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**Research Article**

# ESTIMATING WIND ENERGY POTENTIAL WITH PREDICTING BURR LSM PARAMETERS: A DIFFERENT APPROACH

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**ABSTRACT**

Estimating wind energy potential and wind speed frequency are important for planning wind energy conversion plants. Probability distribution functions are utilized to model wind speed distributions.
In this study, an estimation was model designed by using the least squares method to predict the wind speed density with the Burr distribution, which has not been studied before. To confirm this model, the annual data of eight different weather stations were analysed, and the results were compared with the Weibull distribution model, which is the most popular one in the literature. For predicting the parameters of both models least square method and maximum likely methods were used. Regarding the comparison results, the performance of designed estimation model (Burr LSM) is higher than the Weibull distribution models, especially for the locations with higher average wind speeds. The results show that the Burr LSM is better than the others for seven of eight weather stations in terms of the power density.
**Keywords:** Burr probability distribution function, forecasting, modelling, probability distribution functions, wind energy potential, wind speed distribution.
**Highlights**:

- Formulas to determine the Burr distribution parameters of *a* and *b* based on the least squares method with increasing values of *k* were given for the first time in the literature.
- Frequencies obtained from Burr.pdf LSM (Burr probability density function - Least Squares Method) were compared with the measured wind speed frequencies and Weibull model.
- Wind power densities obtained from Burr.pdf and W.pdf were compared with the measured wind power densities.

## 1. INTRODUCTION

Wind energy is attractive for investors as one of renewable energy sources to increase incomes at a crucial time when fossil fuel reserves rapidly diminish, while demands for energy gradually increases. Thus, investors are forced by this phenomenon to seek alternative energy sources not only for enhancing sustainability but also for avoiding environmental pollution as well [1]. Today it is an obvious fact that energy consumption tends to increase due to

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technological developments, population growth, and increasing living standards. Being one of the fast-growing country, Turkey's electrical energy demand also grows by about 8 % each year [2]. To meet the energy demands of Turkey, therefore, power capacity should be increased. The installed power capacity of Turkey by sources for last 5 years is given in Table 1. In Turkey, the installed wind power was about 4,561 GW by the end of year 2015 [3, 4], yet now Turkey's wind energy potential is 48 GW. Hence the market opportunity for wind energy investors is higher in Turkey.

Characterization of the wind speed is important for the investments in this area, as well as for the utilization of this tremendous wind energy source [1]. To get this, wind speed measurements must be made every year. Calculation and analysis of the wind speed distribution, assessment of wind energy potential and forecasting wind energy are important for identification and design of wind farms [1, 5].

Thus, distribution functions related to the wind speed frequency provide vital information for wind energy applications [1, 5]. There have been many studies on distribution functions and wind energy potential [1, 2, 4, 5, 7-11]. Feasibility of installing wind turbines and estimating wind power potential have been investigated for Turkey [2, 7-12], Greece [13, 14], Germany [15], Italy [5], Spain [16], Iran [17- 23] and Saudi Arabia [24, 25]. Particularly studies on probability distribution and Weibull distribution can be divided into three categories. Studies in the first category are mainly on the wind speed frequency characterization and estimating wind energy potential. In the second category, modifications of Weibull distribution (or other distribution functions) are performed. [25], and the estimations of distribution parameters are considered as the third category [9, 11, 27, 28].

**Table 1.** Installed power capacity of Turkey by sources [3, 6]

| Installed Power Capacity | 2011 | 2012 | 2013 | 2014 | 2015 |
|---|---|---|---|---|---|
| Thermic Energy | 33931.1 | 35027.2 | 38648 | 41800.7 | 41848.6 |
| Hydraulic Energy | 17137.1 | 19609.4 | 22289 | 23640.9 | 26137.2 |
| Wind Energy | 1728.70 | 2260.5 | 2759.60 | 3629.7 | 4561.4 |
| Geothermal Energy | 114,20 | 162.2 | 310.80 | 404.9 | 979.8 |
| Solar Energy | -- | -- | -- | 40.18 | 327.6 |
| Sum | 52911.1 | 57059.4 | 64.007.5 | 69516.4 | 73854.6 |

Weibull distribution model is commonly used for the wind speed distribution analysis, but many of data obtained from weather stations show that values were incorrectly estimated. For example, the high number of calm sample values and bimodal sample values cannot be analysed for a true estimation [29]. Thus, various probability density functions are used for better solutions. Burr distribution model is one of promising models but with the distribution parameters (shape and scale), it is hard to predict [5,16].

In this study, least squares method (LSM) was used to calculate Burr distribution parameters as a new approach. The formulas were proposed to predict Burr distribution parameters with this method (Burr.pdf LSM). As most used distribution models, Weibull [9, 11, 27, 28] and Burr probability density functions were evaluated and compared, while modelling wind speed frequency was used with the data of eight different weather stations: Karabük City Centre as residential potential, Zonguldak as a Black Sea coast, Osmaniye as a Mediterranean Sea coast, Söke weather station in Aydın [33], Karabük Kahyalar an efficient point in Karabük zone, Loras weather station located on the Mountain Loras in Konya [34], Mersinkoy as a Aegean Sea coast in Izmir and Gelibolu weather station just below the Marmara Sea [35]. These stations are located at different distances throughout Turkey.

First, mathematical models of Weibull and B urr distribution functions were introduced. As a new approach, the formulas of Burr.pdf LSM were proposed to predict Burr distribution parameters with least squares method. Also, the prediction method of model parameters was described. Then the data of eight weather stations were used to estimate the distribution parameters. And the wind power density generated by the model of probability density distribution functions were compared with real data, which were obtained from the weather stations. Then, distributions of Weibull and Burr are evaluated according to the coefficient of determination ($R^2$) and root mean square error (*RMSE*). The results show that Burr LSM is better than the others for seven of eight weather stations in terms of power density. And root mean square error (RMSE) results show that Burr LSM is better than the others for five of eight weather stations in terms of wind speed distribution. Also wind power densities of eight locations were estimated with both models, and it has been found that Burr LSM is better than Weibull LSM except Gelibolu station. Therefore, Burr LSM can be considered a new model to estimate wind Energy potential.

