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Research Article

SOFT FIXED POINT THEOREMS IN TERMS OF SOFT ALTERING DISTANCE FUNCTION

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ABSTRACT

The target of this study is to introduce some fixed point theorems in soft metric spaces which are the generalizations of Banach fixed point theorem of soft mappings. For this reason, we define the notion of a soft altering distance function. Then we consider some fixed point theorems in soft metric spaces in terms of these functions by giving an illustrative example.
Keywords: Soft mapping, soft metric, completeness, fixed point.

1. INTRODUCTION

Banach contraction principle in metric spaces is one of the most important results in fixed point theory and nonlinear analysis in general. So far, according to its importance and simplicity, many authors ([7], [8]) have obtained interesting extensions and generalizations of the Banach contraction principle.

In 1999, the concept of soft set was introduced by Molodtsov [10], is a new mathematical tool for dealing with uncertainties. Soft set is a parameterized general mathematical tool which deal with a collection of approximate descriptions of objects. Works on soft set theory has been progressing rapidly since Maji et al. [9] introduced several operations of soft sets. Since then, Pei and Miao [11] and Ali et al. [3] introduced and studied several soft set operations as well. Soft set theory has also potential applications in many fields.

Das and Samanta [5,6] introduced the notions of soft element, soft real number and soft point, and discussed their properties. Based on these notions, they introduced the concept of a soft metric [6]. Wardowski [12] defined the concept of a soft mappings and obtained some fixed point results. After, Abbas [1] gave the similar definitions about soft cartesian products and soft mappings with Wardowski. Abbas et al. [1,2] introduced the notion of soft contraction mapping based on the theory of soft elements of soft metric spaces. They studied fixed points of soft contraction mappings and obtained among others results.

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2. PRELIMINARIES

Throughout this paper, X refers to an initial universe, and E the set of parameters for X. Let E_1 and E_2 be the non-empty parameter subsets of E. We denote by $\mathcal{P}(X)$ the family of all subsets of X.

Definition 2.1. [10] A pair (F, E) is called a soft set over X if F is a mapping given by $F: E \to \mathcal{P}(X)$. In other words, the soft set is a parametrized family of subsets of the set X. Every set $F(e), e \in E$, from this family may be considered as the set of $e - elements$ of the soft set (F, E), or as the set of $e - approximate$ elements of the soft set.

For any soft set (F, E_1), we can extend the soft set (F, E_1) to the soft set $(\bar{F}, E)$ where

$$\bar{F} : E \to \mathcal{P}(X), \bar{F}(e) = \begin{cases} F(e) , & if\ e \in E_1 \\ \emptyset , & if\ e \notin E_1 \end{cases}$$

We denote the collection of soft sets over a common universe X by $S(\tilde{X})$.

Definition 2.2. [9] Let $(F, E_1), (G, E_2) \in S(\tilde{X})$. We say that (F, E_1) is a soft subset of (G, E_2) if $F(e) \subseteq G(e)$ for all $e \in E_1$. We write $(F, E_1) \tilde{\subseteq} (G, E_2)$. Two soft sets (F, E_1) and (G, E_2) over a common universe X are said to be soft equal if (F, E_1) is a soft subset of (G, E_2) and (G, E_2) is a soft subset of (F, E_1).

Definition 2.3. [4] The complement of a soft set $(F, E_1) \in S(\tilde{X})$ is denoted by $(F, E_1)^c = (F^c, E_1)$, where $F^c : E_1 \to \mathcal{P}(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$, for all $e \in E_1$.

Definition 2.4. [9] (1) (Null Soft Set) A soft set $(F, E_1) \in S(\tilde{X})$ is said to be a null soft set denoted by Φ, if $F(e) = \Phi$ for all $e \in E_1$.

(2) (Absolute Soft Set) A soft set (F, E_1) over X is said to be an absolute soft set denoted by $\tilde{X}$, if $F(e) = X$ for all $e \in E_1$.

Definition 2.5. [4] (1) The union of two soft sets $(F, E_1), (G, E_2) \in S(\tilde{X})$ is the soft set (H, E_3), where $E_3 = E_1 \cup E_2$ and $H(e) = F(e) \cup G(e)$ for all $e \in E_3$. We express it as $(F, E_1) \tilde{\cup} (G, E_2) = (H, E_3)$.

(2) The intersection of two soft sets $(F, E_1), (G, E_2) \in S(\tilde{X})$ is the soft set (H, E_3), where $E_3 = E_1 \cap E_2$ and $H(e) = F(e) \cap G(e)$ for all $e \in E_3$. We express it as $(F, E_1) \tilde{\cap} (G, E_2) = (H, E_3)$.

(3) The difference (H, E) of two soft sets $(F, E), (G, E) \in S(\tilde{X})$, denoted by $(F, E) \setminus (G, E)$, is defined by $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 2.6. [5] Let $\mathbb{R}$ be the set of real numbers and $\mathcal{B}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E taken as a set of parameters. Then a mapping $F : E \to \mathcal{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F, E), or simply by F. If F is a single valued mapping on E taking values in $\mathbb{R}$ then the pair (F, E) or simply F, is called a soft element of $\mathbb{R}$ or a soft real number. If F is a single valued mapping on E taking values in $\mathbb{R}^+$ then F is called a nonnegative soft real number. We shall denote the set of all nonnegative soft real numbers by $\mathbb{R}(E)^*$.

We use notations $\tilde{r}, \tilde{s}, \tilde{t}$ to denote soft real numbers whereas $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r$ for all $\lambda \in E$, which is called a constant soft real number. For example $\bar{0}$ is the soft real number where $\bar{0}(\lambda) = 0$ for all $\lambda \in E$.

