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Research Article

AN EMPIRICAL STUDY FOR STATISTICAL EVALUATION OF PEAK RESPONSE DISPLACEMENTS

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ABSTRACT

In this study, elastic response analyses of single degree of freedom (SDOF) structures under an extensive database of real earthquake excitation are investigated to clarify two particular issues: First is to determine whether the statistical distribution of ordered peaks in response displacement time histories follows Rayleigh distribution as it is a theoretical assumption valid for narrow band response of SDOF system with low damping value. Second is to look for a functional form to define amplitudes of the peaks in response displacement time histories as a function of structural period and response spectral displacement (Sd). Probability density functions (PDFs) were calculated for structural periods between 0.1-5 sec. using order statistics approach. Numerical results support that Rayleigh distribution describes the probabilistic characteristics of ordered peaks in elastic earthquake response adequately. Scale parameters of the distribution were calculated. It was observed that there is a clear logarithmic relationship between the peak number and the normalized peaks' amplitudes in displacement time histories. Hence, a nonlinear regression model was proposed to define the peaks' amplitudes in response displacement time histories of a SDOF system with known response Sd values and structural periods. In another words the model is capable of estimating number of times of a Sd that can be repeated by a structure without any damage. It is observed that even in the elastic systems, response values that are quite close to the value of the highest peak displacement are repeated many times. In general, for SDOF systems having natural period of vibration greater than or equal to 0.9 sec., oscillation having 90 % of the value of highest peak is repeated two times under the earthquake excitation. SDOF systems having natural period of vibration vary between 1 and 1.7 sec., ratio between second largest peak and Sd is about 95 %.Those calculations are essential for determining start of low-cycle fatigue behavior particularly for design of column-beam joints of steel structures or bridges.
Keywords: Elastic single degree of freedom systems, earthquake response analysis, regression analysis, order statistics approach, amplitudes of the local maximum values, low-cycle fatigue.

1. INTRODUCTION

The earthquake-resistant design philosophy permits reductions in the linear design earthquake forces due to economic reasons. Structures with sufficient ductility are expected to withstand inelastic excursions as required by ground motion without any failure. Most of the time, the design practices based on this philosophy require assessment of the maximum response value of structures and ignore the rest of inelastic excursions, which differ in number, sequence and

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relative amplitudes, in their seismic responses. Considering overall damage, performance of structure is closely related with amplitudes of remaining peaks that are realized several times in the whole duration of ground motion. To have knowledge of these repeated inelastic excursions is helpful in understanding e.g. the progression of damage in a structure, the probability of exceeding or not exceeding specified response levels, the effective response durations and the relationship between the whole response peaks' amplitudes to the characteristics of structural system. Therefore, while motivation for this study comes from the need to study the connection between damage and inelastic excursions of SDOF structures during deformation, at the first stage it is required to portray the ordered response peaks' statistics for the elastic response of SDOF structure.

Several researchers have studied statistics of maxima in literature. The pioneering attempt was made by [1, 2]. In this study, theory of probability distribution of a random function and its zeros and maxima are described. The study of [3] extended the work of [1, 2]. They presented the distribution function, the expected and the most probable values of the peaks of a random function which find useful application in earthquake engineering. Many investigators have applied and developed the results of the above mentioned studies. Among them, [4] determined the response spectrum of a damped oscillator through statistics of the largest peak of the response. Using random-vibration theory method, earthquake response spectra of bilinear hysteretic systems were estimated by [5]. Another work which was generalization and extension of the theory of [3] was carried out by [6, 7]. They showed some new results on the distribution of the maximum peak amplitudes in the responses of structures to earthquake excitation. A further study was made by [8], to refine the theoretical distribution function for amplitudes of all peaks in the random response function by following the order statistics approach. According to [9], the study of [6, 7] have substantial errors for higher order peaks when compared to the study of [8] because they did not use order statistics approach. In order to assess performance of three approximate formulations owing to [6], [8] and [9], [10] carried out numerical simulation experiments for ordered peaks' probability distributions.

In summary, the review of the literature on ordered statistics revealed the efficacy of the method but, PDFs of maxima have not been compared with vast amount of recorded ground motions within the knowledge of authors. Hence, one objective of this research is to define statistical distribution of ordered peaks in linear earthquake response displacement time histories by using a worldwide ground motion database. It is approved that Rayleigh distribution best represents the PDFs of maxima in response. As a second goal a new nonlinear regression equation model which has capable of estimating all of the local maximum values of a structure of which its Sd and vibration period (T) are already known is proposed. Once the key parameters of the Rayleigh distribution are determined and regression model is proposed, expected and most probable values of the N^{th} order peaks of the response of linear, viscously damped SDOF system can be described, the relationship between all local response maxima, their number and amplitudes to the physical characteristics of the designed structural system can be evaluated. Moreover, how many times certain response levels may be exceeded can be calculated and the probabilities of exceeding or not exceeding particular response peak amplitudes can be estimated.

