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Research Article

FIXED POINT RESULTS FOR $\tilde{A}$-CONTRACTIONS IN SOFT METRIC SPACES

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ABSTRACT

In the present paper, we aim to establish a new class of soft mappings called soft $\tilde{A}$-contraction mappings, and also obtain fixed point and common fixed point results in soft metric spaces by using these mappings.
Keywords: Soft point, soft metric space, contraction mapping, (common) fixed point.

1. INTRODUCTION

Fixed point theory which was introduced by Brouwer in 1912, plays a very important role and is applied in many different fields such as optimization theory, differential equations, variational inequalities, complemantary problems, equalibrum theory, game theory, economics theory and so on. Brouwer's work was followed by Banach's contraction principle in 1922. After that, lots of researchers have defined different types of contraction/contractive mappings to investigate the (common) fixed point results in metric spaces and metric-like spaces.

The notion of soft sets was initiated by Molodtsov [12] in 1999 as a new mathematical tool for dealing with uncertainties. Soft set is a parameterized general mathematical tool which deal with a collection of approximate descriptions of objects. Many authors have considered the notion of soft sets in different aspects and applied them to different directions [3,4,5,6,13,14,15]. Das and Samanta [7,8] introduced the notions of soft real numbers and soft points, and discussed their properties. Based on these notions, they introduced the concept of a soft metric with its basic properties.

Nowadays, fixed point theory is developed for soft metric spaces. Wardowski [16] defined the concept of a soft mappings and obtained some fixed point results in the framework of soft topological spaces. Abbas et al. [1] extended the Banach and Kannan contraction mappings to the soft metric spaces by using the soft points. Hosseinzadeh [9] defined the notion of soft metric spaces in a different way and also gave an application of a Banach fixed point theorem proposed for this spaces, to the dynamic programming.

In this work, we define a family of special kind of mappings as an extension of Akram et al. [2] and we define the notion of a soft $\tilde{A}$-contraction mapping in soft metric spaces which is a generalization of the soft contractions studied by Abbas et al.[1]. In conclude, we prove some

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(common) fixed point theorems in soft metric spaces by using soft points and soft $\tilde{A}$-contraction mappings.

2. PRELIMINARIES

Throughout this paper, X refers to an initial universe, and E the set of parameters for X. Let E_1 and E_2 be the non-empty parameter subsets of E. We denote by $\mathcal{P}(X)$ the family of all subsets of X.

Definition 2.1. [12] A pair (F, E) is called a soft set over X if F is a mapping given by $F: E \rightarrow \mathcal{P}(X)$. In other words, the soft set is a parametrized family of subsets of the set X. Every set $F(e), e \in E$, from this family may be considered as the set of $e - elements$ of the soft set (F, E), or as the set of $e - approximate$ elements of the soft set.

For any soft set (F, E_1), we can extend the soft set (F, E_1) to the soft set $(\bar{F}, E)$ where

$$\bar{F} : E \rightarrow \mathcal{P}(X)\, , \bar{F}(e) = \begin{cases} F(e) \;, & if\ e \in E_1 \\ \emptyset \;, & if\ e \notin E_1 \end{cases}$$

Example 2.2. [11] Let X be the set of houses under consideration and E be the set of parameters. Each parameter is a word or a sentence. E = {expensive, beautiful, wooden, cheap, in the green surroundings, modern, in good repair, in bad repair}.

In this case, to define a soft set means to point out expensive houses, beautiful houses, and so on. The soft set (F, E) describes the “attractiveness of the houses” which Mr. A is going to buy. Suppose that there are six houses in the universe X given by $X = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$:

Where e_1 stands for the parameter ‘expensive’, e_2 stands for the parameter ‘beautiful’, e_3 stands for the parameter ‘wooden’, e_4 stands for the parameter ‘cheap’, e_5 stands for the parameter ‘in the green surroundings’.

Suppose that $F(e_1) = \{h_2, h_4\}$, $F(e_2) = \{h_1, h_3\}$, $F(e_3) = \{h_3, h_4, h_5\}$, $F(e_4) = \{h_1, h_3, h_5\}$, $F(e_5) = \{h_1\}$.

The soft set $(F, E) = \{(e_1, \{h_2, h_4\}),\ (e_2, \{h_1, h_3\}), (e_3, \{h_3, h_4, h_5\}),\ (e_4, \{h_1, h_3, h_5\}), (e_5, \{h_1\})\}$ is a parametrized family $\{F(e_i), i = 1,2,3,4, \ldots, 8\}$ of subsets of the set X and gives us a collection of approximate descriptions of an object.

Definition 2.3. [11] For two soft sets (F, E_1) and (G, E_2) over X, we say that (F, E_1) is a soft subset of (G, E_2) if $F(e) \subseteq G(e)$ for all $e \in E_1$. We write $(F, E_1) \tilde{\subseteq} (G, E_2)$. Two soft sets (F, E_1) and (G, E_2) over X are said to be soft equal if (F, E_1) is a soft subset of (G, E_2) and (G, E_2) is a soft subset of (F, E_1).

Definition 2.4. [5] The complement of a soft set (F, E_1) is denoted by $(F, E_1)^c = (F^c, E_1)$ where $F^c : E_1 \rightarrow \mathcal{P}(X)$ is a mapping given by $F^c(e) = X \setminus F(e)$, for all $e \in E_1$.

