Sigma J Eng & Nat Sci 9 (4), 2018, 395-403

Publications Prepared for Transist 2017,
10th International İstanbul Transport Congress

Research Article

MULTIPLE-SINK SHORTEST PATH NETWORK INTERDICTION PROBLEM

Ertuğrul AYYILDIZ*[1], Gökhan ÖZÇELİK[2], Emrullah DEMİRCİ[3]

[1]*Yildiz Technical University, Dept. of Industrial Eng., Esenler-İSTANBUL;* ORCID:0000-0002-6358-7860
[2]*Karadeniz Technical University, Dept. of Industrial Engineering, TRABZON;* ORCID:0000-0001-5314-8576
[3]*Karadeniz Technical University, Dept. of Industrial Engineering, TRABZON;* ORCID:0000-0002-3951-4493

Received: 04.10.2018 Revised: 20.11.2018 Accepted: 21.12.2018

ABSTRACT

This paper defines and studies the multiple-sink shortest path problem (*MS-NIP*). The MS-NIP corresponds to many real-life problems relating to especially terrorist actions. In the MS-NIP, a network user seeks the shortest path to meet the demands of sink nodes while an interdictor aims to maximize the shortest path of a network user by interdicting the arc(s). In this study, we formulate exact mathematical models of the network user and interdictor for the MS-NIP and apply them on an illustrative example. We test the model depending on the different interdiction budget levels and discuss the obtained results.

Keywords: OR in defense, shortest path problem, network interdiction problem, mixed-integer programming.

1. INTRODUCTION

Network structures are used in a significant part of daily life. It is possible to see different network structures used in all areas of life such as infrastructure networks such as electricity, water, natural gas, telecommunication networks used for communication between people, transportation and logistics networks used for transfer between two points, rapidly developing information networks. There is a physical flow in network structures. However, this flow sometimes may not be desirable. For example, the transported material must be prevented by the authorities in a smuggling network. On the other hand, terrorists may want to create unrest in society by damaging networks (*water, electricity, natural gas, etc.*) that provide critical infrastructure services to people. Such situations are defined as network interdiction problems (*NIP*) in the literature. NIPs involve two opposing forces, an interdictor (*leader*) and a network user (*follower*), who are engaged in a warlike conflict. The network user operates a network in order to optimize some objective functions such as moving a supply convoy through the network as quickly as possible or maximizing the amount of material transported through the network. The interdictor attempts to limit the network user's achievable objective values by interdicting arcs or nodes, for example, by attacking arcs or nodes to destroy them, to slow travel over arcs or to reduce arcs' or nodes' capacity [1]. NIPs are designed under the assumption that both sides have perfect information about the other side.

---

* Corresponding Author: e-mail: ertugrulayyildiz@ktu.edu.tr, tel: (554) 906 60 27

NIPs are usually modeled as the two-player game. One of the players, called as an interdictor, attacks to maximize the minimum flow within the network or to maximize the shortest path within the network with the limited resource. The other player, called as a network user, aims to strengthen network components to be less affected by attacks [2]. This leader-follower relationship is similar to the Stackelberg game in literature [3]. From the point of view of the network user, it is important to know that the sensitive components (*node or arcs*) for ensuring to continue network's operations against any attacks. He wants to know how the results will occur in attacks and what actions should be taken. From the interdictor's perspective, it is important to know that which components of the network should be interdicted to give the greatest damage to the network operated by the other side. In the literature, the first studies related to the network interdiction are usually carried out for determining the most important component(s) in a network. Many authors studied to identify the most important arc(s) on the network [4-11]. In fact, almost all studies before Wood's study [12] are specific to the application and are not extendible to more general contexts. Wood [12] developed a min–max formulation of maximum flow network interdiction problem and then converted it to an integer programming model. Wood's study [12] has become a key study for many authors. Since then, NIP is well studied in many different scopes such as minimizing the maximum flow on a network [13-15]; maximizing the shortest path on a network [16-18]; minimizing the likelihood of escape from the smuggler by placing sensors [19-21]; vulnerability analyses which are related to detection of grid elements that need to be strengthened in order to get the least damage. [22-24]; defending of critical infrastructure networks such as electricity, natural gas and water [25-27]; extending the project completion time by blocking critical activities [28-29]; preparing the operational plans of military units [30]; disruptions in the procurement process of two competing buyer firms [31-32].