2. STRONG GROUND MOTION (SGM) DATABASE AND SYSTEM PROPERTIES

SGM records are obtained from the Next Generation Attenuation (NGA) database [11], which is an extended version of the Pacific Earthquake Engineering Research Center (PEER) strong motion database [12]. The database used for this study (see Table 1.) covers 317 pairs of horizontal records from 51 earthquakes with M_w changing between 4.5-7.6 and their epicentral distances are within range of 6 to 230 kilometers. The data set includes regions of NEHRP site classes B, C, D and E. SGM records are not classified in regard to magnitudes, source-to-site distances, and style of faulting and local site conditions but classified so as to include broad frequency range of engineering interest in the calculation. Ground motion records with 0.2 Hz lowest usable frequency for both horizontal components are considered. The SDOF systems having periods ranging from 0.1 to 5 sec. with 0.1 sec. intervals are considered for statistical evaluation. In response calculations, an elastic model with 5 % viscous damping is employed.

Table 1. Database of strong motion records

Record No	Earthquake Name	Earthquake Magnitude (M_w)	Number of stations
1	Helena, Montana-01	6	1
2	Imperial Valley-02	6.95	1
3	Kern County	7.36	1
4	Parkfield	6.19	5
5	Borrego Mtn	6.63	1
6	San Fernando	6.61	8
7	Point Mugu	5.65	1
8	Friuli, Italy-01	6.5	2
9	Coyote Lake	5.74	2
10	Imperial Valley-06	6.53	4
11	Livermore-01	5.8	2
12	Mammoth Lakes-04	5.7	1
13	Mammoth Lakes-06	5.94	1
14	Victoria, Mexico	6.33	4
15	Irpinia, Italy-01	6.9	2
16	Taiwan SMART1 (5)	5.9	1
17	Coalinga-01	6.36	40
18	Coalinga-02	5.09	2
19	Coalinga-03	5.38	1
20	Coalinga-04	5.18	1
21	Taiwan SMART1 (25)	6.5	2
22	Borah Peak, ID-02	5.1	1
23	Morgan Hill	6.19	12
24	Taiwan SMART1 (33)	5.8	5
25	Taiwan SMART1 (40)	6.32	8
26	N. Palm Springs	6.06	4
27	Chalfant Valley-03	5.65	1
28	Kalamata, Greece-02	5.4	1
29	San Salvador	5.8	1
30	Whittier Narrows-01	5.99	1
31	Superstition Hills-01	6.22	1
32	Loma Prieta	6.93	34
33	Georgia, USSR	6.2	3
34	Big Bear-01	6.46	5
35	Northridge-01	6.69	26
36	Dinar, Turkey	6.4	3
37	Chi-Chi, Taiwan	7.62	2

38	Sierra Madre	5.61	1
39	Northridge-06	5.28	1
40	Hector Mine	7.13	1
41	Yountville	5	1
42	Big Bear-02	4.53	1
43	Mohawk Val, Portola	5.17	3
44	Anza-02	4.92	14
45	Gilroy	4.9	2
46	Big Bear City	4.92	3
47	Chi-Chi, Taiwan-02	5.9	2
48	Chi-Chi, Taiwan-03	6.2	4
49	Chi-Chi, Taiwan-04	6.2	16
50	Chi-Chi, Taiwan-05	6.2	36
51	Chi-Chi, Taiwan-06	6.3	41
Total			**317**

3. STATISTICAL EVALUATION OF MAXIMA OF STRUCTURAL RESPONSE

3.1. Ordered Peak Statistics

Order statistics approach is based on arranging the amplitudes of peaks in decreasing order. This approach has been successfully utilized in several theoretical predictions [8, 9, and 10]. This approach has been applied to calculate the distribution functions for amplitudes of the maxima of structural response. Absolute peak amplitudes of response displacement time histories are arranged in decreasing order for all SDOF structures that are different in their vibration periods and are similar in their damping ratios. For instance, Figure 1 exhibits the response displacement for a 5 % damped SDOF system with a period of 2 second subjected to the 2001 Anza earthquake recorded at Seven Oaks Dam Downstream Surf. Station. In Figure 1, amplitudes of the response peaks are also prepared in decreasing order such as required by order statistics approach. In Figure 2, absolute response peaks'(second, third, fourth and so on) amplitudes with structural period and number of crossings are plotted by using 3D Response Spectra for SDOF systems with fundamental periods changing between 0.1-5 sec. subjected to same excitation. In the figure, standard response spectra curve is marked by a thicker line and corresponds to peak number (N=1).