Definition 2.7. [6] The orderings between soft real numbers $\tilde{r}, \tilde{s}$ are defined as follows :

i. $\tilde{r} \tilde{\leqq} \tilde{s}$ if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$ for all $\lambda \in E$.
ii. $\tilde{r} \tilde{\geqq} \tilde{s}$ if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$ for all $\lambda \in E$.
iii. $\tilde{r} \tilde{<} \tilde{s}$ if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$ for all $\lambda \in E$.
iv. $\tilde{r} \tilde{>} \tilde{s}$ if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$ for all $\lambda \in E$.

Definition 2.8. [6] (1) A soft set $(P,E) \in S(\tilde{X})$ is said to be a soft point if there is exactly one $\lambda \in A$ such that $P(\lambda) = \{x\}$ for some $x \in X$ and $P(\mu) = \emptyset$, for all $\mu \in E \setminus \{\lambda\}$. It will be denoted by P_λ^x. The collection of all soft points of $\tilde{X}$ is denoted by $SP(\tilde{X})$.

(2) A soft point P_λ^x is said to belongs to a soft set (F,E) if $\lambda \in E$ and $P(\lambda) = \{x\} \subset F(\lambda)$. We write it by $P_\lambda^x \tilde{\in} (F,E)$.

(3) Two soft points P_λ^x, P_μ^y are said to be equal if $\lambda = \mu$ *and* $P(\lambda) = P(\mu)$. i.e., $x = y$. Thus $P_\lambda^x \neq P_\mu^y$ if $fx \neq y$ *or* $\lambda \neq \mu$.

Proposition 2.1. [6] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it. i.e., $(F,E) = \bigcup_{P_\lambda^x \tilde{\in} (F,E)} P_\lambda^x$.

Proposition 2.2. [6] For two soft sets $(F,E),(G,E) \in S(\tilde{X})$, $(F,E) \tilde{\subset} (G,E) \Leftrightarrow P_\lambda^x \tilde{\in} (F,E) \Rightarrow P_\lambda^x \tilde{\in} (G,E)$ and hence $(F,E) = (G,E)$ if and only if $P_\lambda^x \tilde{\in} (F,E) \Leftrightarrow P_\lambda^x \tilde{\in} (G,E)$.

Proposition 2.3. [6] For two soft sets $(F,E),(G,E) \in S(\tilde{X})$ and a soft point $P_\lambda^x \in SP(\tilde{X})$,

(i) $P_\lambda^x \tilde{\in} (F,E) \Leftrightarrow P_\lambda^x \tilde{\notin} (F,E)^c$.
(ii) $P_\lambda^x \tilde{\in} (F,E) \tilde{\cup} (G,E) \Leftrightarrow P_\lambda^x \tilde{\in} (F,E)$ *or* $P_\lambda^x \tilde{\in} (G,E)$.
(iii) $P_\lambda^x \tilde{\in} (F,E) \tilde{\cap} (G,E) \Leftrightarrow P_\lambda^x \tilde{\in} (F,E)$ *and* $P_\lambda^x \tilde{\in} (G,E)$.

Definition 2.9. [1] Let $(F,E),(G,E) \in S(\tilde{X})$. A soft cartesian product of (F,E) and (G,E), is denoted by $(F,E) \tilde{\times} (G,E)$, is defined as

$$(F,E) \tilde{\times} (G,E) = \{\left((e_1,e_2), F(e_1) \times G(e_2)\right) : e_1, e_2 \in E\}.$$

Example 2.10. [1] Suppose that $X = \{h_1,h_2,h_3\}$ and $E = \{e_1,e_2,e_3\}$. Define soft sets (F,E) and (G,E) as follows; $(F,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_2,h_3\}),(e_3,\{h_1\})\}$, $(G,E) = \{(e_1,\{h_1\}),(e_2,\{h_1,h_3\}),(e_3,\{h_1,h_2\})\}$

Then

$$(F,E) \tilde{\times} (G,E) = \left\{ \begin{array}{c} \left((e_1,e_1),\{h_1,h_2\} \times \{h_1\}\right), \left((e_1,e_2),\{h_1,h_2\} \times \{h_1,h_3\}\right), \\ \left((e_1,e_3),\{h_1,h_2\} \times \{h_1,h_2\}\right), \left((e_2,e_1),\{h_2,h_3\} \times \{h_1\}\right), \\ \left((e_2,e_2),\{h_2,h_3\} \times \{h_1,h_3\}\right), \left((e_2,e_3),\{h_2,h_3\} \times \{h_1,h_2\}\right), \\ \left((e_3,e_1),\{h_1\} \times \{h_1\}\right), \left((e_3,e_2),\{h_1\} \times \{h_1,h_3\}\right), \\ \left((e_3,e_3),\{h_1\} \times \{h_1,h_2\}\right) \end{array} \right\}$$

Definition 2.11. [1] Let $(F,E),(G,E) \in S(\tilde{X})$. A soft relation R is a soft set such that $(R, E \times E) \tilde{\subseteq} (F,E) \tilde{\times} (G,E)$. i.e.,

$$(R, E \times E) = \left\{\left((e_1,e_2), U_{e_1} \times U_{e_2}\right) : e_1, e_2 \in E, U_{e_1} \subseteq F(e_1), U_{e_2} \subseteq G(e_2)\right\}$$

We will denote $\left((e_1,e_2), U_{e_1} \times U_{e_2}\right) \in (R, E \times E)$ as $(e_1, U_{e_1}) R (e_2, U_{e_2})$.