Now, let's look at some remarkable studies about the shortest path NIP in the literature. Fulkerson and Harding [33] studied maximizing the shortest single source-sink path under budget constraint with a linear cost function. Corley and David [34] modeled traditional shortest path problem in which there are one interdictor and one system operator with contradictory objectives. They developed an algorithm for finding the most important arc and node in their defined network. Israeli and Wood [1] developed a bi-level mathematical model for the shortest path NIP. They formulated this model as a one-level mixed-integer programming model which can be readily solved. Cappanera and Scaparra [35]; Sefair and Smith [36]; Lozano and Smith [37]; Sadeghi et al. [38] developed tri-level mathematical models for the different shortest path NIPs.

In this study, unlike previous shortest path NIPs in the literature, the multiple-sink shortest path problem (*MS-NIP*) is presented. To give an example of this problem: let us assume that there are terrorist actions at the same time at different nodes. In this case, these nodes need support teams. The aim of the support teams is to reach to demand nodes as soon as possible by starting from a specific support point. However, terrorist wants to extend the shortest path of the support teams. In this case, the support teams want to minimize total distance traveled between source and demand nodes to meet requirements of the demand nodes while the terrorist attempts to maximize the total shortest path achieved by support teams.

In the MS-NIP, the sides engage in a two-step and sequential game process: the interdictor initially interdicts arc(s) to maximize the total shortest path (*achieved by the network user*) depending on his limited budget or resources, later; the network user tries to find the shortest path in order to meet the demands of the sink nodes using the uninterdicted arcs. Clearly, the interdictor's main goal is to try to explicitly maximize the total shortest path by interdicting the arc(s) (*deleting the link(s) between the nodes*). Moreover, it is assumed that the sides have sufficient information about each other. The aim of the study is to introduce the exact mathematical formulation of the MS-NIP that provides the information of the interdicted arc(s) on a network. Thereby, risky arcs (*the most vital arc(s)*) are identified on the network. Terrorists (*interdictor*) tend to interdict these arcs since interdicted arcs are on the shortest paths.

The rest of the paper is organized as follows: In the next section, exact mathematical models for the network user and interdictor are presented, separately. In Section 3, a numerical example is illustrated to show applicability of the model. Finally, the study is concluded in Section 4.

2. METHODOLOGY

In the MS-NIP, the interdictor aims to maximize the total traveled shortest path starting from the source node while the network user travels to satisfy all the demands of the sink nodes within the capacities of the source nodes.

MS-NIP is defined on an undirected network $G = (N, A)$ with node set N and arc set A consisting of pairs of different nodes. Arc (i, j) is used for traveling from node i to node j or from node j to node i. In the network consisting of n nodes, the lengths of the arcs $d_{ij}(i, j \epsilon N)$ are positive. There are two sides in the problem, one is interdictor (*leader*) and the other is the network user (*the one who runs the network /uses the shortest path*). In order to avoid using the arcs interdicted by the interdictor, the lengths of the interdicted arcs are extended by C units. Here, C is the length of the penalty added to the interdicted arc and greater than lengths of all the arcs in the network. There is also a set of sink nodes indicated by F and F is also subset of N. The number of elements of the F is p and $p < n$. Unlike the traditional shortest path NIP, the network user aims to go to all nodes (*p-nodes*) within the set of sink nodes defined by the F set from starting node after the interdictor make interdictions. Also, the other nodes that are not in the F set are defined as D set.

In the following sub-sections, four different mathematical models are presented. Two of these are network user's models: shortest path mathematical model (*NU-Model*) and the corresponding dual model (*NU-Model(D)*). The others relate to interdictor: bi-level mathematical model (*I-Model*) and exact mixed-integer mathematical model (*I-Model(F)*).