Accordingly, at any period, response spectrum can be explained as the maximum response level that will be crossed only once with positive slope by the oscillator [13].

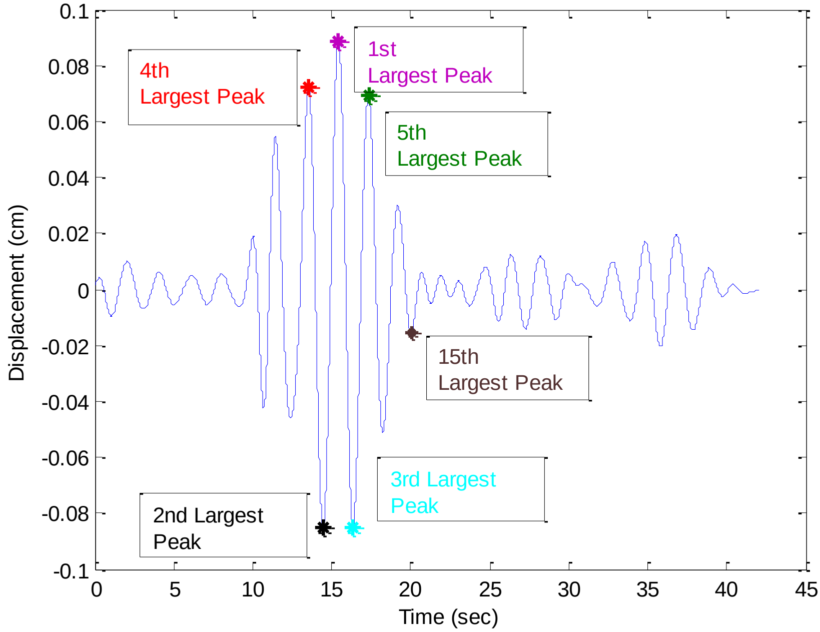

Figure 1. An example of displacement response time history of a 2-second oscillator with absolute response peaks under 2001, Anza earthquake

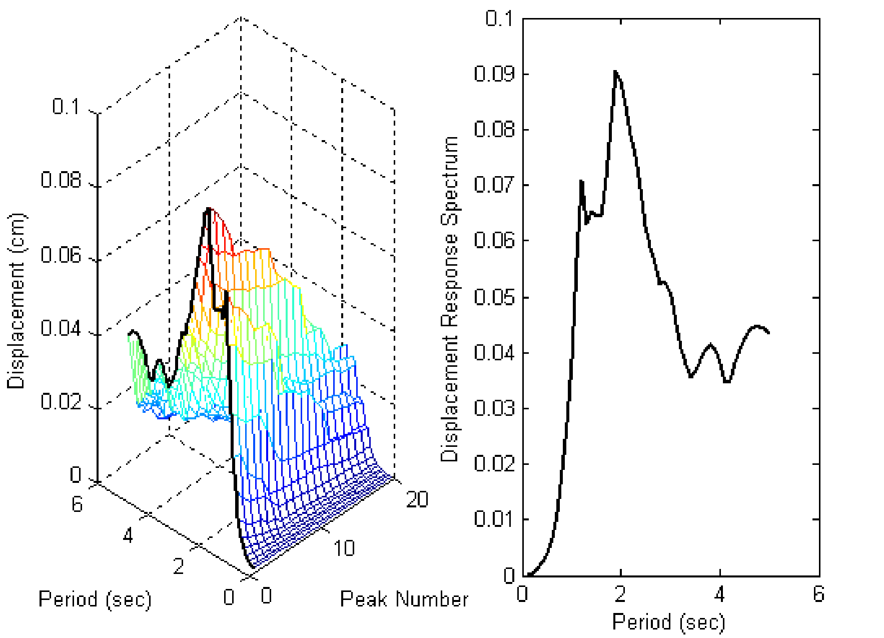

Figure 2. 3D Response Spectra and Standard Response Spectra of a series of linear single degree of freedom systems under 2001, Anza earthquake

In general, earthquake ground motion is characterized as "wide band" since it includes all frequencies and amplitude fluctuations, whereas response of a SDOF system with low damping ($\xi<1$) is "narrow band" because the frequency of response is restricted to the system' natural frequency although amplitude fluctuation persists [14].Two commonly used probability distributions namely; Normal (for a wide band response) and Rayleigh (for a narrow band response) are accepted in literature to provide an improved fit for overall statistical characterization of peaks.

