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Research Article

COMPARISION OF HIGH STRENGTH AND ORDINARY REINFORCED CONCRETE SLABS UNDER FIXED BOUNDARY CONDITIONS BY YIELD LINE THEORY

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ABSTRACT

In this study, one of the fairly known practical plate problems was investigated, a square plate laterally loaded with single concentrated load at mid-span under all edges clamped. Clamping was made continuously along the edges with a small quantity of rotation. This type of clamping could be called as partially fixed. The primal objective of the present study is to investigate the behavior of two slabs with different dimensions (660x660x40 mm and 1080x1080x40 mm), produced with high-strength and ordinary concrete. High-strength reinforced concrete plates containing 8 mm orthogonal reinforcement with 100 mm spacing were constructed and tested. Load-deflection relationships were investigated. Mechanical properties of high-strength and ordinary concrete were examined. Fracture patterns of plates were presented. Comparisions were made both for high strength and ordinary concrete slabs.

Keywords: Slab, reinforced concrete slab, high strength concrete, yield line theory.

1. INTRODUCTION

Plates are structural members with flat surfaces. The thickness is quite small as compared to the other two dimensions (length and width). They provide living spaces in buildings. Plates are defined to mid-plane separating the plate into two halves along the thickness of the plate. They bear loads perpendicular to that surface. The load acting on a small area could be called as concentrated load.

"High strength concrete" in relevant standards is defined as above 50 MPa in Turkish Standard of TS 500 [1] ,between minimum 60MPa and maximum 130 MPa by CEB/FIB [2]. ACI [3] specifies 41 MPa as the ultimate strength limit for Standard concrete. In this study, compressive strengths over 50 MPa is referred to as "high strength concrete". Behavior of high strength concrete under load is more brittle than ordinary concrete. Therefore, exact applicability of relationships observed for ordinary concrete into high strength reinforced concrete slabs is always open for discussion.

Yield line method, which based on yield strength of steel, ultimate bearing capacity could be determined, realistically. This method uses possible collapse mechanism by means of trial-error process [4,5,6]. Structural analysis on slabs with yield line method is based on equilibrium

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equations for “n” number of members constituting the failure mechanism. By this manner, “3n” number of equilibrium equations could be obtained. [7,8]. The publications [9-13] are considered with collapse mechanisms in slabs are investigated.

1.1. Objective of the study

This study includes slabs under fixed boundary conditions with a single concentrated load at midspan. The primal objective of the present study is to investigate the behavior of two slabs with different dimensions (660x660x40 mm and 1080x1080x40 mm), produced with high-strength and ordinary concrete. All slabs have same reinforcement of 100 mm spacing in both directions. Applicability of yield line method on high strength reinforced concrete slabs was proved for slabs having simply supported boundary conditions [14]. Comparisions were made both for high strength and ordinary concrete slabs. Failure mechanisms and yield line solutions were given and variations was tried to explain.

2. EXPERIMENTAL STUDY

2.1. Materials used in test samples

Reinforced concrete test samples were produced from both high-strength and ordinary concrete. Maximum aggregate grain size diameter was 16 mm. Physical characteristics were provided in Table 1. Compositions of concretes were provided in Table 2.

Table 1. Aggregate physical characteristics

Size	Loose Unit Weight (kg/m^3)	Specific Gravity (kg/m^3)		Water Absorption (%)
		Dry	Saturated	
Coarse (>4mm)	1435	2712	2692	0,49
Fine (<4mm)	1486	2668	2685	0,55

Table 2. Concrete composition

Concrete	Cement Type	Cement Dose (kg/m^3)	W/C	Aggregate (kg/m^3)	Silica Fume (kg/m^3)	Superplasticiser %	Saturation Water %
High Strength	Cem I 42,5 R	500	0,30	1737	50	2	1,52
Ordinary	Cem III 32,5R	350	0,50	1737	-	2	1,52

Control samples taken from produced slab concretes were cured in accordance with relevant standards [15] and concrete strengths were determined. Concrete mechanical characteristics were given in Table 3 and 4. Average stress-strain curves were presented in Fig. 1 and 2. Slabs were also cured until the test date by continuous wetting.

