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Research Article

ANALYSIS OF AXIALLY SYMMETRIC SHELL STRUCTURES

Ezgi ÖZTORUN*[1], Erdem DAMCI[2], Namık Kemal ÖZTORUN[3]

[1]*Istanbul University - Cerrahpasa, Department of Civil Eng., ISTANBUL;* ORCID:0000-0002-9882-0933
[2]*Istanbul University - Cerrahpasa, Department of Civil Eng., ISTANBUL;* ORCID:0000-0003-2295-1686
[3]*Istanbul University - Cerrahpasa, Department of Civil Eng., ISTANBUL;* ORCID:0000-0003-1393-733X

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ABSTRACT

In the present study; by using symmetry, anti-symmetry, axial symmetry properties and maintaining the feature of being the exact solution of the problem; model minimization and more accurate and detailed information acquisition techniques in the minimized model are discussed with examples on the analysis by the Finite Element method. On the other hand, the computer programs "ESKA-2" and "ESKA-4" (Axisymmetric Shell Analysis) which are formulated on the classical shell structure theory and developed within the scope of the study are introduced. By means of these programs, model modifications can be made in an extremely practical way compared to the Finite Element Method in axially symmetric shell structures; load and load combinations, boundary conditions and so on. In addition to all these conveniences, more detailed and more accurate analysis results can be obtained than the Finite Element Method, which can only analyse with a limited number of unknowns with the capacity of computers. ESKA-2 has been developed for the analysis of systems (high walled) where the analysis of axial symmetric wall with two integral constants is suitable. ESKA-4 is formulated with four integral constants and gives accurate results regardless of the wall height.
Keywords: Axially symmetric shell structure, classic shell structure theory, Finite Element method, post tensioning, structural optimization.

1. INTRODUCTION

Shell structures have geometries that are smaller in thickness than other sizes and principal radius of curvature. There are many degrees of freedom for such structures in analysis with prevalent Finite Element Method. For this reason, about the solution of the unknowns, the number of equations, that push the limits of computers and software, becomes the subject. In this case, technical problems are experienced frequently. Some of these problems are confronted by the limitations of the computer program, or the operating system used, or the inadequacies of the computer hardware. These limitations often require a large number of dynamic storage areas, (RAM and similar units) which may be insufficient during operations of these variables, for dimensioned variables such as vectors and matrices during the solution of the problem. In addition to the development of computer technologies and software; progression in analysis methods, developed in matrix operations, (for example all hard disks and similar magnetic or digital storage areas may be defined to take on a task as RAM, or only the values of the

* Corresponding Author: e-mail: ezgi.oztorun@istanbul.edu.tr, tel: (212) 473 70 70 / 17920

symmetric half terms and diagonal terms of the square system stiffness matrix, which are different from zero, may be stored in a vector; thus inverse and other matrix operations can be made by means of this vector) are inadequate because of engineering problems that are increasing and pushing the limits of the same subject. On the other hand, with the technological advancement, expectatio ns about the analysis of more complex systems also augment. In this case, analysis with the models, that does not exceed the limits, is made do.

One of the system analysis methods used prevalently in the field of civil engineering and has a high transaction volume is Finite Element Method. Although some assumptions are concerned, the general-purpose Finite Element Method is used as the exact solution. The method provides very successful results in solving problems related to structural systems in building types as well as other structural systems such as dams, suspension bridges, closed conduits, tunnels, shell structures and even parts and / or bodies of aircraft, submarines, space ships and similar mechanical systems. However, in the analysis of the shell structure and similar problems where the cross-sectional effects may show non-local alterations, the Finite Element Method has problems with the above-mentioned limitations, the inevitable changes in the geometry due to the complexity of the problem, a large number of alternative model analysis is required; therefore, the method is not practical. According to the Finite Element formulation; the criteria that must be satisfied in terms of the aspect ratio corner angles, the member connectivity, in relation to the element geometry in the finite element types often result in an increase in the number of elements and nodal point and consistently the degree of freedom. Since these conditions are not satisfied time to time; it is possible to cause some errors such as run-time, syntax and round off. If not checked, the system stiffness matrix and the errors that occur during the solution of the equations result in distortion of the matrices or equations and very incorrect analysis results may be obtained. In civil engineering problems, especially for building type structures, analysis is carried out for different loading and load combinations by keeping data such as system geometry, cross-section and material properties, boundary conditions and external loads constant. Even only for the optimization of the cross-section geometry, a large number of model changes are involved. On the other hand, in systems where precise solution methods are inevitable, analyses of different boundary conditions and different geometric details are often important. Additionally, even modification of the points and/or areas of load application, requires the preparation of a large number of different computer models. One of the engineering fields of study, in which aforementioned problems are frequently encountered, is the shell structures.

