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Research Article

A SEQUENTIAL APPROACH BASED DESIGN OF MULTIPLE TUNED MASS DAMPERS UNDER HARMONIC EXCITATION

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ABSTRACT

This study evaluates the response reduction effect of single-degree-of-freedom (SDOF) primary systems with multiple tuned mass dampers (MTMDs) under harmonic excitation. To design MTMD, TMD properties are re-calibrated based on the natural frequencies of the system on which the number of TMDs is one less than the current system. This is a sequential approach that does not require any iteration for each step. Instead it requires starting from the case with single TMD, and then increasing the numbers of TMDs one more at a time until the desired number is reached. Using the obtained design parameters, the effectiveness and robustness of the MTMDs are studied in comparison to MTMDs designed by previous works available in the literature. As a result, the proposed design procedure produces an effective multiple tuned mass damper to be utilized in a SDOF system under harmonic excitation. Additionally, the proposed approach provides a simple way to design the MTMD system than the traditional optimization methods, thus it significantly reduces the computation effort in the design process.
Keywords: Harmonic excitation, optimal parameters, robustness analysis, tuned mass dampers, vibration control.

1. INTRODUCTION

The reducing the vibrations induced on tall buildings and other civil engineering structures due to wind, seismic excitations and traffic loads have attracted great interest of many researchers. The efforts on vibration control of structures have resulted in developing various control devices [1, 2]. Among them, tuned mass dampers (TMDs) consisting of a mass, a spring and a viscous damper is one of the simplest, reliable and low-cost control devices.

In 1909, the first application of TMD consisting of a mass and a spring is introduced by Frahm [3]. It has a narrow operation region, and its performance reduces significantly when the exciting frequency varies. Since then, many efforts have been made to obtain the optimum parameters of TMDs. Den Hartog [4] proposed a closed form solution to minimize the dynamic response of undamped main system under harmonic loads. Later, Warburton [5] derived expressions for optimum parameters for undamped system under harmonic and white noise excitations. Asami et al. [6] gave a series solution for the H_∞ optimization and an exact solution for the H_2 optimization. Since their solution is excessively complicated, they proposed an

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approximate solution for practical use. Ghosh and Basu [7] presented a closed-form expression for optimal tuning ratio of TMDs based on the fixed-point theory of Den Hartog. Brown and Singh [8] proposed a mini-max procedure to design TMDs in the presence of uncertainties in the forcing frequency. Anh and Nguyen [9] carried out an approximate analytical solution for optimal tuning ratio of TMDs by using the equivalent linearization method. Yu et al. [10] proposed a reliability based robust design optimization for the tuned mass damper in passive vibration control. Chun et al. [11] investigated a H_∞ optimal design of TMDs variant for suppressing high-amplitude vibrations of damped primary systems. Dell'Elce et al. [12] proposed a new tuning strategy of the linear absorber based on the concept of robust equal peaks for suppressing a specific resonance of an uncertain mechanical system.

All studies mentioned above are concerned with tuning TMD to a dominant frequency of the main system. However, single TMD is very sensitive to any change in the frequency of TMD or the main system, which is so-called the detuning. To overcome the detuning due to the frequency deviation, Xu and Igusa [1] proposed to use multiple tuned mass dampers (MTMDs) instead of the classic single TMD. They indicated that the use of MTMDs with distributed natural frequencies in a frequency bandwidth can be more effective than that of a single TMD with the same total mass. This kind of MTMD system has also been studied by references in Farshidianfar and Soheili [13], Salvi and Rizzi [14], Yazdi et al. [15], Bekdaş and Nigdeli [16], Zuo et al. [17] and Bozer and Özsarıyıldız [18]. The main difference in these studies is the methods and criteria adopted for obtaining the optimal design variables.

Although there have been various techniques to obtain optimal tuning parameters, they are not simple as much as Den Hartog's approach developed for a single TMD based on the fixed-point theory. The most useful aspect of Den Hartog's approach is to give simpler, closed form formulas for optimal parameters of classic TMD. In addition, it is also more effective in reducing dynamic responses on the damped structural systems (Kwon et al. [19]; Luu et al. [20]; Bekdaş and Nigdeli [21]). The purpose of this paper is to extend the method proposed by Den Hartog to obtaining optimal MTMDs. For this aim, properties of each absorber in MTMD are re-calibrated based on the natural frequencies of the system on which the number of TMDs is one less than the current system. This is a sequential procedure that does not require any iteration for each step. Instead it requires starting from the case with a single TMD, and then increasing the numbers of TMDs one more at a time until the desired number is reached. Accuracy and efficiency of the proposed method is demonstrated by some numerical examples.

2. GOVERNING EQUATIONS

Consider a single-degree-of-freedom (SDOF) system with a MTMD shown in Fig. 1. As seen, the MTMD device attached to the primary structure is composed of a set of different TMD units. The natural frequencies of TMD units are tuned to a frequency range in the vicinity of the natural frequency of the main structure. Note that total DOFs of the coupled system is $n + 1$ where n is the number of TMD units. The equation of motion for the main structure with MTMD under harmonic excitation is

$$m_s\ddot{x}_s + c_s\dot{x}_s + k_s x_s + \sum_{j=1}^{n} c_j(\dot{x}_s - \dot{x}_j) + k_j(x_s - x_j) = Pe^{i\omega t} \tag{1}$$

and the vertical motion of the jth TMD is

$$m_j\ddot{x}_j + c_j(\dot{x}_j - \dot{x}_s) + k_j(x_j - x_s) = 0 \quad j = 1, 2, \cdots, n \tag{2}$$

where over dot denotes differentiation with respect to time t. m, c and k are the mass, damping coefficient and stiffness, respectively. Subscripts s and j denote the primary structure, the jth TMD, respectively. x_s and x_j indicate the vertical displacements.