2.1. The formulation of the network user's problem and its dual form

The exact formulation of the network user's problem corresponds to the formulation multiple-sink shortest path. *NU-Model* consists of objective function (1) and constraints (2-5). The objective function (1) minimizes the sum of distance traveled. Constraints (2) and (3) relate to source node and sink nodes, respectively. Constraints (4) are balance constraints for other nodes. Constraints (5) are non-negativity constraints. The x_{ij} decision variable is the binary variable indicating whether or not the (i, j) arc is used. The network user's model (*NU-Model*) is presented as follows:

$$NU - Model : Z^* = min \sum_{i=1}^{n} \sum_{j=1}^{n} x_{ij} d_{ij} \tag{1}$$

s.t.

$$\sum_{i=1}^{n} x_{1i} - \sum_{i=1}^{n} x_{i1} = p \tag{2}$$

$$\sum_{i=1}^{n} x_{if} - \sum_{i=1}^{n} x_{fi} = 1; \quad (\forall f \in F) \tag{3}$$

$$\sum_{i=1}^{n} x_{ik} - \sum_{i=1}^{n} x_{ki} = 0; \quad (\forall k \in D) \tag{4}$$

$$x_{ij} \geq 0; \quad (\forall i, j \in N) \tag{5}$$

The dual form of the *NU-Model* will help to ensure reducing to single level of the interdictor's bi-level interdiction model. In dual model (*NU-Model(D)*), Y_is are dual variables related to the constraints (2), (3) and (4). There are n $(i = 1,2, \dots, n)$ dual variables defined on dual model since there are n constraints in the network user's model. The exact formulation of the dual of network user's problem consists of objective function (6) and dual constraints (7-10). The dual model (*NU-Model(D)*) is presented as follows:

$$NU - Model(D): Z^* = max \; pY_1 + \sum_{f=1}^{p} Y_f \tag{6}$$

s.t.

$$Y_1 + Y_j \leq d_{1j}; \quad (\forall j \in N - \{1\}) \tag{7}$$

$$-Y_i + Y_j \leq d_{ij}; \quad (\forall i \in N - \{1\}\ and\ j \in N - \{1\}) \tag{8}$$

$$-Y_i \leq d_{i1}; \quad (\forall i \in N - \{1\}) \tag{9}$$

$$Y_i : unresticted\ \ (\forall i \in N) \tag{10}$$

2.2. The formulation of the interdictor's problem

In this subsection, the final version of the *I-Model* is presented as an exact mixed-integer mathematical model after the interdictor's problem is modeled as a bi-level integer model (*I-Model*). In order to avoid using the arcs interdicted by the interdictor in the inner minimization problem in the objective function (11), the lengths of the interdicted arcs are extended by C units. The objective function (11) maximizes the sum of distance traveled with respect to interdicted arc's length. In the mathematical model, when the decision variable k_{ij} takes the value 1 (*which means arc d_{ij} is interdicted by interdictor*), the path is blocked and thus the new length is expressed as $d_{ij} + C$. In the model, a constraint (15) is added which adjust the total number of interdicts within interdicting budget (*capacity constraint*). T represents the interdiction budget. In addition to the constraints included in the network user model (12-14). Constraints (16) are non-negativity and (17) shows the binary decision variable of the interdictor. The bi-level interdictor' model (*I-Model*) is presented as follows:

$$I - Model: Z^* = maxmin\ \sum_{i=1}^{n}\sum_{j=1}^{n} x_{ij}(d_{ij} + Ck_{ij}) \tag{11}$$

s.t.

$$\sum_{i=1}^{n} x_{1i} - \sum_{i=1}^{n} x_{i1} = p \tag{12}$$

$$\sum_{i=1}^{n} x_{if} - \sum_{i=1}^{n} x_{fi} = 1; \quad (\forall f \in F) \tag{13}$$

$$\sum_{i=1}^{n} x_{ik} - \sum_{i=1}^{n} x_{ki} = 0; \quad (\forall k \in D) \tag{14}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij} = T \tag{15}$$

$$x_{ij} \geq 0; \quad (\forall i, j \in N) \tag{16}$$

$$k_{ij} \in \{0, 1\}; \quad (\forall i, j \in N) \tag{17}$$

The interdictor's bi-level model is reduced to a one-level mixed-integer mathematical model taking the dual problem of inner minimization problem. Accordingly, it is obtained the mixed-integer "*max–max*" model, which is simply a maximization model by fixing k temporarily and then releasing k. The final version of the interdictor' model (*I-Model(F)*) is presented as follows:

$$I - Model(F): Z^* = max\ pY_1 + \sum_{f=1}^{p} Y_f \tag{18}$$

s.t.