## 2. MATHEMATICAL MODELLING

### 2.1. Weibull Probability Distribution

Weibull probability distribution function is known through the descriptions; *f(v)* is probability related to the wind speed data; $k$ is dimensionless shape parameter; $c$ is scale parameter (m/s) and $v$ is observed wind speed data as seen below [1];

$$f(v) = \frac{k}{c}\left(\frac{v}{c}\right)^{k-1} exp\left(-\left(\frac{v}{c}\right)^{k}\right) \ , \qquad k > 0,\ c > 0,\ v > 0 \tag{1}$$

Commonly used techniques for parameter estimation are least squares method (LSM) and maximum likelihood method (MLM) [1, 9, 28].

#### 2.1.1. Least Square Method (LSM)

The least square method (W.pdf LSM, Burr.pdf LSM) and maximum likelihood method (W.pdf MLM, Burr.pdf MLM) are used to predict the Weibull distribution parameters and Burr distribution parameters as well.

LSM is one of the most common method in statistical estimation field [9, 16, 28]. The cumulative distribution function of two-parameters W.pdf is represented $F_i$ cumulative frequency, with $v_i$ wind speed intervals at $i^{th}$ position [16, 28]. Eq. (3) is obtained by taking logarithm Eq. (2) as linearized form. To minimize squares of error sum, which is seen on Eq. (4), Weibull distribution parameters $k$ and $c$ can be calculated by Eq. (5) and Eq. (6) [9, 28].

$$F(v_i) = 1 - exp\left[-\left(\frac{v_i}{c}\right)^{k}\right] \tag{2}$$

$$\text{Ln}\big(-\ln\big(1 - F(v_i)\big)\big) = -k\ln c + k\ln v_i \tag{3}$$

$$\sum_{i=1}^{n}\Big[\ln\big(-\ln\big(1 - F(v_i)\big)\big) - [-k\ln c + k\ln v_i]\Big]^2 \tag{4}$$

$$k = \frac{n\sum_{i=1}^{n}(\ln v_i)\big[\ln\big(-\ln\big(1-F(v_i)\big)\big)\big] - \sum_{i=1}^{n}\ln v_i \sum_{i=1}^{n}\ln\big[-\ln\big(1-F(v_i)\big)\big]}{n\sum_{i=1}^{n}\ln(v_i{}^2) - \big[\sum_{i=1}^{n}\ln v_i\big]^2} \tag{5}$$

$$c = exp\left[\frac{k\sum_{i=1}^{n}\ln v_i - \sum_{i=1}^{n}\ln\big(-\ln\big(1-F(v_i)\big)\big)}{n.k}\right] \tag{6}$$

#### 2.1.2. Maximum Likelihood Method (MLM)

MLM is based on maximizing likelihood function. For two-parameter Weibull distribution, the likelihood function is seen on Eq. (7) [28].

The values of *k* and *c* to maximize Eq. (7), can be calculated by taking natural logarithm of Weibull Maximization likelihood function Eq. (7). Then the result is seen on Eq. (8). By partial derivatives of Eq. (8) according to *c* and *k*, Eq. (9) and Eq. (10) were obtained to reach the values of *k* (Eq. (11)) and *c* (Eq. (12)). Standard iterative techniques or Newton Raphson method should be used to solve Eq. (11).

Eq. (13) was obtained by Newton Raphson method iterations for the solution of $k_m$ value, for $m^{th}$ iteration step [31] with first iteration value, $k_0$ is following as seen on Eq. (14) [28].

$$L(v,c,k) = \prod_{i=1}^{n} kc^{-k} v_i^{k-1} exp\left(-c^{-k} v_i^k\right) \quad (7)$$

$$\ln L\,(v,c,k) = \sum_{i=1}^{n} \ln k - \sum_{i=1}^{n} k \ln c + (k-1) \sum_{i=1}^{n} \ln v_i - c^{-k} \sum_{i=1}^{n} {v_i}^k \quad (8)$$

$$\frac{\partial \ln L\,(v;c,k)}{\partial c} = -nkc^{-1} + kc^{-(k+1)} \sum_{i=1}^{n} {v_i}^k \quad (9)$$

$$\frac{\partial\, ln\, L\,(v;c,k)}{\partial k} = nk^{-1} - n\, ln\, c + \sum_{i=1}^{n} ln\, v_i - c^{-k} \sum_{i=1}^{n} v^k\, ln\, v_i + c^{-k}\, ln\, c \sum_{i=1}^{n} v^k \quad (10)$$

$$\frac{\sum_{i=1}^{n} {v_i}^k \ln v_i}{\sum_{i=1}^{n} {v_i}^k} - \frac{\sum_{i=1}^{n} \ln v_i}{n} - \frac{1}{k} = 0 = f_k \quad (11)$$

$$c = \left(\frac{\sum_{i=1}^{n} v_i^k}{n}\right)^{\frac{1}{k}} \quad (12)$$

$$k_{m+1} = k_m - \frac{\left[\frac{\sum_{i=1}^{n} {v_i}^{k_m} \ln v_i}{\sum_{i=1}^{n} {v_i}^{k_m}} - \frac{\sum_{i=1}^{n} \ln v_i}{n} - \frac{1}{k_m}\right]}{\left.\frac{df_k}{dx}\right|_{k_m}} \quad (13)$$

$$k_0 = \left[\frac{\frac{6}{\pi^2}\left[\sum_{i=1}^{n} (\ln v_i)^2 - \frac{\left(\sum_{i=1}^{n} \ln v_i\right)^2}{n}\right]}{n-1}\right]^{-\frac{1}{2}} \quad (14)$$