Definition 2.5. [11] (1) (Null Soft Set) A soft set (F, E_1) over X is said to be a null soft set denoted by Φ, if for all $e \in E_1, F(e) = \Phi$.

(2) (Absolute Soft Set) A soft set (F, E_1) over X is said to be an absolute soft set denoted by $\tilde{X}$, if for all $e \in E_1, F(e) = X$.

Definition 2.6. [5] (1) The union of two soft sets (F, E_1) and (G, E_2) over X is the soft set (H, E_3), where $E_3 = E_1 \cup E_2$ and for all $e \in E_3$, $H(e) = F(e) \cup G(e)$. We express it as $(F, E_1) \tilde{\cup} (G, E_2) = (H, E_3)$.

(2) The intersection of two soft sets (F, E_1) and (G, E_2) over X is the soft set (H, E_3), where $E_3 = E_1 \cap E_2$ and for all $e \in E_3$, $H(e) = F(e) \cap G(e)$. We express it as $(F, E_1) \tilde{\cap} (G, E_2) = (H, E_3)$.

(3) The difference of two soft sets (F,E) and (G,E) over X is the soft set (H,E), which is denoted by $(F,E) \setminus (G,E)$, is defined by $H(e) = F(e) \setminus G(e)$ for all $e \in E$.

Definition 2.7. [10] Let (F,E_1) and (G,E_2) be two soft sets over X and Y, respectively and let φ_ψ be a soft mapping from $(\mathcal{P}(X))^{E_1}$ into $(\mathcal{P}(Y))^{E_2}$.

(1) The image of (F,E_1) under the soft mapping φ_ψ is the soft set over Y, defined by

$$\varphi_\psi\big((F,E_1)\big)(k) = \bigcup_{e\in\psi^{-1}(k)} \varphi((F,E_1)(e)) \quad for\ all\ k \in E_2$$

(2) The pre-image of (G,E_2) under the soft mapping φ_ψ is the soft set over X, defined by

$$\varphi_\psi^{-1}\big((G,E_2)\big)(e) = \varphi^{-1}\Big((G,E_2)\big(\psi(e)\big)\Big)\ for\ all\ e \in E_1$$

Definition 2.8. [7] Let $\mathbb{R}$ be the set of real numbers and $\mathcal{B}(\mathbb{R})$ be the collection of all non-empty bounded subsets of $\mathbb{R}$ and E taken as a set of parameters. Then a mapping $F : E \to \mathcal{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F,E), or simply by F. If F is a single valued mapping on E taking values in $\mathbb{R}$ then the pair (F,E) or simply F, is called a soft element of $\mathbb{R}$ or a soft real number. If F is a single valued mapping on E taking values in $\mathbb{R}^+$, then F is called a nonnegative soft real number. We shall denote the set of all nonnegative soft real numbers by $\mathbb{R}(E)^*$.

We use notations $\tilde{r},\tilde{s},\tilde{t}$ to denote soft real numbers whereas $\bar{r},\bar{s},\bar{t}$ will denote a particular type of soft real numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in E$, which is called a constant soft real number. For example $\bar{0}$ is the soft real number where $\bar{0}(\lambda) = 0$, for all $\lambda \in E$.

Definition 2.9. [8] The orderings between soft real numbers $\tilde{r}$ and $\tilde{s}$ are defined as follows:

i. $\tilde{r} \,\tilde{\leq}\, \tilde{s}$ if $\tilde{r}(\lambda) \leq \tilde{s}(\lambda)$ for all $\lambda \in E$.
ii. $\tilde{r} \,\tilde{\geq}\, \tilde{s}$ if $\tilde{r}(\lambda) \geq \tilde{s}(\lambda)$ for all $\lambda \in E$.
iii. $\tilde{r} \,\tilde{<}\, \tilde{s}$ if $\tilde{r}(\lambda) < \tilde{s}(\lambda)$ for all $\lambda \in E$.
iv. $\tilde{r} \,\tilde{>}\, \tilde{s}$ if $\tilde{r}(\lambda) > \tilde{s}(\lambda)$ for all $\lambda \in E$.

Definition 2.10. [8] (1) A soft set (P,E) over X is said to be a soft point if there is exactly one $\lambda \in A$ such that $P(\lambda) = \{x\}$ for some $x \in X$ and $P(\mu) = \emptyset$, for all $\mu \in E \setminus \{\lambda\}$. It will be denoted by P_λ^x.

(2) A soft point P_λ^x is said to belongs to a soft set (F,E) if $\lambda \in E$ and $P(\lambda) = \{x\} \subset F(\lambda)$. We write it by $P_\lambda^x \,\tilde{\in}\, (F,E)$.

(3) Two soft points P_λ^x, P_μ^y are said to be equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$. i.e., $x = y$. Thus $P_\lambda^x \neq P_\mu^y\ iff\ x \neq y\ or\ \lambda \neq \mu$.

The collection of all soft points of $\tilde{X}$ is denoted by $SP(\tilde{X})$.

Proposition 2.1. [8] The union of any collection of soft points can be considered as a soft set and every soft set can be expressed as union of all soft points belonging to it. i.e., $(F,E) = \bigcup_{P_\lambda^x \tilde{\in} (F,E)} P_\lambda^x$.