Table 3. Average mechanical characteristics of concrete

	Ordinary Concrete (OC)	High-Strength Concrete (HSC)	Modules of Elasticity (MPa)	Poisson Ratio
f_{cm} (MPa)	41	74,68	22500 (OC) 34000 (HSC)	0,251 (OC) 0,236 (HSC)
Std. Dev.	5,8	6,51		
f_{ck} (MPa)	33,6	66,4		
			f_{cm}: average compressive strength f_{ck} :characteristic compressive strength	

Table 4. Average concrete compressive strengts of slab specimens

Material	Slab	f_{cm} (MPa)	Standard deviation	f_{ck} (MPa)
High Strength Concrete	A4	73,5	6,8	65,3
	AD4	73,4	6,5	65,1
	AB4	76,2	6,1	68,4
Ordinary Concrete	AG4	40,6	6,0	32,9
	AGD4	42,0	5,4	35,1
	AGB4	40,5	6,0	32,8

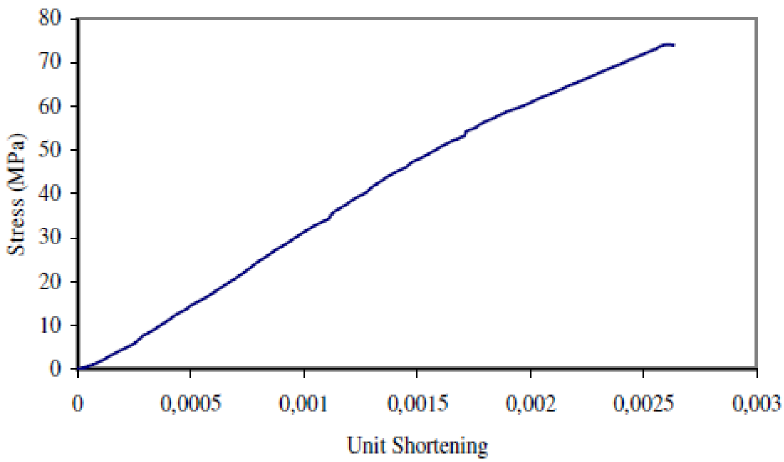

Figure 1. Stress-Strain curve for high-strength concrete

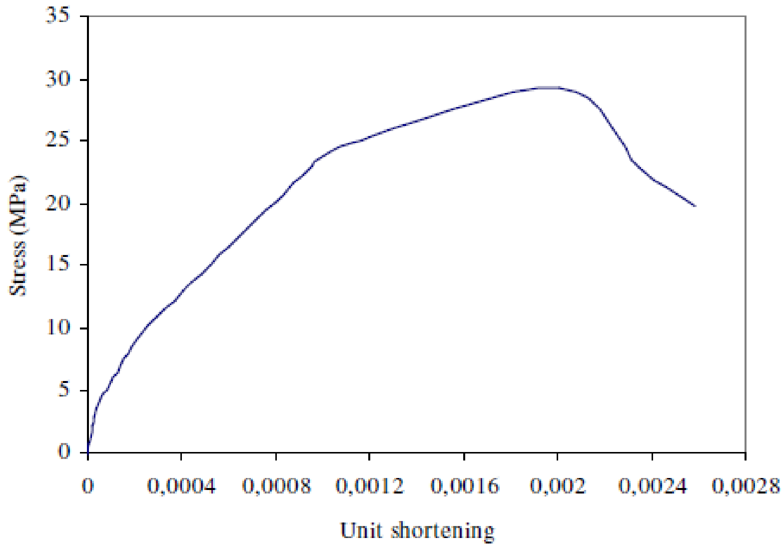

Figure 2. Stress-Strain curve for ordinary concrete

8 mm reinforcement with 100 mm spacing was used in test samples of reinforced concrete slab (Fig. 3). Mechanical characteristics of the reinforcement were provided in Table 5.

Figure 3. Reinforcement in slabs

Table 5. Mechanical characteristics of reinforcement

	Average Tension Strength (N/mm^2)	Average Yield Strength (N/mm^2)	Rupture Strain (%)
8	619	430	21

2.2. Reinforced concrete slabs and testing assembly

Experimental set up was prepared by using U140 (h=140 mm) steel profiles. Profiles were perforated at 400 mm spacing along their axis of symmetry to attach the profiles on to steel rods. Plates were placed between two U140 profiles along the plate edges (Fig. 4). Characteristics of reinforced concrete slabs are given in Table 6.

Table 6. Characteristics of reinforced concrete slabs

Slab	Dimensions (mm)	Free-Span (mm)	Reinforcement Diameter/Spacing (mm)
A4	900x900x40	660x660	$\phi 8/100$
AD4	900x1300x40	660x1060	$\phi 8/100$
AB4	1300x1300x40	1060x1060	$\phi 8/100$
AG4	900x900x40	660x660	$\phi 8/100$
AGD4	900x1300x40	660x1060	$\phi 8/100$
AGB4	1300x1300x40	1060x1060	$\phi 8/100$

2.3. Deflection measurements

Load-Deflection relationships were measured. One displacement transducer was placed at the mid-span, the others were on the symmetry axes and far from the boundaries by one-fourth of the length of plate. Deflection measurements were made at 5 points in 660x660mm free-span plates , 7 points in 660x1060 mm free-span plates and 9 points in 1060x1060 mm free-span plates. (Fig. 4, 5 and 6).

