Sigma J Eng & Nat Sci 36 (3), 2018, 587-608

Sigma Journal of Engineering and Natural Sciences
Sigma Mühendislik ve Fen Bilimleri Dergisi

Research Article

ANALYTICAL AND NUMERICAL STUDY OF HYDRODYNAMIC NANO FLUID FLOW IN A TWO –DIMENSIONAL SEMI-POROUS CHANNEL WITH TRANSVERSE MAGNETIC FIELD

Nozar AKBARI[1], Mosayeb GHOLINIA*[2], Saber GHOLINIA[3], Shoeil DABBAGHIAN[4], Davood Domiri GANJI[5]

[1]*Shahid Sattari University of Aeronautical Engineering, Department of Aerospace Engineering, Tehran-IRAN;* ORCID:0000-0003-0924-8363
[2]*Mazandaran University of Science and Technology, Department of Mechanical Engineering, Babol-IRAN;* ORCID:0000-0001-8291-8824
[3]*Babol University of Technology, Dep. of Mechanical Eng., Babol-IRAN;* ORCID:0000-0003-4597-2279
[4]*Babol University of Technology, Department of Mechanical Engineering, Babol, IRAN;* ORCID:0000-0002-4774-8054
[5]*Babol University of Technology, Department of Mechanical Engineering, Babol, IRAN;* ORCID:0000-0002-4293-5993

Received: 18.04.2018 Revised: 18.06.2018 Accepted: 27.06.2018

ABSTRACT

In this research, we used Akbari-Ganji's Method (AGM) to solve the issue of laminar Nano fluid flow in a semi-porous channel in the presence of latitudinal magnetic field. The effectual viscosity and thermal conductivity of Nano fluid flow are computed by Brinkman and Maxwell–Garnetts (MG) models, respectively. Also, the concept of Akbari-Ganji's Method is briefly employed and introduced to derive solutions of nonlinear equations. The received outcomes of AGM are compared with those of acquired from Numerical Method (fourth-order Runge–Kutta method), Collocation Method (CM), Homotopy Perturbation Method (HPM) and Flex-PDE software to check the precision of the considered manner. In the present perusal, the impact of the three dimensionless numbers like the Nano fluid volume fraction, Reynolds number and Hartmann number on non-dimensional velocity profiles are examined. Outcomes show when Ha is tiny, the impact of Re number is very sensible on the velocity profiles but in Ha large, Re number is less impact. In addition, this study shows AGM is strong manner to solve nonlinear differential equations.
Keywords: Akbari-Ganji's Method (AGM), semi-porous channel, collocation method (CM), uniform magnetic, flex-PDE software, laminar nano fluid flow.

1. INTRODUCTION

In fluid mechanics, we study the particles' behavior at any point within the range of different physical conditions. Mathematical models are used to explain physical phenomena in fluid mechanics for a variety of fluids. Most engineering problems in heat transfer and fluid mechanics problems are inherently nonlinear. For this reason, resolving these difficult problems has been a

* Corresponding Author: e-mail: m.gholinia1371@gmail.com, tel: 011-32024159

controversial issue for mathematicians, physicists and engineers. Some equations are solved by Numerical solu tions, some are solved using different analytical methods. Methods that can be introduced to examine the nonlinear problems such as the Differential Transform Method (DTM) [1], Optimal Homotopy Analysis Method (OHAM) [2-3], Homotopy perturbation method (HPM) [4-7], Exp-Function Method (EFM) [8-9], Since- Cosine Function Method [10-11], F-Expansion Method (FEM) [12-13], Tanh- Coth Method [14-15], many methods are not considered in this study because of brevity. Akbari-Ganji's Method (AGM) is a modern manner that be used for research of nonlinear issues. A synopsis of AGM advantages compared to other manners is as follows: Boundary conditions are required in accordance with the order of differential equations in the resolving manner however when the number of boundary conditions is lesser than the order of the differential equation, this approach can engender additional new boundary conditions in regard to the own differential equation and its derivatives. Therefore, AGM is a strong manner for solving the nonlinear differential equations like presented equation in this perusal. In this paper, we have applied AGM to discover the proximate solutions of nonlinear differential equations governing the Nano fluid flow in a semi-porous channel in the attendance of latitudinal magnetic field. Flow issue in a porous channel or tube received much consideration in recent years due to its different applications in medical engineering, for instance in the dialysis of blood in artificial kidney [16], in the flow of blood in the capillaries [17], in the flow in blood oxygenators [18], also, in most other engineering areas for instance gaseous diffusion [19], in transpiration cooling boundary layer control [20] and the sketching of filters [21]. In 1953, Berman [22] explained an accurate solution of the Navier–Stokes equation for steady two-dimensional laminar flow of an incompressible and viscose fluid in a channel with rigid, parallel, porous walls Steamy by uniform, steady injection or suction at the walls. This mass transfer is Prevalent in several industrial processes. More recently, Fakour and et al. [23] fastidiousness of underdeveloped heat conduction and Nano fluid magneto hydro dynamic (MHD) flow in a channel with porous walls investigated. Sheikholeslami et al. [24] analyzed the influence of a magnetic field on the Nano fluid flow in a porous channel via weighted residual methods called Galerkin method. As well as, impacts of sectional slip and diffusion-thermo and thermal-diffusion on steady magneto hydro dynamic (MHD) convective flow owing to a rotating disk studied by Rashidi et al. [25]. Chandran and Sacheti [26] analyzed the impact of a magnetic field on the thermodynamic flow bygone a continuously moving porous sheet. Hamad et al. [27] investigated the steady magneto hydro dynamic free convection boundary layer flow bygone a vertical semi-infinite flat plane embedded in water stuffed with a Nano fluid has been theoretically. They found that Ag and Cu nanoparticles proved to have the highest cooling efficiency for the sake this issue. The Nano fluid flow and heat transfer due to a stretching cylinder in the attendance of magnetic field investigated by Ashorynejad et al. [28]. The subject of laminar gooey flow in a semi-porous channel in the attendance of latitudinal magnetic field was studied with Sheikholeslami et al. [29]. They showed that optimal homotopy asymptotic method (HAM) was a strong approach to solving nonlinear differential equations. Sheikholeslami et al. [30] studied the natural convection heat transfer in a cavity with sinusoidal wall filled with CuO-water Nano fluid in attendance of magnetic field. In addition, the effect of a magnetic field on natural convection in an inclined half-annulus enclosure filled with Cu-water Nano fluid using CVFEM studied by Sheikholeslami et al. [31]. Also, several authors analyzed about heat transfer and Nano fluid flow [32–35]. The main purpose of this work is to present the effects of the three dimensionless numbers: Reynolds number, Hartmann number and the Nano fluid volume fraction on non-dimensional velocity profiles. The comparison of the outcomes of Homotopy Perturbation Method (HPM), AGM, the Numerical Method (fourth-order Runge-Kutta), Flex-PDE software and Collocation Method (CM) outcomes indicates excellent complying in solving this nonlinear problem. Furthermore, the timings of the aforementioned manners are demonstrated in this research for a better use of the methods for solving nonlinear issues.

2. PROBLEM DESCRIPTION

Consider the laminar two-dimensional stationary flow of an electrical conducting incompressible viscous fluid in a semi-porous channel made by a lengthy rectangular plate with a length of L_x in uniform translation in x^* direction and an infinite porous plate. The interval between the two plates is h. We observe a normal velocity q on the porous wall. A monotonic magnetic field B is assumed to be applied towards direction y^* (Fig. 1) [36].