Definition 2.11. [8] A mapping $d : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ is said to be a $soft\ metric$ on the soft set $\tilde{X}$ if d satisfies the following conditions :

(M1) $d(P_\lambda^x, P_\mu^y) = \bar{0}$ if and only if $P_\lambda^x = P_\mu^y$.
(M2) $d(P_\lambda^x, P_\mu^y) = d(P_\mu^y, P_\lambda^x)$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$.
(M3) $d\big(P_\lambda^x, P_\mu^y\big) \,\tilde{\leq}\, d(P_\lambda^x, P_\gamma^z) + d(P_\gamma^z, P_\mu^y)$ for all $P_\lambda^x, P_\mu^y, P_\gamma^z \in SP(\tilde{X})$.

The soft set $\tilde{X}$ with a soft metric d on $\tilde{X}$ is called a $soft\ metric\ space$ and denoted by the triplet $(\tilde{X},d,E)$ or $(\tilde{X},d)$, for short.

Example 2.12. [8] Let X be a non-empty set and E be the non-empty set of parameters. Let $\tilde{X}$ be the absolute soft set. The mapping $d : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ defined by

$$d(P_\lambda^x, P_\mu^y) = \begin{cases} \bar{1}, & if\ P_\lambda^x \neq P_\mu^y \\ \bar{0}, & if\ P_\lambda^x = P_\mu^y \end{cases}$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ is a soft metric on the soft set $\tilde{X}$. d is called the discrete soft metric on the soft set $\tilde{X}$ and $(\tilde{X}, d)$ is said to be the discrete soft metric space.

Example 2.13. [8] Let $X \subset \mathbb{R}$ be a non-empty set and $E \subset \mathbb{R}$ be the non-empty set of parameters. Let $\tilde{X}$ be the absolute soft set and $\bar{x}$ be denotes the soft real number such that $\bar{x}(\lambda) = x$ for all $\lambda \in E$. The mapping $d : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ defined by $d(P_\lambda^x, P_\mu^y) = |\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|$, for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ is a soft metric on $\tilde{X}$.

Definition 2.14. [8] (Convergent Sequence) Let $(\tilde{X}, d)$ be a soft metric space and $\{P_{\lambda_n}^{x_n}\}$ be a sequence of soft points in $\tilde{X}$. The sequence $\{P_{\lambda_n}^{x_n}\}$ is said to be convergent in $(\tilde{X}, d)$ if there exists a soft point $P_\mu^y \in SP(\tilde{X})$ such that $d\left(P_{\lambda_n}^{x_n}, P_\mu^y\right) \to \bar{0}$ as $n \to \infty$. This means that for every $\tilde{\varepsilon} \tilde{>} \bar{0}$, there exists a natural number $N = N(\tilde{\varepsilon})$ such that $d(P_{\lambda_n}^{x_n}, P_\mu^y) \tilde{<} \tilde{\varepsilon}$ whenever $n > N$. This is denoted by $P_{\lambda_n}^{x_n} \to P_\mu^y$ as $n \to \infty$.

Definition 2.15. [8] (Cauchy Sequence) Let $(\tilde{X}, d)$ be a soft metric space and $\{P_{\lambda_n}^{x_n}\}$ be a sequence of soft points in $\tilde{X}$. The sequence $\{P_{\lambda_n}^{x_n}\}$ is said to be a Cauchy sequence in $(\tilde{X}, d)$ if for each $\tilde{\varepsilon} \tilde{>} \bar{0}$ there exists $n_0 \in \mathbb{N}$ such that $d\left(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m}\right) \tilde{<} \tilde{\varepsilon}$ for all $m, n \geq n_0$.

Definition 2.16. [8] (Complete Soft Metric Space) A soft metric space $(\tilde{X}, d)$ is called complete if every Cauchy sequence in $\tilde{X}$ converges to some soft point of $\tilde{X}$.

Definition 2.17. [1] Let $(\tilde{X}, d, E)$ and $(\tilde{Y}, \rho, E^*)$ be two soft metric spaces. A soft mapping $\varphi_\psi : (\tilde{X}, d, E) \to (\tilde{Y}, \rho, E^*)$ is said to be soft continuous at a soft point $P_\lambda^x \in SP(\tilde{X})$, if for every $\tilde{\varepsilon} \tilde{>} \bar{0}$ there exists a $\tilde{\delta} \tilde{>} \bar{0}$ such that $\rho\left(\varphi_\psi(P_\lambda^x), \varphi_\psi(P_\mu^y)\right) \tilde{<} \tilde{\varepsilon}$ whenever $d(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{\delta}$ for all $P_\mu^y \in SP(\tilde{X})$. If φ_ψ is soft continuous at every soft point of $\tilde{X}$, we say that φ_ψ is soft continuous on $\tilde{X}$.

Proposition 2.2. Let $(\tilde{X}, d, E)$ and $(\tilde{Y}, \rho, E^*)$ be two soft metric space and $\varphi_\psi : (\tilde{X}, d, E) \to (\tilde{Y}, \rho, E^*)$ be a soft mapping. For each soft point $P_\lambda^x \in SP(\tilde{X})$, $\varphi_\psi(P_\lambda^x) \left(= P_{\psi(\lambda)}^{\varphi(x)}\right)$ is a soft point of $SP(\tilde{Y})$.