### 2.2. Burr Distribution (Singh Maddala Distribution)

In the last decade, the Burr distribution given by Eq. (15) has been applied to estimate the wind speed frequency, and it has given well performed results [5].

$$f(v,a,b,k) = \frac{a.k.\left(\frac{v}{b}\right)^{a-1}}{b\left(1+\left(\frac{v}{b}\right)^a\right)^{k+1}} \quad (15)$$

Three-parameters Burr cumulative distribution function with *a* is the shape parameter, while *b* and *k* are the scale parameters, and *v* represents the wind speed as given by Eq. (16).

$$F(v,a,b,k) = 1 - \left(1 + \left(\frac{v}{b}\right)^a\right)^{-k} \quad (16)$$

#### 2.2.1. Least Squares Method (LSM)

Eq. (20) is obtained by linearizing Eq. (16) with algebraic steps Eq. (17) – Eq. (19).

$$1 + \left(\frac{v}{b}\right)^a = (1-P)^{-\frac{1}{k}} \quad (17)$$

$$\left(\frac{v}{b}\right)^a = (1-P)^{-\frac{1}{k}} - 1 \quad (18)$$

$$a\, ln\left(\frac{v}{b}\right) = ln\left[(1-P)^{-\frac{1}{k}} - 1\right] \tag{19}$$

$$a\, ln\, v - a\, ln\, b = ln\left[(1-P)^{-\frac{1}{k}} - 1\right] \tag{20}$$

Eq. (20) can be revised to Eq. (21) with the assumptions,

$$a \ln v_i = ax_i, -a \ln b = B \text{ and } \ln\left[(1-P)^{-\frac{1}{k}} - 1\right] = Y_i.$$

So, Eq. (21) can be accepted as follows,

$$ax_i + B = Y_i \tag{21}$$

We offered a new method, which would be re-calculated by using $a$ and $b$ iteratively and increasing values of $k$ parameter, to obtain Burr parameters $a$ and $b$. Thus, we would be able to choose optimum values of $a$, $b$, and $k$ parameters by Eq. (22) – Eq. (25).

$$E(a, B) = \sum e_i^2 = \sum (Y_i - ax_i - B)^2 \tag{22}$$

$$a = \frac{n\sum x_i y_i - \sum y_i \sum x_i}{n\sum x_i^2 - (\sum x_i)^2} \tag{23}$$

$$B = \frac{\sum x_i^2 \sum y_i - \sum x_i y_i \sum x_i}{n\sum x_i^2 - (\sum x_i)^2} \tag{24}$$

$$b = \exp\left(\frac{-B}{a}\right) \tag{25}$$

### 2.2.2. Maximum Likelihood Method (MLM)

Maximum likelihood function for the Burr distribution is given on Eq. (26). And Eq. (27) is obtained by taking the logarithm of Eq. (26). Then Eq. (28) - Eq. (30) are obtained respectively by taking partial derivatives of Eq. (27) and equalizing to zero to calculate the parameters of $a$, $b$ and $k$ [5].

The values of the parameters $a$, $b$ and $k$, which maximize Eq. (26), can be calculated via Eq. (28) - Eq. (30). Newton-Raphson method, which is commonly solved method of Eq. (28) - Eq. (30), follows Eq. (31). Jacobian matrix is formed by taking partial derivatives of the expressions $S_1, S_2$ and $S_3$. Then $a, b$ and $k$ can be calculated via Eq. (31) iteratively.

$$L(v, a, b, k) = \prod_{i=1}^{n} \frac{a.k\left(\frac{v}{b}\right)^{a-1}}{b\left[1+\left(\frac{v}{b}\right)^{a}\right]^{k+1}} \tag{26}$$

$$\ln L(v, a, b, k) = \sum\left[\ln(a.k.b^{-1}) + (a-1)\ln\left(\frac{v}{b}\right) - (k+1)\left(\ln\left[1+\left(\frac{v}{b}\right)^{a}\right]\right)\right] \tag{27}$$

$$S_1 = \frac{\partial LL(v,a,b,k)}{\partial a} = \sum\left[\frac{1}{a} + ln\left(\frac{v_i}{b}\right) - \frac{(k+1)\, ln\left(\frac{v_i}{b}\right)}{\left(1+\left(\frac{b}{x}\right)^{a}\right)}\right] \tag{28}$$

$$S_2 = \frac{\partial LL(v,a,b,k)}{\partial b} = \sum\left(-\frac{1}{k+1} + \left[1+\left(\frac{b}{v_i}\right)^{a}\right]^{-1}\right) \tag{29}$$

$$S_3 = \frac{\partial LL(v,a,b,k)}{\partial k} = \sum\left(-\frac{1}{k} + \ln\left[1+\left(\frac{v_i}{b}\right)^{a}\right]\right) \tag{30}$$

$$\begin{bmatrix} a_{i+1} \\ b_{i+1} \\ k_{i+1} \end{bmatrix} = \begin{bmatrix} a_i \\ b_i \\ k_i \end{bmatrix} - \begin{bmatrix} \frac{\partial S_1}{\partial a} & \frac{\partial S_1}{\partial b} & \frac{\partial S_1}{\partial k} \\ \frac{\partial S_2}{\partial a} & \frac{\partial S_2}{\partial b} & \frac{\partial S_2}{\partial k} \\ \frac{\partial S_3}{\partial a} & \frac{\partial S_3}{\partial b} & \frac{\partial S_3}{\partial k} \end{bmatrix}^{-1}_{(a_i, b_i, c_i)} . \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix}_{(a_i, b_i, c_i)} \tag{31}$$

## 3. APPROACHES TO WIND DATA AND ANALYSIS

The weather stations must consist of two or three anemometers, two direction sensors, a humidity meter, a pressure gauge, a temperature measurement device, a data logger and a data transfer modem, regarding the measurement standards [31]. Wind speed data were analysed in eight different weather stations for one year for this survey. The descriptive statistics of the wind speed data were presented on Fig. 1..