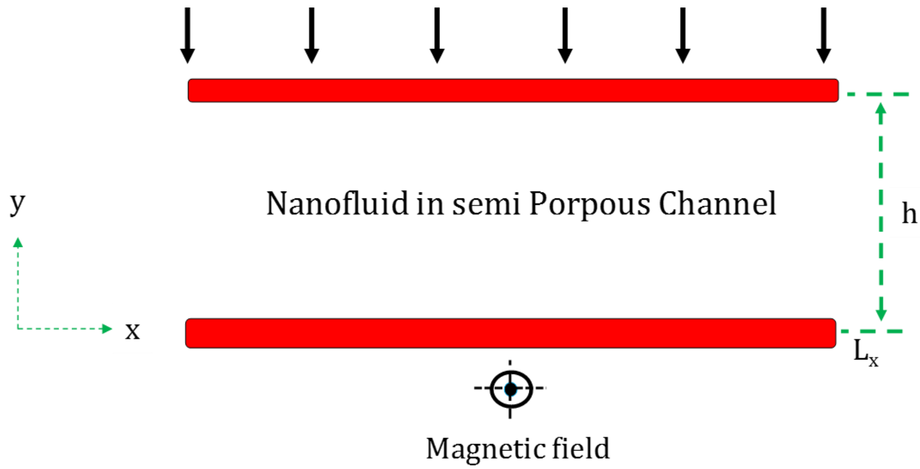

Figure 1. Physical of the issue (Nano fluid in a porous media between parallel sheets and magnetic field).

Table 1. Thermo-physical confidants of water and nanoparticles [39].

Material	Density (kg/m^3)	Cp (J/kg.k)	K (w/m.k)	$\sigma(\frac{\Omega^{-1}}{k^{-1}})$
Pure water	997.1	4179	0.613	0.05
Silver	10500	235	429	6.30×10^7

In case of a short flow to neglect the electrical field, perturbations to the basic natural field and without gravity forces, the governing equations are:

$$\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0, \tag{1}$$

$$u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial x^*}+\frac{\mu_{nf}}{\rho_{nf}}(\frac{\partial^2 u^*}{\partial x^{*2}}+\frac{\partial^2 u^*}{\partial y^{*2}})-u^*\frac{\sigma_{nf}B^2}{\rho_{nf}}, \tag{2}$$

$$u^*\frac{\partial v^*}{\partial x^*}+v^*\frac{\partial v^*}{\partial y^*}=-\frac{1}{\rho_{nf}}\frac{\partial P^*}{\partial y^*}+\frac{\mu_{nf}}{\rho_{nf}}(\frac{\partial^2 v^*}{\partial x^{*2}}+\frac{\partial^2 v^*}{\partial y^{*2}}), \tag{3}$$

The boundary conditions for the velocity are as follows:

$$y^*=0:\ u^*=u_0^*, v^*=0, \tag{4}$$

$$y^* = h:\ u^* = 0, v^* = -q, \tag{5}$$

Calculating a mean velocity U with the relation:

$$y^* = 0:\ u^* = u_0^*, v^* = 0, \tag{6}$$

We consider the following transformations:

$$x = \frac{x^*}{L_x}; y = \frac{y^*}{h}, \tag{7}$$

$$u = \frac{u^*}{U};\ v = \frac{v^*}{q},\ P_y = \frac{P^*}{\rho_f q^2} \tag{8}$$

Afterwards, we can discuss two dimensionless numbers: the Reynolds number Re for dynamic forces and the Hartman number Ha for the description of magnetic forces [37]:

$$Ha = Bh\sqrt{\frac{\sigma_f}{\rho_f . v_f}}, \tag{9}$$

$$Re = \frac{hq}{\mu_{nf}} \rho_{nf}. \tag{10}$$

Where the effective density (ρ_{nf}) is specified as [38]:

$$\rho_{nf} = \rho_f (1-\phi) + \rho_s \phi \tag{11}$$

Where $\emptyset$ is the solid volume fraction of nanoparticles. The dynamic viscosity of the Nano fluids given by Brinkman [38] is

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}, \tag{12}$$

The effective thermal conductivity can be modeled via the Maxwell–Garnetts as [39]:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi(k_f - k_s)}{k_s + 2k_f + \phi(k_f - k_s)}, \tag{13}$$

The effective electrical conductivity of Nano fluid was presented with Maxwell [39] as follows:

$$\frac{\sigma_{nf}}{\sigma_f} = 1 + \frac{3(\frac{\sigma_s}{\sigma_f} - 1)\phi}{(\frac{\sigma_s}{\sigma_f} + 2) - (\frac{\sigma_s}{\sigma_f} - 1)\phi}, \tag{14}$$

The thermo physical properties of the Nano fluid are given in Table 1[39]. Therefore, we can evolve the dimensionless equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{15}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\varepsilon^2\frac{\partial P_y}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\frac{1}{hp}(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})-u\frac{Ha^2}{Re}\frac{B^*}{A^*}, \tag{16}$$

$$u\frac{\partial \upsilon}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial P_y}{\partial x}+\frac{\mu_{nf}}{\rho_{nf}}\frac{1}{hp}(\varepsilon^2\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}), \tag{17}$$

Where A^* and B^* are constant parameters:

$$A^*=(1-\phi)+\frac{\rho_s}{\rho_f}\phi,$$
$$B^*=1+\frac{3(\frac{\sigma_s}{\sigma_f}-1)\phi}{(\frac{\sigma_s}{\sigma_f}+2)-(\frac{\sigma_s}{\sigma_f}-1)\phi}, \tag{18}$$

Amount of ε is defined as the aspect ratio between a characteristic length L_x and spacing h of the slider. This ratio is normally tiny. Berman's similarity transformation is used to be free from the aspect ratio of ε:

$$v=-V(y);u=\frac{u^*}{U}=u_0U(y)+x\frac{dV}{dy}. \tag{19}$$

Introducing Eq. (19) in the second momentum Eq. (17) shows that amount $\partial P_y/\partial y$ does not depend on the longitudinal variable x. With the first momentum equation, we as well as observe that $\partial^2 P_y/\partial x^2$ is independent of x. We omit star for simplicity. So a separation of variables leads to [36]:

$$V'^2-VV''-\frac{1}{Re\,A^*(1-\phi)^{2.5}}V'''+\frac{Ha^2B^*}{Re\,A^*}V'=\varepsilon^2\frac{\partial^2 P_y}{\partial x^2}=\varepsilon^2\frac{\partial P_y}{(\partial x)x}, \tag{20}$$

$$UV'-VU'=\frac{1}{Re\,A^*(1-\phi)^{2.5}}[U''-Ha^2B^*(1-\phi)^{2.5}U], \tag{21}$$

The right-hand side of Eq. (20) is constant. Therefore, we derive this equation with respect to x. This gives:

$$V^{IV}=Ha^2B^*(1-\phi)^{2.5}V''+Re\,A^*(1-\phi)^{2.5}[V'V''-VV''']. \tag{22}$$

Where primes denote differentiation with respect to y and star have been omitted. The dynamic boundary conditions are:

$$\begin{cases} y=0:U=1;\ V=0;V'=0, \\ y=1:U=0;V=1;V'=0. \end{cases} \tag{23}$$

3. MATHEMATICAL PROCEDURES:

In this section three methods have been examined:

3.1. Akbari-Ganji's Method (AGM)

Initial conditions and boundary conditions are needed differential equation conforming to the physic of the moot point. Therefore, we can solve every differential equation with any degree. In order to understand the given manner in this research, two differential equations ruling on engineering operations will be solved in this new method. The nonlinear differential equation of p which is a function of u(which is a function of x), and their derivatives are considered as follows:

$$p_k : f(u, u', u'', \ldots u^m) = 0 \;\; ; u = u(x), \tag{24}$$

Boundary conditions:

$$\begin{cases} u(0) = u_0, u'(0) = u_1, \ldots u^{(m-1)}(0) = u_{m-1} \\ u(L) = u_{L0}, u'(L) = u_{L1}, \ldots u^{(m-1)}(L) = u_{Lm-1} \end{cases} \tag{25}$$

To solver the first differential equation, with attention to the boundary conditions in $x = L$ in Eq. (25), the series of letters in the $n\ th$ order by constant coefficients, which is the reply of the first differential equation, is considered as follows:

$$u(x) = \lim_{n\to\infty} \sum_{i=0}^{n} a_i x^i = \lim_{n\to\infty} (a_0 + a_1 x^1 + a_2 x^2 + \ldots a_n x^n), \tag{26}$$

Boundary conditions are applied to the function as follows:

a) The use of the boundary conditions for the reply of differential Eq. (26) is in the form of

If $x = 0$

$$\begin{cases} u(0) = a_0 = u_0 \\ u'(0) = a_1 = u_1 \\ u''(0) = a_2 = u_2 \\ \vdots \quad \vdots \quad \vdots \end{cases} \tag{27}$$

And when $x = L$

$$\begin{cases} u(0) = a_0 + a_1 L + a_2 L^2 + \ldots + a_n L^n = u_{L_0} \\ u'(0) = a_1 + 2a_2 L + 3a_3 L^2 + \ldots + n a_n L^{n-1} = u_{L_1} \\ u''(0) = 2a_2 + 6a_3 L + 12a_4 L^2 + \ldots + n(n-1) a_n L^{n-2} = u_{L_{m-1}} \\ \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots \end{cases} \tag{28}$$

b) After substituting Eq. (28) into Eq. (24), the use of the boundary conditions on differential Eq. (24) is done according to the following manner:

$$
\begin{aligned}
&p_0 : f(u(0), u'(0), u''(0), \dots u^{(m)}(0)) \\
&p_1 : f(u(L), u'(L), u''(L), \dots u^{(m)}(L)) \\
&\vdots \quad \vdots \quad \vdots \quad \vdots \quad \vdots
\end{aligned}
\tag{29}
$$

With attention to the selection of n; (n < m) sentences from Eq. (26) and in order to creation a collection of equations which is consisted of $(n + 1)$ equations and $(n + 1)$ unknowns, we confront with a number of extra unknowns which are indeed the same coefficients of Eq. (26). So, to delete this issue, we should derive m times from Eq. (24) with a view to the extra unknowns in the afore-mentioned collection differential equations and then this is the time to apply the boundary conditions of Eq. (25) on them.

$$
\begin{aligned}
&p_k{}' : f'(u', u'', u''', \dots u^{(m+1)}) \\
&p_k{}'' : f''(u'', u''', u^{IV}, \dots u^{(m+2)}) \\
&\vdots \quad \vdots \quad \vdots \quad \vdots
\end{aligned}
\tag{30}
$$

c) Usage of the boundary conditions on the derivatives of the differential equation P_k in Eq. (30) is done in the form of

$$
p_k{}' : \begin{cases} f'(u'(0), u''(0), u'''(0), \dots, u^{(m+1)}(0)) \\ \\ f'(u'(L), u''(L), u'''(L), \dots, u^{(m+1)}(L)) \end{cases}
\tag{31}
$$

$$
p_k{}'' : \begin{cases} f''(u''(0), u'''(0), \dots, u^{(m+2)}(0)) \\ \\ f''(u''(L), u'''(L), \dots, u^{(m+2)}(L)) \end{cases}
\tag{32}
$$

The $(n + 1)$ equations can be built from Eq. (27) to Eq. (32) so that $(n + 1)$ unknown coefficients of Eq. (26) for instance, $a_0 + a_1 + a_2 + a_3 + \cdots a_n$ can be calculated. The reply of the nonlinear differential Eq. (24) will be gained by determining coefficients of Eq. (26).

3.2. Homotopy Perturbation Method (HPM)

To explain the basic ideas of this manner, we consider the following nonlinear differential equation:

$$
A(u) - f(r) = 0, \quad r \in \Omega,
\tag{33}
$$

With the boundary condition of:

$$
B(u, \frac{\partial u}{\partial n}), \quad r \in \Gamma,
\tag{34}
$$

Where $f\,(r)$ a known analytical function, A is a general differential operator, B a boundary operator, $(\partial u/\partial n)$ denotes differentiation along the normal drawn outwards from (Ω) and (Γ) is the boundary of the domain(Ω).

A can be divided into two parts which are L and N, where L is linear part and N is nonlinear part. Eq. (33) can therefore be rewritten as follows:

$$L(u)+N(u)-f(r)=0, \tag{35}$$

Homotopy perturbation structure is shown as follows:

$$H(\nu,p)=L(\nu)+L(u_0)+pL(u_0)+p(N(\nu)-f(r))=0, \tag{36}$$

Where,

$$\nu(r,p): \quad \Omega\times[0,1]\rightarrow R, \tag{37}$$

In Eq. (36), u_0 is the first approximation that satisfies the boundary condition and $p \in [0,1]$ is an embedding parameter. We can assume that the solution of Eq. (36) can be written as a power series in P, as following:

$$\nu=\nu_0+p\nu_1+p^2\nu_2+\ldots \tag{38}$$

and the best approximation for solution is:

$$u=\lim_{p\rightarrow 1}\nu=\nu_0+\nu_1+\nu_2+\ldots \tag{39}$$

3.3. Collocation Method (CM)

Weighted residual method was first introduced by Ozisk [40] to solve the differential equation in heat transfer, Collocation and Galerkin method are analytical methods that are based on the weighted residual method [41]. Suppose a differential operator D, is applied on a function u to produce a function P

$$D(u(x))=p(x), \tag{40}$$

basic functions chosen from a linearly independent set. That is:

$$u\cong\tilde{u}=\sum_{i=1}^{n}c_i\phi_i \tag{41}$$

Now, when substituted into the differential operator, D the result of the operations is not, in general, $P(x)$. Hence an error or residual will exist as

$$E(x)=R(x)=D(\tilde{u}(x))-p(x)\neq 0 \tag{42}$$

The main idea of the CM is to force the residual to zero in some average sense over the domain. That is:

$$\int_X R(x)W_i(x)=0, \quad i=1,2,3\ldots n \tag{43}$$

Where the number of weight functions W_i is exactly equal to the number of unknown constants C_i in $\tilde{u}$ function. The result is a set of n algebraic equations for the unknown constants C_i. For collocation method, the weighting functions are taken from the family of Dirac δ functions in the domain. That is, $W_i(x)=\delta(x-x_i)$. The Dirac δ function has the property of:

$$\delta(x-x_i)=\begin{cases}1 & \text{if } x=x_i \\ 0 & \text{otherwise}\end{cases}$$

Also, the residual function in Eq. (42) must be forced to be zero at specific points.