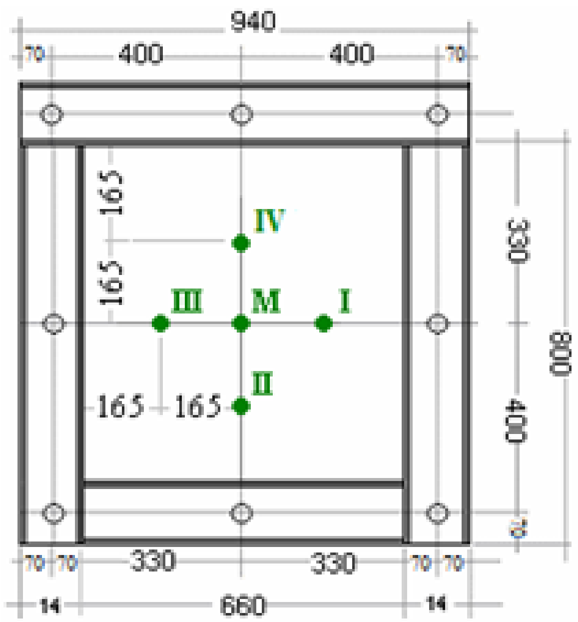

Figure 4. Deflection measurement points for 660x660 mm free-span slabs [16]

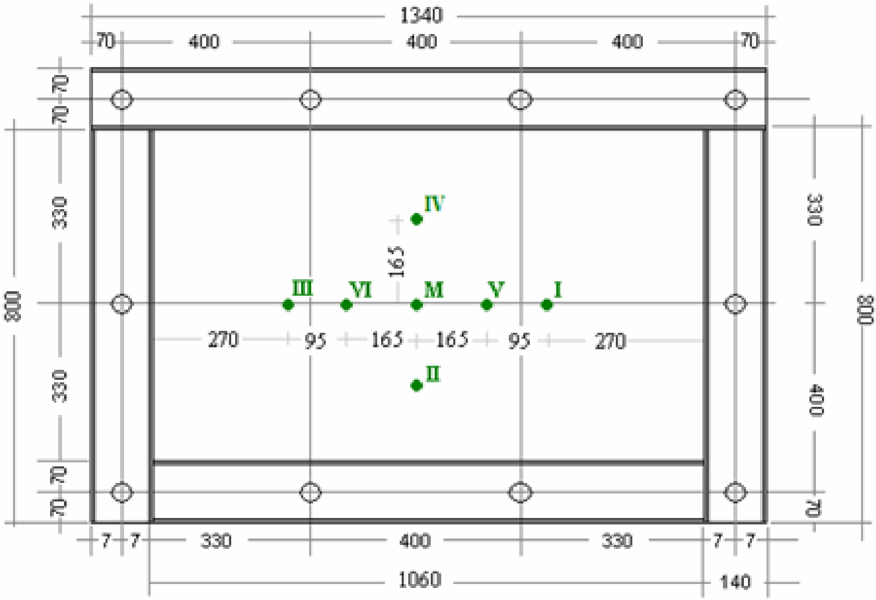

Figure 5. Deflection measurement points for 660x1060 mm free-span slabs [16]