Definition 2.18. [1] Let $(\tilde{X}, d)$ be a soft metric space and $\varphi_\psi : (\tilde{X}, d) \to (\tilde{X}, d)$ be a soft self-mapping. If there exists a soft point $P_\lambda^x \in SP(\tilde{X})$ such that $\varphi_\psi(P_\lambda^x) = P_\lambda^x$, then P_λ^x is called a fixed point of φ_ψ.

Definition 2.19. Let $(\tilde{X}, d)$ be a soft metric space and $\varphi_\psi, \sigma_\omega : (\tilde{X}, d) \to (\tilde{X}, d)$ be two soft self-mappings. If there exists a soft point $P_\lambda^x \in SP(\tilde{X})$ such that $\varphi_\psi(P_\lambda^x) = P_\lambda^x = \sigma_\omega(P_\lambda^x)$, then P_λ^x is called a common fixed point of φ_ψ and σ_ω.

3. FIXED POINT THEOREMS IN SOFT METRIC SPACES

Let E be the non-empty finite parameter set and $\tilde{A}$ be the set of all functions $\alpha : \mathbb{R}(E)^* \times \mathbb{R}(E)^* \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ satisfying the following properties:

1. α is sequentially continuous for each coordinates, i.e., $\widetilde{r_n} \xrightarrow{\mathbb{R}(E)^*} \tilde{r}$, $\widetilde{s_n} \xrightarrow{\mathbb{R}(E)^*} \tilde{s}$, $\widetilde{t_n} \xrightarrow{\mathbb{R}(E)^*} \tilde{t}$ $implies\ \alpha(\widetilde{r_n}, \widetilde{s_n}, \widetilde{t_n}) \xrightarrow{\mathbb{R}(E)^*} \alpha(\tilde{r}, \tilde{s}, \tilde{t})\ on\ the\ set\ (\mathbb{R}(E)^*)^3$.

2. $\tilde{a} \ \tilde{\leqq}\ \bar{k}\ .\tilde{b}$ for some $\bar{0} \ \tilde{\leqq}\ \bar{k} \ \tilde{<}\ \bar{1}$ whenever $\tilde{a} \ \tilde{\leqq}\ \alpha(\tilde{a}, \tilde{b}, \tilde{b})$ or $\tilde{a} \ \tilde{\leqq}\ \alpha(\tilde{b}, \tilde{a}, \tilde{b})$ or $\tilde{a} \ \tilde{\leqq}\ \alpha(\tilde{b}, \tilde{b}, \tilde{a})$ for all $\tilde{a}, \tilde{b}\ \tilde{\in}\ \mathbb{R}(E)^*$.

Definition 3.1. A soft self-mapping φ_ψ on a soft metric space $(\tilde{X}, d)$ is said to be a soft $\tilde{A}$-contraction if it satisfies the condition

$$d\left(\varphi_\psi(P_\lambda^x), \varphi_\psi(P_\mu^y)\right) \ \tilde{\leqq}\ \alpha\left(d(P_\lambda^x, P_\mu^y), d\left(P_\lambda^x, \varphi_\psi(P_\lambda^x)\right), d\left(P_\mu^y, \varphi_\psi(P_\mu^y)\right)\right)$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ and some $\alpha \in \tilde{A}$.

In [1], Abbas et al. established a Kannan fixed point theorem for complete soft metric spaces as follows :

Theorem 3.1. [1] Let $(\tilde{X}, d)$ be a complete soft metric space with a finite parameter set E. Suppose that the soft self-mapping $\varphi_\psi : (\tilde{X}, d) \to (\tilde{X}, d)$ satisfies

$$d\left(\varphi_\psi(P_\lambda^x), \varphi_\psi(P_\mu^y)\right) \ \tilde{\leqq}\ \bar{c}\left[d\left(P_\lambda^x, \varphi_\psi(P_\lambda^x)\right) + d\left(P_\mu^y, \varphi_\psi(P_\mu^y)\right)\right]$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ where $\bar{0} \ \tilde{\leqq}\ \bar{c} \ \tilde{<}\ \frac{\bar{1}}{2}$. Then φ_ψ has a unique fixed point in $\tilde{X}$.

Theorem 3.2. Every soft contraction mapping which satisfies the inequality given in the above theorem is also a soft $\tilde{A}$-contraction mapping.

Theorem 3.3. Let φ_ψ be a soft $\tilde{A}$-contraction on a complete soft metric space $(\tilde{X}, d)$. Then φ_ψ has a unique fixed soft point in $\tilde{X}$ such that the sequence $\{\varphi_\psi^n(P_{\lambda_0}^{x_0})\}$ of soft points converges to the fixed soft point of φ_ψ, for any $P_{\lambda_0}^{x_0} \in SP(\tilde{X})$.

