Sigma J Eng & Nat Sci 36 (1), 2018, 191-206

Sigma Journal of Engineering and Natural Sciences
Sigma Mühendislik ve Fen Bilimleri Dergisi

Research Article

PARAMETRIC EIGENVALUE ANALYSIS OF MINDLIN PLATES RESTING ON WINKLER FOUNDATION WITH SECOND ORDER FINITE ELEMENT

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Received: 15.12.2017 Accepted: 23.02.2018

ABSTRACT

The purpose of this paper is to study free vibration analysis of thick plates resting on Winkler foundation using Mindlin's theory with shear locking free fourth order finite element, to determine the effects of the thickness/span ratio, the aspect ratio, subgrade reaction modulus and the boundary conditions on the frequency paramerets of thick plates subjected to free vibration. In the analysis, finite element method is used for spatial integration. Finite element formulation of the equations of the thick plate theory is derived by using second order displacement shape functions. A computer program using finite element method is coded in C++ to analyze the plates free, clamped or simply supported along all four edges. In the analysis, 8-noded finite element is used. Graphs are presented that should help engineers in the design of thick plates subjected to earthquake excitations. It is concluded that 8-noded finite element can be effectively used in the free vibration analysis of thick plates. It is also concluded that, in general, the changes in the thickness/span ratio are more effective on the maximum responses considered in this study than the changes in the aspect ratio
Keywords: Free vibration parametric analysis, **t**hick plate, mindlin's theory, second order finite element, winkler foundation.

1. INTRODUCTION

Plates are structural elements which are commonly used in the building industry. A plate is considered to be a thin plate if the ratio of the plate thickness to the smaller span length is less than 1/20; it is considered to be a thick plate if this ratio is larger than 1/20 [1].

The dynamic behavior of thin plates has been investigated by many researchers [2- 17]. There are also many references on the behavior of the thick plates subjected to different loads. The studies made on the behavior of the thick plates are based on the Reissner-Mindlin plate theory [18, 19, 20, 21]. This theory requires only C^0 continuity for the finite elements in the analysis of thin and thick plates. Therefore, it appears as an alternative to the thin plate theory which also requires C^1 continuity. This requirement in the thin plate theory is solved easily if Mindlin theory is used in the analysis of thin plates. Despite the simple formulation of this theory, discretization of the plate by means of the finite element comes out to be an important parameter. In many cases, numerical solution can have lack of convergence, which is known as "shear-locking". Shear locking can be avoided by increasing the mesh size, i.e. using finer mesh, but if the

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thickness/span ratio is "to o small", convergence may not be achieved even if the finer mesh is used for the low order displacement shape functions.

In order to avoid shear locking problem, the different methods and techniques, such as reduced and selective reduced integration, the substitute shear strain method, etc., are used by several researchers [22, 23, 24, 25, 26]. The same problem can also be prevented by using higher order displacement shape function [27, 28]. Wanji and Cheung [29] proposed a new quadrilateral thin/thick plate element based on the Mindlin-Reissner theory. Soh *et al*. [30] improved a new element ARS-Q12 which is a simple quadrilateral 12 DOF plate bending element based on Reissner-Mindlin theory for analysis of thick and thin plates. Brezzi and Marini [31] developped a locking free nonconforming element for the Reissner-Mindlin plate using discontinuous Galarkin techniques. Belounar and Guenfound [32] improved a nev rectangular finite element based on the strain approach and the Reissner-Mindlin theory is presented for the analysis of plates in bending either thick or thin. Vibration analysis made by Senjanovic et al. [33], in the paper the modified Mindlin theory is used for the construction of the dynamic stiffness matrix, the flexibility matrix, and the transfer matrix of a thick plate simply supported at two opposite edges. They presented natural frequencies and modes of panel Mindlin plates. Si et al. [16] studied vibration analysis of rectangular plates with one or more guided edges via bicubic B-spline method, Cen *et al.* [34] developped a new high performance quadrilateral element for analysis of thick and thin plates. This distinguishing character of the new element is that all formulations are expressed in the quadrilateral area co-ordinate system. Shen *et al*. [35] studied free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundations. Woo *et al*. [36] found accurate natural frequencies and mode shapes of skew plates with and without cutouts by p-version finite element method using integrals of Legendre polynomial for p=1-14. Qian *et al.* [37] studied free and forced vibrations of thick rectangular plates using higher-order shear and normal deformable plate theory and meshless Petrov-Galarkin method. Özdemir and Ayvaz [38] studied shear locking free earthquake analysis of thick and thin plates using Mindlin's theory. GuangPeng *et al.* [39] studied free vibration analysis of plates on Winkler elastic foundation by boundary element method. Fallah *et. al.* [40] analyzed free vibration of moderately thick rectangular FG plates on elastic foundation with various combinations of simply supported and clamed boundary conditions. Governing equations of motion were obtained based on the Mindlin plate theory. Jahromi *et al.* [41] analyzed free vibration analysis of Mindlin plates partially resting on Pasternak foundation. The governing equations which consist of a system of partial differential equations are obtained based on the first-order shear deformation theory. Özgan and Daloğlu [42] studied free vibration analysis of thick plates on elastic foundations using modified Vlasov model with higher order finite elements, also same writers [43] studied the effects of various parameters such as the aspect ratio, subgrade reaction modulus and thickness/span ratio on the frequency parameters of thick plates resting on Winkler elastic foundations.