Definition 2.12. [1] Let $(F,E),(G,E) \in S(\tilde{X})$. A soft relation $(T, E \times E) \tilde{\subseteq} (F,E) \tilde{\times} (G,E)$ is called a soft mapping from (F,E) to (G,E) if for each soft point $P_\lambda^x \tilde{\in} (F,E)$ there exists only one soft point $P_\mu^y \tilde{\in} (G,E)$ such that $P_\lambda^x T P_\mu^y$. We will denote $P_\lambda^x T P_\mu^y$ by $T(P_\lambda^x) = P_\mu^y$. If $(T, E \times E) \tilde{\subseteq} (F,E) \tilde{\times} (G,E)$ is soft mapping from (F,E) to (G,E), then we write it as $T: (F,E) \tilde{\rightarrow} (G,E)$.

Definition 2.13. [1] Let $(F,E),(G,E) \in S(\tilde{X})$ and $T: (F,E) \tilde{\rightarrow} (G,E)$ be a soft mapping.

a) The image of $(H,E) \tilde{\subseteq} (F,E)$ under the soft mapping T is a soft set, denoted by $T((H,E))$, defined as follows :

$$T\left((H,E)\right) = \tilde{\cup} \{T(P_\lambda^x) : P_\lambda^x \tilde{\in} (H,E)\}$$

b) The inverse of $(K,E) \tilde{\subseteq} (G,E)$ under the soft mapping T is a soft set, denoted by $T^{-1}((K,E))$, defined as follows :

$$T^{-1}\big((K,E)\big) = \tilde{\cup}\,\{P_\lambda^x : P_\lambda^x \tilde{\in} (F,E), T(P_\lambda^x) \tilde{\in} (K,E)\}$$

Definition 2.14. [6] A mapping $d : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ is said to be a $soft\ metric$ on the soft set $\tilde{X}$ if d satisfies the following conditions :

(M1) $d(P_\lambda^x, P_\mu^y) \tilde{\geq} \bar{0}$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$.
(M2) $d(P_\lambda^x, P_\mu^y) = \bar{0}$ if and only if $P_\lambda^x = P_\mu^y$.
(M3) $d(P_\lambda^x, P_\mu^y) = d(P_\mu^y, P_\lambda^x)$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$.
(M4) $d(P_\lambda^x, P_\mu^y) \tilde{\leq} d(P_\lambda^x, P_\gamma^z) + d(P_\gamma^z, P_\mu^y)$ for all $P_\lambda^x, P_\mu^y, P_\gamma^z \in SP(\tilde{X})$.

The soft set $\tilde{X}$ with a soft metric d on $\tilde{X}$ is called a $soft\ metric\ space$ and denoted by the triplet $(\tilde{X}, d, E)$ or $(\tilde{X}, d)$, for short.

Example 2.15. [6] Let $X \subset \mathbb{R}$ be a non-empty set and $E \subset \mathbb{R}$ be the non-empty set of parameters. Let $\tilde{X}$ be the absolute soft set and $\bar{x}$ be denotes the soft real number such that $\bar{x}(\lambda) = x$ for all $\lambda \in E$. The mapping $d : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ defined by $d(P_\lambda^x, P_\mu^y) = |\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$, is a soft metric on $\tilde{X}$.

Definition 2.16. [6] Let $\{P_{\lambda_n}^{x_n}\}$ be a sequence of soft points in a soft metric $(\tilde{X}, d)$.

(a) The sequence $\{P_{\lambda_n}^{x_n}\}$ is said to be convergent in $(\tilde{X}, d)$ if there exists a soft point $P_\mu^y \in SP(\tilde{X})$ such that $d\left(P_{\lambda_n}^{x_n}, P_\mu^y\right) \to \bar{0}$ as $n \to \infty$.

(b) The sequence $\{P_{\lambda_n}^{x_n}\}$ is said to be a Cauchy sequence in $(\tilde{X}, d)$ if for each $\tilde{\varepsilon} \tilde{>} \bar{0}$ there exists $n_0 \in \mathbb{N}$ such that $d\left(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m}\right) \tilde{<} \tilde{\varepsilon}$ for all $m, n \geq n_0$.

Definition 2.17. [6] (Complete Soft Metric Space) A soft metric space $(\tilde{X}, d)$ is called complete if every Cauchy sequence in $\tilde{X}$ converges to some point of $\tilde{X}$.

Definition 2.18. [1] Let $(\tilde{X}, d, E)$ and $(\tilde{Y}, \rho, E^*)$ be two soft metric spaces. A soft mapping $T : (\tilde{X}, d, E) \tilde{\to} (\tilde{Y}, \rho, E^*)$ is said to be soft continuous at a soft point $P_\lambda^x \in SP(\tilde{X})$, if for every $\tilde{\varepsilon} \tilde{>} \bar{0}$, there exists a $\tilde{\delta} \tilde{>} \bar{0}$ such that $\rho\left(T(P_\lambda^x), T(P_\mu^y)\right) \tilde{<} \tilde{\varepsilon}$ whenever $d(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{\delta}$ for all $P_\mu^y \in SP(\tilde{X})$. If T is soft continuous at every soft point of $\tilde{X}$, we say that T is soft continuous on $\tilde{X}$.

Proposition 2.4. Let $(\tilde{X}, d, E)$ and $(\tilde{Y}, \rho, E^*)$ be two soft metric spaces and $T : (\tilde{X}, d, E) \tilde{\to} (\tilde{Y}, \rho, E^*)$ be a soft mapping. For each soft point $P_\lambda^x \in SP(\tilde{X}), T(P_\lambda^x)$ is a soft point of $SP(\tilde{Y})$.