Proof. Fix $P_{\lambda_0}^{x_0} \in SP(\tilde{X})$ and define the iterative sequence $\{P_{\lambda_n}^{x_n}\}$ of soft points by $P_{\lambda_n}^{x_n} = \varphi_\psi^n(P_{\lambda_0}^{x_0})$ (equivalently $P_{\lambda_{n+1}}^{x_{n+1}} = \varphi_\psi(P_{\lambda_n}^{x_n})$) where φ_ψ^n stands for the soft mapping obtained by n-times composition of φ_ψ with φ_ψ. Since φ_ψ is a soft $\tilde{A}$-contraction,

$$d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right) = d\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right)\right) = d\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right)\right)\right)$$

$$\tilde{\leqq}\ \alpha\left(d\left(P_{\lambda_0}^{x_0}, \varphi_\psi\left(P_{\lambda_0}^{x_0}\right)\right), d\left(P_{\lambda_0}^{x_0}, \varphi_\psi\left(P_{\lambda_0}^{x_0}\right)\right), d\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right)\right)\right)$$

$$= \alpha\left(d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right)\right) \ \tilde{\leqq}\ \bar{k}\ d(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1})$$

for some $\bar{0} \ \tilde{\leqq}\ \bar{k} \ \tilde{<}\ \bar{1}$, because $\alpha \in \tilde{A}$. Again

$$d\left(P_{\lambda_2}^{x_2}, P_{\lambda_3}^{x_3}\right) = d\left(\varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi^3\left(P_{\lambda_0}^{x_0}\right)\right) = d\left(\varphi_\psi\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right)\right), \varphi_\psi\left(\varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right)\right)\right)$$

$$\tilde{\leqq}\ \alpha\left(d\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right)\right), d\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right)\right), d\left(\varphi_\psi^2\left(P_{\lambda_0}^{x_0}\right), \varphi_\psi^3\left(P_{\lambda_0}^{x_0}\right)\right)\right)$$

$$= \alpha\left(d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right), d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right), d\left(P_{\lambda_2}^{x_2}, P_{\lambda_3}^{x_3}\right)\right) \ \tilde{\leqq}\ \bar{k}\ d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right) \ \tilde{\leqq}\ \bar{k}^2\ d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right)$$

Proceeding in this way, we get $d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) \ \tilde{\leqq}\ \bar{k}^n\ d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right)$ for some $\bar{0} \ \tilde{\leqq}\ \bar{k} \ \tilde{<}\ \bar{1}$. Thus $\{P_{\lambda_n}^{x_n}\}$ is a Cauchy sequence in $\tilde{X}$. Since $\tilde{X}$ is complete, there exists $P_e^a \in SP(\tilde{X})$ such that $P_{\lambda_n}^{x_n} \to P_e^a$ as $n \to \infty$. Since φ_ψ is a soft $\tilde{A}$-contraction,

$$d\left(\varphi_\psi(P_e^a), P_{\lambda_{n+1}}^{x_{n+1}}\right) = d\left(\varphi_\psi(P_e^a), \varphi_\psi\left(P_{\lambda_n}^{x_n}\right)\right)$$
$$\tilde{\leqq} \alpha\left(d\left(P_e^a, P_{\lambda_n}^{x_n}\right), d\left(P_e^a, \varphi_\psi(P_e^a)\right), d\left(P_{\lambda_n}^{x_n}, \varphi_\psi\left(P_{\lambda_n}^{x_n}\right)\right)\right)$$
$$=\alpha\left(d\left(P_e^a, P_{\lambda_n}^{x_n}\right), d\left(P_e^a, \varphi_\psi(P_e^a)\right), d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right)\right)$$

for all $n \in \mathbb{N}$ .Taking limit as $n \to \infty$, we get

$$d\big(\varphi_\psi(P_e^a), P_e^a\big) \ \tilde{\leqq}\ \alpha\left(d(P_e^a, P_e^a), d\left(P_e^a, \varphi_\psi(P_e^a)\right), d(P_e^a, P_e^a)\right) \ \tilde{\leqq}\ \bar{k}.\bar{0} = \bar{0}$$

Thus $\varphi_\psi(P_e^a) = P_e^a$. i.e., P_e^a is a fixed point of φ_ψ.

To prove the uniqueness of P_e^a, let P_f^b be another fixed point of φ_ψ. Then

$$d\big(P_e^a, P_f^b\big) = d\left(\varphi_\psi(P_e^a), \varphi_\psi\big(P_f^b\big)\right) \tilde{\leqq} \alpha\left(d\big(P_e^a, P_f^b\big), d\left(P_e^a, \varphi_\psi(P_e^a)\right), d\left(P_f^b, \varphi_\psi\big(P_f^b\big)\right)\right)$$
$$= \alpha\left(d\big(P_e^a, P_f^b\big), d(P_e^a, P_e^a), d\big(P_f^b, P_f^b\big)\right) \ \tilde{\leqq}\ \bar{k}.\bar{0} = \bar{0}$$

So that $P_e^a = P_f^b$ and thus the uniqueness is proved.