The purpose of this paper is to study free vibration analysis of thick plates resting on Winkler foundation using Mindlin's theory with second order finite element, to determine the effects of the thickness/span ratio, the aspect ratio, subgrade reaction modulus and the boundary conditions on the frequency paramerets of thick plates subjected to free vibration. A computer program using finite element method is coded in C++ to analyze the plates free, clamped or simply supported along all four edges. In the program, the finite element method is used for spatial integration. Finite element formulation of the equations of the thick plate theory is derived by using higher order displacement shape functions. In the analysis, 8-noded finite element is used to construct the stiffness and mass matrices of the thick and thin plates [27].

2. MATHEMATICAL MODEL

The governing equation for a flexural plate (Fig. 1) subjected to free vibration without damping can be given as

$$[M]\{\ddot{w}\}+[K]\{w\}=0 \tag{1}$$

where [K] and [M] are the stiffness matrix and the mass matrix of the plate, respectively, w and $\ddot{w}$ are the lateral displacement and the second derivative of the lateral displacement of the plate with respect to time, respectively.,

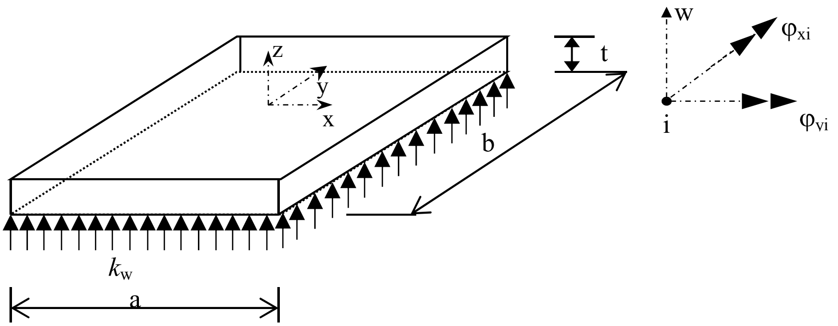


Figure 1. The sample plate used in this study

The total strain energy of plate-soil-structure system (see Fig. 1) can be written as;

$$\Pi = \Pi_p + \Pi_s + V \tag{2}$$

where Π_p is the strain energy in the plate,

$$\Pi_p = \frac{1}{2}\int_A \left(-\frac{\partial \varphi_x}{\partial x} \quad \frac{\partial \varphi_y}{\partial y} \quad -\frac{\partial \varphi_x}{\partial y}+\frac{\partial \varphi_y}{\partial x} \right)^T E_\kappa \left(-\frac{\partial \varphi_x}{\partial x} \quad \frac{\partial \varphi_y}{\partial y} \quad -\frac{\partial \varphi_x}{\partial y}+\frac{\partial \varphi_y}{\partial x} \right) d_A$$

$$+\frac{k}{2}\int_A \left(-\varphi_x+\frac{\partial w}{\partial x} \quad \varphi_y+\frac{\partial w}{\partial y} \right)^T E_\gamma \left(-\varphi_x+\frac{\partial w}{\partial x} \quad \varphi_y+\frac{\partial w}{\partial y} \right) d_A \; - \tag{3}$$

where Π_s is the strain energy stored in the soil,

$$\Pi_s = \frac{1}{2}\int_0^H \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \sigma_{ij}\varepsilon_{ij} \tag{4}$$

and V is the potential energy of the external loading;

$$V=-\int_A \overline{q} w d_A \tag{5}$$

In this equation E_κ *and* E_γ are the elasticity matrix and these matrices are given below at Eq. (17), $\overline{q}$ shows applied distributed load.