Combining Eqs. (1) and (2), the equations of motion can be given in the following matrix form:

$$\mathbf{M}\ddot{\mathbf{X}}+\mathbf{C}\dot{\mathbf{X}}+\mathbf{K}\mathbf{X}=\mathbf{F} \tag{3}$$

where **M**, **C**, **K** are the mass, damping and stiffness matrices, $\ddot{\mathbf{X}}$, $\dot{\mathbf{X}}$ and $\mathbf{X}$ are the acceleration, velocity and displacement vectors, respectively, **F** is the external force vector, that can be defined as:

$$\mathbf{M}=diag(m_s \quad m_1 \quad m_2 \quad \cdots \quad m_n] \tag{4}$$

$$\mathbf{C}=\begin{bmatrix} c_s+\sum_{j=1}^{n}c_j & -c_1 & -c_2 & \cdots & -c_n \\ -c_1 & c_1 & 0 & \cdots & 0 \\ -c_2 & 0 & c_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -c_n & 0 & 0 & \cdots & c_n \end{bmatrix} \tag{5}$$

$$\mathbf{K}=\begin{bmatrix} k_s+\sum_{j=1}^{n}k_j & -k_1 & -k_2 & \cdots & -k_n \\ -k_1 & k_1 & 0 & \cdots & 0 \\ -k_2 & 0 & k_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ -k_n & 0 & 0 & \cdots & k_n \end{bmatrix} \tag{6}$$

$$\mathbf{X}=\{x_s \quad x_1 \quad x_2 \quad \cdots \quad x_n\}^{\mathrm{T}}, \quad \mathbf{F}=e^{i\omega t}\{P \quad 0 \quad 0 \quad \cdots \quad 0\}^{\mathrm{T}} \tag{7}$$

In order to obtain the normalized displacement amplitude or dynamic magnification factor (DMF) for the primary structure under harmonic excitation, the harmonic solution can be assumed as

$$\mathbf{X}=e^{i\omega t}[x_s \quad x_1 \quad x_2 \quad \cdots \quad x_n]^{\mathrm{T}} \tag{8}$$

Substituting Eqs. (4)–(8) into Eq. (3), the displacement amplitude of the structure x_s can be written as

$$x_s=\frac{P}{k_s}\frac{1}{\mathrm{Re}(z)+\mathrm{Im}(z)\mathrm{i}} \tag{9}$$

where

$$\begin{aligned} \mathrm{Re}(z)&=1-\beta^2-\beta^2\sum_{j=1}^{n}\mu_j\frac{\dfrac{\beta_j^2}{\beta^2}-1+4\xi_j^2}{\left(\dfrac{\beta_j}{\beta}-\dfrac{\beta}{\beta_j}\right)^2+4\xi_j^2}, \\ \mathrm{Im}(z)&=2\beta\xi_s+\beta^2\sum_{j=1}^{n}\mu_j\frac{\dfrac{2\beta\xi_j}{\beta_j}}{\left(\dfrac{\beta_j}{\beta}-\dfrac{\beta}{\beta_j}\right)^2+4\xi_j^2} \end{aligned} \tag{10}$$

Then, the amplitude of the displacement of the structure is finally obtained in the normalized form as follow

$$\mathrm{DMF} = \frac{1}{\sqrt{\mathrm{Re}(z_1)^2 + \mathrm{Im}(z_1)^2}} \tag{11}$$

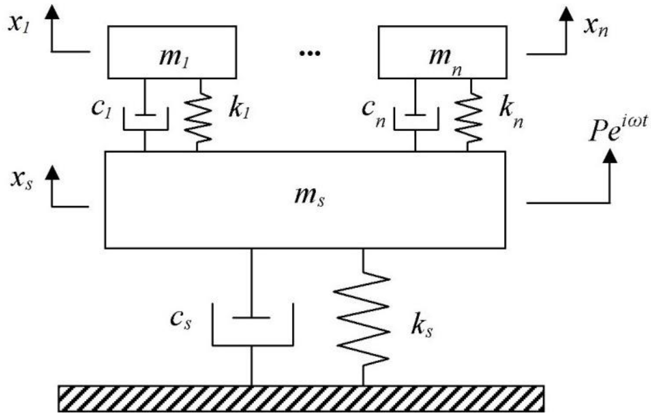

Figure 1. Primary structure – MTMD coupled system

where $\beta = \omega / \omega_s$ is the frequency ratio between the external force and the structure, ξ_s is the damping ratio of the structure, $\mu_j = m_j / m_s$ is the mass ratio, $\beta_j = \omega_j / \omega_s$ is the frequency ratio, and $\xi_j = c_j / 2m_j\omega_j$ is the damping ratio of *j*th TMD.

3. PROPOSED METHOD

The below analytical expressions are proposed by Den Hartog [4] for obtaining the optimum parameters of a single TMD by depending on the mass ratio used to reduce the vibration of SDOF main system under harmonic excitation.

$$f_{opt} = \frac{1}{1+\mu}, \quad \xi_{opt} = \sqrt{\frac{3\mu}{8(1+\mu)}} \tag{12}$$

where f_{opt} is the optimal tuning ratio, ξ_{opt} is the optimal damping ratio and μ is the mass ratio of TMD.

Unlike Den Hartog's approach, the proposed sequential method in this paper is a extension of Den Hartog's tuning approach for a single TMD to the case of MTMDs. In addition, Den Hartog's tuning approach is used together with fundamental modes of the primary structure with TMDs to obtain the design parameters of a MTMD system. This method is extensively examined in the following sections.