In this study, PDFs were calculated for each structural period between 0.1-5 sec. using order statistics approach as mentioned in the introduction part. PDFs for SDOF system with 2 sec. structural period are presented in Figure 3 as an example. Notice that the horizontal axis represents the normalized and averaged values of the maximum 50 response peaks. To make the amplitudes of the maxima in earthquake responses of SDOF systems comparable, their values are scaled to the Sd value of the oscillator. For a general evaluation of the maxima in earthquake response (earthquake-independent amplitudes), the normalized values are averaged. Quality of the fittings was provided based on the Chi-Square test results at a 5 % significance level. The Chi-Square goodness-of-fit test results are presented for determination of each oscillator's response peaks' distribution in Figure 4. It is obvious that the test statistics have minimum value for the Rayleigh distribution when compared to the Normal distribution. In all periods PDFs of the maxima can sufficiently be characterized by the Rayleigh distribution of the following form:

$$f(x) = \frac{x}{\sigma^2} \exp\left(\frac{-x^2}{2\sigma^2}\right); \qquad x \in [0;\infty) \tag{1}$$

where the only free parameter is σ (the so called scale parameter). Key parameters, σ, calculated for all periods presented in Table 2. Instead of theoretical derivations, key parameters that define the statistical distribution of the structural response maxima are presented for all SDOF structures performing Maximum Likelihood Estimation (MLE) method.

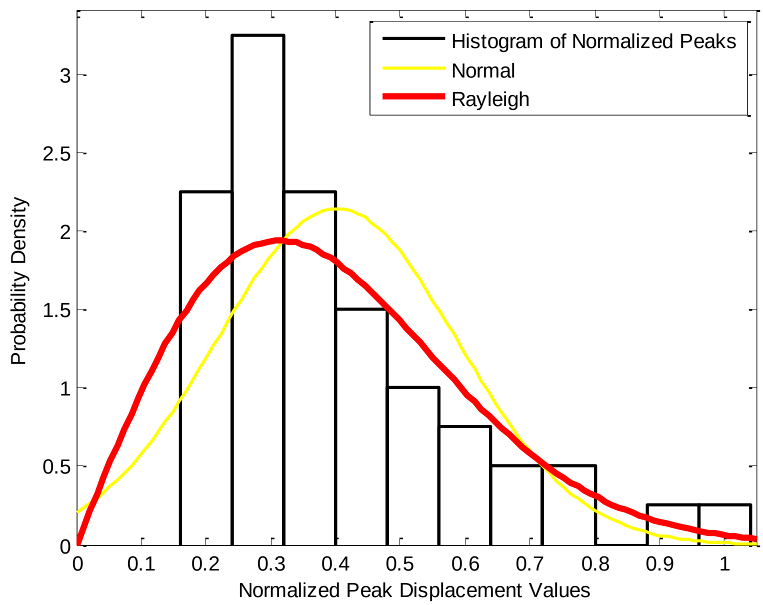

Figure 3. Probability density functions for normalized displacement peaks of a 2 second oscillator

Table 2. Key parameter which defines the distribution of different oscillators' response peaks

Oscillator Period (sec.)	Key Parameter	Oscillator Period (sec.)	Key Parameter
0.1	0.309811	**2.6**	0.325796
0.2	0.310082	**2.7**	0.322428
0.3	0.310619	**2.8**	0.32108
0.4	0.311535	**2.9**	0.319832
0.5	0.312727	**3**	0.318351
0.6	0.314298	**3.1**	0.317008
0.7	0.320729	**3.2**	0.315483
0.8	0.327006	**3.3**	0.312732
0.9	0.347457	**3.4**	0.309117
1	0.365031	**3.5**	0.306659
1.1	0.371144	**3.6**	0.303794
1.2	0.370542	**3.7**	0.300933
1.3	0.370201	**3.8**	0.2982
1.4	0.373587	**3.9**	0.296297
1.5	0.373348	**4**	0.294197
1.6	0.365057	**4.1**	0.292077
1.7	0.356595	**4.2**	0.289831
1.8	0.352473	**4.3**	0.288019
1.9	0.34639	**4.4**	0.28653
2	0.341033	**4.5**	0.285162
2.1	0.34024	**4.6**	0.283739
2.2	0.338313	**4.7**	0.282482
2.3	0.33419	**4.8**	0.28172
2.4	0.332178	**4.9**	0.280975
2.5	0.329485	**5**	0.279875

3.2. Regression Analysis Procedure

Nonlinear regression analysis was performed to predict the amplitudes of the local maxima of response as a function of the elastic Sd, the number of peaks and the system parameters.