2.1. Evaluation of the stiffness matrix

The total strain energy of the plate-soil system according to Eq. (2) is;

$$U_e=\frac{1}{2}\int_A\left(-\frac{\partial\varphi_x}{\partial x}\quad\frac{\partial\varphi_y}{\partial y}\quad-\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\right)^T E_\kappa\left(-\frac{\partial\varphi_x}{\partial x}\quad\frac{\partial\varphi_y}{\partial y}\quad-\frac{\partial\varphi_x}{\partial y}+\frac{\partial\varphi_y}{\partial x}\right)d_A\ +$$

$$\frac{k}{2}\int_A\left(-\varphi_x+\frac{\partial w}{\partial x}\quad\varphi_y+\frac{\partial w}{\partial y}\right)^T E_\gamma\left(-\varphi_x+\frac{\partial w}{\partial x}\quad\varphi_y+\frac{\partial w}{\partial y}\right)d_A\ +$$

$$\frac{1}{2}\int_A\left(w_{x,y}\right)^T K\left(w_{x,y}\right)d_A \tag{6}$$

At this equation the first and second part gives the conventional element stiffness matrix of the plate, $[k_p^e]$, differentiation of the third integral with respect to the nodal parameters yields a matrix, $[k_w^e]$, which accounts for the axial strain effect in the soil. Thus the total energy of the plate-soil system can be written as;

$$U_e=\frac{1}{2}\{w_e\}^T\left(\left[k_p^e\right]+\left[k_w^e\right]\right)\{w_e\}d_A \tag{7}$$

where

$$\{w_e\}=\left[w_1\quad\varphi_{y1}\quad\varphi_{x1}\quad\cdots\quad w_n\quad\varphi_{yn}\quad\varphi_{xn}\right]^T \tag{8}$$

Assuming that in the plate of Fig. 1 u and v are proportional to z and that w is the independent of z [21], one can write the plate displacement at an arbitrary x, y, z in terms of the two slopes and a displacement as follows;

$$u_i=\{w, v, u\}=\{w_0(x,y,t), z\varphi_y(x,y,t), -z\varphi_x(x,y,t)\} \tag{9}$$

where w_0 is average displacement of the plate, and φ_x and φ_y are the bending slopes in the x and y directions, respectively.

The nodal displacements for 8-noded finite element (MT8) (Fig. 2) can be written as follows;

$$w=\sum_1^8 h_i w_i\ ,\ v=z\varphi_y=z\sum_1^8 h_i\varphi_{yi}\ ,\ u=-z\varphi_x=-z\sum_1^8 h_i\varphi_{xi}\qquad i=1,\ldots,8\ \text{for 8 - noded element}, \tag{10}$$

The displacement function chosen for this element is;

$$w=c_1+c_2r+c_3s+c_4r^2+c_5rs+c_6s^2+c_7r^2s+c_8rs^2 \tag{11}$$

From this assumption, it is possible to derive the displacement shape function to be [38];

$$h=[h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8] \tag{12}$$

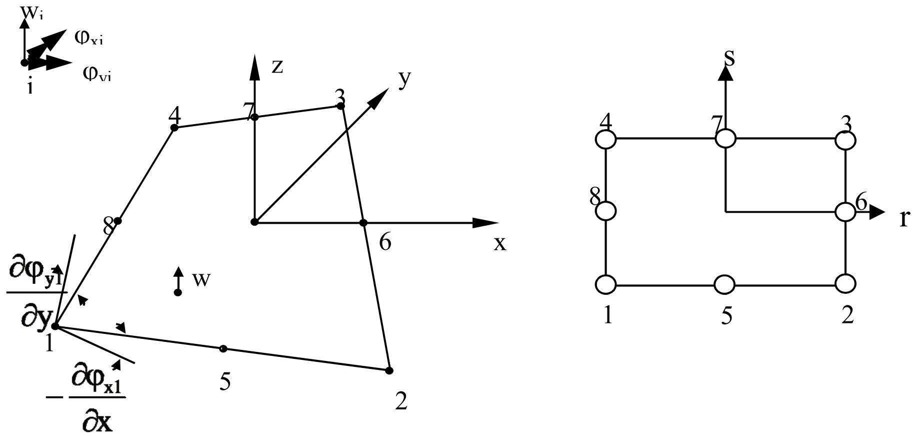


Figure 2. 8- noded finite element(second order),used in this study [47].