4. APPLICATION OF DESCRIBED METHODS IN THE PROBLEM

4.1. Akbari-Ganji's Method (AGM)

First of all, we rewrite the problem Eqs. (21) – (22) in the following order:

$$K(y)=V^{IV}-Ha^2B^*(1-\phi)^{2.5}V''+\mathrm{Re}\,A^*(1-\phi)^{2.5}(V'V''-VV''')=0,$$
$$M(y)=UV'-VU'-\frac{1}{\mathrm{Re}\,A^*(1-\phi)^{2.5}}(U''-Ha^2B^*(1-\phi)^{2.5}U)=0, \tag{44}$$

In AGM, the answer of the differential equation is considered as a finite series of polynomials with constant coefficients, as follows:

$$V(y)=\sum_{k=0}^{7}a_ky^k=a_0+a_1y+a_2y^2+a_3y^3+a_4y^4+a_5y^5+a_6y^6+a_7y^7, \tag{45}$$

$$U(y)=\sum_{k=0}^{5}d_ky^k=d_0+d_1y+d_2y^2+d_3y^3+d_4y^4+d_5y^5. \tag{46}$$

The given answer function has the constant coefficients a_0 to a_7 and d_0 to d_5 which can easily be computed by applying the initial conditions from Eq. (23). It is notable that the more numbers of series sentence of Eqs. (45) - (46), the more precise the answer, and the answer is tended to the exact solution [42]. For example, solving to differential Eqs. (21) - (22) by used Akbari-Ganji's Method with ($Re=1, Ha=1, \emptyset=0.04$).

In AGM, the boundary conditions are applied in two ways:

a) Applying the boundary conditions on Eqs. (45) - (46) is expressed as follows:

$$V=V(BC),\qquad U=U(BC). \tag{47}$$

So the boundary conditions are applied with respect to Eq. (47) as follows:

$$\begin{aligned}
&U(0)=1\rightarrow d_0=1\\
&V(0)=0\rightarrow a_0=0\\
&V'(0)=0\rightarrow a_1=0\\
&U(1)=0\rightarrow d_5+d_4+d_3+d_2+d_1+d_0=0\\
&V(1)=1\rightarrow a_7+a_6+a_5+a_4+a_3+a_2+a_1+a_0=1\\
&V'(1)=0\rightarrow a_1+2a_2+3a_3+4a_4+5a_5+6a_6+7a_7=0
\end{aligned} \tag{48}$$

b) Boundary conditions are applied on Eq. (44), shown by $K\ (y)$ and $M\ (y)$, and also on their derivatives as

$$\begin{aligned}
&K(V(y))\rightarrow K(V(BC))=0,\ K'(V(BC))=0,\ldots\\
&M(U(y))\rightarrow M(U(BC))=0, M'(U(BC))=0,\ldots
\end{aligned} \tag{49}$$

Eq. (49) means that the answer functions are substituted into the set of Eq. (44) instead of the dependent parameters U and V, and then the boundary conditions are applied on them as follows:

$$K(V(0))\rightarrow -1.203+24a_4-2.022a_2=0, \tag{50}$$

$$K(V(1)) \rightarrow 797.523a_7 + 329.65a_6 + 99.77a_5 + 11.86a_4 - 6.06a_3 - 2.02a_2 - 1.203 = 0, \quad (51)$$

$$M(U(0)) \rightarrow d_0.a_1 - a_0.d_1 - 1.66d_2 + 0.84d_0 = 0, \quad (52)$$

$$\begin{aligned} &M(U(1)) \rightarrow (d_5 + d_4 + d_3 + d_2 + d_1 + d_0).(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7) \\ &-(a_7 + a_6 + a_5 + a_4 + a_3 + a_2 + a_1 + a_0).(5d_5 + 4d_4 + 3d_3 + 2d_2 + d_1) - 15.77d_5 \\ &-9.129d_4 - 4.144d_3 - 0.82d_2 + 0.84d_1 + 0.84d_0 = 0, \end{aligned} \quad (53)$$

Applying the boundary conditions on the derivatives of the set of differential equations is done in the following forms:

$$K'(V(0)) \rightarrow 120a_5 - 6.06a_3 = 0, \quad (54)$$

$$K'(V(1)) \rightarrow 2307.61a_7 + 598.63a_6 + 59.31a_5 - 24.27a_4 - 6.06a_3 = 0, \quad (55)$$

$$M'(U(0)) \rightarrow d_1.a_1 + 2(d_0.a_2) - a_1.d_1 - 2(a_0.d_2) - 4.98d_3 + 0.84d_1 = 0, \quad (56)$$

$$\begin{aligned} &M'(U(1)) \rightarrow (5d_5 + 4d_4 + 3d_3 + 2d_2 + d_1).(a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7) \\ &+(d_5 + d_4 + d_3 + d_2 + d_1 + d_0).(2a_2 + 6a_3 + 12a_4 + 20a_5 + 30a_6 + 42a_7) - (a_1 + 2a_2 \\ &+3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7).(5d_5 + 4d_4 + 3d_3 + 2d_2 + d_1) - (a_7 + a_6 + a_5 + a_4 + a_3 \\ &+a_2 + a_1 + a_0).(2d_2 + 6d_3 + 12d_4 + 20d_5) - 45.64d_5 - 16.57d_4 - 2.46d_3 + 1.680d_2 + 0.84d_1 = 0, \end{aligned} \quad (57)$$

By solving a set of algebraic equations which is consisted of fourteen equations with fourteen unknowns from Eq. (48) and Eqs. (50)- (57), the constant coefficients of Eqs. (45)- (46) can easily be gained.

$$\begin{aligned} &a_0 = 0, a_1 = 0, a_2 = 3.099, a_3 = -2.3021, a_4 = 0.311, a_5 = -0.116, a_6 = 0.01, a_7 = -0.0024, \\ &d_0 = 1, d_1 = -1.750, d_2 = 0.505, d_3 = 0.948, d_4 = -0.98, d_5 = 0.281 \end{aligned} \quad (58)$$

By substituting the achieved constant coefficients into Eqs. (45) – (46), the solution of the set of coupled nonlinear differential equation is gained as follows:

$$\begin{aligned} &V(y) = [-0.00243y^7 + 0.0102y^6 - 0.116y^5 + 0.311y^4 - 2.302y^3 + 3.099y^2], \\ &U(y) = [0.281y^5 - 0.985y^4 + 0.948y^3 + 0.505y^2 - 1.75y + 1]. \end{aligned} \quad (59)$$

4.2. Homotopy Perturbation Method (HPM)

In this section, we will apply the HPM to nonlinear ordinary differential Eqs. (21)– (22). According to the HPM, we construct a homotopy suppose the solution of Eqs. (21)– (22) has the form:

$$\begin{aligned} &H(V,p) = (1-p)(\frac{d^4}{dy^4}V(y)) + p(\frac{d^4}{dy^4}V(y) - (Ha^2).B.((1-\phi)^{2.5}).(\frac{d^2}{dy^2}V(y)) \\ &+R.A.((1-\phi)^{2.5})(V(y).(\frac{d}{dy}V(y)) - V(y).(\frac{d^3}{dy^3}V(y)))) = 0, \end{aligned} \quad (60)$$

$$H(U,p)=-\frac{(1-p)(\frac{d^2}{dy^2}U(y))}{R.A.((1-\phi)^{2.5})}+p(U(y).(\frac{d}{dy}V(y))-V(y).(\frac{d}{dy}U(y)) \tag{61}$$
$$-\frac{\frac{d^2}{dy^2}U(y)-(Ha^2).B.((1-\phi)^{2.5}).U(y)}{R.A.((1-\phi)^{2.5})})=0.$$

We consider $V(y)$ and $U(y)$ as follows:

$$V(y)=\sum_{i=0}^{m}p^iV_i(y)=V_0(y)+pV_1(y)+p^2V_2(y)+p^3f_3(y)+\ldots \tag{62}$$

$$U(y)=\sum_{i=0}^{m}p^iU_i(y)=U_0(y)+pU_1(y)+p^2U_2(y)+p^3U_3(y)+\ldots \tag{63}$$