**Table 2.** Annual measured wind speed distribution

| | Vr | Karabük | Zonguldak | Osmaniye | Söke | Kahyalar | Loras | Mersinkoy | Gelibolu |
|---|---|---|---|---|---|---|---|---|---|
| 0-0.5 | 0 | 0.0348 | 0.0305 | 0.0605 | 0.0836 | 0.0040 | 0.0882 | 0.0064 | 0.0047 |
| 0.5-1.5 | 1 | 0.7445 | 0.3686 | 0.3577 | 0.2314 | 0.1811 | 0.0994 | 0.0491 | 0.0401 |
| 1.5-2.5 | 2 | 0.1463 | 0.3546 | 0.2596 | 0.1674 | 0.2405 | 0.1337 | 0.0898 | 0.0668 |
| 2.5-3.5 | 3 | 0.0416 | 0.1505 | 0.1118 | 0.1913 | 0.2071 | 0.1414 | 0.1281 | 0.0901 |
| 3.5-4.5 | 4 | 0.0172 | 0.0614 | 0.0784 | 0.1761 | 0.1249 | 0.1248 | 0.1276 | 0.0537 |
| 4.5-5.5 | 5 | 0.0102 | 0.0209 | 0.0603 | 0.1035 | 0.0809 | 0.0956 | 0.1100 | 0.0825 |
| 5.5-6.5 | 6 | 0.0041 | 0.0072 | 0.0447 | 0.0346 | 0.0588 | 0.0763 | 0.1024 | 0.0808 |
| 6.5-7.5 | 7 | 0.0002 | 0.0038 | 0.0168 | 0.0084 | 0.0417 | 0.0595 | 0.0957 | 0.0784 |
| 7.5-8.5 | 8 | 0.0006 | 0.0016 | 0.0058 | 0.0019 | 0.0254 | 0.0473 | 0.0847 | 0.0750 |
| 8.5-9.5 | 9 | 0.0002 | 0.0007 | 0.0028 | 0.0009 | 0.0173 | 0.0340 | 0.0630 | 0.0725 |
| 9.5-10.5 | 10 | 0.0001 | 0.0001 | 0.0008 | 0.0006 | 0.0090 | 0.0262 | 0.0441 | 0.0730 |
| 10.5-11.5 | 11 | 0.0001 | 0.0001 | 0.0002 | 0.0002 | 0.0045 | 0.0215 | 0.0324 | 0.0608 |
| 11.5-12.5 | 12 | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0027 | 0.0155 | 0.0243 | 0.0545 |
| 12.5-13.5 | 13 | 0.0000 | 0.0000 | 0.0003 | 0.0000 | 0.0011 | 0.0124 | 0.0156 | 0.0427 |
| 13.5-14.5 | 14 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0005 | 0.0082 | 0.0106 | 0.0352 |
| 14.5-15.5 | 15 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0003 | 0.0062 | 0.0066 | 0.0225 |
| 15.5-16.5 | 16 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0040 | 0.0038 | 0.0173 |
| 16.5-17.5 | 17 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0029 | 0.0029 | 0.0137 |
| 17.5-18.5 | 18 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0014 | 0.0017 | 0.0114 |
| 18.5-19.5 | 19 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0007 | 0.0007 | 0.0081 |
| 19.5-20.5 | 20 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0003 | 0.0063 |
| 20.5-21.5 | 21 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0001 | 0.0043 |
| 21.5-22.5 | 22 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0002 | 0.0000 | 0.0025 |
| 22.5-23.5 | 23 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0014 |
| 23.5-24.5 | 24 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0011 |
| 24.5-25.5 | 25 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0000 | 0.0001 | 0.0000 | 0.0006 |

The weather stations are located in Karabük city center with the altitude of 278 m., and Zonguldak as coast city which is neighbor city of Karabük 110 km away, and Osmaniye with the altitude of 120m., which is 20 km away from Mediterranean Sea coast [36], and Söke with the altitude 44 m., and Kahyalar which was established by KARES Mall on a location in Kahyalar Village of Karabük with an altitude of 610 m and 10 km away from the city centre. Others are located on the Mountain Loras in Konya by Konya water and sewage administration (KOSKİ), and Mersinkoy as an Aegean Sea coast and Gelibolu coast as well on the peninsula in Marmara [32].

Annual average wind speeds of the weather stations were calculated as seen on Fig. 1. Besides, annual standard deviation and skewness wind speeds of the weather stations can be seen on the Fig. 1. Maximum wind speeds of Gelibolu, Mersinkoy Izmir, Karabuk Loras Mountain in Konya, Kahyalar in Karabuk, Söke in Aydın, Osmaniye, Zonguldak, Karabuk City were observed as 25 m/s, 21.5 m/s, 25.96 m/s, 18.488 m/s, 10.92 m/s 13.1 m/s, 9.8 m/s and 10.8 m/s respectively.

Additionally, wind speed frequencies were displayed on Table 2., which are needed to estimate the parameters of the distribution functions.