Then, the strain-displacement matrix [B] for this element can be written as follows [44]:

$$[B]=\begin{bmatrix} 0 & 0 & -\frac{\partial h_i}{\partial x} & \dots \\ 0 & \frac{\partial h_i}{\partial y} & 0 & \dots \\ 0 & \frac{\partial h_i}{\partial x} & -\frac{\partial h_i}{\partial y} & \dots \\ \frac{\partial h_i}{\partial x} & 0 & -h_i & \dots \\ \frac{\partial h_i}{\partial y} & h_i & 0 & \dots \end{bmatrix}_{5x21} \qquad i=1,...,8 \text{ for 8 - noded element} \tag{13}$$

The stiffness matrix for MT8 element can be obtained by the following equation [44]:

$$[K]=\int_A [B]^T [D][B] dA. \int_{-1}^{1}\int_{-1}^{1} [B]^T [D][B]|J| drds \tag{14}$$

which must be evaluated numerically [26].

As seen from Eq. (14), in order to obtain the stiffness matrix, the strain-displacement matrix, [B], and the flexural rigidity matrix, [D], of the element need to be constructed

The flexural rigidity matrix, [D], can be obtained by the following equation.

$$[D]=\begin{bmatrix} E_k & 0 \\ 0 & E_\gamma \end{bmatrix}. \tag{15}$$

In this equation, [E_k] is of size 3x3 and [E_γ] is of size 2x2. [E_k], and [E_γ] can be written as follows [45, 47]:

$$[E_k]=\frac{t^3}{12}\begin{bmatrix} \frac{E}{(1-\nu^2)} & \frac{\nu E}{(1-\nu^2)} & 0 \\ \frac{\nu E}{(1-\nu^2)} & \frac{E}{(1-\nu^2)} & 0 \\ 0 & 0 & \frac{E}{2(1-\nu)} \end{bmatrix};\ [E_\gamma]=k\,t\begin{bmatrix} \frac{E}{2.4(1+\nu)} & 0 \\ 0 & \frac{E}{2.4(1+\nu)} \end{bmatrix} \tag{16}$$

where E, υ, and t are modulus of the elasticity, Poisson's ratio, and the thickness of the plate, respectively, k is a constant to account for the actual non-uniformity of the shearing stresses. By assembling the element stiffness matrices obtained, the system stiffness matrix is obtained.

2.2. Evaluation of the mass matrix

The formula for the consistent mass matrix of the plate may be written as

$$M=\int_\Omega H_i^T \mu H_i d\Omega\ . \tag{17}$$

In this equation, μ is the mass density matrix of the form [46]:

$$\mu=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}, \tag{18}$$

where $m_1=\rho_p t$, $m_2=m_3=\frac{1}{12}(\rho_p t^3)$, and ρ_p is the mass densities of the plate. and H_i can be written as follows,

$$H_i=[dh_i/dx \quad dh_i/dy \quad h_i] \qquad i=1\ldots 8.. \tag{19}$$

It should be noted that the rotation inertia terms are not taken into account. By assembling the element mass matrices obtained, the system mass matrix is obtained.

2.3. Evaluation of frequency of plate

The formulation of lateral displacement, w, can be given as motion is sinusoidal;

$$w= W\sin \omega t \tag{20}$$

Here ω is the circular frequency. Substitution of Eq. (20) and its second derivation into Eq. (1) gives expression as;

$$[K-\omega^2 M]\,\{W\}=0 \tag{21}$$

Eq. (21) is obtained to calculate the circular frequency, ω, of the plate. Then natural frequency can be calculated with the formulation below;

$$f=\omega/2\pi \tag{22}$$

3. NUMERICAL EXAMPLES

3.1. Data for numerical examples

In the light of the results given in references [27, 28, 38], the aspect ratios, b/a, of the plate are taken to be 1, 1.5, and 2.0. The thickness/span ratios, t/a, are taken as 0.01, 0.05, 0.1, 0.2, and 0.3 for each aspect ratio. The shorter span length of the plate is kept constant to be 10 m. The mass density, Poisson's ratio, and the modulus of elasticity of the plate are taken to be 2.5 kN s^2/m^2, 0.2, and $2.7x10^7$ kN/m^2. Shear factor k is taken to be 5/6. The subgrade reaction modulus of the Winkler-type foundation is taken to be 500 and 5000 kN/m^3.