Axially symmetric shell structures consist of combinations of structural components such as spherical parabolic or partial spherical dome, axially symmetrical wall, circular beam at the top and/or bottom of the wall, or circular plate. In particular, reinforced concrete shell structures, which can be exposed to loads such as water pressure inside, are preferred not to be exposed to tensile stresses in order to ensure their structural safety and prevent water leakage. The system should be under compression for all types of load and load combinations. It is considered appropriate to apply post tensioning to keep the system under constant compression, to reduce the possible tensile stresses in the system, or even to convert them to reasonable compression stresses if necessary. It is a preferred method to apply points of load application, load sizes, generally staggered load application system, gradual load applications according to friction losses, and application of horizontal and vertical tension loads determined by detailed analyses. However, even optimization of the locations of horizontal post tensioning cables requires the preparation of multiple (sometimes hundreds) different mathematical models. Cross-sectional distributions may show non-local changes [1]. On the other hand, the principal cross-sectional forces, that constitute the basis for calculations, can be reduced by utilizing the eccentric settlements at the connections of the structural components or the stiffness distribution at the boundary conditions for the optimum design. This necessity also requires the preparation of additional mathematical models. All these studies cannot be done practically with the Finite Element Method. Moreover, often the problem is not analysed in the desired detail due to the limitations mentioned before. Minimizing the number of unknowns of the problem by reducing, increases the risk of sudden and non-local stress distributions. For this and similar reasons, analysis methods formulated with the classical shell theory, are suggested as very practica l and more accurate alternative methods for the analysis of shell structures. Practical methods are also reminded and presented for the dynamic interaction with the structure and the fluid in it.

Öztorun and Utku [2], suggested an analytical solution that uses the superposition method in the analysis of axially symmetric cylindrical walls. In their studies, water tanks consisting of structural members as dome, circumferential beam and cylindrical wall, were analysed using flexibility method. Analysed water reservoirs were built in Saudi Arabia [1, 3-5]. Öztorun et al. in another study [6], they performed cylindrical wall analyses using five moment equations.

In this study, computer programs "ESKA-2" and "ESKA-4" (Axisymmetric Shell Analysis), formulated on the classical shell structure theory and developed within the scope of the study, are introduced. An example of axially symmetric shell structure is analysed by means of these programs. Later, the results obtained from analyses on "ESKA-2", "ESKA-4" programs for this axially symmetric shell structure are compared with the results of same axially symmetric shell structure analysed by using Finite Element Method. Subsequently, the advantages of classical shell structure theory over Finite Element Method are discussed.

2. MATERIALS AND METHODS

2.1. Field Equations and Formulation

For the Shell that provides long wall criteria, in the analysis carried out by the Billington method [7], the general displacement expression (2.1) of a cylindrical wall is defined by equation [8].

$$D_w \frac{d^4 w}{dy^4} + \frac{E_w \cdot t_w}{r_w^2} w = p \tag{2.1}$$

In equation (2.1) terms are given as D_w, bending stiffness of the wall, t_w, wall thickness, r_w, average radius of the wall, E_w, elasticity of the wall, w linear displacement and p radial compression.

