frac{\varepsilon q^-(1-\xi)}{2}\left\|A^{1/2}u\right\|^2\\&-\varepsilon q^-(1-\xi)\int_\Omega \frac{1}{q(x)}|u|^{q(.)}dx,\end{aligned} \tag{21}$$

where $0<\xi<1$.

Adding and subtracting (21) into (20), we obtain

$$\begin{aligned}\Psi'(t)&\geq(1-\sigma)H^{-\sigma}(t)H'(t)+\varepsilon q^-(1-\xi)H(t)\\&+\varepsilon\left(\frac{q^-(1-\xi)}{2}+1\right)\|u_t\|^2+\varepsilon\left(\frac{q^-(1-\xi)}{2}-1\right)\left\|A^{1/2}u\right\|^2\\&+\varepsilon\xi\int_\Omega |u|^{q(.)}dx-\varepsilon\int_\Omega uu_t|u_t|^{p(.)-2}dx.\end{aligned} \tag{22}$$

Then, for ξ small enough, we get

$$\Psi'(t) \geq \varepsilon\beta\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$+(1-\sigma)H^{-\sigma}(t)H'(t) - \varepsilon\int_\Omega uu_t|u_t|^{p(.)-2}dx \tag{23}$$

where

$$\beta = min\left\{q^-(1-\xi),\ \varepsilon\xi,\ \frac{q^-(1-\xi)}{2} - 1,\ \frac{q^-(1-\xi)}{2} + 1\right\} > 0$$

and

$$\rho_{q(.)}(u) = \int_\Omega |u|^{q(.)}dx.$$

In order to estimate the last terms in (23), we make use the following Young inequality $XY \leq \frac{\delta^k X^k}{k} + \frac{\delta^{-l}Y^l}{l}$,

where $X, Y \geq 0,\ \delta > 0,\ k, l \in R^+$ such that $\frac{1}{k} + \frac{1}{l} = 1$. Consequently, applying the previous we have

$$\int_\Omega u|u_t|^{p(.)-1}dx \leq \int_\Omega \frac{1}{p(x)}\delta^{p(x)}|u|^{p(x)}dx + \int_\Omega \frac{p(x)-1}{p(x)}\delta^{-\frac{p(x)}{p(x)-1}}|u_t|^{p(x)}dx$$
$$\leq \frac{1}{p^-}\int_\Omega \delta^{p(x)}|u|^{p(x)}dx + \frac{p^+-1}{p^+}\int_\Omega \delta^{-\frac{p(x)}{p(x)-1}}|u_t|^{p(x)}\,dx \tag{24}$$

where δ is constant depending on the time t and specified later. Inserting estimate (24) into (23), we get

$$\Psi'(t) \geq \varepsilon\beta\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$+(1-\sigma)H^{-\sigma}(t)H'(t) - \varepsilon\frac{1}{p^-}\int_\Omega \delta^{p(x)}|u|^{p(x)}dx$$
$$-\varepsilon\frac{p^+-1}{p^+}\int_\Omega \delta^{-\frac{p(x)}{p(x)-1}}|u_t|^{p(x)}dx. \tag{25}$$

Therefore, by taking δ so that $\delta^{-\frac{p(x)}{p(x)-1}} = kH^{-\sigma}(t)$, where $k > 0$ is specified later, we obtain

$$\Psi'(t) \geq \varepsilon\beta\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$+(1-\sigma)H^{-\sigma}(t)H'(t) - \varepsilon\frac{1}{p^-}\int_\Omega k^{1-p(x)}H^{\sigma(p(x)-1)}(t)|u|^{p(x)}dx$$
$$-\varepsilon\frac{p^+-1}{p^+}\int_\Omega kH^{-\sigma}(t)|u_t|^{p(x)}dx$$
$$\geq \varepsilon\beta\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$+(1-\sigma)H^{-\sigma}(t)H'(t) - \varepsilon\frac{k^{1-p^-}}{p^-}H^{\sigma(p^+-1)}(t)\int_\Omega |u|^{p(x)}dx$$
$$-\varepsilon\left(\frac{p^+-1}{p^+}\right)kH^{-\sigma}(t)\int_\Omega |u_t|^{p(x)}dx$$
$$\geq \varepsilon\beta\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$+\left((1-\sigma) - \varepsilon\left(\frac{p^+-1}{p^+}\right)k\right)H^{-\sigma}(t)H'(t)$$
$$-\varepsilon\frac{k^{1-p^-}}{p^-}H^{\sigma(p^+-1)}(t)\int_\Omega |u|^{p(x)}dx. \tag{26}$$

By using (14) and (17), we get

$$H^{\sigma(p^+-1)}(t)\int_\Omega |u|^{p(x)}dx \le H^{\sigma(p^+-1)}(t)\left[\int_{\Omega_-}|u|^{p^-}dx + \int_{\Omega_+}|u|^{p^+}dx\right]$$
$$\le H^{\sigma(p^+-1)}(t)c\left[\left(\int_{\Omega_-}|u|^{q^-}dx\right)^{\frac{p^-}{q^-}} + \left(\int_{\Omega_+}|u|^{q^-}dx\right)^{\frac{p^+}{q^-}}\right]$$
$$\le H^{\sigma(p^+-1)}(t)c\left[\|u\|_{q^-}^{p^-} + \|u\|_{q^-}^{p^+}\right]$$
$$\le c\left(\frac{1}{q^-}\rho_{q(.)}(u)\right)^{\sigma(p^+-1)}\left[\left(\rho_{q(.)}(u)\right)^{\frac{p^-}{q^-}} + \left(\rho_{q(.)}(u)\right)^{\frac{p^+}{q^-}}\right]$$
$$= c_1\left[\left(\rho_{q(.)}(u)\right)^{\frac{p^-}{q^-}+\sigma(p^+-1)} + \left(\rho_{q(.)}(u)\right)^{\frac{p^+}{q^-}+\sigma(p^+-1)}\right] \tag{27}$$

where $\Omega_- = \{x \in \Omega: |u| < 1\}$ and $\Omega_+ = \{x \in \Omega: |u| \ge 1\}$.