For the sake of accuracy in the results, rather than starting with a set of a finite element mesh size, the mesh size required to obtain the desired accuracy were determined before presenting any results. This analysis was performed separately for the mesh size. It was concluded that the results have acceptable error when equally spaced 4x4 mesh size for 17-noded elements are used for a 10 m x 10 m plate. Length of the elements in the x and y directions are kept constant for different aspect ratios as in the case of square plate.

In order to illustrate that the mesh density used in this paper is enough to obtain correct results, the first six frequency parameters of the thick plate with b/a=1 and t/a=0.05 is presented in Table 1 by comparing with the result obtained SAP2000 program and the results Özgan and Daloğlu [2015]. In this study Özgan and Daloğlu used 4-noded and 8-noded quadrilateral finite element with 10x10 and 5x5 mesh size. It should be noted that the results presented for MT8 element are obtained by using equally spaced 2x2, 4x4 and 8x8 mesh sizes. As seen from Table 1, the results obtained by using 8-noded quadrilateral finite element have excellent agreement with the results obtained by Özgan and Daloğlu [2015] and SAP200 even if 2x2 mesh size is used for MT8 element.

3.2. Results

The first six frequency parameters of thick plate resting on Winkler foundation with free edges are compared with the same thick plate modeled by Ozgan and Daloğlu (2010) and SAP2000 program and it is presented in Table1. The subgrade reaction modulus of the Winkler-type foundation for this example is taken to be 5000 kN/m^3. This thick plate is modeled with MT8 element 8x8 mesh size for b/a=1.0, t/a=0.05 ratios.

As seen from Tables 1, the values of the frequency parameters of these analyses are so close even if this study mesh size is so poor. And also SAP2000 analysis made by first order finite element, writer is use second order finite element as before explained. Then writers enlarged parameters of aspect ratio, b/a, thickness/span ratio, t/a, for help the researchers.

The first six frequency parameters of thick plates resting on Winkler foundation considered for different aspect ratio, b/a, thickness/smaller span ratio, t/a, are presented in Table 2 for the with free edges , in Table 3 for thick clamped plates. In order to see the effects of the changes in these parameters better on the first six frequency parameters, they are also presented in Figs 3-4 for the thick free plates, in Figs 5-6 for the thick clamped plates.

Table 1. The first five natural frequency parameters of plates for b/a=0.1 and t/a=0.05.

$\lambda_i=\omega^2$	Ozgan and Daloğlu, 2015 PBQ8(FI)	This Study			SAP2000
		MT8 (4 element)	MT8 (16 element)	MT8 (64 element)	
1	3990.42	3997.94	3851.64	3962.35	4000.00
2	3990.42	3997.94	3862.49	3962.35	4000.00
3	4000.40	4004.04	3862.51	3963.71	4000.00
4	8676.00	8813.62	8489.34	8597.96	8619.60
5	13957.64	16817.97	13672.78	13798.20	13292.31
6	17252.34	22889.28	16822.21	16967.53	16380.24

As seen from Tables 2, and Figs. 3, except the value of t/a 0.05,the values of the first six frequency parameters for a constant value of *t/a* decreases as the aspect ratio, *b*/*a*, increases. For t/a 0.05 the results are the opposite.

Table 2. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates resting on elastic foundation.

a)Subgrade reaction modulus k=500

k	b/a	t/a	$\lambda=\omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	1.0	0.05	373	390	390	5042	10254	13428
		0.10	218	218	225	17542	37640	49454
		0.20	69	69	73	58673	126701	164565
	2.0	0.05	369	377	386	1087	1532	5752
		0.10	218	223	226	2999	4558	20459
		0.20	67	72	72	10634	15462	68549

b)Subgrade reaction modulus k=5000

k	b/a	t/a	$\lambda=\omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5000	1.0	0.05	3962	3962	3964	8598	13798	16968
		0.10	1996	1996	2006	19316	39376	51187
		0.20	937	937	967	59525	127495	165364
	2.0	0.05	3962	3962	3963	4664	5102	9318
		0.10	1993	2002	2003	4770	6335	22226
		0.20	934	959	964	11498	16326	69399