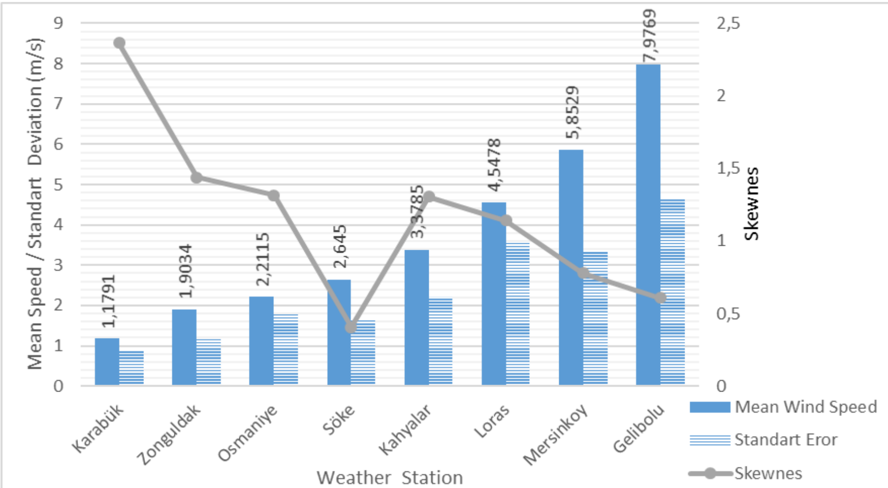


**Figure 1.** Yearly mean wind speed of the weather stations

### 3.1. Wind Power and Energy Density

*P*, which represents wind power density per square meter, can be estimated by Eq. (32) with the variables: density of weather, ρ and wind speed, *v*:

$$P = \frac{1}{2}\rho v^3 \tag{32}$$

Periodical mean wind power density is formulated by Eq. (33) with the function frequency of wind speed $f(v)$ [23];

$$P_{w,d} = \frac{1}{2}\rho \int_0^{v_{max}} v^3 f(v)\, dv \tag{33}$$

The feasibility criterion of power density is classified in 4 levels as follows [22]:

- Weak Resource ($P_{w,d}$<100 W/m$^2$)
- Weakly Good (100 W/m$^2$< $P_{w,d}$< 300 W/m$^2$)
- Good (300 W/m$^2$ < $P_{w,d}$< 700 W/m$^2$)
- Very Good ($P_{w,d}$> 700 W/m$^2$)

The weather stations, whose data were used in our survey, can be named according to this classification as seen on Table 3.

The highest mean power density values were 1084.8 W/m2 in Gelibolu, 433.1992 W/m$^2$ in Izmir Mersinkoy, 307 W/m2 in Konya Loras. Gelibolu, Mersinkoy and Loras can be accepted as good energy resource to establish a wind power plant. Power density of Kahyalar village was 103.74 W/m$^2$ which it is between 100 W/m$^2$ - 300 W/m$^2$ annually, it is accepted as weak good resource for the wind power classification, means it can be used for small-scale applications.

The mean power density values were 43.8 W/m$^2$ in Aydın Söke, 40.55 W/m$^2$, 18.23 W/m$^2$ Zonguldak and the lowest average power density was 10.36 W/m$^2$ in Karabuk city centre. Karabuk City centre, Zonguldak, Osmaniye and Söke are not feasible wind energy source for generating electricity to meet all the energy needs in the region. However, it can be considered for utilization of small-scale wind energy applications in Söke, Osmaniye, Zonguldak and Karabuk city centre for rural areas such as traffic warning signs, wireless internet gateways, battery chargers, and water pumps.

**Table 3.** Annual mean wind speeds, standard deviation and power densities

| Stations | Mean Speed (m/s) | Std Deviation (m/s) | Measured (W/m$^2$) | Class of Power Density |
|---|---|---|---|---|
| Karabük | 1,3636 | 0,7773 | 5,4334 | Weak |
| Zonguldak | 1,903 | 1,128 | 16,7663 | Weak |
| Osmaniye | 2,3024 | 1,6625 | 40,5524 | Weak |
| Söke | 2,6610 | 1,5163 | 43,8541 | Weak |
| Kahyalar | 3,3914 | 2,2136 | 103,7443 | Weakly Good |
| Loras | 4,5339 | 3,2443 | 307,9826 | Good |
| Mersinkoy | 5,9045 | 3,3138 | 433,1992 | Good |
| Gelibolu | 8,0085 | 4,5895 | 1084,8000 | Very Good |

### 3.2. Estimating the Parameters and Comparison with Real Wind Data

We computed Weibull *[shape (k) and scale (c)*] and Burr [*shape (a) and scale (b, k)*] distribution parameters with LSM and MLM equations [*Eq. (5) –Eq (31)*] as seen on Table 4. by using wind speed frequencies from Table 2.. Then we used predicted Weibull and Burr parameters to estimate power density values. Measured values and those estimated values by using models were compared on Table 6. As seen on Table 5., Burr LSM is best fitting distribution for 5 stations (Söke, Kahyalar, Loras, Mersinkoy and Gelibolu) and Burr MLM is second better fitting distribution for remaining’s (Karabük, Zonguldak and Osmaniye).

**Table 4.** Estimation parameters of Weibull pdf and Burr pdf

| | | Weibull | | Burr | | |
|---|---|---|---|---|---|---|
| | | k | c | a | b | K |
| Karabük | LSM | 0,3474 | 0,2357 | 7,0906 | 0,1347 | 0,116 |
| | MLM | 1,9554 | 4,7375 | 12,7352 | 1,0054 | 0,0494 |
| Zonguldak | LSM | 0,7925 | 1,6045 | 1,3684 | 1,5102 | 1,459 |
| | MLM | 1,9554 | 4,7375 | 12,7352 | 1,0054 | 0,0494 |
| Osmaniye | LSM | 0,7105 | 2,2266 | 1,7046 | 1,0421 | 0,72 |
| | MLM | 1,9109 | 5,4733 | 13,9945 | 1,0041 | 0,0412 |
| Söke | LSM | 1,005 | 3,3186 | 1,0972 | 16,0902 | 6,315 |
| | MLM | 1,9554 | 4,7375 | 12,735 | 1,0054 | 0,0494 |
| Kahyalar | LSM | 1,1572 | 3,2089 | 1,6052 | 5,31 | 2,981 |
| | MLM | 1,8408 | 7,305 | 16,9304 | 1,0026 | 0,0292 |
| Loras | LSM | 0,9245 | 6,6764 | 1,3438 | 5,0246 | 1,272 |
| | MLM | 1,7873 | 9,8575 | 20,5795 | 1,0025 | 0,0209 |
| Mersinkoy | LSM | 1,5665 | 6,3241 | 1,6022 | 50,6195 | 29 |
| | MLM | 1,8138 | 8,4002 | 18,5453 | 1,0024 | 0,025 |
| Gelibolu | LSM | 1,5677 | 8,3438 | 1,6017 | 66,8203 | 29 |
| | MLM | 1,7819 | 10,2214 | 20,9996 | 1,0029 | 0,0202 |