To obtain optimal parameters of TMD according to the Den Hartog's approach, the fundamental mode of the primary structure is considered. On the other hand, it can be obviously seen in Table 1 that the natural frequency of the primary structure without TMD is split into two independent modes, when a TMD (n = 1) is attached to it. From this observation, if n TMD are installed on the primary structure, its natural frequency without TMD will be split into n + 1 independent modes. Similarly, if n TMD are installed on the primary structure, its response curve without TMD will be split into n + 1 independent resonant peaks, as shown in Fig. 2. The structural response is plotted for Δ_n = 0.2, μ = 0.01 and f_T = 1 [22].

In the approach proposed, MTMD properties are re-calibrated based on the natural frequencies of the system on which the number of TMDs is one less than the current system. This is a sequential procedure, and it does not require any iterations for each step. Instead it requires to start from the system with single TMD, and then to increase in number of TMD units one more at a time until the desired number is reached. For example, when n = 1, the frequency of the TMD is tuned to that of the structure without TMD. When n = 2, each TMD is tuned to those of the coupled SDOF primary - TMD (n = 1) system. Similarly, if MTMD system is composed of n TMDs, n - 1 independent modes must be considered in design process. Therefore, the following expressions can be proposed for the optimal tuning parameters of each TMD unit based on the formulas given by Eqs. (12):

$$\begin{aligned}
&\omega_{j,n} = \frac{\bar{\omega}_{j,n-1}}{1+\mu_T}, \quad \xi_{j,n} = \xi_T = \sqrt{\frac{3\mu_T}{8(1+\mu_T)}}, \\
&\omega_T = \sum_{j=1}^{n} \frac{\omega_{j,n}}{n}, \quad f_T = \frac{\omega_T}{\bar{\omega}_{1,0}}, \quad \Delta_n = \frac{\omega_{n,n} - \omega_{1,n}}{\omega_T}, \\
&\beta_{j,n} = \frac{\omega_{j,n}}{\bar{\omega}_{1,0}}, \quad \mu_j = \mu_T = \frac{\mu}{n}, \quad \mu = \frac{1}{m_s}\sum_{j=1}^{n} m_{j,n}
\end{aligned} \tag{13}$$

where $\omega_{j,n}$ and $\xi_{j,n}$ is the natural frequency and the damping ratio of the jth TMD in MTDM composed of n TMDs, μ is the MTMD/primary structure mass ratio, ω_T is the average frequency, f_T is the average frequency ratio, and Δ_n is the non-dimensional frequency bandwidth of MTMD. $\beta_{j,n}$ is the frequency ratio of the jth TMD, and $\bar{\omega}_{1,0}$ is the natural frequency of the primary structure without TMD. $\bar{\omega}_{j,n-1}$ denotes the jth natural frequency of the primary structure carrying (n – 1) TMD units. In the proposed approach, MTMD system is designed with the identical mass ratio, μ_T , i.e., $m_{1,n} = m_{2,n} = \cdots = m_{n,n}$, and the stiffness of each TMD unit is adjusted based on $\omega_{j,n} = \sqrt{k_{j,n} / m_{j,n}}$. We assume the damping ratio ξ_T is constant for all TMD units. The MTMD model considered herein is validated for structures under the ground acceleration by Li [23].

Table 1. Natural frequencies (rad/s) of a SDOF main system with MTMD system (m_s = 10kg, k_s = 1kN/m, μ = 0.01)

Mode	w/o TMD	n = 1	n = 2	n = 3	n = 4	n = 5	n = 6
1	10.00	9.4653	9.2533	9.1290	9.0435	8.9793	8.9284
2	-	10.4603	9.9380	9.6894	9.5346	9.4256	9.3430
3	-	-	10.6599	10.1775	9.9265	9.7625	9.6438
4	-	-	-	10.7808	10.3326	10.0874	9.9217
5	-	-	-	-	10.8651	10.4444	10.2070
6	-	-	-	-	-	10.9287	10.5303
7	-	-	-	-	-	-	10.9791

If the mass ratio μ of MTMD system is known, the proposed procedure can be summarized as follows:

Step 1: Design single TMD (n = 1):

First, the first natural frequency of the main system without TMD is obtained, then TMD is tuned to the first natural frequency $\bar{\omega}_{1,0}$. Then, the first two natural frequencies of the main system with TMD $\bar{\omega}_{i,1}\,(i=1,2)$ are obtained. Here μ_T is get as $\mu/1$, and the optimal frequencies and damping ratios for TMD are given as follows:

$$\omega_{1,1}=\frac{\bar{\omega}_{1,0}}{1+\mu_T},\quad f_{1,1}=\frac{\omega_{1,1}}{\bar{\omega}_{1,0}},\quad \xi_{1,1}=\sqrt{\frac{3\mu_T}{8(1+\mu_T)}} \tag{14}$$