Table 3. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick clamped plates resting on elastic foundation.

a)Subgrade reaction modulus k=500

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
500	1.0	0.05	29759	120226	120226	252899	377489	382001
		0.10	102044	376857	376857	745179	1050457	1069678
		0.20	278637	864190	864190	1568800	2053714	2110595
	2.0	0.05	14250	23434	4562	90194	92812	113158
		0.10	49981	81613	155628	295807	296925	357465
		0.20	278307	861812	863825	1566356	2045759	2103845

b)Subgrade reaction modulus k=5000

k	b/a	t/a	$\lambda = \omega^2$					
			λ_1	λ_2	λ_3	λ_4	λ_5	λ_6
5000	1.0	0.05	33343	123791	123791	256447	381027	385540
		0.10	103782	378568	378568	746871	1052140	1071364
		0.20	279493	865025	865025	1569623	2054527	2111418
	2.0	0.05	17840	27018	49199	93760	96382	116723
		0.10	51728	83350	157352	297524	298634	359176
		0.20	146447	231679	418269	694223	742664	829834

As also seen from Tables 2, and Fig.. 3, the values of the first three frequency parameters for a constant value of *b/a* decrease as the thickness/span ratio, *b/a*, increases up to the 3rd frequency parameters, but after the 3rd frequency parameters, the values of the frequency parameters for a constant value of *b/a* increase as the thickness/span ratio, t/*a*, increases.

The decreases in the frequency parameters with increasing value of b/a for a constant t/a ratio gets less for larger values of b/a up to the 3rd frequency parameters. After the 3rd frequency parameters, the decrase in the frequency parameters with increasing value of b/a for a constant t/a ratio gets also less for larger values of b/a.

The changes in the frequency parameters with increasing value of b/a for a constant t/a ratio is larger for the smaller values of the b/a ratios. Also, the changes in the frequency parameters with increasing value of b/a for a constant t/a ratio is is less than that in the frequency parameters with increasing increasing t/a ratios for a constant valueof b/a.

These observations indicate that the effects of the change in the t/a ratio on the frequency parameter of the plate are generally larger than those of the change in the b/a ratios considered in this study.

