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Research Article

INVESTIGATION OF MECHANICAL PROPERTIES OF RANDOM HETEROGENEOUS MODEL

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ABSTRACT

Various local variations of the material microstructures lead mesoscale homogenization techniques considering capability in computer analysis. In this study moving window generalized method of cells technique is used to capture small scale material heterogeneity and randomness. Elastic and inelastic behavior considering yield criterion of plasticity are investigated. This mesoscale representation regarding the material constitutive models described is implemented into the finite-element model to which loading will be applied, is considered the macroscale model. Numerical simulations are demonstrated through comprehensive comparison with the examples in the literature.
Keywords: Effective properties, heterogeneous materials, method of cells, microstructure.

1. INTRODUCTION

Composite materials subjected to different kind of loading conditions might have various responses, which is based on the characterization of the material at the macroscopic level in terms of effective properties. The identification of this characterization can be more complicated if local response is considered because it is most likely that there is randomness in their microstructure of the composite materials. There can be various type of randomness in the material matrix such as shape, size, spatial distribution of inclusions. Characterization of the heterogeneous media need to be described regarding mechanical properties to analyze the critical behavior of the structure. The objective of this study is to characterize composite materials that have inherent randomness.

Random heterogeneous material has become interesting to study in civil, mechanical and aerospace engineering and especially material science. The effects of microstructural randomness on the local material behavior have been investigated in [1-3], in which analytical is hard to manage to explore random material structure. Whereas computational simulations of the microstructural configurations based on the considering probability density functions have been studied by [4-6].

The micromechanical analysis on a representative volume element (RVE) of the composite has been performed by Murthy and Chamis [7], who developed a computer code (Integrated CompositeAnalyzer—ICAN) for the simulations of the strength properties of a composite. The

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calculation is used a geometry, where one cell contains pure resin and another cell contains fiber. Aboudi [8,9] developed method of cells to model the continuously reinforced, unidirectional fibrous compositeas a doubly periodic array of fibers embedded in a matrix phase. The unit cell consists of a single cell surrounded by three matrix subcells.

In this study, the moving window generalized method of cells (moving window GMC) technique is utilized for the evaluation of the local properties of the material in a specified microstructural area. A heterogeneous area is divided into small windows by using moving window technique and is homogenized by performing generalized method of cells (GMC). Therefore, effective material properties can be obtained. Moving window GMC is applied to the heterogeneous random medium considering that; first, each moving window that is taken from the digital image is subjected to a regular boundary conditions and second, unit cell, which is reflected by each window, has a feature of repeating so that it constructs the entire composite structure.

2. BASIC EQUATIONS

Let $\omega \in \Omega$ denote a realization of a random medium, sampled from probability space Ω . This random medium occupies a region D in R^3 with the random stiffness tensor C(x,ω) described by a homogeneous stochastic tensor field. The coordinates y in the fine scale of microstructure may be defined as $\frac{x}{\varepsilon}$, where ε is a scale parameter representing the ratio between the real length of a unit vector in the microscopic coordinates *y* and macroscopic coordinates *x*. If we select a large enough unit cell Y, then we can assume that the boundaries of the unit cell behave εY -periodically. The constitutive properties can then be described by $\mathbf{C}(y,\omega) \equiv \mathbf{C}\left(\frac{x}{\varepsilon},\omega\right) \equiv \mathbf{C}^{\varepsilon}\left(x,\omega\right)$. According to the theory of stochastic processes, an appropriate length scale describing microscopic random fields is the correlation length *l*, defined as

$$l = \left[\int_0^\infty \rho(\tau)d\tau\right]^{1/d} \quad (1)$$

where q(s) is the correlation function describing the random tensor field. $\mathbf{C}(y,\omega) \equiv \mathbf{C}^{\varepsilon}\left(x,\omega\right)$ and *d* is the number of spatial dimensions of the random field.