In the analysis, the same dataset is used and 5 % damped SDOF systems with periods ranging from 0.01 to 2.5 sec. with 0.1 sec. intervals are considered. The first twenty highest peaks of computed responses are evaluated.

The correlations between the number of response displacement peaks, the normalized and averaged amplitudes of peaks and the structural period are evaluated separately and presented in Figure 5.

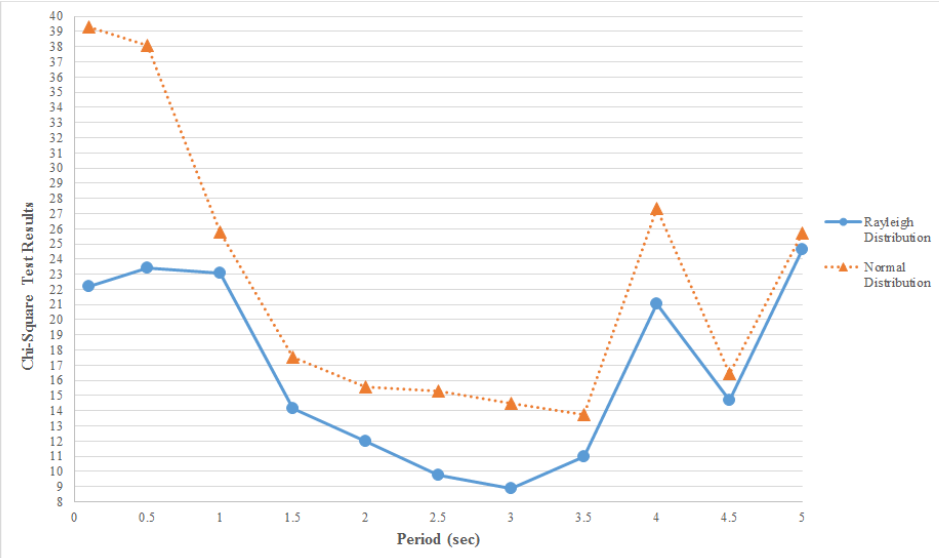

Figure 4. Chi-Square test results for Rayleigh and Normal distribution functions

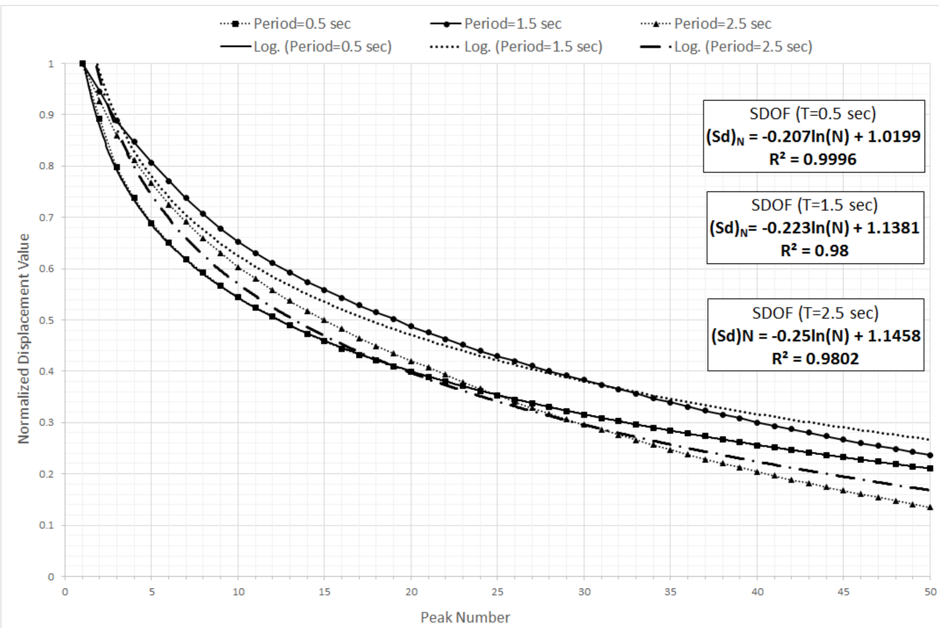

Figure 5. Variations of the normalized and averaged amplitudes of peaks with the number of peaks for five single degree of freedom systems having periods equal to 0.5, 1.5 and 2.5 seconds