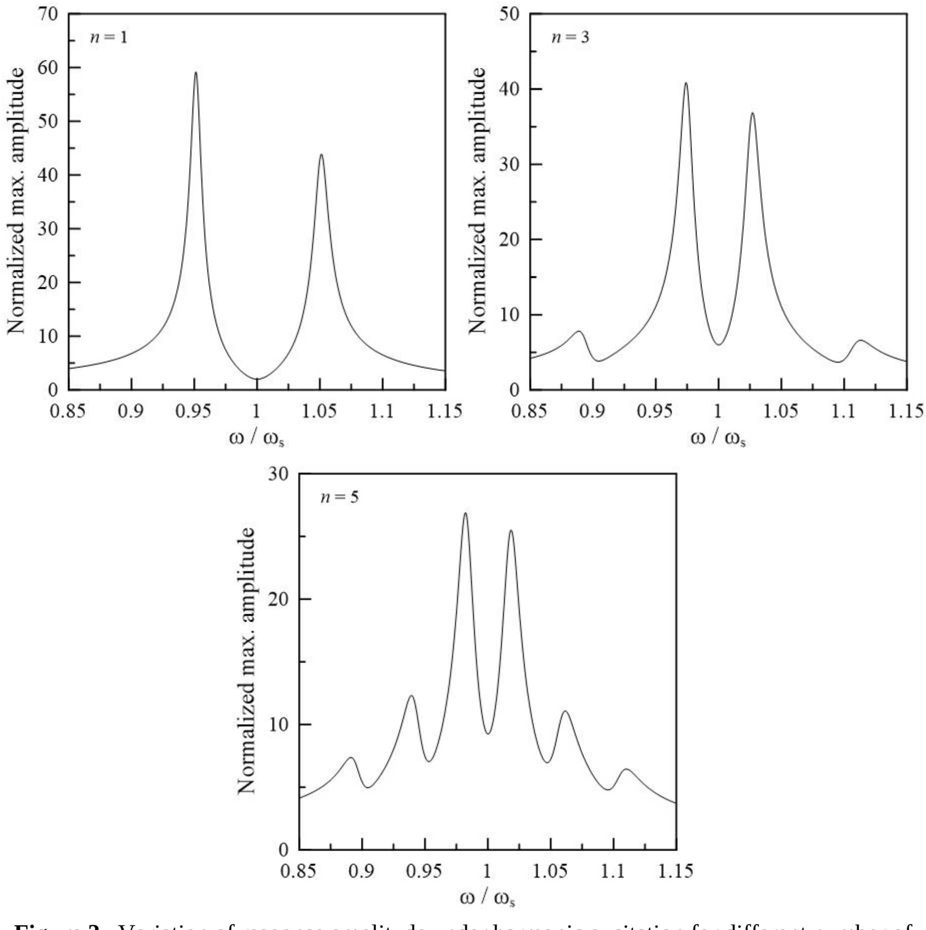

Figure 2. Variation of response amplitude under harmonic excitation for different number of TMD units

where $\bar{\omega}_{1,0}$ first natural circular frequency of the main system without TMD, $f_{1,1}$ is the optimal frequency ratio, $\xi_{1,1}$ is the optimal damping ratio, μ_T is the mass ratio of TMD, $\omega_{1,1}$ is the natural circular frequency for TMD.

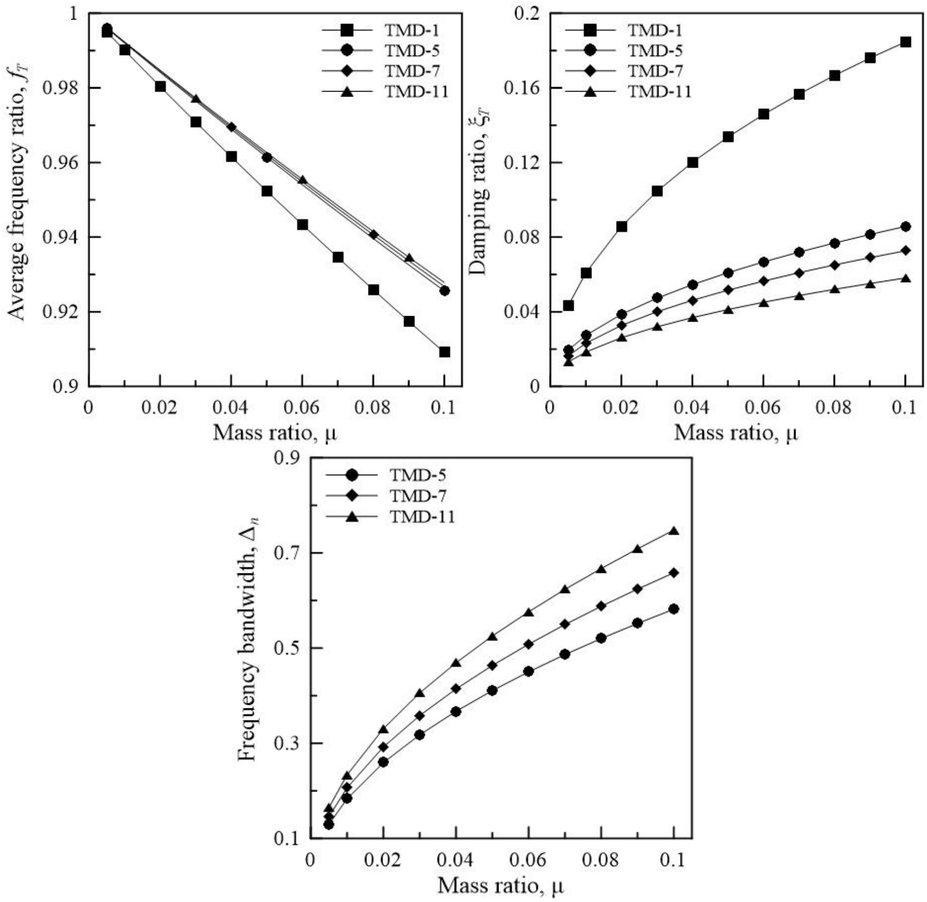

Figure 3. Variation of the average frequency ratio, the damping ratio and the non-dimensional frequency bandwidth of MTMDs with the mass ratio