Without loss of generality, we study elasticity problems with the following governing stochastic elliptic equations, constitutive law and boundary conditions:

$$\partial_j \sigma_{ij}^e\left(x,\omega\right) + f_i(x) = 0 \quad in\ D \quad (2)$$

$$\sigma_{ij}^{\varepsilon}(x,\omega) = C_{ijkl}(\mathbf{y},\omega)e_{kl}^{\varepsilon}(x,\omega) \quad in\ D \quad (3)$$

$$u_i^{\varepsilon}(.,\omega) = \overline{u}_i \quad on\ \partial_1 D \quad (4)$$

$$\sigma_{ij}^{\varepsilon}(\cdot,\omega)n_j = \overline{t}_i \quad on\ \partial_2 D \quad (5)$$

where $u_i^{\varepsilon}, \sigma_{ij}^{\varepsilon}, e_{kl}^{\varepsilon}$ and f_i are the displacement vector, the stress tensor, the strain tensor and the body force vector, respectively.In (4) and (5), $\partial_1 D$ and $\partial_2 D$ denote the boundaries of the domain *D* corresponding to displacement and stress boundary conditions, respectively. Note that *fi*, u_i and $\overline{t_i}$ are macroscopic quantities, independent of ε. The fourth order random tensor field C_{ijkl} $(\mathbf{y},\omega)$ satisfies symmetry and positive-definiteness. The strain–displacement relation describing $e_{kl}(y,\omega) \equiv e_{kl}\left(\frac{x}{\varepsilon},\omega\right) \equiv e_{kl}^{\varepsilon}\left(x,\omega\right)$ is

$$e_{kl}^{\varepsilon}\left(x,\omega\right) = \tfrac{1}{2}\left(\partial_l^x u_k^{\varepsilon}\left(x,\omega\right) + \partial_k^x u_l^{\varepsilon}\left(x,\omega\right)\right) + \tfrac{1}{2}\tfrac{1}{\varepsilon}\left(\partial_l^x u_k^{\varepsilon}\left(x,\omega\right) + \partial_k^x u_l^{\varepsilon}\left(x,\omega\right)\right) \quad (6)$$

where $\partial_l^x = \partial / \partial x_l, \partial_l^y = \partial / \partial y_l$.

Applying the following asymptotic expansions of u_i^{ε} and $\sigma_{ij}^{\varepsilon}$ about scale parameter e, series expressions for displacement and stress are found to be

$$u_i^{\varepsilon}(x,\omega) = u_i^{(0)}(x,\omega) + \varepsilon u_i^{(1)}(x,y,\omega) + \varepsilon^2 u_i^{(2)}(x,y,\omega) + \cdots \quad (7)$$

$$\sigma_{ij}^{\varepsilon}(x,\omega) = \varepsilon^{-1}\sigma_{ij}^{(-1)}(x,y,\omega) + \sigma_{ij}^{(0)}(x,y,\omega) + \varepsilon\sigma_{ij}^{(1)}(x,y,\omega) + \cdots \quad (8)$$

Substituting (8) into (2), (5)–(7) into (3) and (4), and equating powers of e [1,2], we obtain the stochasticPDE's.

$$\partial_j^y\left[C_{ijmn}(y,\omega)e_{mn}^y(y,\omega)\right] = -\partial_j^y\left[C_{ijkl}(y,\omega)e_{kl}^x(x,\omega)\right] \quad (9)$$

To solve local problem (9) in a unit cell with practical homogenization techniques, unit global strain and periodic boundary conditions are prescribed in [8]. Note the local strain $e_{mn}^y(y,\omega)$ depends on the global strain $e_{kl}^y(x,\omega)$ that is defined by

$$e_{kl}^x(x,\omega) = \tfrac{1}{2}\left[\partial_l^x u_k^{(0)}(x,\omega) + \partial_k^x u_l^{(0)}(x,\omega)\right] \quad (10)$$

Based on the relationship between local strain and global strain defined by (9) it is straightforward to derive an expression for the local stres

$$\sigma_{ij}^y(y,\omega) = C_{ijkl}(y,\omega)e_{kl}^x(x,\omega) + C_{ijmn}(y,\omega)e_{mn}^y(y,\omega) \quad (11)$$

3. NUMERICAL ANALYSIS

Studying the response of composites under simple shear loading is necessary to build up the composite yield function. Two previously introduced unit cell containing rectangular and circular fibers are studied. The study excludes the out-of-plane shear loadings. Fig.1 shows both the conventional and higher order boundary conditions of simple shear imposed on the unit cell with rectangular fibers. Plastic strain is set to be zero at the fiber matrix interfaces due to plastic flow suppression.