$$Y_1 + Y_j \leq d_{1j} + Ck_{1j}; \quad (\forall j \in N - \{1\}) \tag{19}$$

$$-Y_i + Y_j \leq d_{ij} + Ck_{ij}; \quad (\forall i \in N - \{1\}\ and\ j \in N - \{1\}) \tag{20}$$

$$-Y_i \leq d_{i1} + Ck_{i1}; \quad (\forall i \in N - \{1\}) \tag{21}$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n} k_{ij} = T \tag{22}$$

$$k_{ij} \in \{0, 1\}; \quad (\forall i, j \in N) \tag{23}$$

$$Y_i : unresticted\ \ (\forall i \in N) \tag{24}$$

The objective function (18) gives the total shortest path by network user after interdictions under the constraints created by the addition of penalties for interdicting dual constraints (19-21). A constraint (22) is added to ensure that the total number of interdicts is equal to the interdiction budget. (23) shows the binary decision variable of the interdictor and (24) relates to the dual variables. All decision variables except k_{ij} (interdiction decision) are defined as continuous variables because integer decision variables are harder to handle in the model.

3. NUMERICAL APPLICATION

In this section, *I-Model(F)* is tested to show its applicability using a specific network which consists of 11 nodes and 35 arcs. The arc lengths are shown on the network. (See: Figure 1). In the network, node 1 is source node while node 10 and 11 are sink nodes. The network user tries to reach the both node 10 and node 11 from the node 1 via shortest path. The interdictor aims to maximize the shortest path of the network user by making interdictions on 35 arcs depending on his budget.

These tests are performed on a work station with 2.5 GHz, i5 7200U processor and 4 GB of RAM by using the solver CPLEX 12.7. The interdiction budget is increased until the solution becomes infeasible. This means that there is no suitable arc(s) that the network user can use to reach sink nodes. In this case, the value of the objective function takes a rather large value depending on the penalties added to the arcs. If the optimal objective function value is extremely large, the network user cannot meet demands on a feasible basis since the interdictor interdicts all possible arcs which ensure the reaching the sink nodes. Transfers between the nodes without connecting arcs are impossible and all arcs are undirected. Moreover, it is assumed that the interdiction cost of each arc is 1 unit in the problem.

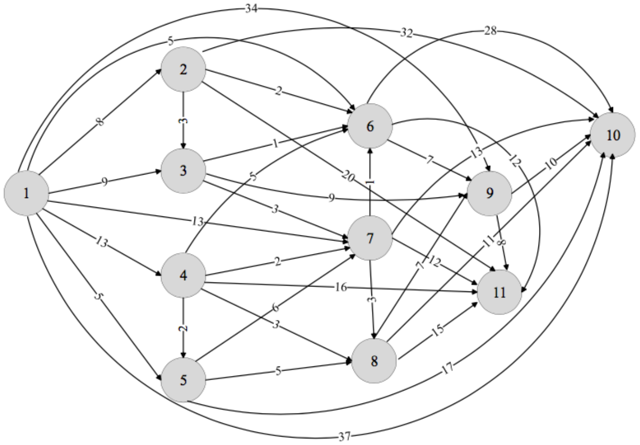

Figure 1. The network used in study $\boldsymbol{G(11,35)}$

If the network user's model (*NU-Model*) is solved with no interdiction, the network user follows the route of 1→5→8→10 to reach the node 10, and also follows the route of 1→6→11 to reach node 11. The network user, reaching a total of 38 units distance, has reached both sink nodes. In Figure 2, the route that the network user uses to reach the sink nodes is indicated by colored lines. Each travel is represented by different colors.