Example 3.4. Let $X = \mathbb{R}^+, E = \{0, 1, 2\}$. According to Example 2.13, it is known that the mapping $d: SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(E)^*$ defined by $d\big(P_\lambda^x, P_\mu^y\big) = |\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ is a soft metric on $\tilde{X}$. Furthermore, it is easy to testify that $(\tilde{X}, d)$ is a complete soft metric space. Define the mapping $\alpha: \mathbb{R}(E)^* \times \mathbb{R}(E)^* \times \mathbb{R}(E)^* \to \mathbb{R}(E)^*$ as $\alpha\big(\tilde{a}, \tilde{b}, \tilde{c}\big) = \frac{\bar{1}}{4}(\tilde{a} + \tilde{b} + \tilde{c})$ for all $\tilde{a}, \tilde{b}, \tilde{c}\ \tilde{\in}\ \mathbb{R}(E)^*$. It is clear that $\alpha \in \tilde{A}$. Let $\varphi_\psi : (\tilde{X}, d) \to (\tilde{X}, d)$ be a soft mapping such that $\varphi_\psi(P_\lambda^x) = P_1^1$ for all $\lambda \in E, x \in X$. Now, we show that φ_ψ satisfies the conditions of Theorem 3.3.

For any given $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$, we obtain that

$$d\left(\varphi_\psi(P_\lambda^x), \varphi_\psi\big(P_\mu^y\big)\right) = d(P_1^1, P_1^1) = \bar{0}, \quad d\big(P_\lambda^x, P_\mu^y\big) = |\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}|,$$
$$d\left(P_\lambda^x, \varphi_\psi(P_\lambda^x)\right) = d(P_\lambda^x, P_1^1) = |\bar{x} - \bar{1}| + |\bar{\lambda} - \bar{1}|,$$
$$d\left(P_\mu^y, \varphi_\psi\big(P_\mu^y\big)\right) = |\bar{y} - \bar{1}| + |\bar{\mu} - \bar{1}|.$$

Also, we have that

$$\alpha\left(d\big(P_\lambda^x, P_\mu^y\big), d\left(P_\lambda^x, \varphi_\psi(P_\lambda^x)\right), d\left(P_\mu^y, \varphi_\psi\big(P_\mu^y\big)\right)\right)$$
$$= \frac{\bar{1}}{4}\big(|\bar{x} - \bar{y}| + |\bar{\lambda} - \bar{\mu}| + |\bar{x} - \bar{1}| + |\bar{\lambda} - \bar{1}| + |\bar{y} - \bar{1}| + |\bar{\mu} - \bar{1}|\big) \tilde{\geqq} \bar{0} = \left(\varphi_\psi(P_\lambda^x), \varphi_\psi\big(P_\mu^y\big)\right).$$

Therefore, φ_ψ satisfies the conditions of Theorem 3.3. In fact, P_1^1 is the unique fixed point of φ_ψ.

Theorem 3.4. Let $\alpha \in \tilde{A}$ and $\left\{(\varphi_\psi)_n\right\}_{n=1}^{\infty}$ be a sequence of soft self-mappings on the complete soft metric space $(\tilde{X}, d)$ such that

$$d\left((\varphi_\psi)_i(P_\lambda^x), (\varphi_\psi)_j\big(P_\mu^y\big)\right) \ \tilde{\leqq}\ \alpha\left(d\big(P_\lambda^x, P_\mu^y\big), d\left(P_\lambda^x, (\varphi_\psi)_i(P_\lambda^x)\right), d\left(P_\mu^y, (\varphi_\psi)_j\big(P_\mu^y\big)\right)\right)$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$. Then $\left\{(\varphi_\psi)_n\right\}_{n=1}^{\infty}$ has a unique common fixed point in $\tilde{X}$.

Proof. Let $P_{\lambda_0}^{x_0} \in SP(\tilde{X})$. For each $n \in \mathbb{N}$, we define $P_{\lambda_n}^{x_n} = (\varphi_\psi)_n(P_{\lambda_{n-1}}^{x_{n-1}})$. Since $\alpha \in \tilde{A}$, we get

$$d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right) = d\left(\left(\varphi_\psi\right)_1\left(P_{\lambda_0}^{x_0}\right), \left(\varphi_\psi\right)_2\left(P_{\lambda_1}^{x_1}\right)\right)$$
$$\widetilde{\leq} \alpha\left(d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), d\left(P_{\lambda_0}^{x_0}, \left(\varphi_\psi\right)_1\left(P_{\lambda_0}^{x_0}\right)\right), d\left(P_{\lambda_1}^{x_1}, \left(\varphi_\psi\right)_2\left(P_{\lambda_1}^{x_1}\right)\right)\right)$$
$$= \alpha\left(d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right)\right) \widetilde{\leq} \bar{k}.d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right)$$

for some $\bar{0} \widetilde{\leq} \bar{k} \widetilde{<} \bar{1}$. Similarly,

$$d\left(P_{\lambda_2}^{x_2}, P_{\lambda_3}^{x_3}\right) = d\left(\left(\varphi_\psi\right)_2\left(P_{\lambda_1}^{x_1}\right), \left(\varphi_\psi\right)_3\left(P_{\lambda_2}^{x_2}\right)\right)$$
$$\widetilde{\leq} \alpha\left(d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right), d\left(P_{\lambda_1}^{x_1}, \left(\varphi_\psi\right)_2\left(P_{\lambda_1}^{x_1}\right)\right), d\left(P_{\lambda_2}^{x_2}, \left(\varphi_\psi\right)_3\left(P_{\lambda_2}^{x_2}\right)\right)\right)$$
$$= \alpha\left(d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right), d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right), d\left(P_{\lambda_1}^{x_1}, P_{\lambda_3}^{x_3}\right)\right) \widetilde{\leq} \bar{k}.d\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right) \widetilde{\leq} \bar{k}^2 d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right)$$