By using $\beta^4 = \frac{E_w . t_w}{4 . r_w^2 . D_w}$ notation in the equation (2.1), it becomes as eq. (2.2)

$$\frac{d^4 w}{dy^4} + 4\beta^4 = \frac{p_z}{D_w} \tag{2.2}$$

This obtained expression is the same as the equation (2.3) obtained for a prismatic bar with a bending stiffness D_w, which is subjected to a load p_z and placed on continuous elastic foundation.

$$EI \frac{d^4 w}{dy^4} + kw = p_z \tag{2.3}$$

In this equation, $EI \frac{d^4 w}{dy^4}$ is the bending rigidity of the beam, k_w is the reaction is the foundation stiffness. When both equations are examined together k is equal to $E_w t_w / r_w^2$. This term can be named as equivalent foundation modulus of the cylindrical wall.

$$D_w = \frac{E_w \cdot t_w^3}{12(1-\nu_w^2)} \tag{2.4}$$

$$\beta^4 = \frac{E_w \cdot t_w}{4 \cdot r_w^2 \cdot D_w} = \frac{3(1-\nu_w^2)}{r_w^2 \cdot t_w^2} \tag{2.5}$$

The general solution of this equation;

$$w_y = e^{\beta y}(C_1.\cos\beta y + C_2.\sin\beta y) + e^{-\beta y}(C_3.\cos\beta y + C_4.\sin\beta y) + f(y)$$

Particular solution *f(y)* based on the load in the equation above, is composed of membrane solution $w = \frac{p_z \cdot r_w^2}{E_w \cdot t_w}$.

In the formula above; C_1, C_2, C_3, C_4 are constants of integration depending on the boundary conditions of the cylinder wall. They correspond to linear and angular displacements because of shear force at the top and bottom of the wall Q_0 and bending moment M_0. Since each of these axially symmetrical loads is self-balancing, according to the Saint-Venant principle, it is possible to draw conclusion that effects of these forces will remain in areas close to the sides. If the cylinder wall is long (high) enough so that the effect of the forces Q_0 and M_0 can be neglected at the other end, both ends of the wall can be analysed independently of each other. On the other hand, it can be clearly seen that $e^{\beta y}$ increases with *y*. Because of this reason, even if C_1 and C_2 are not zero, the effect will be dampened and will approach zero. Under the circumstances, the constants can be taken as zero and the function can be solved within practical limits. In this case, the formula becomes,

$$w_y = e^{-\beta y}(C_3.\cos\beta y + C_4.\sin\beta y) + f(y) \tag{2.6}$$

In the equation (2.6), since *f(y)* is not taken into account, only the effects of the end forces can be calculated.

$$M_0 = X_2 = \left[-D_w \cdot \left(\frac{d^2 w}{dy^2}\right)\right]_{y=0} = M_y \tag{2.7}$$

$$Q_0 = X_1 = \left[-D_w \cdot \left(\frac{d^3 w}{dy^3}\right)\right]_{y=0} = Q_y \tag{2.8}$$

$$\frac{dw}{dy} = -\beta \cdot e^{-\beta y}\left[C_3 \cdot (\cos\beta y + \sin\beta y) - C_4 \cdot (\cos\beta y - \sin\beta y)\right] \tag{2.9}$$

$$\frac{dw^2}{dy^2} = 2\beta^2 \cdot e^{-\beta y} \cdot \left[C_3 \cdot \sin\beta y - C_4 \cdot \cos\beta y\right] \tag{2.10}$$

$$\frac{d^3 w}{dy^3} = 2\beta^3.e^{-\beta y}\left[C_3 \cdot (\cos\beta y - \sin\beta y) + C_4 \cdot (\cos\beta y + \sin\beta y)\right] \tag{2.11}$$

can be obtained. In the general method developed by Öztorun and Utku for long and short wall analyses [5], *H* is on the point of being the height of the wall,

$$X_{1w} = \left[-D_w \cdot \left(\frac{d^3 w}{dy^3} \right) \right]_{y=0}$$

$$X_{2w} = \left[-D_w \cdot \left(\frac{d^2 w}{dy^2} \right) \right]_{y=0}$$

$$X_{3w} = \left[-D_w \cdot \left(\frac{d^3 w}{dy^3} \right) \right]_{y=H}$$

$$X_{4w} = \left[-D_w \cdot \left(\frac{d^2 w}{dy^2} \right) \right]_{y=H}$$

By this formula for unknown forces at the points where *y= 0* and *y= H*, the first coefficient matrix [*KM*1] with dimensions 4x4 which includes solutions of the unknowns X_1, X_2, X_3, X_4 can be obtained. {*C*} is the vector that includes the integral constants related to the boundary conditions of the cylinder wall.