### 3.3. Performance Evaluation

The performances of Weibull and Burr were evaluated according to the coefficient of determination ($R^2$) and root mean square error (*RMSE*). $R^2$ was calculated with Eq. (34) by using predicted probability distribution value $f_i$ and observed frequency value $p_i$ [31].

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(f_i - p_i)^2}{\sqrt{(\sum_{i=1}^{n}(f_i - \bar{f})^2)(\sum_{i=1}^{n}(p_i - \bar{p})^2)}} \tag{34}$$

A smaller *RMSE* value shows the better model [24]. *RMSE* values were calculated by Eq. (35).

$$RMSE = \left[\frac{1}{n}\sum_{i=1}^{n}(f_i - p_i)^2\right]^{0,5} \tag{35}$$

*RMSE* and $R^2$ values of distribution models were compared on Table 5. Performance criteria of Burr LSM are best fitting distribution for 5 stations (Osmaniye, Söke, Kahyalar, Loras and Mersinkoy) by considering *RMSE*, and second better fit value for remaining's (Karabük, Zonguldak and Gelibolu). Burr LSM and MLM are best fitting distribution for 4 stations (Karabük, Zonguldak, Osmaniye and Mersinkoy) by considering$R^2$. As a result, Burr LSM and Burr MLM are better than Weibull LSM and Weibull MLM, so that Burr distribution model values are better than that of Weibull except Söke, Kahyalar and Loras, even similar (Table 5.).

**Table 5.** Performance of models to estimate wind speed frequencies

| Stations | Criteria | Weibull LSM | Weibull MLM | Burr LSM | Burr MLM |
|---|---|---|---|---|---|
| Karabük | *RMSE* | 0,1939 | 0,2147 | 0,1796 | **0,1404** |
| | $R^2$ | 0,5177 | 0,7375 | 0,4544 | 0,**7533** |
| Zonguldak | *RMSE* | 0,073 | 0,1221 | 0,0637 | **0,055** |
| | $R^2$ | 0,8949 | 0,9249 | 0,8827 | 0,**9295** |
| Osmaniye | *RMSE* | 0,0523 | 0,0999 | **0,0295** | 0,0321 |
| | $R^2$ | 0,9738 | 0,9726 | 0,9699 | 0,**974** |
| Söke | *RMSE* | 0,0378 | 0,0551 | **0,0374** | 0,0477 |
| | $R^2$ | 0,**9647** | 0,9516 | 0,964 | 0,9543 |
| Kahyalar | *RMSE* | 0,024 | 0,0643 | **0,0209** | 0,0347 |
| | $R^2$ | 0,**9835** | 0,9811 | 0,982 | 0,9816 |
| Loras | *RMSE* | 0,0189 | 0,0357 | **0,0162** | 0,0227 |
| | $R^2$ | 0,**9953** | 0,9929 | 0,9949 | 0,993 |
| Mersinkoy | *RMSE* | 0,0107 | 0,0183 | **0,0106** | 0,0222 |
| | $R^2$ | 0,9959 | 0,9925 | **0,9959** | 0,9928 |
| Gelibolu | *RMSE* | **0,0102** | 0,0094 | 0,0107 | 0,0258 |
| | $R^2$ | 0,9975 | 0,9944 | **0,9975** | 0,9948 |

**Table 6.** Comparison of measured and estimated power densities

| Stations | Measured (W/m$^2$) | Weibull LSM | Weibull MLM | Burr LSM | Burr MLM |
|---|---|---|---|---|---|
| Karabük | **7,1011** | 19,6904 | 138,1251 | **14,7921** | 86,6853 |
| Zonguldak | **18,2386** | 40,1808 | 138,1251 | **30,9681** | 86,6853 |
| Osmaniye | **40,5524** | 95,8914 | 217,6516 | **67,3531** | 130,4146 |
| Söke | **43,8541** | 99,8224 | 138,1251 | ***93,4616*** | **88,8139** |
| Kahyalar | **103,7443** | 117,2848 | 536,9436 | **94,9225** | 301,5609 |
| Loras | **307,9826** | 986 | 1360.8 | **742** | 882.7 |
| Mersinkoy | **433,1992** | 453.8787 | 829.0359 | **445,488** | 733.9691 |
| Gelibolu | **1084,8000** | **1019.** | 1522.1 | 998.7 | 1197.5 |

### 3.4. Graphical Analysis

Wind speed frequency graphs, which were obtained by Weibull and Burr distribution parameters for eight weather stations, were drawn on Fig. 2. - Fig. 9. one by one, and all of them were compared with real wind speed frequencies. Burr LSM graphs are best fitting graphs with real measured values for eight stations as seen on Fig. 2-9..