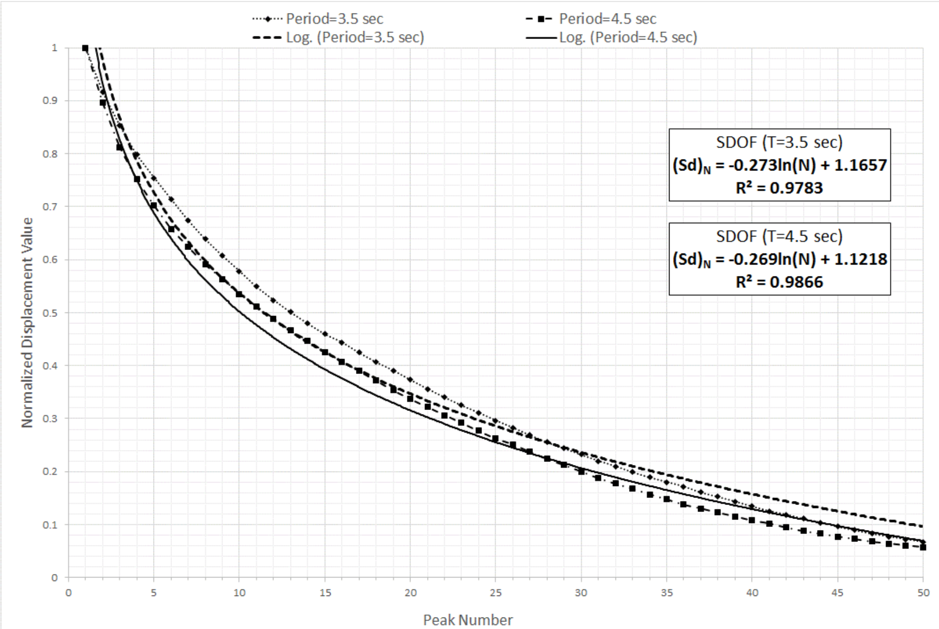

Figure 5 (Cont.) Variations of the normalized and averaged amplitudes of peaks with the number of peaks for five single degree of freedom systems having periods equal to 3.5 and 4.5 seconds

It can be inferred that there is a clear logarithmic relation between these parameters. In Figure 5 the logarithmic trend lines (Log.) correspond to each structural period. Equations in the figure define the normalized and averaged amplitudes of peaks for given values of the number of peaks. The functional form of the regression model is given by Eq. (2).

$$f(x,y) = a - b*\ln(x) + e*\exp(-(x*d)) + c*\exp(-(w*y)^2) - 0.08 \tag{2}$$

To determine the coefficients of the functional form, the Levenberg-Marquardt algorithm in Matlab (Mathworks Inc., Natick, MA, USA) software is employed. A nonlinear least-squares formulation is used to adapt a nonlinear model to data. Basic matrix techniques are insufficient to evaluate the coefficients of nonlinear models. Hence, an iterative improvement to parameter values in order to reduce the sum of the squares of the errors between the function and the measured data points is required [15]. The following final equation results from the regression analysis;

$$(Sd)_N = 1.055 - a*\ln N + d*e^{-cN} + b*e^{-(fT)^2} \tag{3}$$

where a, b, c, d, and f are coefficients fitted by regression, $(Sd)_N$ is the normalized and averaged amplitudes of peaks, T is the structural period and N is the number of response displacement peaks.

The results of the goodness-of-fit statistics for proposed model (in Figure 6) are given in Table 3. Regression coefficients are given in Table 4. with 95% confidence bounds.

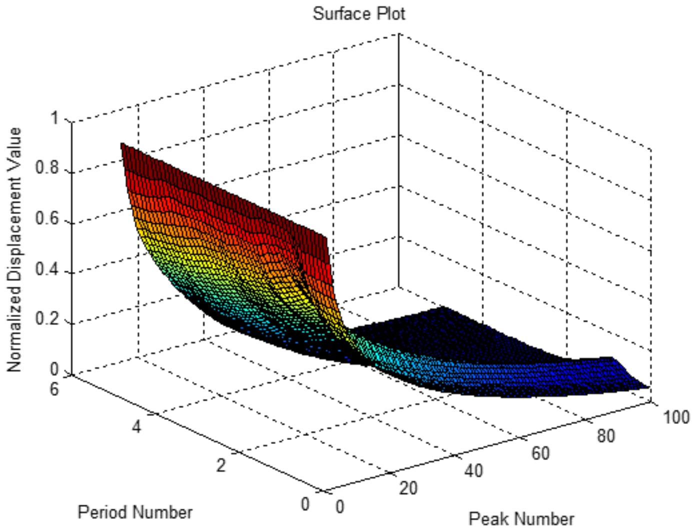

Figure 6. 3D displacement response spectra of the 317 pairs of earthquake records

Table 3. Results of the goodness-of-fit statistics

SSE	R^2	Adjusted R-Square	% RMSE
0.1597	0.9987	0.9987	0.008003

Table 4. Regression Coefficients

a	b	c	d	f
0.3045	0.2519	0.3625	-0.4301	0.2039

The relative differences between the proposed model and the real data are calculated by Eq. (4),

$$\%\ \text{Re}\,lative\ \ Difference = \frac{(\text{D}_{model} - D_{real})}{D_{real}} \tag{4}$$

where D_{model} and D_{real} represent the results of the proposed mathematical model and the real data for amplitudes of local maxima, respectively. The relative difference values are shown in Figure 7.