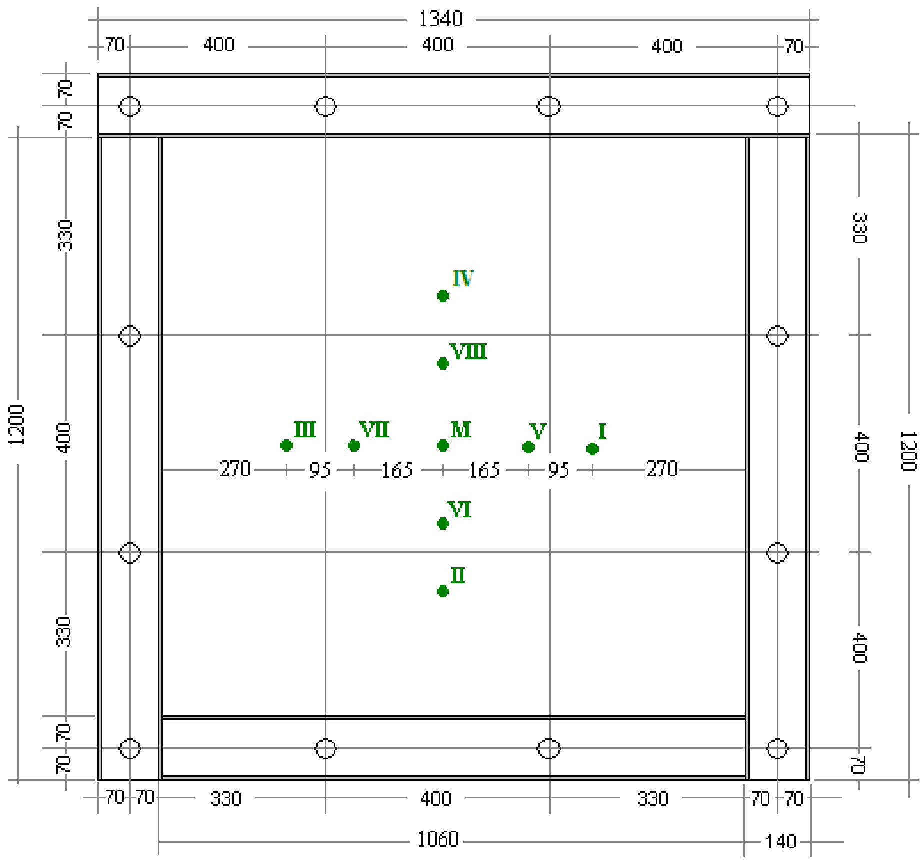

Figure 6. Deflection measurement points for 1060x1060 mm free-span slabs [16]

3. RESULTS

Load deflection curves for high strength slabs of (A4, AD4 and AB4) are given in Fig. 7, 9 and 11. Corresponding failure mechanisms for A4, AD4 and AB4 are given in Fig. 8, 10 and 12.