In general, we get $d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) \widetilde{\leq} \bar{k}^n d\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right)$ for some $\bar{0} \widetilde{\leq} \bar{k} \widetilde{<} \bar{1}$. Thus $\{P_{\lambda_n}^{x_n}\}$ is a Cauchy sequence in $\tilde{X}$. Since $\tilde{X}$ is complete, there exists $P_e^a \in SP(\tilde{X})$ such that $P_{\lambda_n}^{x_n} \to P_e^a$ as $n \to \infty$. Since d is a soft metric, by (M3)

$$d\left(P_e^a, \left(\varphi_\psi\right)_n(P_e^a)\right) \leq d\left(P_e^a, P_{\lambda_{m+1}}^{x_{m+1}}\right) + d\left(P_{\lambda_{m+1}}^{x_{m+1}}, \left(\varphi_\psi\right)_n(P_e^a)\right)$$
$$= d\left(P_e^a, P_{\lambda_{m+1}}^{x_{m+1}}\right) + d\left(\left(\varphi_\psi\right)_{m+1}\left(P_{\lambda_m}^{x_m}\right), \left(\varphi_\psi\right)_n(P_e^a)\right)$$
$$\widetilde{\leq} d\left(P_e^a, P_{\lambda_{m+1}}^{x_{m+1}}\right) + \alpha\left(d\left(P_{\lambda_m}^{x_m}, P_e^a\right), d\left(P_{\lambda_m}^{x_m}, \left(\varphi_\psi\right)_{m+1}\left(P_{\lambda_m}^{x_m}\right)\right), d\left(P_e^a, \left(\varphi_\psi\right)_n(P_e^a)\right)\right)$$
$$= d\left(P_e^a, P_{\lambda_{m+1}}^{x_{m+1}}\right) + \alpha\left(d\left(P_{\lambda_m}^{x_m}, P_e^a\right), d\left(P_{\lambda_m}^{x_m}, P_{\lambda_{m+1}}^{x_{m+1}}\right), d\left(P_e^a, \left(\varphi_\psi\right)_n(P_e^a)\right)\right)$$

for all $m, n \in \mathbb{N}$. Taking limit as $m \to \infty$, the above inequality gives that

$$d\left(P_e^a, \left(\varphi_\psi\right)_n(P_e^a)\right) \widetilde{\leq} d(P_e^a, P_e^a) + \alpha\left(d(P_e^a, P_e^a), d(P_e^a, P_e^a), d\left(P_e^a, \left(\varphi_\psi\right)_n(P_e^a)\right)\right)$$
$$\widetilde{\leq} \alpha\left(\bar{0}, \bar{0}, d\left(P_e^a, \left(\varphi_\psi\right)_n(P_e^a)\right)\right) \widetilde{\leq} \bar{k}.\bar{0} = \bar{0}$$

for some $\bar{0} \widetilde{\leq} \bar{k} \widetilde{<} \bar{1}$ and hence $\left(\varphi_\psi\right)_n(P_e^a) = P_e^a$ for all $n \in \mathbb{N}$.

For uniqueness of the fixed point P_e^a, we suppose that $\left(\varphi_\psi\right)_n\left(P_f^b\right) = P_f^b$, for some $P_f^b \in SP(\tilde{X})$. From hypothesis,

$$d\left(P_e^a, P_f^b\right) = d\left(\left(\varphi_\psi\right)_i(P_e^a), \left(\varphi_\psi\right)_j\left(P_f^b\right)\right)$$
$$\widetilde{\leq} \alpha\left(d\left(P_e^a, P_f^b\right), d\left(P_e^a, \left(\varphi_\psi\right)_i(P_e^a)\right), d\left(P_f^b, \left(\varphi_\psi\right)_j\left(P_f^b\right)\right)\right) = \alpha\left(d\left(P_e^a, P_f^b\right), \bar{0}, \bar{0}\right) \widetilde{\leq} \bar{k}.\bar{0} = \bar{0}$$

for some $\bar{0} \widetilde{\leq} \bar{k} \widetilde{<} \bar{1}$ and this implies $P_e^a = P_f^b$.

Theorem 3.5. Let d and δ be the soft metrics on $\tilde{X}$ which verify the following conditions:

i. $d\left(P_\lambda^x, P_\mu^y\right) \widetilde{\leq} \delta(P_\lambda^x, P_\mu^y)$ for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$.
ii. $(\tilde{X}, d)$ is a complete soft metric space.
iii. $\varphi_\psi, \sigma_\omega$ are soft self-mappings on $\tilde{X}$ such that φ_ψ is continuous with respect to d and

$$\delta\left(\varphi_\psi(P_\lambda^x), \sigma_\omega\left(P_\mu^y\right)\right) \widetilde{\leq} \alpha\left(\delta\left(P_\lambda^x, P_\mu^y\right), \delta\left(P_\lambda^x, \varphi_\psi(P_\lambda^x)\right), \delta\left(P_\mu^y, \sigma_\omega\left(P_\mu^y\right)\right)\right)$$

for all $P_\lambda^x, P_\mu^y \in SP(\tilde{X})$ and some $\alpha \in \tilde{A}$. Then φ_ψ and σ_ω have a unique common fixed point in X.