$$\{X_w\} = [KM1]\,\{C\} \tag{2.12}$$

Following the same path, the cylinder displacements (angular and linear displacements) can be written as,

$$(w)_{y=0} = C_1 + C_3$$

$$\left(\frac{dw}{dy} \right)_{y=0} = C_1 \cdot \beta + C_2 \cdot \beta - C_3 \cdot \beta + C_4 \cdot \beta$$

$$(w)_{y=H} = C_1 \cdot e^{\beta H} \cos\beta H + C_2 \cdot e^{\beta H} \sin\beta H + C_3 \cdot e^{-\beta H} \cos\beta H + C_4 \cdot e^{-\beta H} \sin\beta H$$

$$\left(\frac{dw}{dy} \right)_{y=H} = C_1 \cdot e^{\beta H} [\cos\beta H - \sin\beta H] + C_2 \cdot e^{\beta H} [\cos\beta H + \sin\beta H]$$

$$- C_3 \cdot \beta\, e^{-\beta H} [\cos\beta H + \sin\beta H] + C_4 \cdot e^{-\beta H} [\cos\beta H - \sin\beta H]$$

When the coefficients of the above equations are written in matrix form, a matrix of 4x4 dimensions is obtained. This second coefficient matrix in dimensions 4x4 is named as [*KM*2]. Displacement vector {*D*},

$$\{D\} = [KM2]\,\{C\} \tag{2.13}$$

With the equation (2.13),

$$\{D\}^T = \left[(w)_{y=0} \quad \left(\frac{dw}{dy} \right)_{y=0} \quad (w)_{y=H} \quad \left(\frac{dw}{dy} \right)_{y=H} \right]$$

can be written. Flexibility matrix [F_w], in dimensions 4x4 and which belongs to the wall, can be obtained by using (2.12) and (2.13).

$$[F_w] = [KM2] \times [KM1]^{-1} \tag{2.14}$$

Similarly, the 1, 2, 3 and 4 terms of the unknowns, obtained by multiplying the inverse of the flexibility matrix and the displacement vector, are the unknowns of the wall.

$$\{X\} = [F_W]^{-1} \{D\} \tag{2.15}$$

3.RESULTS

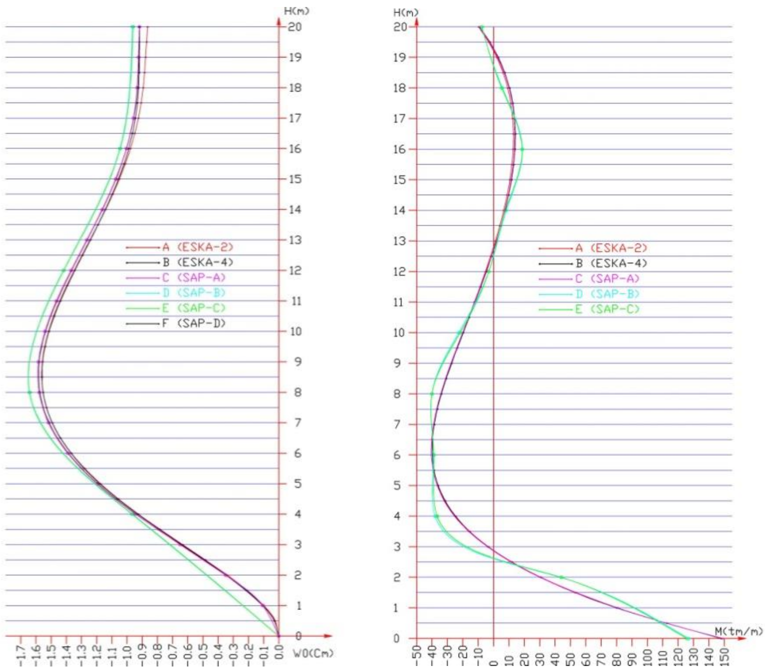
The present study is continuation of ref. [9] where the modelling details of the analysed axially symmetric shell structure is given.