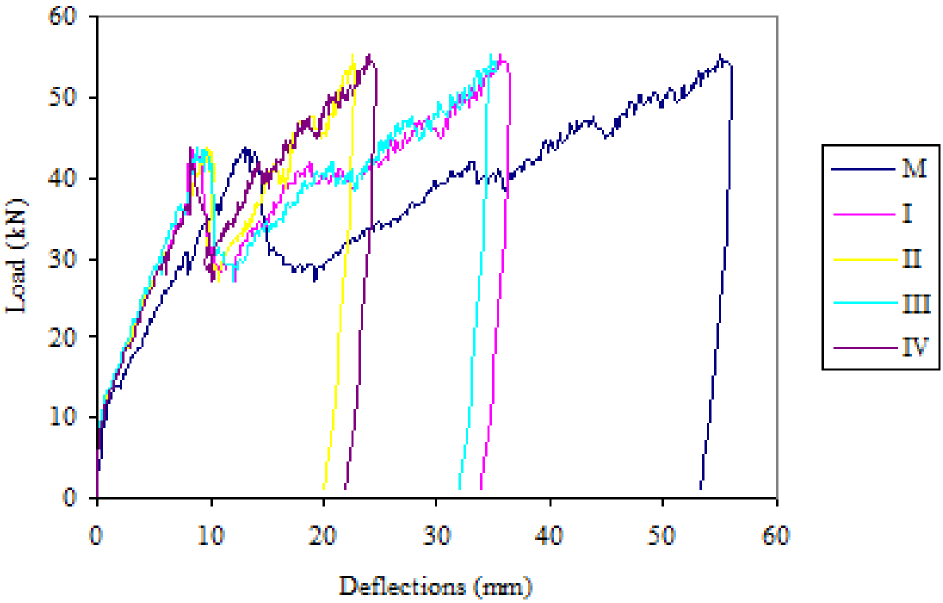

Figure 7. Load Deflection curve of A4 slab

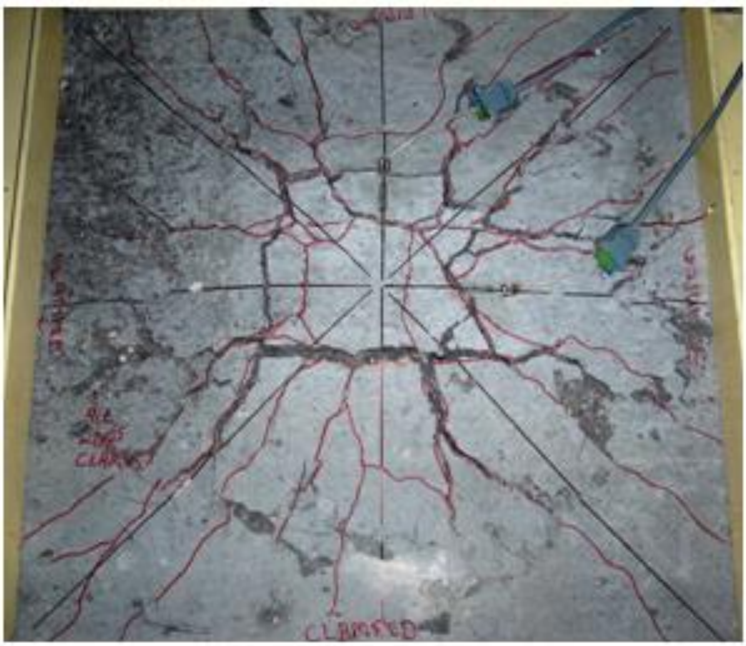

Figure 8. Failure pattern of A4 slab

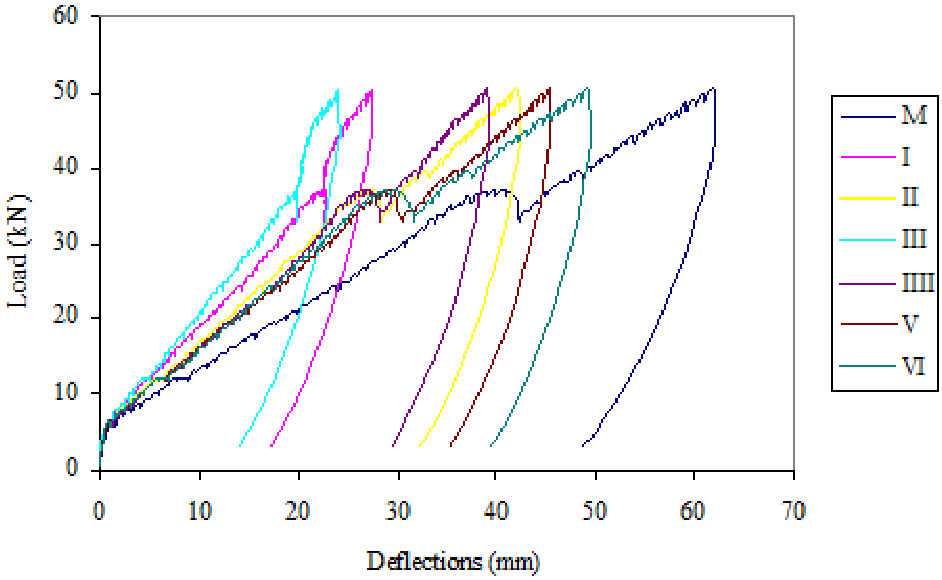

Figure 9. Load Deflection curve of AD4 slab

Figure 10. Failure pattern of AD4 slab

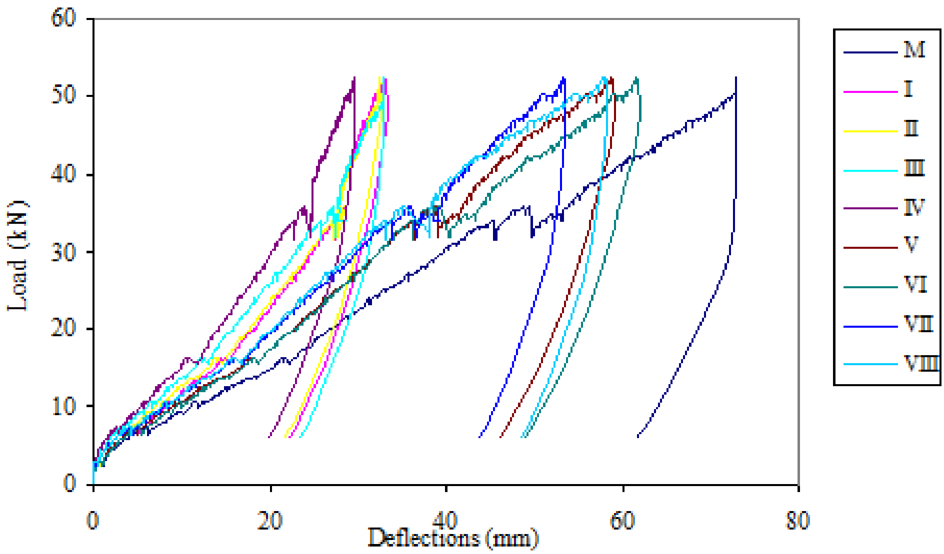

Figure 11. Load Deflection curve of AB4 slab

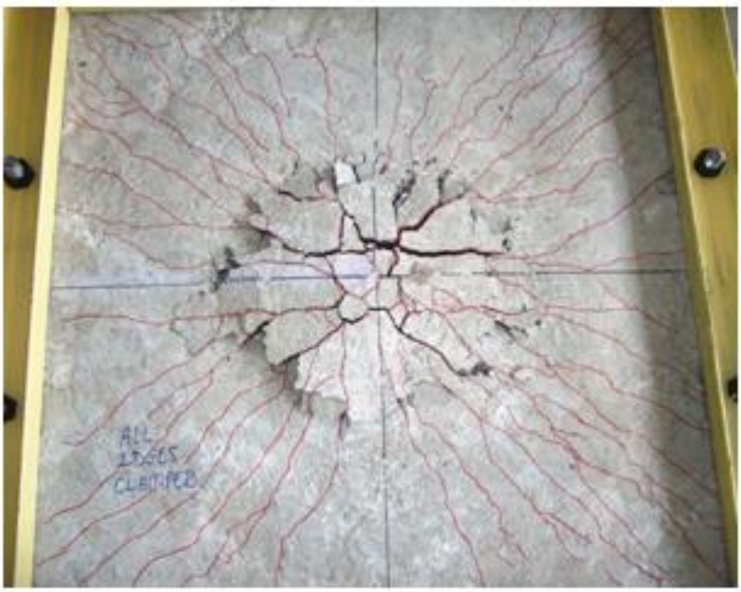

Figure 12. Failure pattern of AB4 slab

Load deflection curves for slabs made with ordinary concrete of (AG4, AGD4 and AGB4) are given in Fig. 13, 15 and 17. Corresponding failure mechanisms for AG4, AGD4 and AGB4 are given in Fig. 14, 16 and 18.