Substituting Eqs. (62)– (63), into Eqs. (60)– (61), and some simplification and rearranging on powers of P- terms, we have:

P^0 :

$$V_0^{IV}=0, \tag{64}$$
$$U_0''=0.$$

And boundary conditions are:

$$y=0\ :\quad U_0=1,\ V_0=V_0'=0, \tag{65}$$
$$y=+1\ :\quad U_0=0,\ V_0=1, V_0'=0.$$

P^1 :

$$Ha^2(1-\phi)^{-2.5}B^*V_0''+V_1^{IV}+\operatorname{Re}A^*(1-\phi)^{-2.5}V_0'''V_0+\operatorname{Re}A^*(1-\phi)^{-2.5}V_0''V_0'=0, \tag{66}$$
$$-Ha^2(1-\phi)^{-2.5}B^*U_0''+U_1''-\operatorname{Re}A^*(1-\phi)^{-2.5}V_0'U_0+\operatorname{Re}A^*(1-\phi)^{-2.5}V_0U_0'=0,$$

And boundary conditions are:

$$y=0\ :\quad U_0=0,\ V_0=V_0'=0, \tag{67}$$
$$y=+1\ :\quad U_0=0,\ V_0=0, V_0'=0.$$

Solving Eqs. (64) and (66) with boundary conditions:

$$V_0(y)=-2y^3+3y^2, \tag{68}$$
$$U_0(y)=-y+1,$$

$$V_1(y)=0.05714\operatorname{Re}A^*(1-\phi)^{-2.5}y^7-0.2\operatorname{Re}A^*(1-\phi)^{-2.5}y^6-0.1Ha^2(1-\phi)^{-2.5}B^*y^5$$
$$+0.3\operatorname{Re}A^*(1-\phi)^{-2.5}y^4-0.3857\operatorname{Re}A^*(1-\phi)^{-2.5}y^3-0.2Ha^2(1-\phi)^{-2.5}B^*y^3 \tag{69}$$
$$+0.228\operatorname{Re}A^*(1-\phi)^{-2.5}y^2+0.5Ha^2(1-\phi)^{-2.5}B^*y^2,$$

$$U_1(y) = -0.2\text{Re } A^*(1-\phi)^{-2.5} y^5 - 0.7 \text{ Re } A^*(1-\phi)^{-2.5} y^4 - 0.1666\text{Ha}^2(1-\phi)^{-2.5} B^* y^3 \\ +0.1\text{Re } A^*(1-\phi)^{-2.5} y^3 + 0.5\text{Ha}^2(1-\phi)^{-2.5} B^* y^2 - 0.45\text{Re } A^*(1-\phi)^{-2.5} y \\ +0.3332\text{Ha}^2(1-\phi)^{-2.5} B^* y \tag{70}$$

In the same method, the rest of components were obtained by using the Maple package, that we obtain (Eqs. (62)– (63)) parameters of it. According to HPM, we can conclude:

$$V(y) = f_0(y) + f_1(y) + .. + f_5(y), \\ U(y) = U_0(y) + U_1(y) + ... + U_5(y). \tag{71}$$

4.3. Collocation Method (CM)

Since trial function must satisfy the boundary conditions in Eq. (23), so they will be considered as

$$U(y) = 1 - y + c_1.(-y^2 + y) + c_2.(-y^3 + y), \\ V(y) = c_3.(\frac{1}{2}y^2 - \frac{1}{3}y^3) + c_4.(\frac{1}{2}y^2 - \frac{1}{4}y^4) + c_5.(\frac{1}{2}y^2 - \frac{1}{5}y^5). \tag{72}$$

We select the collocation locations $y = 1/4$ to $3/4$ which are evenly spaced throughout the domain. Introducing these values into the residual Eq. (72).Thus we have five algebraic equations for the determination of the five unknown coefficients c_1 to c_5. For example, using collocation method with ($Re = 0.5, Ha = 0.52, \emptyset = 0.05$), V(y) and U(y) are as follows:

$$U(y) = [1 - 1.4274y + 0.5097y^2 - 0.0822y^3], \\ V(y) = [3.284y^2 - 2.44y^3 + 0.217y^4 - 0.023y^5]. \tag{73}$$

4.4. Solution whit Flex-PDE software:

In this article, we first introduce the Flex-PDE software. Flex-PDE software is simple modeling software based on finite element manner for coding. This software has capability to analyze the wide range of engineering issues like chemical reaction kinetic, tension and modeling of real mathematical and engineering issues [43].

4.4.1 Precision control in Flex-PDE Software:

The advantage of this software is its precision control. Flex-PDE applies checking the compatibility of PDE equations integrals over the grid cells, thus estimating the relative error in the response variables and comparing it with the allowable limits of precision. If any one of the grid cells exceeds the allowable limit of error, that cell can be split and the solution process will be re-applied. Allowable limit error: This is called ERRLIM in this software and its default value is 0.002. This means that Flex-PDE corrects the grid when the estimation error in each variable (in ratio to the range of changes of that variable) is less than 0.2 percent per cell. This shows that this software has much compatibility with Numerical solutions. This software has rarely been used in the field of fluid mechanics and heat transfer so far. On the other hand, as this is an open-source software, there is easy access to the dominant equations and it is possible to apply the desired changes to the equations or the material properties. On the other hand, the main capacity of this simple software is solving the complex non-linear equations, which happens abundantly in the field of fluid mechanics and heat transfer. In this study, we compare the outcomes of Flex-

PDE software with acquired outcomes from VIM, CM and AGM by writing Flex-PDE software codes for Eqs. (21) - (22).

5. RESULTS AND DISCUSSION

5.1. Comparison between AGM, CM, HPM methods and Flex-PDE software.

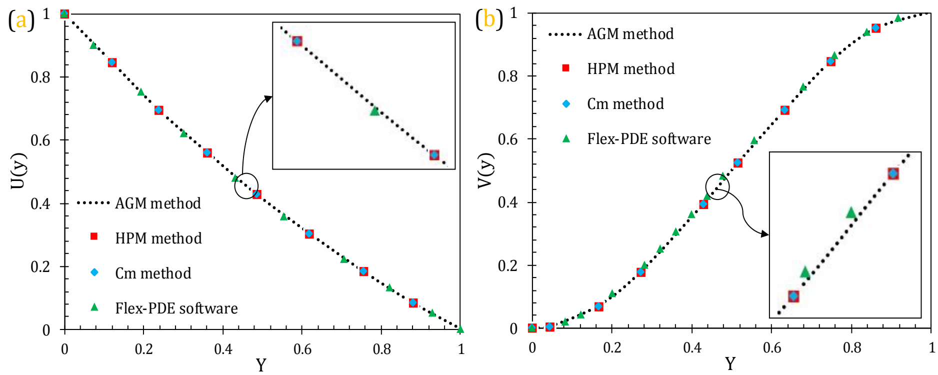

Figure 2. Comparison of AGM, CM, HPM and Flex-PDE outcomes for dimensionless velocities a) U(y) and b) V(y) when Re=0.5, Ha=0.5 and $\emptyset$ =0.04.