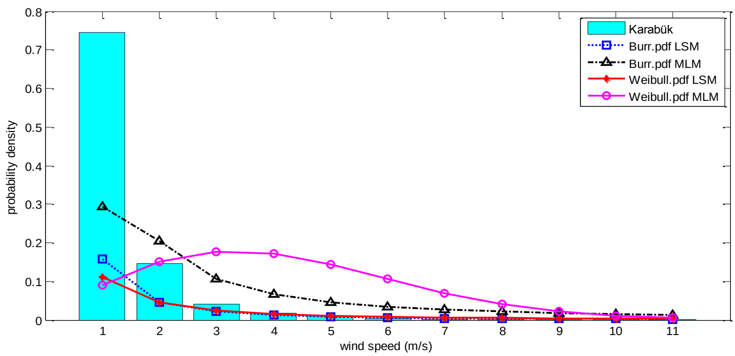


**Figure 2.** Actual wind speed density and the wind speed density produced by distributions for Karabuk city centre

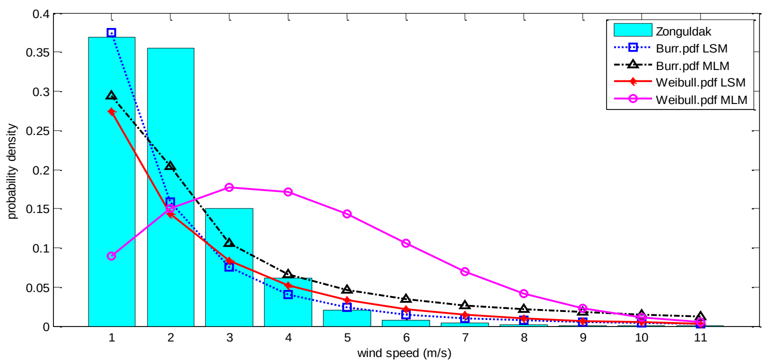


**Figure 3.** Actual wind speed density and the wind speed density produced by distributions for Zonguldak

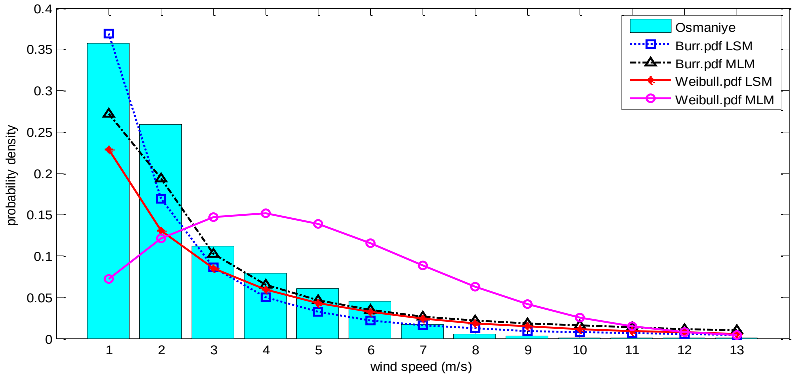


**Figure 4.** Actual wind speed density and the wind speed density produced by distributions for Osmaniye

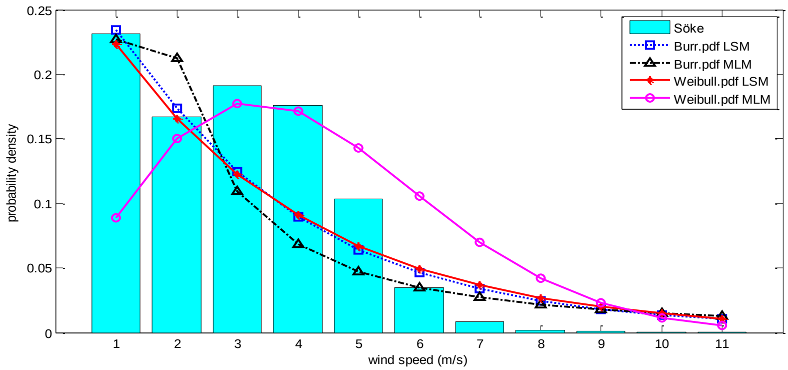


**Figure 5.** Actual wind speed density and the wind speed density produced by distributions for Söke

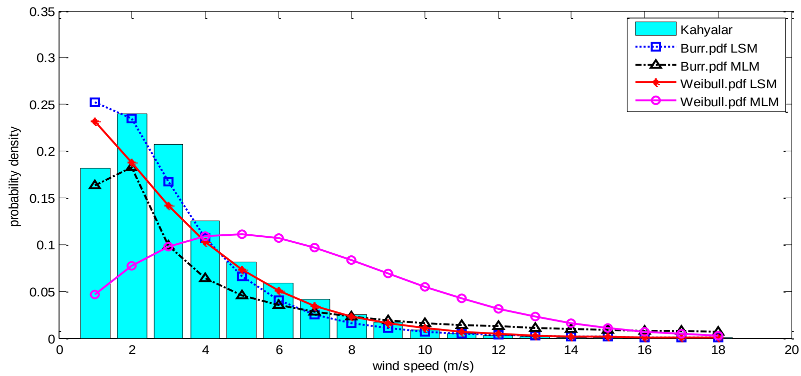


**Figure 6.** Actual wind speed density and the wind speed density produced by distributions for Karabuk Kahyalar

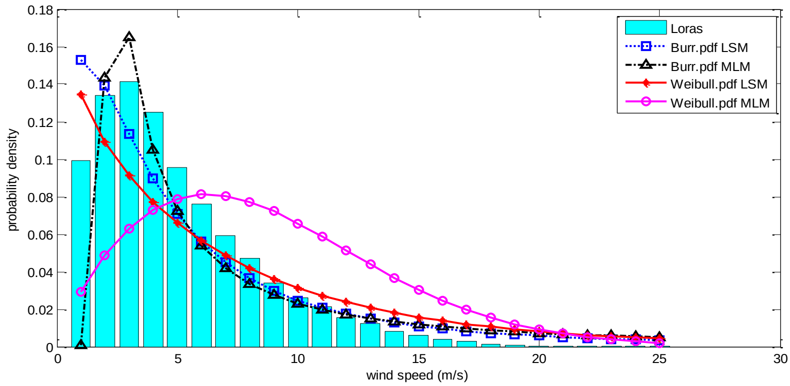


**Figure 7.** Actual wind speed density and the wind speed density produced by distributions for Loras

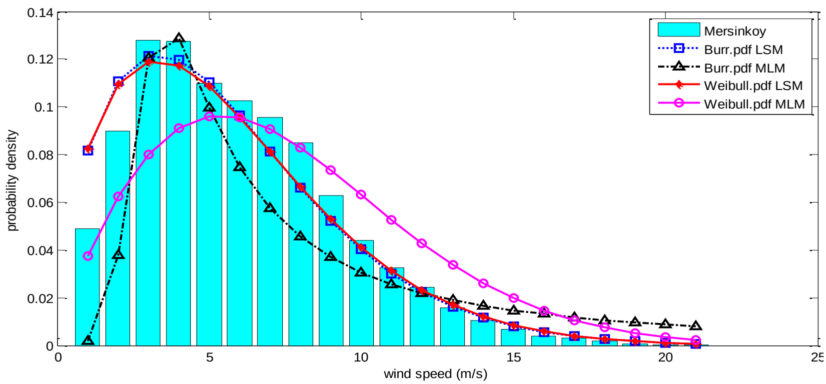


**Figure 8.** Actual wind speed density and the wind speed density produced by distributions for Mersinkoy

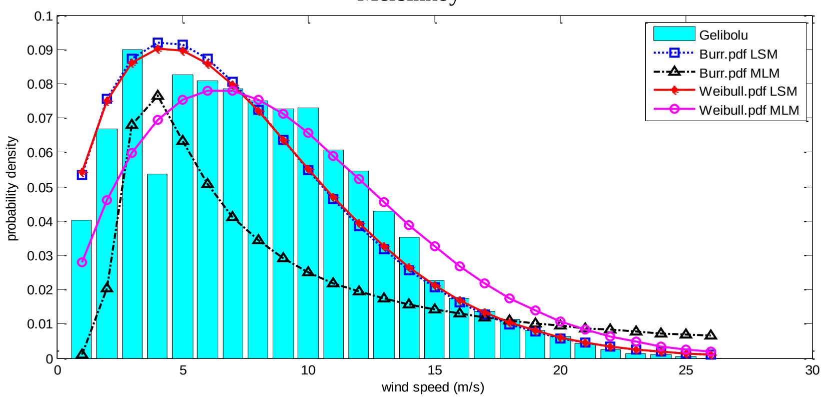


**Figure 9.** Actual wind speed density and the wind speed density produced by distributions for Gelibolu

## 4. RESULTS AND DISCUSSION

Before establishing wind energy conversion plants, it will be useful for an effective planning to know wind energy potential and wind speed frequency estimation process. Probability distribution functions are utilized to model wind speed distributions and power densities. Weibull pdf is most used method for wind power systems. With Burr pdf, more accurate results can be obtained, but distribution parameters (scale and shape) of Burr pdf cannot be calculated easily.

In this study, we used least squares method (LSM) to calculate Burr pdf parameters, which have not been known before. After that, Burr and Weibull probability density functions were compared to model wind speed frequencies of eight different locations that have different average data. Wind power densities were calculated by Weibull and Burr distribution functions and those models compared with observed annual data of the weather stations. LSM and MLM methods were used to predict the Weibull and the Burr distribution parameters. Hence, we can check the accuracies and performances of Weibull and Burr for LSM and MLM both. We tried to categorize appropriate theoretical probability density distributions of wind speed. To evaluate the performance