3. SOFT FIXED POINT THEOREMS IN TERMS OF SOFT ALTERING DISTANCE FUNCTION

Definition 3.1. A soft function $\psi : \mathbb{R}(E)^* \tilde{\to} \mathbb{R}(E)^*$ is called a soft altering distance function if ψ satisfies the following conditions:

a) $\psi(\bar{0}) = \bar{0}$.

b) ψ is sequentially continuous. i.e., $if\ P_{\lambda_n}^{x_n} \to P_\lambda^x,\ then\ \psi\left(P_{\lambda_n}^{x_n}\right) \to \psi(P_\lambda^x)$.

c) ψ is monotone non-decreasing.

Theorem 3.1. Let $(\tilde{X}, d)$ be a complete soft metric space, $\psi : \mathbb{R}(E)^* \tilde{\to} \mathbb{R}(E)^*$ be a soft altering distance function and $T : \tilde{X} \tilde{\to} \tilde{X}$ be a soft self mapping satisfying the following inequality

$$\psi\left(d\left(T(P_\lambda^x),T(P_\mu^y)\right)\right) \tilde{\leqq} \bar{c}.\psi\left(d(P_\lambda^x,P_\mu^y)\right) \quad (1)$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ and for some $\bar{0} \tilde{<} \bar{c} \tilde{<} \bar{1}$. Then T has a unique fixed point in $\tilde{X}$.

Proof. Let $P_{\lambda_0}^{x_0} \tilde{\in} \tilde{X}$ and define $P_{\lambda_{n+1}}^{x_{n+1}} = T(P_{\lambda_n}^{x_n})$, $\widetilde{a_n} = d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right)$ for all $n \in \mathbb{N} \cup \{0\}$.

We first prove that T has a fixed point in $\tilde{X}$. We may assume that $\widetilde{a_n} \tilde{>} \bar{0}$ for each $n \in \mathbb{N} \cup \{0\}$. From the contraction condition (1), we obtain

$$\psi\left(d\left(T\left(P_{\lambda_n}^{x_n}\right),T\left(P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)\right) \tilde{\leqq} \bar{c}.\psi\left(d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$
$$\Rightarrow \psi\left(d\left(P_{\lambda_{n+1}}^{x_{n+1}},P_{\lambda_{n+2}}^{x_{n+2}}\right)\right) \tilde{\leqq} \bar{c}.\psi\left(d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$
$$\Rightarrow \psi(\widetilde{a_{n+1}}) \tilde{\leqq} \bar{c}\,\psi(\widetilde{a_n}) \tilde{<} \psi(\widetilde{a_n}).$$

Since ψ is non-decreasing, $\{\widetilde{a_n}\}$ is a decreasing sequence of soft real numbers. Hence $\{\widetilde{a_n}\}$ has a limit point. We put $\lim_{n\to\infty}\widetilde{a_n} = \tilde{a}$ and suppose that $\tilde{a} \tilde{>} \bar{0}$. Hence $\widetilde{a_n} \tilde{\geqq} \tilde{a}$ implies that $\psi(\tilde{a}) \tilde{\leqq} \bar{c}\,\psi(\tilde{a}) \tilde{<} \psi(\tilde{a})$ which is a contradiction. So $\tilde{a} = \bar{0}$. Therefore $\{\widetilde{a_n}\}$ converges to $\bar{0}$.

Now, we prove that $\left\{P_{\lambda_n}^{x_n}\right\}$ is a Cauchy sequence in $(\tilde{X}, d)$. Suppose it is not a Cauchy sequence. Then, there exists $\tilde{\varepsilon} \tilde{>} \bar{0}$ and two subsequence $\left\{P_{\lambda_{n_k}}^{x_{n_k}}\right\}, \left\{P_{\lambda_{m_k}}^{x_{m_k}}\right\}$ of $\left\{P_{\lambda_n}^{x_n}\right\}$ such that for every $n \in \mathbb{N} \cup \{0\}$, we find that $n_k > m_k \geq n$, $d\left(P_{\lambda_{n_k}}^{x_{n_k}}, P_{\lambda_{m_k}}^{x_{m_k}}\right) \tilde{\geqq} \tilde{\varepsilon}$ and $d\left(P_{\lambda_{n_{k-1}}}^{x_{n_{k-1}}}, P_{\lambda_{m_k}}^{x_{m_k}}\right) \tilde{<} \tilde{\varepsilon}$. For each $n \geq 0$, we put $\widetilde{s_n} = d\left(P_{\lambda_{n_k}}^{x_{n_k}}, P_{\lambda_{m_k}}^{x_{m_k}}\right)$. Then, we have

$$\tilde{\varepsilon} \tilde{\leqq} d\left(P_{\lambda_{n_k}}^{x_{n_k}}, P_{\lambda_{m_k}}^{x_{m_k}}\right) \tilde{\leqq} d\left(P_{\lambda_{n_k}}^{x_{n_k}}, P_{\lambda_{n_{k-1}}}^{x_{n_{k-1}}}\right) + d\left(P_{\lambda_{n_{k-1}}}^{x_{n_{k-1}}}, P_{\lambda_{m_k}}^{x_{m_k}}\right) \tilde{<} \widetilde{a_{n_k-1}} + \tilde{\varepsilon}$$