We then use Lemma 4 and (19), for

$$s = p^- + \sigma q^-(p^+ - 1) \le q^-$$

and

$$s = p^+ + \sigma q^-(p^+ - 1) \le q^-,$$

to deduce, from (27),

$$H^{\sigma(p^+-1)}(t)\int_\Omega |u|^{p(x)}dx \le c_1\left[\left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]. \tag{28}$$

Thus, inserting estimate (28) into (26), we have

$$\Psi'(t) \ge \varepsilon\left(\beta - \frac{k^{1-p^-}}{p^-}c_1\right)\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$+\left[(1-\sigma) - \varepsilon\left(\frac{p^+-1}{p^+}\right)k\right]H^{-\sigma}(t)H'(t). \tag{29}$$

Let us choose k large enough so that $\gamma = \beta - \frac{k^{1-p^-}}{p^-}c_1 > 0$, and picking ε small enough such that $(1-\sigma) - \varepsilon\left(\frac{p^+-1}{p^+}\right)k \ge 0$ and

$$\Psi(t) \ge \Psi(0) = H^{1-\sigma}(0) + \varepsilon\int_\Omega u_0u_1dx > 0, \quad \forall t \ge 0. \tag{30}$$

Consequently, (29) yields

$$\Psi'(t) \ge \varepsilon\gamma\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + \rho_{q(.)}(u)\right]$$
$$\ge \varepsilon\gamma\left[H(t) + \|u_t\|^2 + \left\|A^{1/2}u\right\|^2 + |u|_{q^-}^{q^-}\right], \tag{31}$$

due to (14). Therefore we get

$$\Psi(t) \ge \Psi(0) > 0, \ \ for\ all\ t \ge 0.$$

On the other hand, applying Hölder inequality, we obtain

$$\left|\int_\Omega uu_tdx\right|^{\frac{1}{1-\sigma}} \le \|u\|^{\frac{1}{1-\sigma}}\|u_t\|^{\frac{1}{1-\sigma}}$$
$$\le C\left(\|u\|_{q^-}^{\frac{1}{1-\sigma}}\|u_t\|^{\frac{1}{1-\sigma}}\right).$$

Young inequality gives

$$\left|\int_\Omega uu_tdx\right|^{\frac{1}{1-\sigma}} \le C\left(\|u\|_{q^-}^{\frac{\mu}{1-\sigma}} + \|u_t\|^{\frac{\theta}{1-\sigma}}\right), \tag{32}$$

for $\frac{1}{\mu}+\frac{1}{\theta}=1$. We take $\theta=2(1-\sigma)$, to obtain $\frac{\mu}{1-\sigma}=\frac{2}{1-2\sigma}\leq q^{-}$ by (19). Therefore, (32) becomes

$$\left|\int_{\Omega} uu_t dx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u_t\|^2+\|u\|_{q^-}^{s}\right),$$

where $\frac{2}{1-2\sigma}\leq q^{-}$. By using (13), we get

$$\left|\int_{\Omega} uu_t dx\right|^{\frac{1}{1-\sigma}} \leq C\left(\|u_t\|^2+\|u\|_{q^-}^{q^-}+H(t)\right).$$

Thus,

$$\begin{aligned}
\Psi^{\frac{1}{1-\sigma}}(t) &\leq \left[H^{1-\sigma}(t)+\varepsilon\int_{\Omega} uu_t dx\right]^{\frac{1}{1-\sigma}} \\
&\leq 2^{\frac{\sigma}{1-\sigma}}\left(H(t)+\varepsilon^{\frac{1}{1-\sigma}}\left|\int_{\Omega} uu_t dx\right|^{\frac{1}{1-\sigma}}\right) \\
&\leq C\left(\|u_t\|^2+|u|_{q^-}^{q^-}+H(t)\right) \\
&\leq C\left(H(t)+\|u_t\|^2+\left\|A^{1/2}u\right\|^2+|u|_{q^-}^{q^-}\right) \qquad (33)
\end{aligned}$$

where

$$(a+b)^p \leq 2^{p-1}(a^p+b^p)$$

is used. By combining of (31) and (33), we arrive

$$\Psi'(t) \geq \xi\Psi^{\frac{1}{1-\sigma}}(t), \qquad (34)$$

where ξ is a positive constant.

A simple integration (34) over $(0,t)$ yields $\Psi^{\frac{\sigma}{1-\sigma}}(t)\geq\frac{1}{\Psi^{-\frac{\sigma}{1-\sigma}}(0)-\frac{\xi\sigma t}{1-\sigma}}$, which implies that the solution blows up in a finite time T^*, with

$$T^* \leq \frac{1-\sigma}{\xi\sigma\Psi^{\frac{\sigma}{1-\sigma}}(0)}.$$

This completes the proof of the theorem.

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