- Wall height: 20 m
- Liquid height: 20 m
- Liquid specific weight: 1.0 tonf/m^3
- Wall thickness: 0.65 m
- Radius to the centre of the wall thickness: 50 m
- Material elasticity module: 2.5x10^6 tonf/m^2
- Poisson ratio: 0.2
- Spherical dome thickness: 0.25 m
- Dome radius: 86.02325 m
- Dome horizontal radius (to the middle of the wall thickness): 50.0 m
- Uniformly distributed load over dome: 0.625 tonf/m^2

Definitions of SAP-A, B, C, D groups are given in the previous study [9]. Results of ESKA-2 and ESKA-4 compared to SAP 2000 [10] are given in the Figures 1.a and 1.b. The graphics relate to the strain in the radial direction and the moment distribution along the wall height of the axi-symmetrical cylindrical wall. In accordance with the formulation of the classical shell theory and sign convention, the values of the system in the direction of the negative horizontal axis (-y direction or left side) are presented.

According to the classical shell theory (the flexibility method) unknowns are forces and the number of equations is defined by the degree of unknowns (the degree of indeterminacy). These values are maximum 2 for ESKA-2 and 4 for ESKA-4 for existing structural members. Displacements are obtained by retrospective solution. In the Finite Element Method, the number of unknowns is defined by the degree of freedom. In the analysis of shell structures with Finite Element Method (stiffness method), generally the "shell" element type is used. In this three-dimensional element type formulation, there are 5 degrees of freedom at each node (the sixth degree of freedom is still a research topic and can only be partially incorporated into the formulation with some approaches). In this case, 5 unknowns are considered at each nodal point. It is possible to reduce this number due to boundary conditions in nodal points. However, it can be said that the numeral of unknowns may reach the number approximately 60000. After the solution of the unknowns, nodal point displacements are obtained and cross-section forces are calculated with retrospective solution.

The geometry of the example problem is chosen to push the limits of the long wall criteria ($H \geq \pi/\beta$). It is nearly in the short wall range, which increase the risk of making mistakes for the computer program ESKA-2. Despite these values, the ESKA-2 program gives almost the same results as the ESKA-4 program. Extremely small differences are values that are not essential to be taken into account. The results of both programs are highly successful. Compared to the Finite Element Method, the same success can only be achieved in the model, which is prepared in detail to push the limits of the computer and the software. Deviations in analysis results of the Finite Element (SAP-A, B, C, D) are more evident in Figure 1.b. Errors in the displacement calculations may cause much larger mistakes in the cross sectional force distribution obtained with the retrospective solution.



a. Displacement of the wall in radial direction b. Moment distribution along the wall height

Figure 1. Comparison between results of ESKA-2, ESKA-4 and analysis with Finite Element Method (SAP 2000)

4. CONCLUSIONS

Two computer programs, named as "ESKA-2" and "ESKA-4" (Axisymmetric Shell Analysis), have been developed within the scope of this study. ESKA-2 is developed for the analysis of systems with a high wall where the analysis of the axial symmetric wall with two integral constants is suitable. ESKA-4 is formulated with four integral constants and gives accurate results regardless of the wall height.

By employing these programs, model modifications can be made in an extremely practical way as compared to the Finite Element Method. With other existent ready-to-use general purpose software, that make use of finite element method, direct analysis results at the desired point can only be obtained if there is a defined nodal point in the structural member at that particular point. Apart from that, by using ESKA-2 and ESKA-4; load and load combinations, boundary conditions and other parameters can be easily modified. Another advantage of these programs is that more detailed and accurate analysis results can be obtained with ESKA-2 and ESKA-4 than the Finite Element Method, which can only analyse with a limited number of unknowns due to the finite capacity of computers. As can be seen, ESKA-2 and ESKA-4 give almost the same results although the geometry of the example problem is chosen to invoke and test the limits of the long wall criteria. On the other hand, different results, for the analysis of the same axially shell structure with Finite Element Method, may be obtained depending on different modelling techniques. It is worth-mentioning that the results obtained in this case are the values at the centre of gravity of the finite element part containing the point of interest. For this reason, in the analysis with ready-to-use general purpose finite element software, it is necessary to model the system with more number of finite elements in order to be able to obtain the results at a certain point or for results to be more precise. However, the number of finite elements, that can be defined, depends on the computer programs and the technical capacity of the computer to be used. If the number of defined end elements increases; it becomes a time-consuming job with the increase in the transaction volume at the stages of generation of model, analysis and interpretation of results. Nevertheless, it is very practical to define the structural members with the developed programs ESKA-2 and ESKA-4 to obtain the desired analysis results at any defined point in a short period of time.

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