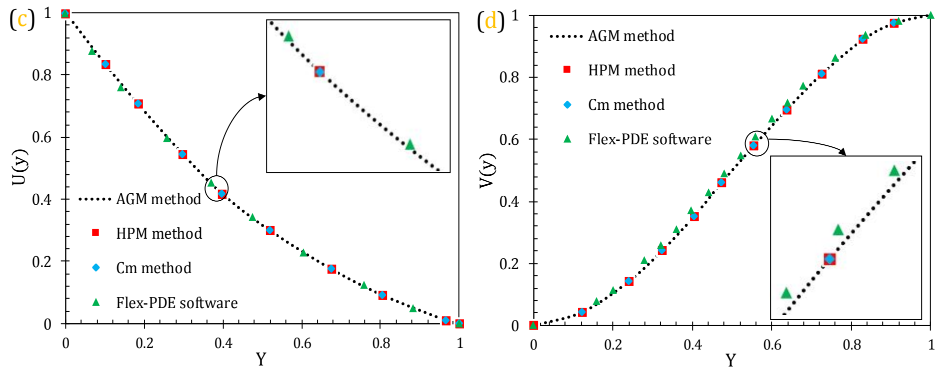

Figure 3. Comparison of AGM, CM, HPM and Flex-PDE results for dimensionless velocities c) U(y) and d) V(y) when Re=1, Ha=1 and $\emptyset$ =0.05.

5.2. Results of AGM

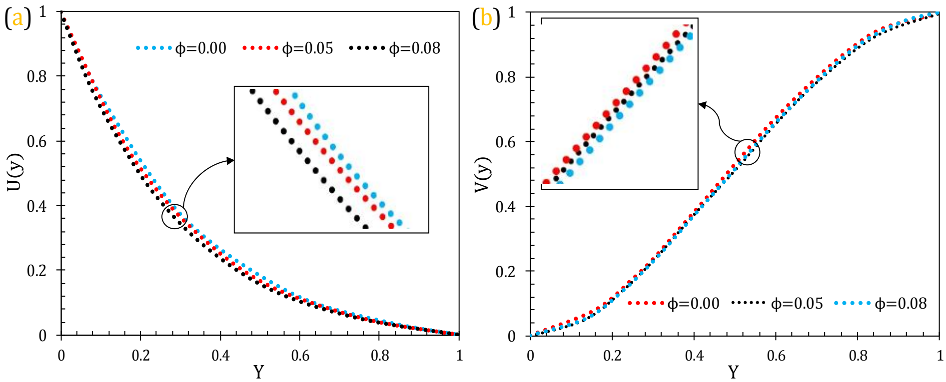

Figure 4. Impact of nanoparticle volume fraction (∅), on (a) U(y) and (b) V(y), for water with silver nanoparticles when Re = 1.5 and Ha = 4.

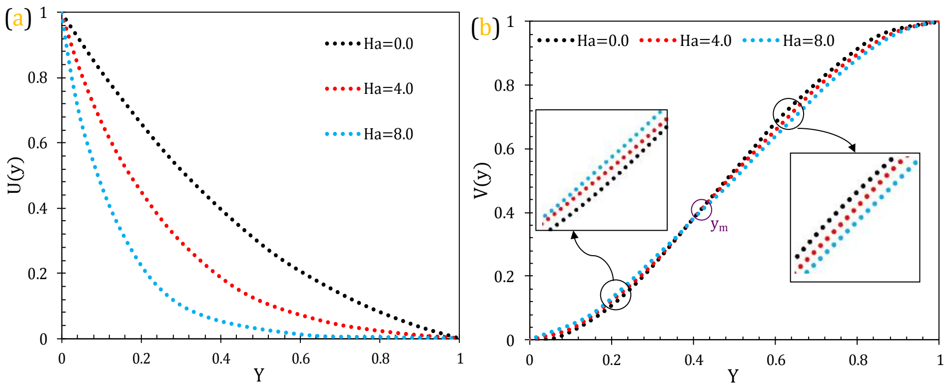

Figure 5. Impact of Hartman number (Ha) on dimensionless velocities for water with silver nanoparticles a) U(y) and b) V(y) when Re=1.5 and ∅ =0.05.

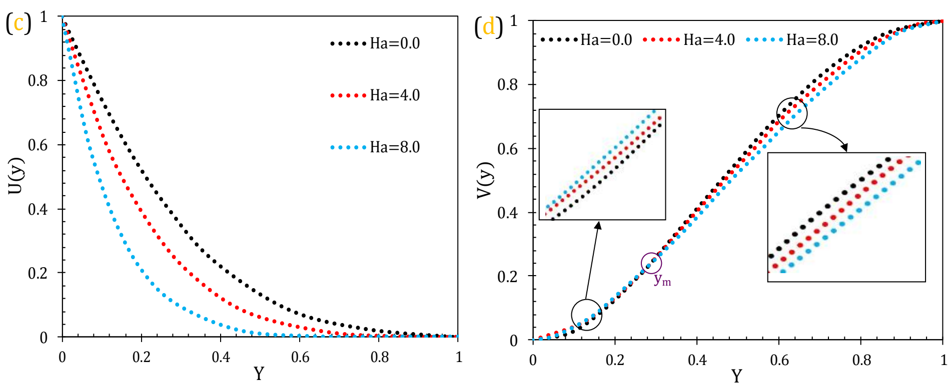

Figure 6. Effect of Hartman number (Ha) on dimensionless velocities for water with silver nanoparticles c) U(y) and d) V(y) when Re=4 and ∅ =0.04.

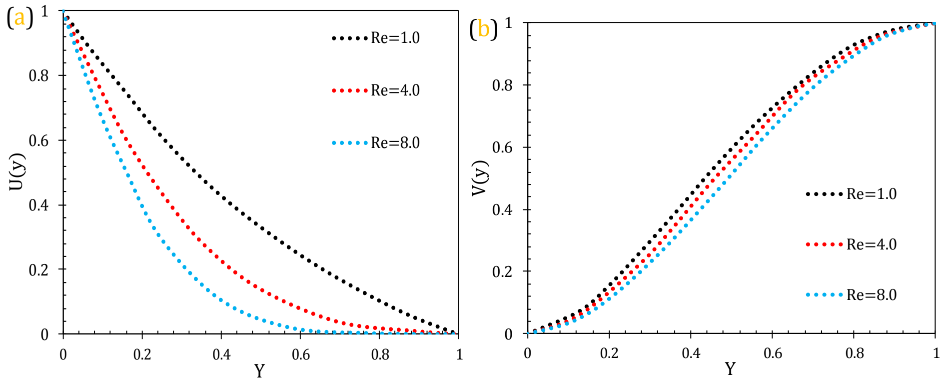

Figure 7. Impact of Reynolds number (Re) on dimensionless velocities for water with silver nanoparticles a) U(y) and b) V(y) when Ha=1 and Ø =0.05.

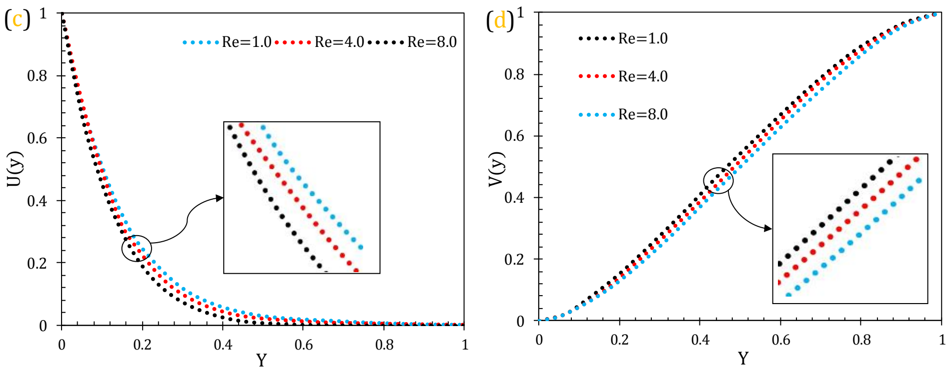

Figure 8. Impact of Reynolds number (Re) on dimensionless velocities for water with silver nanoparticles c) U(y) and d) V(y) when Ha=8 and Ø =0.04.