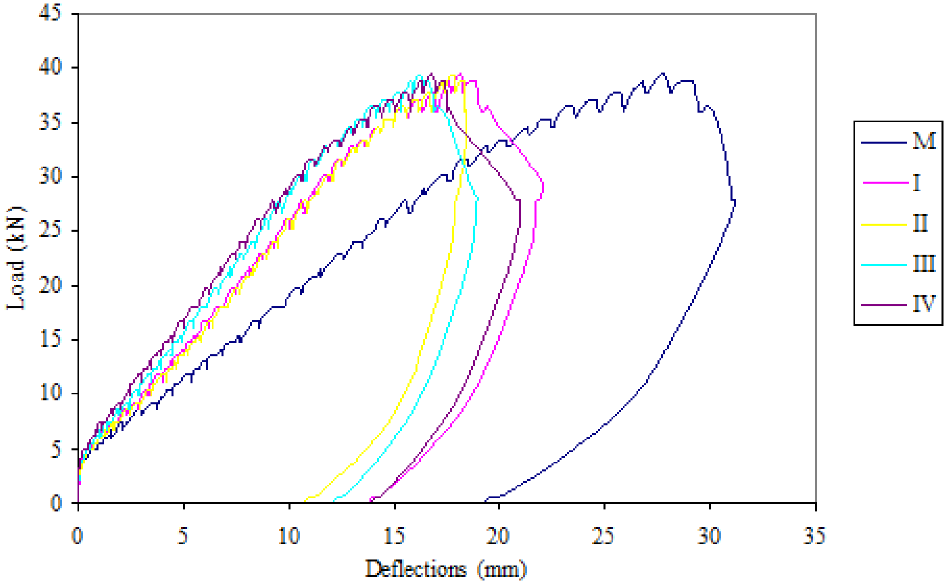

Figure 13. Load Deflection curve of AG4 slab

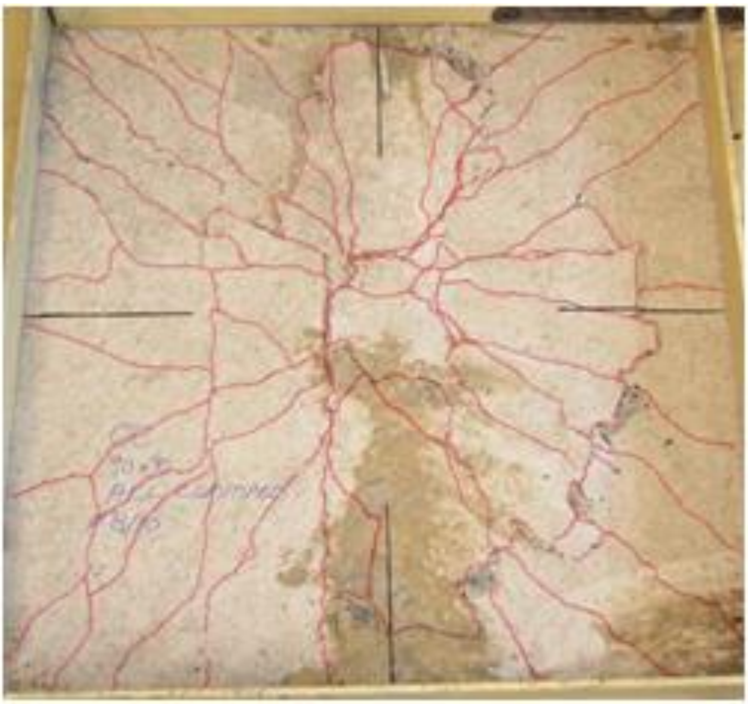

Figure 14. Failure pattern of AG4 slab

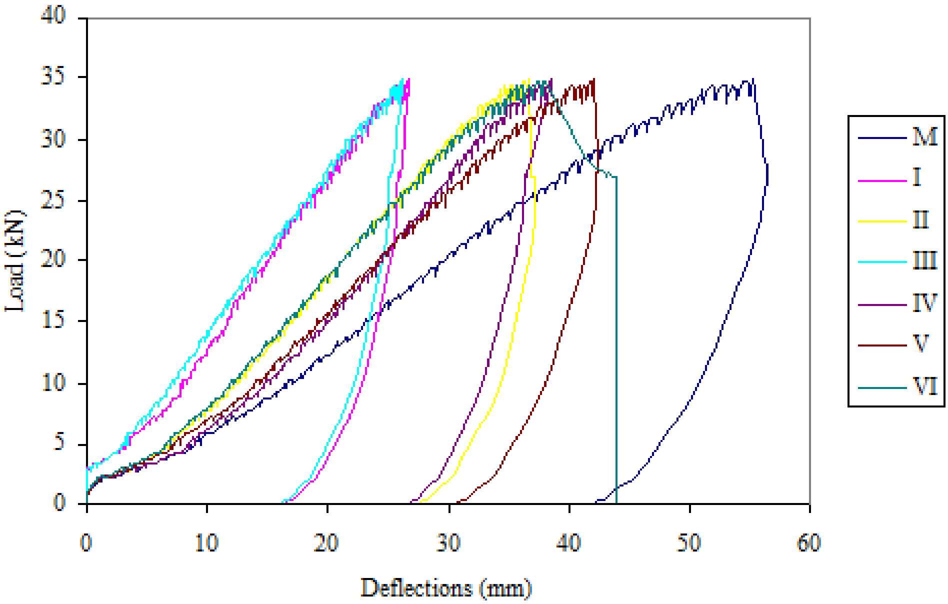

Figure 15. Load Deflection curve of AGD4 slab

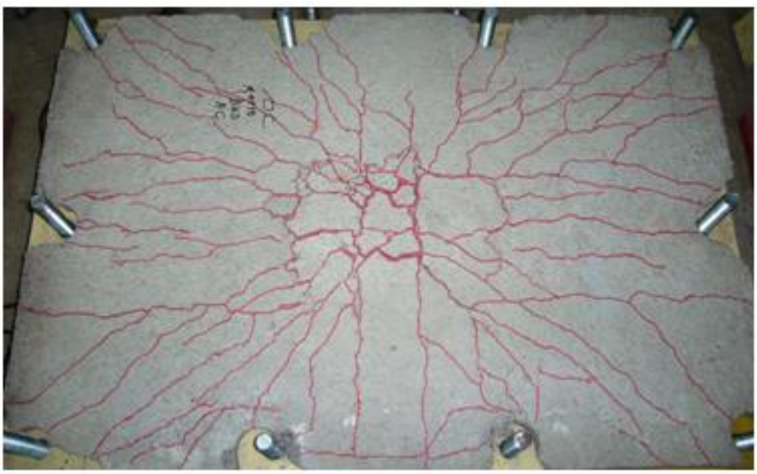

Figure 16. Failure pattern of AGD4 slab

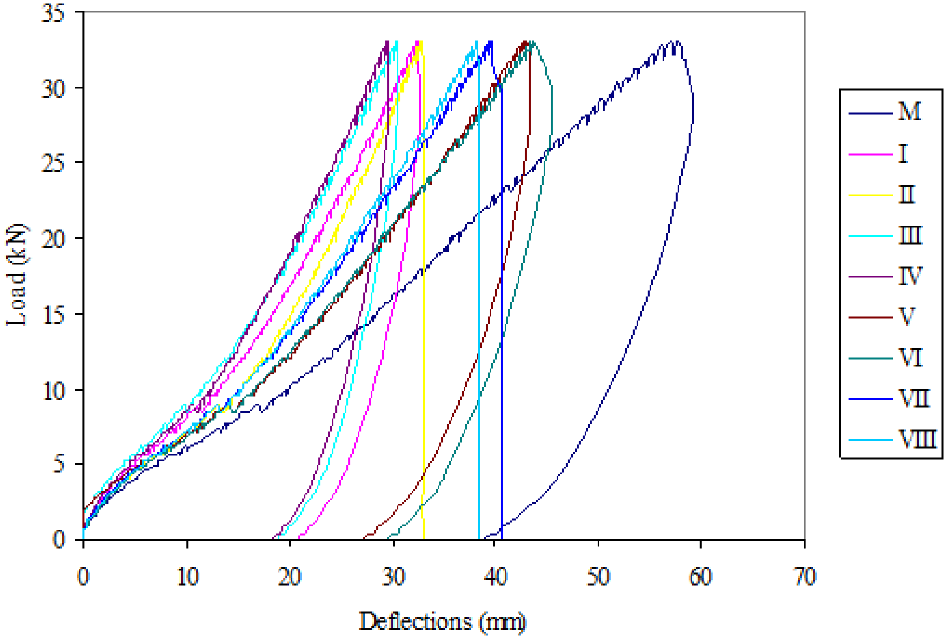

Figure 17. Load Deflection curve of AGB4 slab

Figure 18. Failure pattern of AGB4 slab

Theoretical failure mechanisms of a fixed slabs are presented in Fig. 19. The initially yield lines for these isotropically reinforced slabs are formed at the midspan and then spread along the direction of reinforcement. After that circular cracks composed of several triangular geometries was observed.