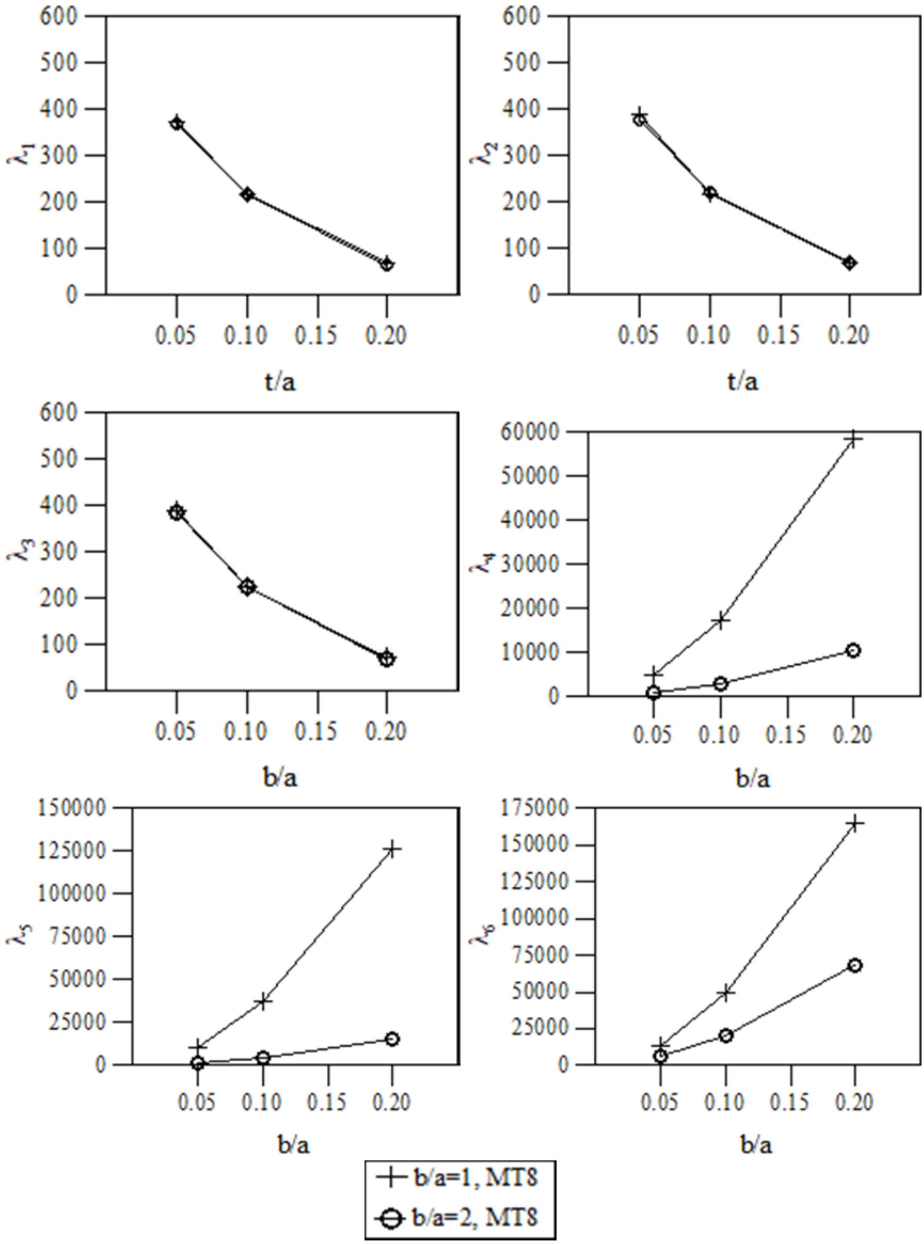


Figure 3. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates with subgrade reaction modulus k=500.

As also seen from Tables 3, and Figure. 4, the curves for a constant value of b/a ratio are fairly getting closer to each other as the value of t/a increases up to the 3rd frequency parameters.

This shows that the curves of the frequency parameters will almost coincide with each other when the value of the ratio of t/a increases more.