According to Figure 7, the relative difference varies between 8 % and 18 % and the maximum relative difference observed in T=1.01 sec. for the twentieth highest peak of computed response. It is indicated that the relative difference values of the first ten highest peaks do not exceed 10 % for all structural periods.

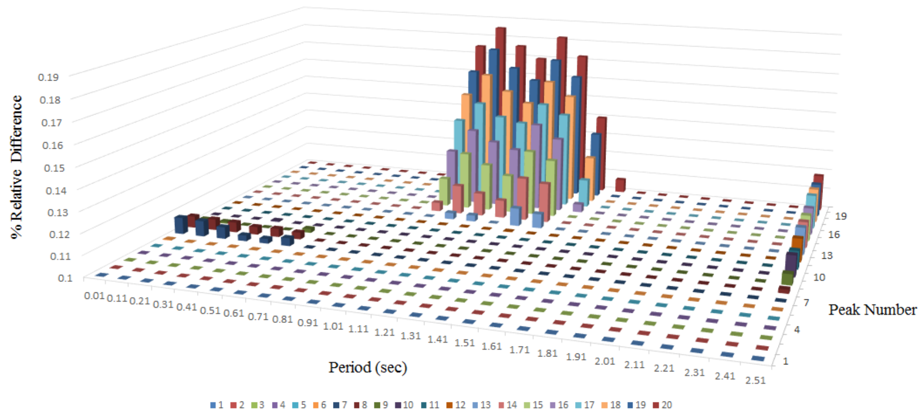

Figure 7. % Relative difference values belongs to relation between the first twenty highest response peaks' amplitudes, natural vibration periods (ranges between 0.01 to 2.5 sec.) and peak number

4. DISCUSSION

Response spectrum concept is of great interest to the seismic design. Although it is an important and widely used tool for description and characterization of SGM amplitudes, it only represents the amplitude of the highest peak of the response. Thus it clearly ignores the much valuable information on overall response time history such as the number, the sequence and the relative amplitudes of excursions beyond its design strength [9]. These repeated excursions have appreciable effects on structural response. Therefore, for better understanding of response characteristics of structures to earthquake motion, it is essential to determine the statistics of response maxima.

Once the statistical distribution of the maxima of the structural displacement response is determined and the prediction of amplitudes of the N^{th} order peak in the seismic response as a function of the elastic Sd, the number of cycles (corresponds to two times peaks) and the system parameters is made, then the following important information can be obtained on the basis of the presented key parameters and proposed regression model.

- Expected and most probable amplitudes of the N^{th} order (first largest, second largest, third largest, etc.) peaks of the response of linear, viscously damped SDOF system (3D Response Spectrum concept e.g., Figure 6)
- The probabilities of exceeding or not exceeding particular response peak amplitudes (PDF plots e.g., Figure 3)
- The number of times certain response levels may be exceeded (histogram plots e.g., Figure 3 and [16,17])
- The effective structural response duration (see Eq. (A.7) in Appendix)
- The relationship between the local maxima of response, their number and amplitudes to the physical characteristics of the designed structural system and the spectral response (Eq. (3))

Those calculations are essential for determining start of low-cycle fatigue behavior particularly for design of column-beam joints of steel structures or bridges. Such kind of information cannot be extracted from the conventionally used response spectra and it is very useful for seismic design as it makes an earthquake's damage capacity more understandable.

Results also reveal that even in the elastic systems, response values that are quite close to the value of the highest peak displacement are repeated many times. In general, for SDOF systems having natural period of vibration greater than or equal to 0.9 sec., oscillation having 90 % of the value of highest peak is repeated two times under the earthquake excitation. SDOF systems having natural period of vibration vary between 1 and 1.7 sec., ratio between second largest peak and Sd is about 95 %. For instance, a SDOF system with 0.5 sec. structural period will still show elastic behavior when 60 % of design spectral level was exceeded 8 times or 40 % of the design level was exceeded 20 times (see Figure 5). Beyond these levels, structure may or may not experience damage. For talking about first initiation of damage experimental studies as well as numerical studies on inelastic behavior of SDOF structures are required to connect the number of cycles to the displacement levels.