Step 2: Design MTMD (n = 2):

TMD units in MTMD are tuned to $\bar{\omega}_{1,1}$ and $\bar{\omega}_{2,1}$, respectively. Then, the first three natural frequencies of the main system with MTMD $\bar{\omega}_{i,2}$ $(i = 1,2,3)$ are obtained. μ_T is get as $\mu / 2$, and the optimal frequency and damping ratios for *j*th TMD unit are given as follows:

$$\omega_{j,2} = \frac{\bar{\omega}_{j,1}}{1+\mu_T}, \quad f_{j,2} = \frac{\omega_{j,2}}{\bar{\omega}_{1,0}}, \quad \xi_{j,2} = \sqrt{\frac{3\mu_T}{8(1+\mu_T)}} \qquad (j = 1,2) \tag{15}$$

where $f_{j,2}$, $\xi_{j,2}$, $\omega_{j,2}$ and μ_T are the optimal frequency ratio, optimal damping ratio, natural circular frequency and mass ratio for *j*th TMD in MTMD system with *n* = 2.

⋮

Step N: Design MTMD (n = N):

TMD units in MTMD are tuned to $\bar{\omega}_{1,N-1}, \bar{\omega}_{2,N-1}, \cdots, \bar{\omega}_{N,N-1}$, respectively. μ_T is get as μ / N, and the optimal frequency and damping ratios for *j*th TMD unit are given as follows:

$$\omega_{j,N} = \frac{\bar{\omega}_{j,N-1}}{1+\mu_T}, \quad f_{j,N} = \frac{\omega_{j,N}}{\bar{\omega}_{1,0}}, \quad \xi_{j,N} = \sqrt{\frac{3\mu_T}{8(1+\mu_T)}} \qquad (j = 1, 2, ..., N) \tag{16}$$

where $f_{j,N}$, $\xi_{j,N}$, $\omega_{j,N}$ and μ_T are the optimal frequency ratio, optimal damping ratio, natural circular frequency and mass ratio for *j*th TMD in MTMD system with $n = N$.

In Figs. 3, variation of the average frequency ratio, damping ratio and non-dimensional frequency bandwidth of MTMDs with the mass ratio are given. Here, n = 1, 5, 7 and 11 are selected. As can be seen, the average frequency ratio decreases with increasing the mass ratio. The average frequency ratio for MTMD devices is greater than that of TMD-1. For greater mass ratio, it is much lower than that of MTMDs. As also seen, the difference between the curves are very small for MTMD devices considered. The damping ratio increases with increasing the mass ratio. This is more notable for the single TMD compared to the MTMD systems. Increasing the number of TMD units results in reduction of the damping ratio. In the last graph of Fig. 3, variation of the non-dimensional frequency bandwidth Δ_n of various MTMDs with the mass ratio is shown. The non-dimensional frequency bandwidth increases with increasing the mass ratio. It becomes larger when the number of TMDs increases.

4. RESULTS AND DISCUSSION

To demonstrate the performance of the proposed design method for MTMDs in vibration control, some illustrative examples are presented. The effectiveness and robustness of the MTMDs (TMD-5, TMD-7, TMD-9, TMD-11 and TMD-20) designed by the proposed method studied.

Fig. 4 gives the normalized maximum displacement amplitudes of the primary system with TMDs in case of the error in the tuning frequency ratio. The abscissa shows the error in the estimated tuning frequencies of TMDs, and the ordinate is the normalized maximum amplitude (DMF) calculated by Eq. (11) for $0.5 \le \beta \le 1.5$. It can be clearly seen that the robustness of the proposed MTMD systems (TMD-5, TMD-7 and TMD-11) are better than that of TMD-1. Here, TMD-1 is designed by Den Hartog's Approach. As seen, we still have more smooth curves for MTMD devices, especially for greater mass ratio, in spite of increasing amount of the frequency detuning. That is, MTMD devices are less sensitive to the frequency detuning compared to the classical TMDs. This figure also indicates that the effectiveness of TMD-1 rapidly decreases for lower mass ratio when the error in the tuning frequency increases. Finally, increasing the total number of TMD units increases the robustness. However, as reported in the literature, there is almost no difference between the curves for the maximum amplitudes for n > 7 as seen in the figure.