Since $\{\widetilde{a_n}\}$ converges to $\bar{0}$, $\{\widetilde{s_n}\}$ converges to $\tilde{\varepsilon}$. Also $\left\{d\left(P_{\lambda_{n_k+1}}^{x_{n_k+1}}, P_{\lambda_{m_k+1}}^{x_{m_k+1}}\right)\right\}$ converges to $\tilde{\varepsilon}$. From the hypothesis, we deduce

$$\psi\left(d\left(T\left(P_{\lambda_{n_k}}^{x_{n_k}}\right),T\left(P_{\lambda_{m_k}}^{x_{m_k}}\right)\right)\right) \tilde{\leqq} \bar{c}\,\psi\left(d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)\right)$$
$$\Rightarrow \psi\left(d\left(P_{\lambda_{n_k+1}}^{x_{n_k+1}},P_{\lambda_{m_k+1}}^{x_{m_k+1}}\right)\right) \tilde{\leqq} \bar{c}\,\psi\left(d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)\right)$$

Letting $k \to \infty$, we obtain that $\psi(\tilde{\varepsilon}) \tilde{\leqq} \bar{c}\,\psi(\tilde{\varepsilon}) \tilde{<} \psi(\tilde{\varepsilon})$ which is a contradiction. Hence, $\left\{P_{\lambda_n}^{x_n}\right\}$ is a Cauchy sequence. By completeness of $(\tilde{X}, d)$, $\left\{P_{\lambda_n}^{x_n}\right\}$ converges to some soft point P_γ^z.

Now, we show that P_γ^z is a fixed point of T. If we substitute $P_\lambda^x = P_{\lambda_{n-1}}^{x_{n-1}}$ and $P_\mu^y = P_\gamma^z$ in (1), we obtain

$$\psi\left(d\left(T\left(P_{\lambda_{n-1}}^{x_{n-1}}\right),T(P_\gamma^z)\right)\right) \tilde{\leqq} \bar{c}\,\psi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}},P_\gamma^z\right)\right)$$
$$\Rightarrow \psi\left(d\left(P_{\lambda_n}^{x_n},T(P_\gamma^z)\right)\right) \tilde{\leqq} \bar{c}\,\psi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}},P_\gamma^z\right)\right).$$

Letting $n \to \infty$ and using the contunity of ψ and contunity of d,

$$\psi\left(d\left(P_\gamma^z,T(P_\gamma^z)\right)\right) \tilde{\leqq} \bar{c}\,\psi\left(d(P_\gamma^z,P_\gamma^z)\right) = \bar{c}\,\psi(\bar{0}) = \bar{0}$$

which implies $\psi\left(d\left(P_\gamma^z,T(P_\gamma^z)\right)\right) = \bar{0}$ that is $T(P_\gamma^z) = P_\gamma^z$.

To prove the uniqueness, we assume that $P_{\gamma_1}^{z_1}, P_{\gamma_2}^{z_2}$ be two different fixed points of T. Then, from (1), we obtain that

$$\psi\left(d\left(T(P_{\gamma_1}^{z_1}), T(P_{\gamma_2}^{z_2})\right)\right) \tilde{\leq} \ \bar{c}\,\psi\left(d(P_{\gamma_1}^{z_1}, P_{\gamma_2}^{z_2})\right)$$
$$\Rightarrow \psi\left(d(P_{\gamma_1}^{z_1}, P_{\gamma_2}^{z_2})\right) \tilde{\leq} \ \bar{c}\,\psi\left(d(P_{\gamma_1}^{z_1}, P_{\gamma_2}^{z_2})\right) \tilde{<} \ \psi\left(d(P_{\gamma_1}^{z_1}, P_{\gamma_2}^{z_2})\right)$$

which is a contradiction. Hence T has a unique fixed point in $\tilde{X}$.

Note. If we consider $\psi(\tilde{t}) = \tilde{t}$, then the above theorems reduces the contraction condition

$$d\left(T(P_\lambda^x), T(P_\mu^y)\right) \tilde{\leq} \bar{c}.\, d(P_\lambda^x, P_\mu^y)$$

where $\bar{0} \ \tilde{\leq} \ \bar{c} \ \tilde{<} \ \bar{1}$, which is given by Abbas [1].

Theorem 3.2. Let $(\tilde{X}, d)$ be a complete soft metric space, $\varphi : \mathbb{R}(E)^* \tilde{\rightarrow} \ \mathbb{R}(E)^*$ be a soft altering distance function with $\varphi(\tilde{r}) \neq \bar{0}$ for all $\tilde{r} \neq \bar{0}$ and $T : \tilde{X} \ \tilde{\rightarrow} \tilde{X}$ be a soft self mapping satisfying the following inequality

$$d\left(T(P_\lambda^x), T(P_\mu^y)\right) \tilde{\leq} d(P_\lambda^x, P_\mu^y) - \varphi\left(d(P_\lambda^x, P_\mu^y)\right) \tag{2}$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$. Then T has a unique fixed point in $\tilde{X}$.

Proof. Let $P_{\lambda_0}^{x_0} \ \tilde{\in} \ \tilde{X}$ and define $P_{\lambda_{n+1}}^{x_{n+1}} = T(P_{\lambda_n}^{x_n})$, $\widetilde{a_n} = d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right)$ for all $n \in \mathbb{N} \cup \{0\}$.