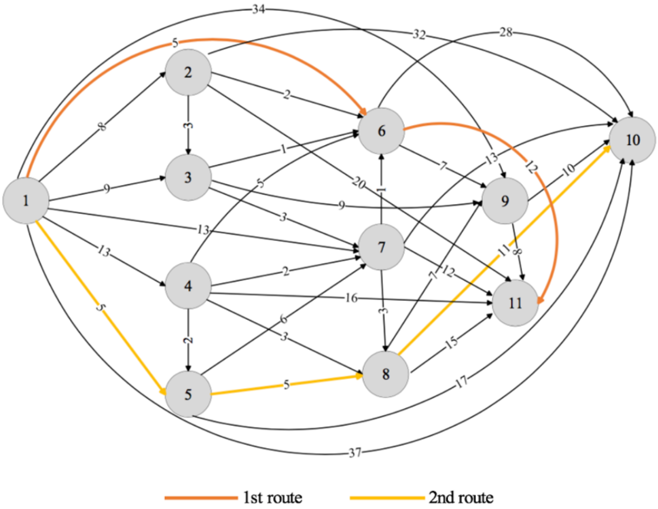

Figure 2. Network user's route with no interdiction

The interdictor makes interdiction in order to maximize shortest path using *I-Model(F)*. To illustrate, if the interdictor has 4 units interdiction budget, he increases the total distance by 13 units interdicting the arcs between node 1 and node 2; node 1 and node 3; node 1 and node 5; node 1 and node 6. The interdicted arcs and new shortest paths are illustrated in Figure 3. The routes that the network user prefers to reach each sink node are indicated by different colored lines while the interdicted arcs are indicated by dashed lines (See: Figure 3). It is seen that the network user goes 2 times from node 1 to node 7. This is made possible by defining the variables x_{ij} as a positive variable instead of a 0-1 binary variable, which is used in the mathematical model.

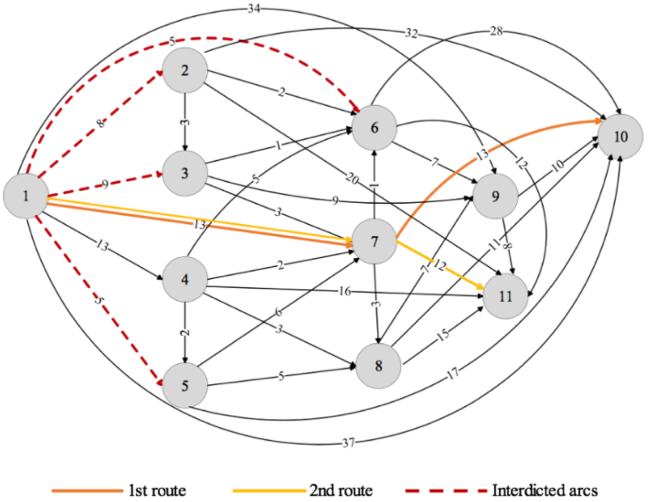

Figure 3. Network user's route for $\boldsymbol{T = 4}$

The analysis is performed for different scenarios (*interdiction budget levels* $(T > 0)$) for the network given in Figure 1. Table 1 summarizes the ways in which the interdictor interdicts and how the network user reaches the sink nodes without using interdicted arcs. When the Table 1 is examined, it can be seen that the shortest path length of the network user increases as the number of interdiction/budget level that the interdictor can adjust. While the budget of the interdictor can make increases between 0 and 5, it is seen that the network user can continuously increase the shortest path length. The interdictor can interdict all possible path(s) with related to interdiction budget (*6 and 8*). There is no optimal solution since the objective function value takes an abnormally value.