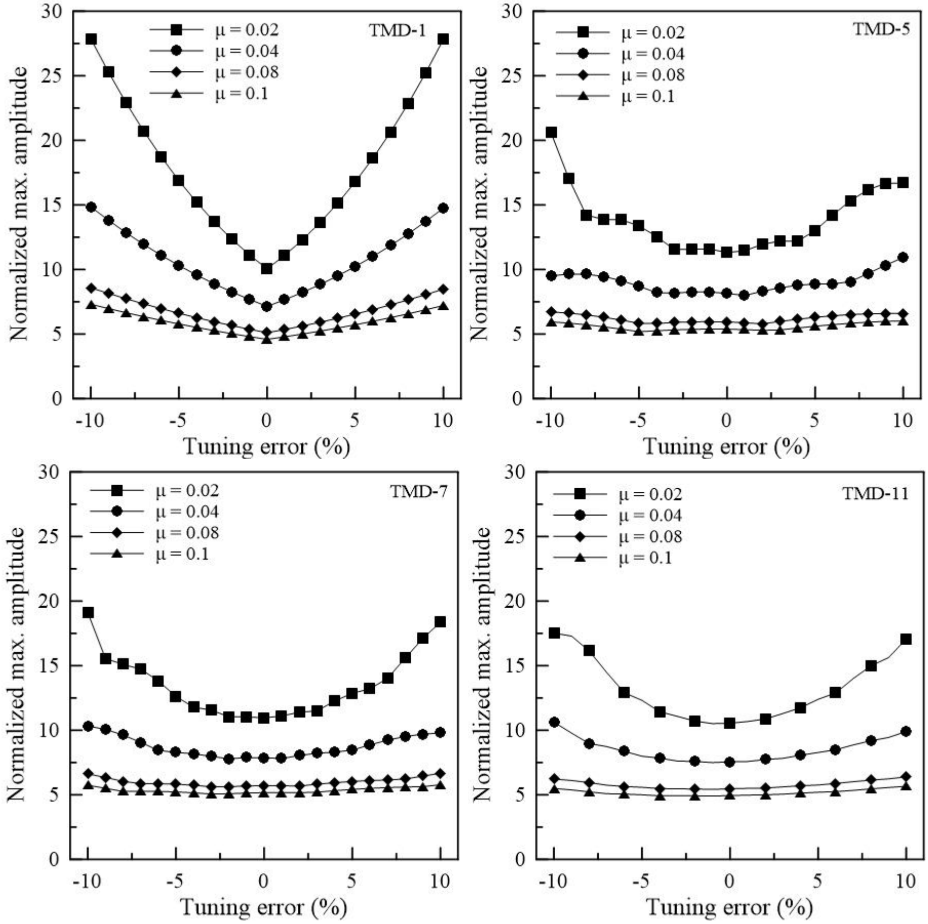

Figure 4. Normalized maximum displacement amplitude vs. tuning error for undamped primary system with different TMDs

In Fig. 5, variation of the amplitude of the displacement of the main system against harmonic excitation frequency is shown for MTMD systems for damped main system. MTMD systems are composed of two different TMD units (i.e., n = 5 and 11) with other parameters as $\mu = 0.01$ and $\xi_s = 0.02$. Optimal parameters for MTMD systems are given in Table 2. In this figure, TMD-5* and TMD-11* represent TMD devices designed by Bandivadekar and Jangid [24]. The reduction of the maximum amplitude with TMD-5, TMD-5*, TMD-11 and TMD-11* are 65.72, 60.95, 66.67 and 62.59 % for the displacement of the main system, respectively. The maximum difference between both design method is found to be less than 5%. This result demonstrates that the proposed MTMD system is also effective for reducing the maximum amplitude of damped main system.

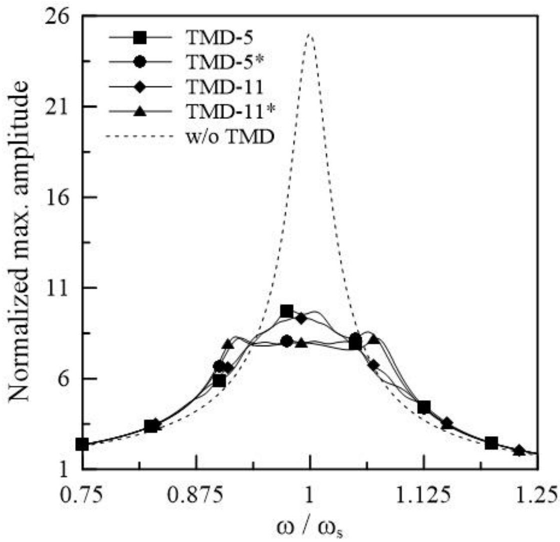

Figure 5. Variation of response amplitude against harmonic excitation frequency with and without MTMD system

Table 2. Optimal tuning parameters of TMD-5 and TMD-11 calculated by the both method for undamped and damped main system

MTMD	μ	ξ_T (%)		f_T		Δ_n	
		Present	[24]	Present	[24]	Present	[24]
TMD-5	0.01	2.74	2.39 (2.57)	0.9921	0.99571 (0.991)	0.1833	0.1113 (0.115)
	0.03	4.73	4.11 (4.35)	0.9765	0.9878 (0.9791)	0.3178	0.1893 (0.198)
TMD-11	0.01	1.85	1.70 (1.82)	0.9923	0.99686 (0.9946)	0.2334	0.1353 (0.1408)
	0.03	3.19	3.00 (3.08)	0.9773	0.9909 (0.9826)	0.406	0.2294 (0.2424)

Numbers in parenthesis are found for $\xi_s = 0.02$.

Optimal tuning parameters of TMD-5 and TMD-11 obtained by the proposed method are shown in Table 2 with comparison to those of Bandivadekar and Jangid [24]. As seen, the present method gives slightly greater damping ratios, and slightly smaller frequency ratios than those of the considered reference for MTMDs considered. The frequency bandwidth obtained from the present study significantly larger than that of the considered reference.

The sensitivity of a system to a certain parameter is determined by comparing the optimal case with those obtained using variations of the parameters of interest. In this study, the robustness of the MTMD system is examined for optimum tuning frequency, i.e. the rest of the parameters except the one examined are the optimum values for MTMD system [24, 25]. The formula of tuning error is expressed by Error (%) = ((f_d - f_{opt}) / f_{opt}). Where, f_{opt} is optimum tuning frequency ratio and f_d is detuned frequency ratio.

Figs. 6 and 7 show comparison of the maximum displacement amplitudes of the primary structure with optimal TMD-5 and TMD-11 devices in case of the frequency detuning. In these figures, TMD-5* and TMD-11* represent TMD devices given by Bandivadekar and Jangid [24]. As can be seen from the figures, the effectiveness of TMD-5* and TMD-11* are better than those designed by the present method when the frequency detuning is rather small. However, the effectiveness of TMD-5* and TMD-11* rapidly decrease as the tuning error increases. When the error greater than $\pm 5\%$, their performance in reducing vibration is worst compared to MTMDs proposed in this study. According to Figs. 6 and 7, the robustness of the proposed MTMDs are also better than that of the considered reference. This is not surprising, since the proposed MTMD covers a wider frequency bandwidth.