We first prove that T has a fixed point. We may assume that $\widetilde{a_n} \ \tilde{>} \ \bar{0}$ for each n. From the contraction condition (2), we obtain

$$d\left(T\left(P_{\lambda_n}^{x_n}\right), T\left(P_{\lambda_{n+1}}^{x_{n+1}}\right)\right) \tilde{\leq} d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) - \varphi\left(d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$
$$\Rightarrow d\left(P_{\lambda_{n+1}}^{x_{n+1}}, P_{\lambda_{n+2}}^{x_{n+2}}\right) \tilde{\leq} d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) - \ \varphi\left(d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$
$$\Rightarrow \widetilde{a_{n+1}} \ \tilde{\leq} \ \widetilde{a_n} - \varphi(\widetilde{a_n}) \ \tilde{<} \ \widetilde{a_n} \tag{2.1}$$

Therefore, $\{\widetilde{a_n}\}$ is a decreasing sequence of soft real numbers. Hence $\{\widetilde{a_n}\}$ has a limit point. We put $\lim_{n\to\infty} \widetilde{a_n} = \tilde{a}$ and suppose that $\tilde{a} \ \tilde{>} \ \bar{0}$. Since φ is non-decreasing, $\widetilde{a_n} \ \tilde{\geq} \ \tilde{a}$ implies that $\varphi(\widetilde{a_n}) \ \tilde{\geq} \ \varphi(\tilde{a}) \ \tilde{>} \ \bar{0}$. By (2.1), we have $\widetilde{a_{n+1}} \ \tilde{\leq} \ \widetilde{a_n} - \varphi(\tilde{a})$.

Thus $\widetilde{a_{n+M}} \ \tilde{\leq} \ \widetilde{a_n} - \bar{M}\varphi(\tilde{a})$ is a contradiction for M large enough. So $\tilde{a} = \bar{0}$. Therefore $\{\widetilde{a_n}\}$ converges to $\bar{0}$. As in the above theoremi it is easy to show that $\left\{P_{\lambda_n}^{x_n}\right\}$ is a Cauchy sequence in $(\tilde{X}, d)$. By completeness of $(\tilde{X}, d)$, $\left\{P_{\lambda_n}^{x_n}\right\}$ converges to some soft point P_γ^z.

Now, we show that P_γ^z is a fixed point of T. If we substitute $P_\lambda^x = P_{\lambda_{n-1}}^{x_{n-1}}$ and $P_\mu^y = P_\gamma^z$ in (2), we obtain

$$d\left(T\left(P_{\lambda_{n-1}}^{x_{n-1}}\right), T(P_\gamma^z)\right) \tilde{\leq} d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right) - \varphi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right)\right)$$
$$\Rightarrow d\left(P_{\lambda_n}^{x_n}, T(P_\gamma^z)\right) \tilde{\leq} d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right) - \ \varphi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right)\right)$$

Letting $n \to \infty$ and using contunity of ψ and contunity of d,

$$d\left(P_\gamma^z, T(P_\gamma^z)\right) \ \tilde{\leq} \ d(P_\gamma^z, P_\gamma^z) - \ \varphi\left(d(P_\gamma^z, P_\gamma^z)\right) = \bar{0}$$

which implies $d\left(P_\gamma^z, T(P_\gamma^z)\right) = \bar{0}$ that is $T(P_\gamma^z) = \ P_\gamma^z$.

To prove the uniqueness, we assume that $P_{\gamma_1}^{z_1}, \ P_{\gamma_2}^{z_2}$ be two different fixed points of T. Then, from (2) we obtain that

$$d\left(T(P_{\gamma_1}^{z_1}),T(P_{\gamma_2}^{z_2})\right) \lesseqqgtr d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2}) - \varphi\left(d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2})\right)$$
$$\Rightarrow d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2}) \lesseqqgtr d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2}) - \varphi\left(d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2})\right)$$
$$\Rightarrow \varphi\left(d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2})\right) \lesseqqgtr \bar{0}$$

which implies $\varphi\left(d(P_{\gamma_1}^{z_1},P_{\gamma_2}^{z_2})\right) = \bar{0}$, that is $P_{\gamma_1}^{z_1} = P_{\gamma_2}^{z_2}$. Hence T has a unique fixed point $\tilde{X}$.

Note. If we consider $\varphi(\tilde{t}) = \bar{k}.\tilde{t},\ \bar{0} \lesssim \bar{k} \lesseqqgtr \bar{1}$, then the above theorem reduces the contraction condition

$$d\left(T(P_\lambda^x),T(P_\mu^y)\right) \lesseqqgtr \bar{c}.d(P_\lambda^x,P_\mu^y)$$

where $\bar{0} \lesseqqgtr \bar{c} \lesssim \bar{1}$, which is given by Abbas [1].

Theorem 3.3. Let $(\tilde{X},d)$ be a complete soft metric space, $\psi,\varphi : \mathbb{R}(E)^* \rightrightarrows \mathbb{R}(E)^*$ be two soft altering distance functions with $\psi(\tilde{r}) \neq \bar{0}$ and $\varphi(\tilde{r}) \neq \bar{0}$ for all $\tilde{r} \neq \bar{0}$ and $T : \tilde{X} \rightrightarrows \tilde{X}$ be a soft self mapping satisfying the following inequality

$$\psi\left(d\left(T(P_\lambda^x),T(P_\mu^y)\right)\right) \lesseqqgtr \psi(d(P_\lambda^x,P_\mu^y)) - \varphi\left(d(P_\lambda^x,P_\mu^y)\right) \quad (3)$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$. Then T has a unique fixed point in $\tilde{X}$.

Proof. Let $P_{\lambda_0}^{x_0} \tilde{\in} \tilde{X}$ and define $P_{\lambda_{n+1}}^{x_{n+1}} = T(P_{\lambda_n}^{x_n})$, $\widetilde{a_n} = d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)$ for all $n \in \mathbb{N} \cup \{0\}$.