of the considered distributions, root-mean-square error (*RMSE*) and determination of the coefficient ($R^2$) were used, too. Graphical comparisons of the distributions have proven mentioned methods as seen on Fig. 2. – Fig. 9. and Table 4. As seen on them, the best modelling of the wind speed frequency distribution is obtained by Burr.pdf LSM. For graphically, the Burr distribution can be preferred as the best-fitting curve for high wind speeds.

As seen on the results of performance criteria's *RMSE* and $R^2$; Burr LSM has minimum RMSE values for 5 weather stations and second minimum values for 2 weather stations. Beside of this RMSE of Burr MLM is minimum for two stations. Only for one stations, Weibull MLM has minimum RMSE. When it comes to $R^2$,Burr MLM is best for 3 stations and Burr LSM and Weibull LSM are equal for remaining 5 stations.

To predict wind power densities, Burr LSM has the best performance for 7 stations and only one station has the best prediction by Weibull LSM. Similar to this, Burr pdf is better for good and very good stations for estimating mean wind speeds, although Weibull is better for weak and weakly good stations.

In conclusion, the calculations and comparisons of annual measurement results of eight weather stations with the proposed methods throughout this study have shown that Weibull LSM is known as graphical method and commonly used, whereas Burr LSM can reach more accurate results. moreover, easy to use with predicting its parameters by least squares method.

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## NOMENCLATURES

| | | |
|---|---|---|
| $f(v)$ | | Probability density function |
| $v, Vr$ | [m/s] | Wind speed |
| $k$ | | Dimensionless shape parameter |
| $c$ | [m/s] | Scale parameter |
| $F_i, F(v_i)$ | | Cumulative distribution function |
| MLM | | Maximum likelihood method |
| LSM | | Least squares method |

| | | |
|---|---|---|
| $v_i$ | | velocity in $i^{th}$ position |
| $\Gamma(.)$ | | Gamma function |
| $\bar{v}$ | [m/s] | Mean wind speed |
| $A$ | | Shape parameters for Burr distribution |
| $b, k$ | | Scale parameters for Burr distribution |
| $R^2$ | | Coefficient of determination |
| $RMSE$ | | Root mean square error |
| $P_{w,d}$ | [W/m$^2$] | Mean wind power density for the period |
| pdf | | Probability Density Function |
| $f_i$ | | predicted pdf value |
| $p_i$ | | observed frequency value |

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