Table 2. Comparison between the AGM solution and Numerical outcomes for V (y).

	Ha = 0.5, Re= 0.5,Ø=0.04			Ha = 1, Re= 1, ,Ø=0.05		
y	NUM	AGM	Error	NUM	AGM	Error
0.0	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
0.2	0.104855	0.104883	-2.800e-05	0.106128	0.106125	3.00e-06
0.4	0.353633	0.353694	-6.100e-05	3.556199	0.355613	6.90e-06
0.6	0.649341	0.649402	-6.100e-05	0.650461	0.650455	6.00e-06
0.8	0.896466	0.896493	-2.700e-05	0.896581	0.896579	2.00e-06
1	1.000000	1.000000	0.000000	1.000000	0.999999	1.00e-06

Table 3. Comparison between the AGM solution and Numerical outcomes for U (y).

	Ha = 0.5, Re= 0.5,∅=0.04			Ha = 1, Re= 1, ,∅=0.05		
y	NUM	AGM	Error	NUM	AGM	Error
0.0	1.000000	1.000000	0.000000	1.000000	1.000000	0.00000
0.2	0.742490	0.739230	0.003260	0.676900	0.670410	0.006490
0.4	0.512451	0.507217	0.005234	0.421168	0.410414	0.010754
0.6	0.314786	0.309633	0.005153	0.233245	0.222850	0.010395
0.8	0.146110	0.142992	0.003118	0.098688	0.092921	0.005767
1	0.000000	0.000000	0.000000	0.000000	4.000e-10	-4.000e-10

Table 4. Comparison between the HPM solution and Numerical outcomes for V (left) and U (Right)

	Ha = 0.5, Re= 0.5,∅=0.04			Ha = 1, Re= 1, ,∅=0.05		
y	NUM	HPM	Error	NUM	HPM	Error
0.0	0.000000	0.000000	0.000000	1.000000	1.000000	0.000000
0.2	0.104855	0.104735	0.000120	0.676900	0.661324	0.015576
0.4	0.353633	0.353422	0.000211	0.421168	0.421434	-0.000266
0.6	0.649341	0.640251	0.009090	0.233245	0.234861	-0.001616
0.8	0.896466	0.895436	0.001030	0.098688	0.098410	0.000278
1	1.000000	1.000000	0.000000	0.000000	0.000000	0.000000

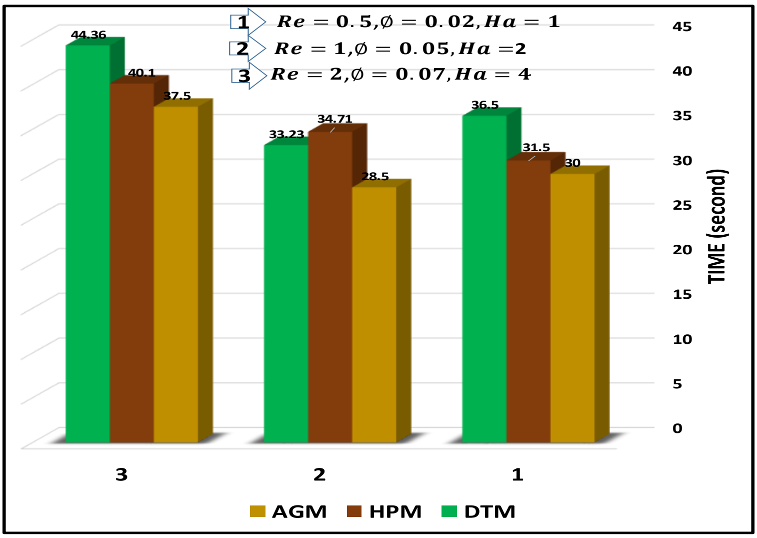

Figure A. Comparison between the timing of the AGM outcomes, HPM and DTM [44] solution.

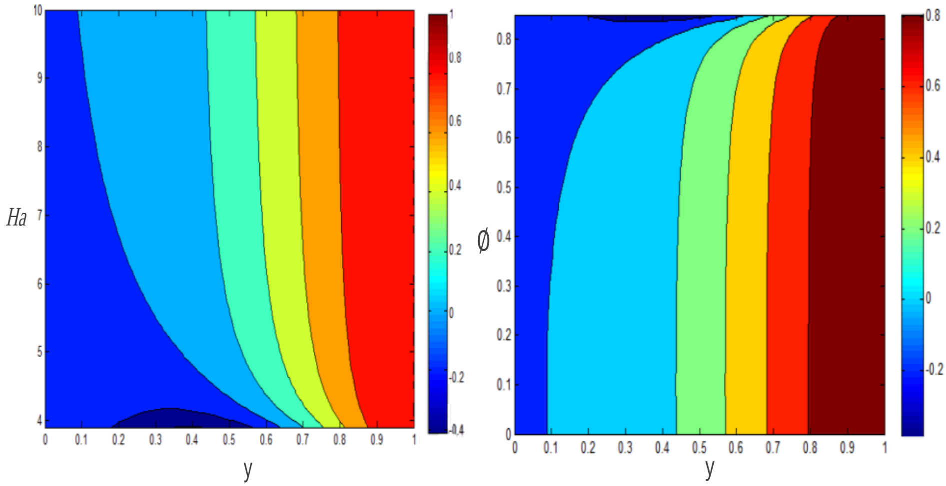

Figure 9. Contour plots of V (y) for Hartman number (Left) and nanoparticles fraction (Right).

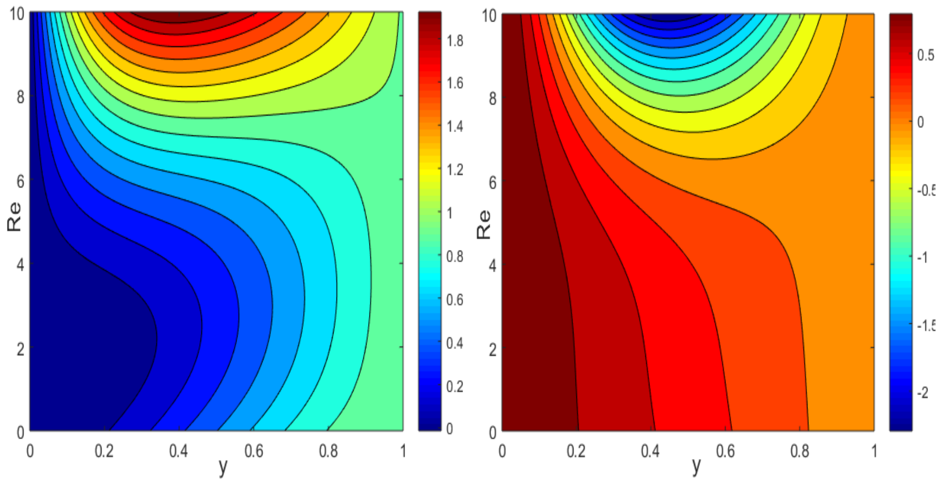

Figure 10. Contour plots of V (left) and U (Right) for Reynolds number (Re).