Furthermore, this study gives opportunity to verify the results of previously conducted studies on this subject of which synthetic (artificial) accelerograms were used by performing the empirical results with worldwide real strong motion data. Instead of theoretical derivations, key parameters which describe the statistical distribution of the structural response maxima are presented.

Current research is limited to displacement response of linear SDOF systems. It is believed that future research is needed in applying order statistic approach in inelastic response of SDOF systems including different hysteretic characteristics, with and without the consideration of stiffness and/or strength degradation. Moreover, for further analysis stages, the relationship between number of cycles, damage and inelastic excursions of SDOF structures during the deformation is planned to be investigated.

5. CONCLUSIONS

In this study, a probabilistic theory, based on order statistics approach has been applied to determine the theoretical distribution functions for the maxima of linear elastic SDOF systems' response under earthquake excitation. Best model and fit are judged by two generally handled probability distributions such as Normal and Rayleigh. For specifying the distributions' parameters, Maximum Likelihood Estimation method is considered. After estimation of each distribution function's related parameters, the Chi-Square goodness of fit test is used to rank the fitted distributions. Our evaluation of histogram plots of the normalized response peaks have showed that the PDFs of the ordered peaks in linear earthquake response can be best represented by a Rayleigh distribution. This result is in good agreement with previously published results. Key parameters are presented for each vibration period to define the distribution of the response maxima.

Based on these findings, a nonlinear regression model is proposed to define amplitudes of the local maximum values in the response of SDOF systems having five percent damping ratio and vibration periods vary between 0.1 and 2.5 sec. as a function of structural period and spectral response. The model has capable of estimating all of the local maximum values of a structure of which its Sd value and vibration period are already known. Thus, the number of times certain response levels may be exceeded can be calculated. In another words the model gives the number of times of a Sd that can be repeated by a structure without any damage.

APPENDIX

Theoretical Derivations of Previous Studies for the Statistical Distribution of Maxima in a Random Process or Response Process

Through generalizations and extensions of the work of [1, 2] and [3] on the theory of probability distributions of a random function which find useful applications in the field of earthquake engineering and strong motion seismology, a random function of time, f(t) may be represented by [8];

$$f(t) = \sum_n C_n \cos(\omega_n t + \Phi_n) \tag{A.1}$$

where ω_n are the circular frequencies, Φ_n are the random phases uniformly distributed between 0 and 2π, t represents time and C_n are the amplitudes related to the energy spectrum, $E(\omega)$, of f(t) by the following relation;

$$\sum_{\omega n=\omega}^{\omega+d\omega} \frac{1}{2} C_n^2 = E(\omega)\, d\omega \tag{A.2}$$

According to [8] and [9]; f(t) may represent the response of a structure to an earthquake excitation.

Using previous definitions, [3] derived the PDF for the distribution of the maxima of f(t):

$$a_{rms} = m_0^{1/2} \tag{A.3}$$

in terms of the root-mean-square value (a_{rms}) of f(t) and

$$\varepsilon = \left[\frac{m_0 m_4 - m_2^2}{m_0 m_4}\right]^{1/2} \tag{A.4}$$

a parameter (ε) which represents a measure of the width of energy spectrum of f(t). In general, the n^{th} moment of the energy spectrum (m_n) given by;

$$m_n = \int_0^{\infty} \omega^n E(\omega)\, d\omega \qquad n = (0,1,2,...) \tag{A.5}$$

The PDF of the maxima of f(t) normalized with respect to root-mean-square value, $m_0^{1/2}$ is given by [3] as:

$$p(\eta) = \frac{1}{\sqrt{2\pi}} \left[\varepsilon\, e^{-\eta^2/2\varepsilon^2} + \left(1-\varepsilon^2\right)^{1/2} \eta\, e^{-\eta^2/2} \int_{-\infty}^{\eta(1-\varepsilon^2)^{\frac{1}{2}}/\varepsilon} e^{-x^2/2}\, dx \right] \tag{A.6}$$

when ε=0, the statistical distribution of the maxima tends to a Rayleigh distribution and when ε approaches its maximum value 1, the distribution of the maxima tends to a Gaussian distribution [18,19 and 20].

Accordingly, the effective structural response duration can then be obtained by [13].

$$T_e = \frac{A_0^2}{64\pi^3 \xi \left(\frac{1}{T}\right)^3 \sigma^2} \tag{A.7}$$

where, T is the natural period, ξ is the viscous damping ratio, A_0 is the averaged Fourier amplitude spectrum, T_e is the effective structural response duration and σ is the key (Rayleigh) parameter.

Acknowledgments

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