Table 1. Scenario Results

Budget (*T*)	Interdicted arc(s)	Routes	Shortest path length
0	-	1→5→8→10 1→6→11	**38**
1	(1-5)	1→6→9→10 1→6→11	**39**
2	(1-5), (1-6)	1→3→7→10 1→2→6→11	**47**
3	(1-5), (6-9), (6-11)	1→3→7→10 1→3→7→11	**49**
4	(1-2), (1-3), (1-5), (1-6)	1→7→10 1→7→11	**51**
5	(1-6), (5-10), (7-10), (8-10), (9-10)	1→10 1→2→6→11	**59**
6	(1-2), (1-4), (6-11), (7-11), (8-11), (9-11)	No optimal solution for node 11	***
8	(1-2), (1-3), (1-4), (1-5), (1-6), (1-7), (1-9), (1-11)	No optimal solution for node 10 and node 11	***

4. CONCLUSION

This paper presents the MS-NIP in which a network user attempts to minimize total distance traveled between source and multiple-sink nodes to meet demands while an interdictor maximizes the total shortest path achieved by network user. The interdictor uses limited interdiction resource or budget to interdict arcs. Accordingly, a computational analysis is done depending on different budget levels after presenting exact formulations for the problem. The results show that *I-Model(F)* optimally solve the generated instances for the handled network. The contributions of the paper to the literature and application can be specified as: (1) a novel shortest path problem which includes multiple sink nodes is introduced; (2) this problem is examined as network interdiction problem and modeled; (3) an application is presented to show the applicability and reliability of the methodology; (4) the proposed method provides opportunities to the governmental and non-governmental organizations to improve their strategies; (5) as far as we know, this is the first study related to the multiple-sink shortest path network interdiction problem. In further research, the demand quantities of the sink nodes and the capacity of the source node can be added to the model by expanding the model. Any meta-heuristic model can be studied in the event that optimal results cannot be obtained for larger sizes of the problem. Moreover, the MS-NIP may be studied by considering the arc lengths as interval-valued to cope with environmental uncertainty.

REFERENCES

[1] Israeli, E. & Wood, R.K., (2002) Shortest-path network interdiction. *Networks*, 40(2), pp.97–111.

[2] Yates, J. & Sanjeevi, S., (2013) "1A length-based, multiple-resource formulation for shortest path network interdiction problems in the transportation sector, *International Journal of Critical Infrastructure Protection*, 6(2), pp.107–119.

[3] Von Stackelberg, H., (1952) *The theory of the market economy*, Oxford.

[4] Wollmer, R.D., (1963) *Some methods for determining the most vital link in a railway network*, California.

[5] Lubore, S. H., Ratliff, H. D., & Sicilia, G. T., (1971) Determining the most vital link in a flow network. *Naval Research Logistics Quarterly*, 18(4), 497-502.

[6] Wollmer, R., (1964) Removing arcs from a network, *Operations Research*, 12(6), pp.934–940.

[7] Ratliff, H.D., Sicilia, G.T. & Lubore, S.H., (1975) Finding the n most vital links in flow networks, *Management Science*, 21(5), pp.531–539.

[8] Malik, K., Mittal, A.K. & Gupta, S.K., (1989) The k most vital arcs in the shortest path problem, *Operations Research Letters*, 8(4), pp.223–227.

[9] Ball, M.O., Golden, B.L. & Vohra, R. V, (1989) Finding the most vital arcs in a network, *Operations Research Letters*, 8(2), pp.73–76.

[10] Lin, K.-C. ve Chern, M.-S., (1993) The fuzzy shortest path problem and its cs, *Fuzzy Sets and Systems*, 58(3), pp. 343–35.

[11] Jiang, Y. ve Hu, A., (2011) Finding the most vital link with respect to the characteristic of network communication, *Journal of Networks*, 6(3), pp. 22-30.

[12] Wood, R.K., (1993), Deterministic network interdiction, *Mathematical and Computer Modelling*, 17(2), pp. 1–18.

[13] Smith, J.C. & Lim, C., (2008) Algorithms for network interdiction and fortification games. *Pareto optimality, game theory and equilibria*, New York, pp. 609–44.

[14] Rad, M.A. & Kakhki, H.T., (2013) Maximum dynamic network flow interdiction problem: new formulation and solution procedures, *Computers & Industrial Engineering*, 65(4), pp.531–36.

[15] Branch, K., (2015) Multi-commodity multi-source-sinks network flow, *Journal of Engineering and Applied Sciences*, Vol. 10, No. 6, pp.118–22.

[16] Israeli, E. & Wood, R.K., (2002) Shortest-path network interdiction, *Networks*, 40(2), pp.97–111.