We first prove that T has a fixed point in $\tilde{X}$. We may assume that $\widetilde{a_n} \gtrsim \bar{0}$ for each $n \in \mathbb{N} \cup \{0\}$.. From the contraction condition (3), we obtain

$$\psi\left(d\left(T\left(P_{\lambda_n}^{x_n}\right),T\left(P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)\right) \lesseqqgtr \psi\left(d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)\right) - \varphi\left(d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$
$$\Rightarrow \psi\left(d\left(P_{\lambda_{n+1}}^{x_{n+1}},P_{\lambda_{n+2}}^{x_{n+2}}\right)\right) \lesseqqgtr \psi\left(d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)\right) - \varphi\left(d\left(P_{\lambda_n}^{x_n},P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$
$$\Rightarrow \psi(\widetilde{a_{n+1}}) \lesseqqgtr \psi(\widetilde{a_n}) - \varphi(\widetilde{a_n}) \lesssim \psi(\widetilde{a_n}) \quad (3.1)$$

Since ψ is non-decreasing, $\{\widetilde{a_n}\}$ is a decreasing sequence of soft real numbers. Hence $\{\widetilde{a_n}\}$ has a limit point. We put $\lim_{n\to\infty} \widetilde{a_n} = \tilde{a}$ and suppose that $\tilde{a} \gtrsim \bar{0}$. Letting $n \to \infty$ in (3.1) and using continuity of ψ, we obtain $\psi(\tilde{a}) \lesseqqgtr \psi(\tilde{a}) - \varphi(a) \lesssim \psi(\tilde{a})$ which is a contradiction. So $\tilde{a} = \bar{0}$. Therefore $\{\widetilde{a_n}\}$ converges to $\bar{0}$.

Now, we prove that $\left\{P_{\lambda_n}^{x_n}\right\}$ is a Cauchy sequence in $(\tilde{X},d)$. Suppose it is not a Cauchy sequence. Then there exists $\tilde{\varepsilon} \gtrsim \bar{0}$ and two subsequence $\left\{P_{\lambda_{n_k}}^{x_{n_k}}\right\},\left\{P_{\lambda_{m_k}}^{x_{m_k}}\right\}$ of $\left\{P_{\lambda_n}^{x_n}\right\}$ such that for every $n \in \mathbb{N} \cup \{0\}$, we find that $n_k > m_k \geq n$, $d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right) \gtreqqless \tilde{\varepsilon}$ and $d\left(P_{\lambda_{n_k-1}}^{x_{n_k-1}},P_{\lambda_{m_k}}^{x_{m_k}}\right) \lesssim \tilde{\varepsilon}$. For each $n \geq 0$, we put $\widetilde{s_n} = d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)$. Then, we have

$$\tilde{\varepsilon} \lesseqqgtr d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right) \lesseqqgtr d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{n_k-1}}^{x_{n_k-1}}\right) + d\left(P_{\lambda_{n_k-1}}^{x_{n_k-1}},P_{\lambda_{m_k}}^{x_{m_k}}\right) \lesssim \widetilde{a_{n_k-1}} + \tilde{\varepsilon}.$$

Since $\{\widetilde{a_n}\}$ converges to $\bar{0}$, $\{\widetilde{s_n}\}$ converges to $\tilde{\varepsilon}$. Also $\left\{d\left(P_{\lambda_{n_k+1}}^{x_{n_k+1}},P_{\lambda_{m_k+1}}^{x_{m_k+1}}\right)\right\}$ converges to $\tilde{\varepsilon}$. From the hypothesis, we deduce

$$\psi\left(d\left(T\left(P_{\lambda_{n_k}}^{x_{n_k}}\right),T\left(P_{\lambda_{m_k}}^{x_{m_k}}\right)\right)\right) \lesseqqgtr \psi\left(d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)\right) - \varphi\left(d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)\right)$$
$$\Rightarrow \psi\left(d\left(P_{\lambda_{n_k+1}}^{x_{n_k+1}},P_{\lambda_{m_k+1}}^{x_{m_k+1}}\right)\right) \lesseqqgtr \psi\left(d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)\right) - \varphi\left(d\left(P_{\lambda_{n_k}}^{x_{n_k}},P_{\lambda_{m_k}}^{x_{m_k}}\right)\right)$$

Letting $k \to \infty$, we obtain that $\psi(\tilde{\varepsilon}) \cong\!\!\!\leq \psi(\tilde{\varepsilon}) - \varphi(\tilde{\varepsilon}) \tilde{<} \psi(\tilde{\varepsilon})$ which is a contradiction. Therefore $\left\{P_{\lambda_n}^{x_n}\right\}$ is a Cauchy sequence. By completeness of $(\tilde{X}, d)$, $\left\{P_{\lambda_n}^{x_n}\right\}$ converges to some soft point P_γ^z.

Now, we show that P_γ^z is a fixed point of T. If we substitute $P_\lambda^x = P_{\lambda_{n-1}}^{x_{n-1}}$ and $P_\mu^y = P_\gamma^z$ in (3), we obtain

$$\psi\left(d\left(T\left(P_{\lambda_{n-1}}^{x_{n-1}}\right), T\left(P_\gamma^z\right)\right)\right) \tilde{\leq} \psi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right)\right) - \varphi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right)\right)$$

$$\Rightarrow \psi\left(d\left(P_{\lambda_n}^{x_n}, T\left(P_\gamma^z\right)\right)\right) \tilde{\leq} \psi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right)\right) - \varphi\left(d\left(P_{\lambda_{n-1}}^{x_{n-1}}, P_\gamma^z\right)\right).$$

Letting $n \to \infty$ and using the contunity of ψ and contunity of d,

$$\psi\left(d\left(P_\gamma^z, T\left(P_\gamma^z\right)\right)\right) \tilde{\leq} \psi\left(d\left(P_\gamma^z, P_\gamma^z\right)\right) - \varphi\left(d\left(P_\gamma^z, P_\gamma^z\right)\right)$$

$$\Rightarrow \psi\left(d\left(P_\gamma^z, T\left(P_\gamma^z\right)\right)\right) \tilde{\leq} \psi(\bar{0}) - \varphi(\bar{0}) = \bar{0}$$

which implies $\psi\left(d\left(P_\gamma^z, T\left(P_\gamma^z\right)\right)\right) = \bar{0}$, that is $T\left(P_\gamma^z\right) = P_\gamma^z$.