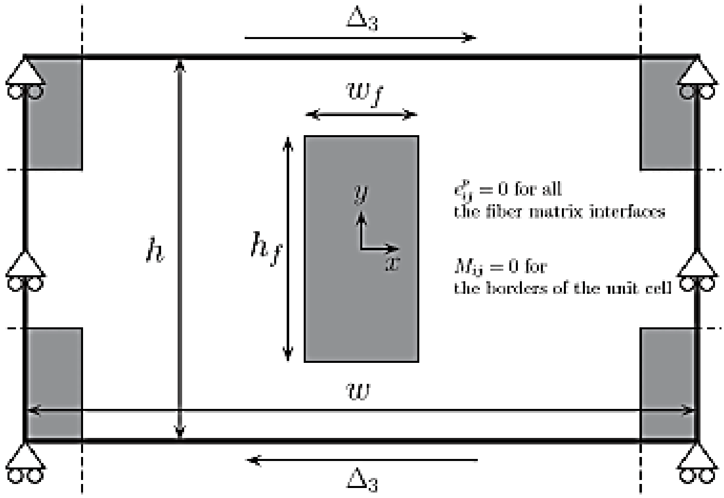

Figure 1. Boundary conditions and geometry of the unit cell of composite with rectangular fibers under simple shear loading

Fig.2 shows the response of the unit cell with rectangular fibers under simple shear loading until $E_{12}=3.25\gamma_y$, where $\gamma_y=\tau_y/G_m$ for different value of the material length scale. The macroscopic elastic shear modulus, $C_{12}=1.24G_m$, is higher than the matrix shear modulus and it seems to be unaffected by the material length scale.

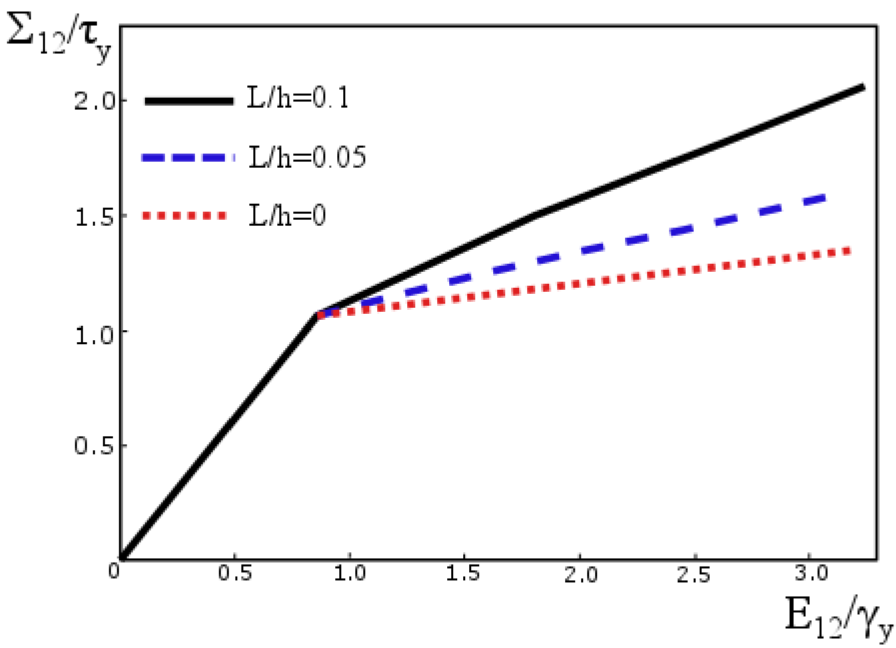

Figure 2. Stress strain response of the unit cell with rectangular fiber under simple shear loading

4. RESULTS

In this paper a stochastic computational model is constructed to assess global effective properties and local probabilistic behavior of random media, based on decomposition of random fields and an iterative numerical scheme for heterogeneous materials. Analogous with representative volume element used in deterministic homogenization problems, a stochastic representative volume element is introduced as a principle of the computational model. Moving window or local homogenization techniques in connection with stochastic simulation is evaluated in this work with a scheme devised for the stochastic homogenization formulation, a closely approximated bound of effective properties can be derived based on solution of auxiliary stochastic equations. While cell structure size is heuristically chosen in this study, we note that a rigorous error study defining the appropriate size relative to correlation length is of big interest for future research.

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