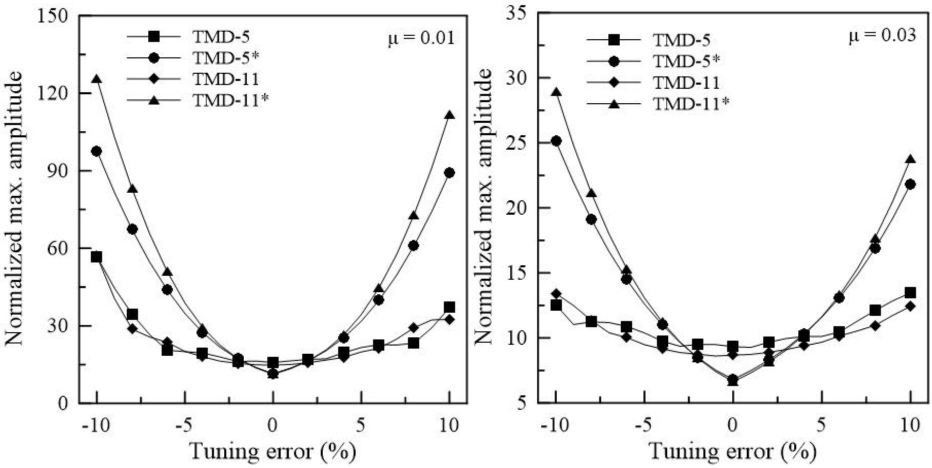

Figure 6. Normalized maximum displacement amplitude vs. tuning error for undamped primary system with different MTMDs

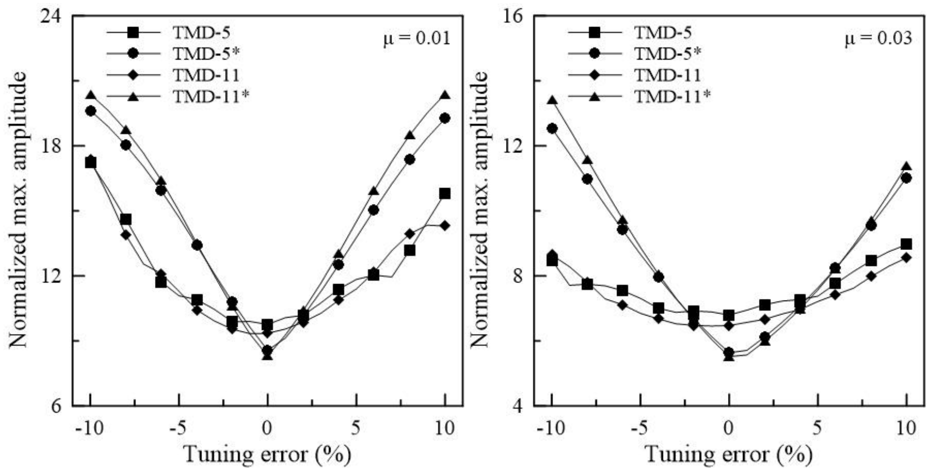

Figure 7. Normalized maximum displacement amplitude vs. tuning error for damped primary system with different MTMDs

In the next example, the robustness of MTMD systems designed by present method is compared with that of MTMD systems available in the literature considering different levels of tuning error in Figs. 8. Here, the damping ratio of the structural system is 0.01 and the total mass ratio of the MTMD system is 0.01 [25]. Optimal tuning parameters for various *n* values obtained by the proposed method are shown in Table 3 with comparison to those of Park and Reed [25]. As seen, the present method gives slightly smaller frequency ratios than those of the considered reference for MTMDs considered. The frequency bandwidth obtained from the present study significantly larger than that of the considered reference. Figs. 8 indicate that the effectiveness of MTMD systems designed by Park and Reed [25] is better than those designed by the present method when the frequency detuning is rather small. Furthermore, the maximum amplitude values of the MTMD systems designed by Park and Reed [25] become even higher than those designed by present method when the offset of the tuning ratio goes beyond certain ranges. As a result, the optimum MTMD system designed by Park and Reed [25] lose its robustness in the presence of detuning, because its bandwidth is not be wide enough to cover the deviated optimum frequencies.