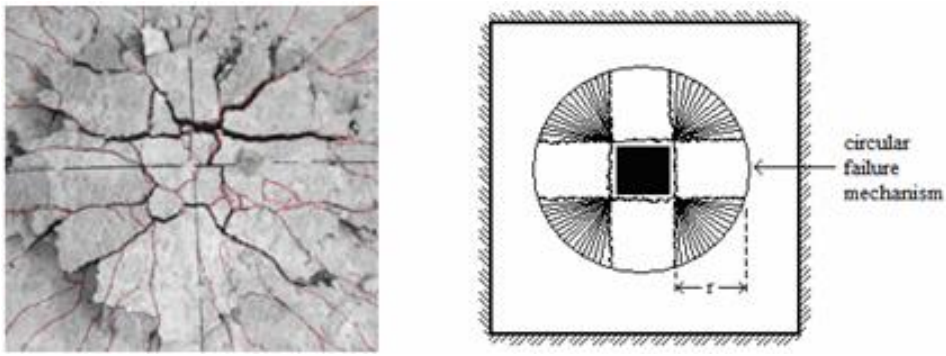

Figure 19. Experimental and theoretical failure mechanisms of a fixed boundary slab

Calculations are performed in accordance with the virtual work principles, in other words, the work done by internal forces is equalized to the work done by external forces as follows [17],

$$P_A\delta = (m+m')2\pi\delta + 2(m+m')(2c)\frac{\delta}{r} \tag{1}$$

Considering the length of one side of the square steel loading plate at the center of the failure mechanism as c=100 mm, the equation (1) turns into the following form;

$$\frac{P_A}{(m+m')} = 2\pi + \frac{400}{r} \tag{2}$$

The slabs constructed with ordinary concrete usually have the same support conditions with the slabs constructed with high-strength concrete. However, there was 10,2% difference in load-bearing capacity of the slabs A4 and AG4, there was 9,5% difference between the slabs AB4 and AGB4. For ordinary slabs, in case of calculation of *m* unit moment with the aid of depth of pressure block for these slabs, the value is obtained as;

$$m = A_s f_y z \cong 3{,}50 \text{ kNm/m} \tag{3}$$

There was 11.1% difference between the average moment of resistance value of high-strength concrete (3,90 kNm/m) and average moment of resistance of ordinary concrete (3,50 kNm/m). In this case, the difference in load bearing capacity of the slabs was attributed to concrete quality since the only different parameter used in both calculations is concrete characteristic compressive strength, f_{ck}.

Load bearing capacity of the slabs with fixed supports increase based on the shape of failure mechanism. Assuming the equation (2) was valid for both high-strength and ordinary concrete slabs, the ratio of load to total moment capacity should be equal to $2\pi + 40/r$ value determined for each one of these slabs. The single parameter of failure mechanism, *r* values, of these slabs were calculated by using yield line method solution for fixed-end slabs with the aid of experimental failure load and experimental moment-bearing capacity and resultant values are provided in Table 7.

Table 7. Determination of failure mechanisms of the slabs with fixed supports

Slab	Experimental Load (kN)	m_r (kNm/m)	r (mm)
A4	43,57	3,89	82
AD4	36,16	3,90	134
AB4	35,85	3,91	138
AG4	39,56	3,48	78,7
AGD4	34,62	3,54	114,6
AGB4	32,76	3,48	127

The slabs with the same dimensions and reinforcement, but with different concrete quality had closer values to each other. The *r* values largely depend on the location of yield lines, bearing capacity and experimental moment capacity of the slabs. Greater values were obtained for high-strength concrete slabs than the ordinary concrete slabs. Such a case was attributed to greater strengths. When the same size slabs were compared, it was observed that small ones had greater rigidity and greater supports rotations were observed since the loads were closer to the edges. These slabs are exposed to greater stresses at limited sections, thus able to bear greater loads. Since the stresses concentrated over the mid-sections, circular failure mechanism had minimum diameters (A4 and AG4 slabs). Cracks spread with increasing dimensions of the structural member and then diameter of circular failure mechanism increases.

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