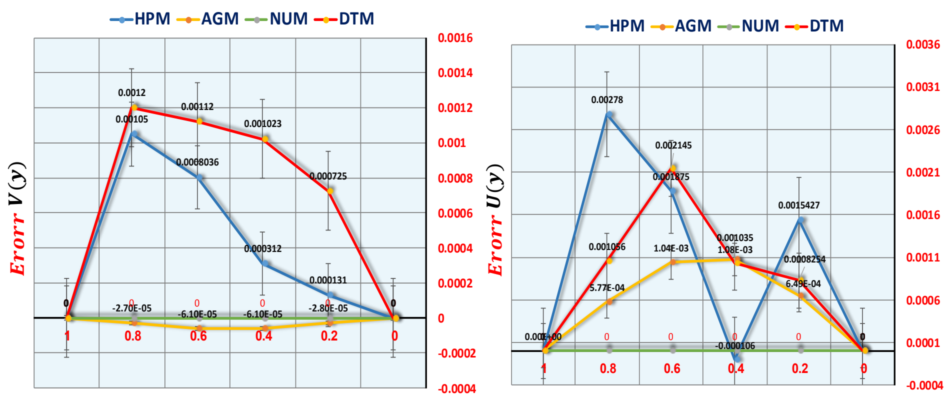

Figure (B). The comparison between Errors of AGM, HPM and DTM [44] for V (y) and U (y) when Ha = 0.7, Re= 1.5,∅=0.06

In this article, Akbari-Ganji's Method (AGM), Collocation Method (CM) and Homotopy Perturbation Method (HPM) are applied to acquire a clear analytic solution of the laminar Nano fluid flow in a semi-porous channel in the attendance of monotone magnetic field (Fig. 1). In order to verify the precision of the present outcomes, we have compared AGM outcomes with Flex-PDE software, CM method, HPM method and Numerical Method (NUM). Comparison between the AGM, HPM outcomes with Numerical Method for different amounts of energetic parameters are shown in Tables (1-3) and Fig. (B). In these tables, for different amounts of y in the range of [0, 1] error rate is very low, which indicates the good complying between the two methods. Plus, Fig(A) is shown the timings of the solution methods. fig (A) indicates that AGM and HPM are high accuracy manners to solve these issues. Figs. (2-3) the comparison between AGM, CM, HPM methods and Flex-PDE software to solve this problem for silver–water Nano fluid has been shown. As seen in this figs, for different amounts of active parameter error rate is very small, the slight error in results indicates that AGM is a high precision method to solve these issues. As well as, in this research the efficacy of the three dimensionless numbers such as the Hartmann number, Reynolds number and Nano fluid volume fraction on non-dimensional velocity profiles are studied. Fig. 4 shows the impact of nanoparticle volume fraction (∅) on V(y) and U(y) for water with silver nanoparticles when Ha = 4 and Re = 1.5. For both cases, velocity profiles thickness declines with the increment of nanoparticle volume fraction. Figs. (5-6) show the impact of Hartman number (Ha) on dimensionless velocities for water with silver. Generally, when the magnetic field is imposed on the enclosure, the velocity field repressed owing to the retarding influence of the Lorenz force. For down Reynolds number (Re), as Hartmann number (Ha) increases V(y) increases for $y < y_m$ but opposite trend is observed for $y > y_m, y_m$ is a meeting dot that all curves common simultaneously at this dot. Also, when Re increases this meeting point small shifts to the solid wall. Figs. (7-8) show the influence of Reynolds number (Re) on dimensionless velocities for water with silver nanoparticles when Ha = 1 and Ha = 8. According to the figs. (7-8), by increasing of *Re* the value of dimensionless velocities decline. Also in this figs, when Ha is small, the impact of Re number is very perceptible but in Ha large, Re number is less impact on the velocity profiles. In addition, it is noteworthy that the *Re* shows the relative importance of the inertia impact compared to the viscous influence. Plus, increasing Re leads to an increase in the amount of the skin friction factor. Furthermore, Contour plots of the impact of nanoparticle volume fraction (∅), Hartmann number (Ha) and Reynolds number (Re) in wide range of data are depicted in Figs. (9-10).

6. CONCLUSION

In the present study, laminar Nano fluid flow in a semi-porous channel in the attendance of uniform magnetic field has been solved using Akbari-Ganji's Method (AGM). The concept of Akbari-Ganji's Method (AGM) is briefly introduced and employed to derive solutions of nonlinear equations. The comparison of the outcomes of AGM with the outcomes of the CM, the Numerical Method (fourth-order Runge-Kutta), HPM method and Flex-PDE software outcomes was conducted. This research shows that AGM is a strong method to solve nonlinear differential equations. Also, the impact of the three dimensionless numbers like the Nano fluid volume fraction, Reynolds number and Hartmann number on non-dimensional velocity profiles are studied. Outcomes show when Ha is small, the impact of Re number is very perceptible on the velocity profiles but in Ha large, Re number is less impact. Also, velocity profiles thickness decreases with the increase of nanoparticle volume fraction ($\emptyset$).

Appendix A.

Parts of Flex-PDE software codes**:**

Title' ***Nano fluid Flow in a Semi-Porous Channel'*** *Coordinates cartesian1 Variables Definitions* $Re = 1$ $Ha = 1$ $phi = 0.04$ $\rho_{of} = 997.1$ $\rho_{os} = 8933$ $\delta_f = 0.05$ $\delta_s = 5.69 \times 10^{(-7)}$

$A = (1-\emptyset) + (\rho_{os}/\rho_{of}) \times \emptyset$ $B=1 + (3 \times \emptyset \times ((\delta_s/\delta_f) - 1))/((\delta_s/\delta_f) + 2) + (((\delta_s/\delta_f) - 1) \times \emptyset)$ *Equations* H: $H = dxx\,(V)$

V: $dxx\,(H) - Ha2 \times B \times (1-\emptyset)^{2.5} \times dxx\,(V) - Re \times A \times (1-\emptyset)^{2.5} \times (dx\,(V) \times dxx\,(V) - V \times dx\,(H)) = 0$

U: $U \times dx\,(V) - V \times dx\,(U) - (1/Re) \times (1/(A \times (1-\emptyset)^{2.5})) \times (dxx\,(U) - Ha2 \times B \times (1-\emptyset)^{2.5} \times U) = 0$

Boundaries *Region 1* *Start* (0) *Point value* (V) =0 *Point load* (H) =0 *Point value* (U) =1 *Line to* (1) *Point value* (V) =1 *Point load* (H) =0 *Point value* (U) =0

Plots

Elevation (V) *from* (0) *to* (1) *Elevation* (U) *from* (0) *to* (1)

End.

Appendix B.

Nomenclature			
$\boldsymbol{A^*}, \boldsymbol{B^*}$	Constant parameter		Greek symbols
$\boldsymbol{P}$	Fluid pressure	$\boldsymbol{\upsilon}$	Kinematic viscosity
$\boldsymbol{q}$	Mass transfer parameter	$\boldsymbol{\sigma}$	Electrical conductivity
$\boldsymbol{x_k}$	General coordinates	$\boldsymbol{\varepsilon}$	Aspect ratio h/Lx
$\boldsymbol{f}$	Velocity function	$\boldsymbol{\mu}$	Dynamic viscosity
$\boldsymbol{\bar{k}}$	Fluid thermal conductivity	$\boldsymbol{\rho}$	Fluid density
$\boldsymbol{n}$	Power law index in temperature distribution		
$\boldsymbol{u}, \boldsymbol{v}$	Dimensionless components velocity in x and y directions, respectively		
$\boldsymbol{u^*}, \boldsymbol{v^*}$	Velocity components in x and y directions respectively		
$\boldsymbol{x}, \boldsymbol{y}$	Dimensionless horizontal, vertical coordinates respectively		
$\boldsymbol{x^*}, \boldsymbol{y^*}$	Distance in x, y directions parallel to the plates		

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