[17] Bayrak, H. & Bailey, M.D., (2008) Shortest path network interdiction with asymmetric information, *Networks*, 52(3), pp.133–40.

[18] Ayyıldız, E., Özçelik, G., & Gencer, C. T. (2019) Determining the most vital arcs on the shortest path for fire trucks in terrorist actions that will cause fire. *Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics*, 68(1), pp. 441-450.

[19] Morton, D.P., Pan, F. & Saeger, K.J., (2007), Models for nuclear smuggling interdiction *IIE Transactions*, 39(1), pp.3–14.

[20] Dimitrov, N. B., Michalopoulos, D. P., Morton, D. P., Nehme, M. V., Pan, F., Popova, E. & Thoreson, G. G. (2011). Network deployment of radiation detectors with physics-based detection probability calculations. *Annals of Operations Research*, *187*(1), 207-228.

[21] Karabulut, E., Aras, N. & Altınel, İ.K., (2016) Optimal sensor deployment to increase the security of the maximal breach path in border surveillance, *European Journal of Operational Research*, 259(1), pp 19-36.

[22] Matisziw, T.C., Murray, A.T. & Grubesic, T.H., (2007) Bounding network interdiction vulnerability through cutset identification. In Critical Infrastructure, *Critical Infrastructure*, 1(1), pp. 243–56.

[23] Grubesic, T. H., Matisziw, T. C., Murray, A. T., & Snediker, D. (2008). Comparative approaches for assessing network vulnerability. *International Regional Science Review*, 31(1), 88-112.

[24] Murray, A. T. (2013). An overview of network vulnerability modeling approaches.

GeoJournal, 78(2), 209-221.

[25] Matisziw, T. C., & Murray, A. T. (2009). Modeling s–t path availability to support disaster vulnerability assessment of network infrastructure. *Computers & Operations Research*, 36(1), 16-26.

[26] Medal, H., Sharp, S. J., Pohl, E., Rainwater, C., & Mason, S. J. (2011). Models for reducing the risk of critical networked infrastructures. *International Journal of Risk Assessment and Management*, 15(2-3), 99-127.

[27] Yates, J., & Casas, I. (2012). Role of spatial data in the protection of critical infrastructure and homeland defense. *Applied Spatial Analysis and Policy*, 5(1), 1-23.

[28] Prince, M. (2013). *Algorithms and reformulation techniques for three-stage interdiction and fortification problems*. University of Florida.

[29] Gutin, E., Kuhn, D., & Wiesemann, W. (2014). Interdiction games on Markovian PERT networks. *Management Science*, 61(5), 999-1017.

[30] Whiteman, P. S. B. (1999). Improving single strike effectiveness for network interdiction. *Military Operations Research*, 15-30.

[31] Lodree Jr, E. J., & Taskin, S. (2008). An insurance risk management framework for disaster relief and supply chain disruption inventory planning. *Journal of the Operational Research Society*, 59(5), 674-684.

[32] Özçelik, G., & Gencer, C. (2018). A goal programming model that ensures efficient usage of limited interdiction budget in the procurement game. *Croatian Operational Research Review*, 9(1), 75-85.

[33] Fulkerson, D. R., & Harding, G. C. (1977). Maximizing the minimum source-sink path subject to a budget constraint. *Mathematical Programming*, 13(1), 116-118.

[34] Corley, H. W., & David, Y. S. (1982). Most vital links and nodes in weighted networks. *Operations Research Letters*, 1(4), 157-160.

[35] Cappanera, P., & Scaparra, M. P. (2011). Optimal allocation of protective resources in shortest-path networks. *Transportation Science*, 45(1), 64-80.

[36] Sefair, J. A., & Smith, J. C. (2016). Dynamic shortest-path interdiction. *Networks*, 68(4), 315-330.

[37] Lozano, L., & Smith, J. C. (2016). A backward sampling framework for interdiction problems with fortification. *INFORMS Journal on Computing*, 29(1), 123-139.

[38] Sadeghi, S., Seifi, A., & Azizi, E. (2017). Trilevel shortest path network interdiction with partial fortification. *Computers & Industrial Engineering*, 106, 400-411.