Proof. Let $P_{\lambda_0}^{x_0} \in SP(\tilde{X})$. For each $n \in \mathbb{N}$, we define

$$P_{\lambda_n}^{x_n} = \begin{cases} \sigma_\omega\left(P_{\lambda_{n-1}}^{x_{n-1}}\right) & , \ if\ n\ is\ even \\ \varphi_\psi\left(P_{\lambda_{n-1}}^{x_{n-1}}\right) & , \ if\ n\ is\ odd \end{cases}$$

Then, from the hypothesis, we get

$$\delta\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right) = \delta\left(\varphi_\psi\left(P_{\lambda_0}^{x_0}\right), \sigma_\omega\left(P_{\lambda_1}^{x_1}\right)\right)$$
$$= \alpha\left(\delta\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), \delta\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right), \delta\left(P_{\lambda_1}^{x_1}, P_{\lambda_2}^{x_2}\right)\right) \lesseqqgtr \bar{k}.\delta\left(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1}\right)$$

for some $\bar{0} \lesseqqgtr \bar{k} \precsim \bar{1}$ and $\alpha \in \tilde{A}$. In general, for any $n \in \mathbb{N}$, we get

$$\delta\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) \lesseqqgtr \bar{k}^n\,\delta(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1})$$

for some $\bar{0} \lesseqqgtr \bar{k} \precsim \bar{1}$. Condition (i) implies the following inequality

$$d\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) \lesseqqgtr \delta\left(P_{\lambda_n}^{x_n}, P_{\lambda_{n+1}}^{x_{n+1}}\right) \lesseqqgtr \bar{k}^n\,\delta(P_{\lambda_0}^{x_0}, P_{\lambda_1}^{x_1})$$

for all $n \in \mathbb{N}$ with $\bar{0} \lesseqqgtr \bar{k} \precsim \bar{1}$. So $\{P_{\lambda_n}^{x_n}\}$ is a Cauchy sequence in $\tilde{X}$ with respect to d and hence by condition (ii), $P_{\lambda_n}^{x_n} \to P_e^a$ as $n \to \infty$ for some $P_e^a \in SP(\tilde{X})$. Since φ_ψ is given to be continuous with respect to d, we have

$$\bar{0} = \lim_{n\to\infty} d\left(P_{\lambda_{2n-1}}^{x_{2n-1}}, P_e^a\right) = \lim_{n\to\infty} d\left(\varphi_\psi\left(P_{\lambda_{2n}}^{x_{2n}}\right), P_e^a\right) = d\big(\varphi_\psi(P_e^a), P_e^a\big)$$

So that $\varphi_\psi(P_e^a) = P_e^a$. Now, by condition (iii)

$$\delta\big(P_e^a, \sigma_\omega(P_e^a)\big) = \delta\left(\varphi_\psi(P_e^a), \sigma_\omega(P_e^a)\right) \lesseqqgtr \alpha\left(\delta(P_e^a, P_e^a), \delta\left(P_e^a, \varphi_\psi(P_e^a)\right), \delta\left(\varphi_\psi(P_e^a), \sigma_\omega(P_e^a)\right)\right)$$
$$\lesseqqgtr \alpha\left(\bar{0}, \bar{0}, \delta\left(\varphi_\psi(P_e^a), \sigma_\omega(P_e^a)\right)\right) \lesseqqgtr \bar{k}.\bar{0} = \bar{0}$$

since $\alpha \in \tilde{A}$ for some $\bar{0} \lesseqqgtr \bar{k} \precsim \bar{1}$. Hence $\sigma_\omega(P_e^a) = P_e^a$. Thus P_e^a is a common fixed point of φ_ψ and σ_ω. For the uniqueness, let P_f^b be another common fixed soft point of φ_ψ and σ_ω in $\tilde{X}$. Then by condition (iii),

$$\delta\big(P_e^a, P_f^b\big) = \delta\left(\varphi_\psi(P_e^a), \sigma_\omega\big(P_f^b\big)\right) \lesseqqgtr \alpha\left(\delta\big(P_e^a, P_f^b\big), \delta\left(P_e^a, \varphi_\psi(P_e^a)\right), \delta\left(P_f^b, \sigma_\omega\big(P_f^b\big)\right)\right)$$
$$= \alpha\big(\delta\big(P_e^a, P_f^b\big), \bar{0}, \bar{0}\big) \lesseqqgtr \bar{k}.\bar{0} = \bar{0}$$

for some $\bar{0} \lesseqqgtr \bar{k} \precsim \bar{1}$. So that $P_e^a = P_f^b$. Hence P_e^a is a unique common fixed point of φ_ψ and σ_ω in $\tilde{X}$.

4. CONCLUSION

Fixed point results in metric spaces and also in metric-like spaces have many practical applications not only in different branches of mathematics such as geometry, functional analysis, numerical analysis and applied mathematics but also in real life situations such as tomography, telecommunications, signal synthesis, filter synthesis and etc. It is well known that the notion of soft sets reflects the real life problems more precisely. So, we hope that the investigations of fixed point results in soft metric spaces will be helpful for these kinds of modellings.

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