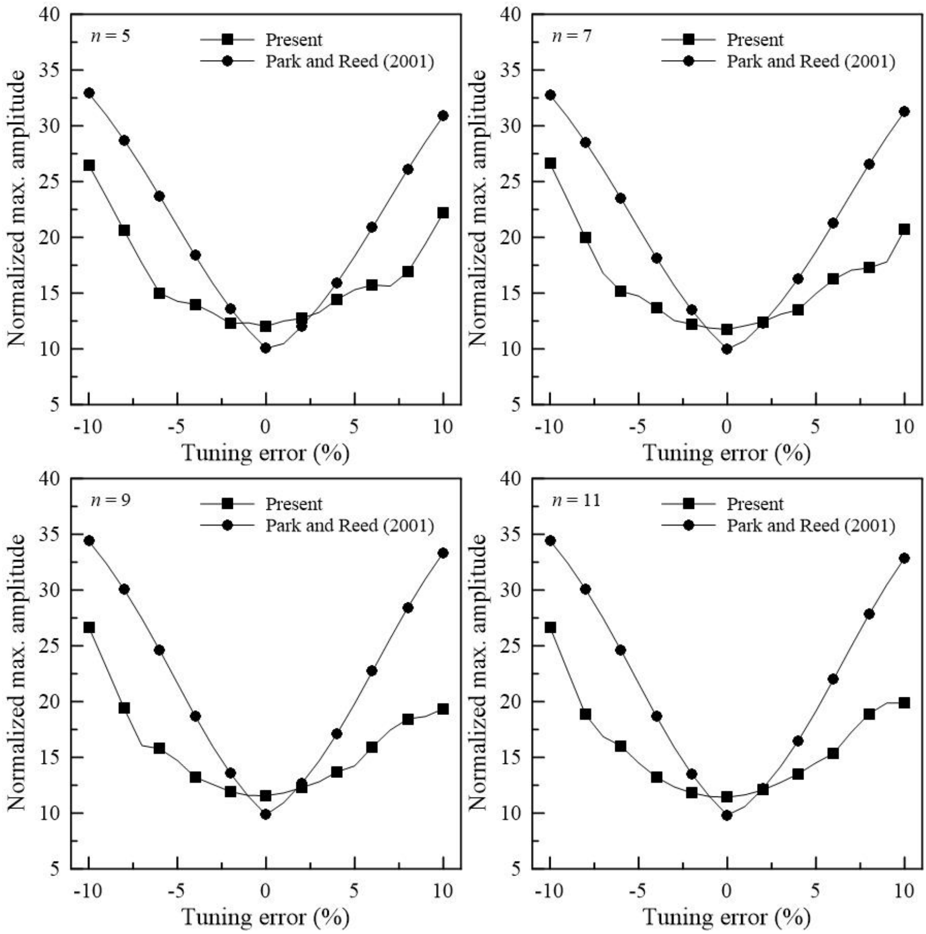

Figure 8. Normalized maximum displacement amplitude vs. tuning error for damped main system with different MTMDs

Table 3. Optimal tuning parameters for damped main system with various number of TMD units (μ = 0.01 and $\xi_s = 0.01$)

MTMD	ξ_T (%)		f_T		Δ_n	
	Present	[25]	Present	[25]	Present	[25]
TMD-5	2.74	2.50	0.9921	0.9912	0.1833	0.1150
TMD-7	2.31	2.50	0.9922	0.9924	0.2064	0.1200
TMD-9	2.04	2.00	0.9923	0.9941	0.2219	0.1300
TMD-11	1.85	2.00	0.9923	0.9932	0.2334	0.1350

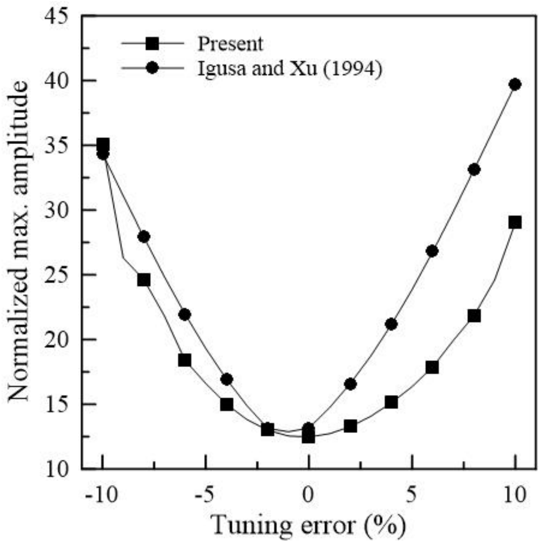

Figure 9. Normalized maximum displacement amplitude vs. tuning error for damped main system with TMD-20 system

As another example, a similar diagram is plotted in Fig. 9, where the robustness test is performed for $\xi_s = 0.5\%$, $\mu = 0.01$, $\mu_T = 0.0005$ and n = 20 in [26]. The average damping ratio $\xi_T = 5\%$ and frequency ratios spanning from 0.944 to 1.056 are chosen by Igusa and Xu [26]. These values are chosen 1.37% and 0.864 to 1.126 for present method, respectively. Compared with Igusa and Xu [26], MTMD system designed by present method are shown to be more effective in reducing the dynamic response of damped main system under harmonic excitation in the existence of detuning effects.

6. CONCLUSIONS

In this paper, a new approach to design MTMD devices with the basis of Den Hartog's optimal criteria is presented. We demonstrated that significantly increased robustness can be obtained when the proposed MTMD is attached to the primary SDOF system. According to the results, we have the following conclusions:

1. The numerical results proved that the proposed MTMD is quite effective for suppressing the maximum amplitudes of the primary system under harmonic excitation. For TMD-5 and

TMD-11 systems, the maximum normalized amplitude is reduced up to 60.95 and 62.59%, respectively.

2. Although the proposed MTMD has the same control effectiveness as the classical single TMD (TMD-1) tuned by Den Hartog's criteria without the frequency detuning, it is more robust to the natural frequency changes in TMD units than TMD-1.

3. Another advantage of the present method is that since design parameters are not obtained based on an optimization routine (i.e., objective function, initial values and boundaries for tuning parameters) for present method, the computational effort required for present method is less than the other optimization techniques.

4. It is interesting to note that the maximum displacement amplitudes of the proposed MTMD is almost insensitive to $\pm$10% changes in the tuning frequency of TMD units with higher mass ratio.

5. According to the results of this study, MTMD systems designed by present method provide higher robustness than all the reference systems considered in suppressing the vibrations of both damped and undamped systems under harmonic excitation.

6. This approach can also be extended to vibration control of short and medium span bridge structures by using MTMDs, because the first natural frequency of these structures is dominant in vibration.

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