Note. In Theorem 3.3, if we particularly take $\varphi(\tilde{t}) = \left(\bar{1} - \bar{k}\right).\psi(\tilde{t})$ for all $\tilde{t} \tilde{>} \bar{0}$ where $\bar{0} \tilde{<} \bar{k} \tilde{<} \bar{1}$, then we obtain the result noted in Theorem 3.1. Again, in Theorem 3.3, if we particularly take $\psi(\tilde{t}) = \tilde{t}$ for all $\tilde{t} \tilde{>} \bar{0}$ where $\bar{0} \tilde{<} \bar{k} \tilde{<} \bar{1}$, then we obtain the result noted in Theorem 3.2.

Example 3.2. Let $X = [0,1]$ and $E = \{0,1\}$. According to Example 2.15, the mapping $d: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ given by $d\left(P_\lambda^x, P_\mu^y\right) = |\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ is a soft metric on $\tilde{X}$. Furthermore, the soft metric space $(\tilde{X}, d)$ is complete [1].

Let $\psi : \mathbb{R}(E)^* \tilde{\to} \mathbb{R}(E)^*$ be defined as $\psi(\tilde{t}) = \begin{cases} \tilde{t} \ , \ if\ \bar{0} \tilde{\leq} \tilde{t} \tilde{\leq} \bar{1} \\ \tilde{t}^2 \ , \ if\ \tilde{t} \tilde{>} \bar{1} \end{cases}$, where $\tilde{t}^2(e) = \left(\tilde{t}(e)\right)^2$ for all $e \in E$ and $\varphi : \mathbb{R}(E)^* \tilde{\to} \mathbb{R}(E)^*$ be defined as $\varphi(\tilde{t}) = \begin{cases} \frac{\bar{1}}{2}\tilde{t}^2 \ , \ if\ \bar{0} \tilde{\leq} \tilde{t} \tilde{\leq} \bar{1} \\ \frac{\bar{1}}{2} \ , \ if\ \tilde{t} \tilde{>} \bar{1} \end{cases}$.

Let $T : \tilde{X} \tilde{\to} \tilde{X}$ be defined as $T(P_0^x) = P_0^0$ and $T(P_1^x) = P_0^{x/2}$ for all $x \in X$. We show that T satisfies all conditions of Theorem 3.3. If $P_\lambda^x = P_\mu^y$, then the contraction condition is satisfied. We assume that $P_\lambda^x \neq P_\mu^y$.

$$\psi\left(d\left(T(P_1^x), T\left(P_1^y\right)\right)\right) = \psi\left(d\left(P_0^{x/2}, P_0^{y/2}\right)\right) = \psi\left(\left|\frac{\bar{x}}{2} - \frac{\bar{y}}{2}\right|\right) = \psi\left(\frac{\bar{1}}{2}|\bar{x} - \bar{y}|\right) = \frac{\bar{1}}{2}|\bar{x} - \bar{y}|$$

$$\tilde{\leq} \left(\bar{1} - \frac{\bar{1}}{2}|\bar{x} - \bar{y}|\right).|\bar{x} - \bar{y}| = |\bar{x} - \bar{y}| - \frac{\bar{1}}{2}|\bar{x} - \bar{y}|^2 = \psi\left(d\left(P_1^x, P_1^y\right)\right) - \varphi\left(d\left(P_1^x, P_1^y\right)\right)$$

$$\psi\left(d\left(T(P_0^x), T\left(P_1^y\right)\right)\right) = \psi\left(d\left(P_0^0, P_0^{y/2}\right)\right) = \psi\left(\left|\frac{\bar{y}}{2}\right|\right) = \frac{\bar{y}}{2} \tilde{\leq} \frac{\bar{1}}{2} \tilde{\leq} \frac{\bar{1}}{2} + |\bar{x} - \bar{y}|^2 + \bar{2}|\bar{x} - \bar{y}|$$

$$= (|\bar{x} - \bar{y}| + \bar{1})^2 - \frac{\bar{1}}{2} = \psi\left(d\left(P_0^x, P_1^y\right)\right) - \varphi\left(d\left(P_0^x, P_1^y\right)\right)$$

$$\psi\left(d\left(T(P_0^x), T(P_1^x)\right)\right) = \psi\left(d\left(P_0^0, P_0^{x/2}\right)\right) = \psi\left(\left|\frac{\bar{x}}{2}\right|\right) = \frac{\bar{x}}{2} \tilde{\leq} \frac{\bar{1}}{2} = \bar{1} - \frac{\bar{1}}{2}$$

$$= \psi(\bar{1}) - \varphi(\bar{1}) = \psi\left(d(P_0^x, P_1^x)\right) - \varphi\left(d(P_0^x, P_1^x)\right)$$

Hence, all conditions of Theorem 3.3 are satisfied. In fact P_0^0 is the unique fixed point of T.

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4. CONCLUSION

In this paper, we have studied some fixed point results which are the generalizations of Banach fixed point theorem in soft metric spaces given by Abbas. With another different contraction conditions, soft metric fixed point theory can be developed further.

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