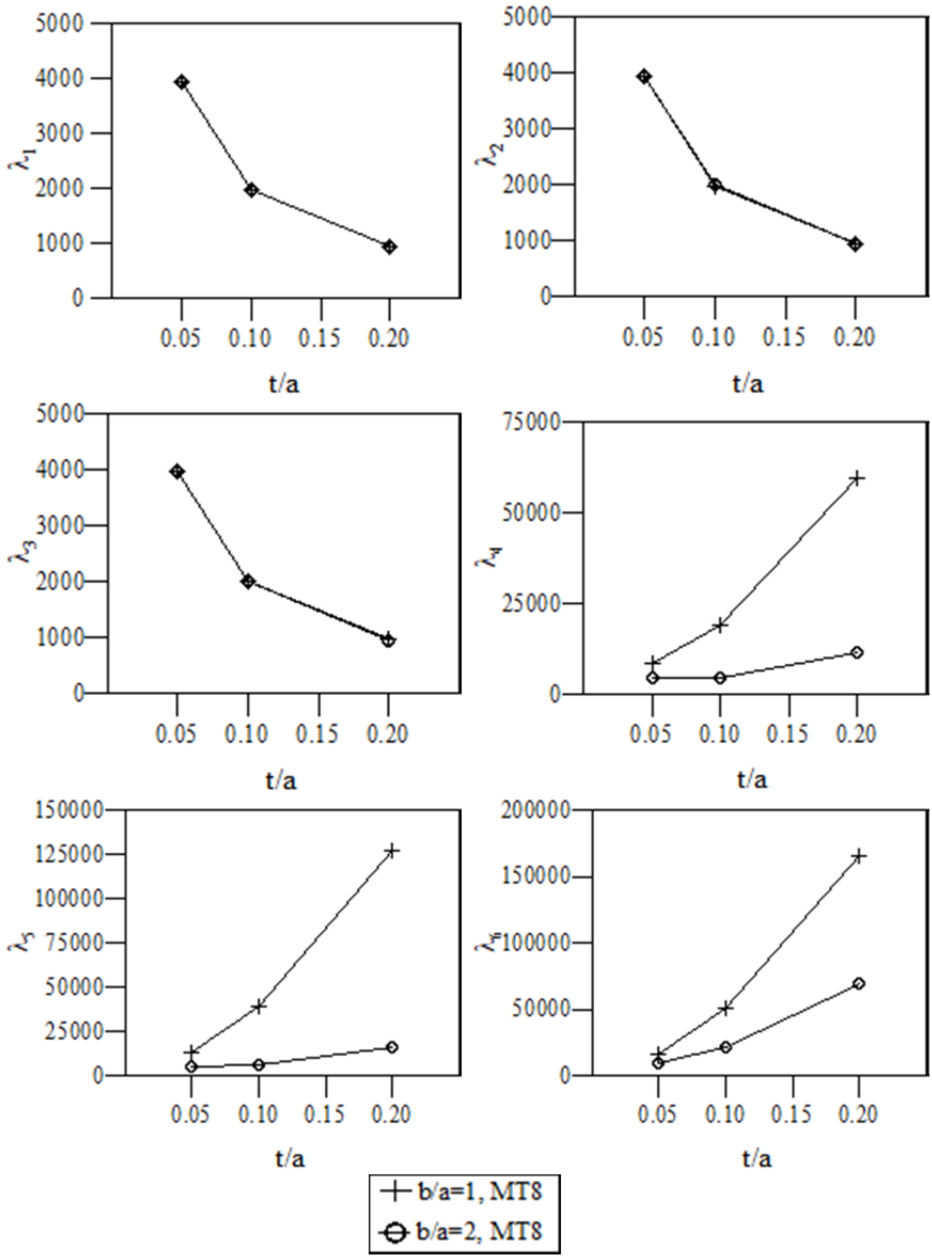


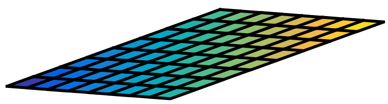
Figure 4. Effects of aspect ratio and thickness/span ratio on the first six frequency parameters of the thick free plates with subgrade reaction modulus k=5000.

After the 3rd frequency parameters, the curves for a constant value of t/a ratio are fairly getting closer to each other as the value of b/a increases. This also shows that the curves of the frequency parameters will almost coincide with each other when the value of the ratio of b/a increases more.

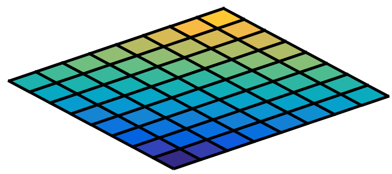
In other words, up to the 3rd ferquency parameters, the increase in the t/a ratio will not affect the frequency parameters after a determined value of t/a. After the 3rd ferquency parameters, the increase in the b/a ratio will not affect the frequency parameters after a determined value of b/a.

As seen from Tables 2, and 3, and Figures. 3, and 4, the values of the frequency parameters for a constant value of t/a decrease as the aspect ratio, b/a, increases. This behavior is understandable in that a thick plate with a larger aspect ratio becomes more flexible and has smaller frequency parameters. The decreases in the frequency parameters with increasing value of b/a ratio gets less for a constant value of t/a.

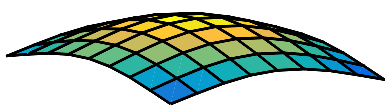
As seen from Tables 3, and Figures. 4,, the values of the frequency parameters for a constant value of b/a increase as the thickness/span ratio, t/a, increases. This behavior is also understandable in that a thick plate with a larger thickness/span ratio becomes more rigid and has larger frequency parameters. The increases in the frequency parameters with increasing value of t/a ratio gets larger for a constant value of b/a.



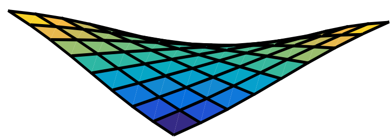
The first mode shape
(λ_1=3962.35)



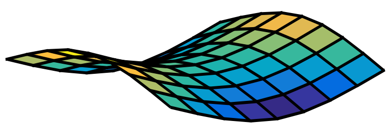
The second mode shape
(λ_2=3962.35)



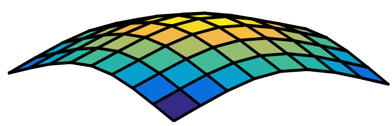
The third mode shape
(λ_3=3963.71)



The fourth mode shape
(λ_4=8597.96)



The fifth mode shape
(λ_5=13798.20)

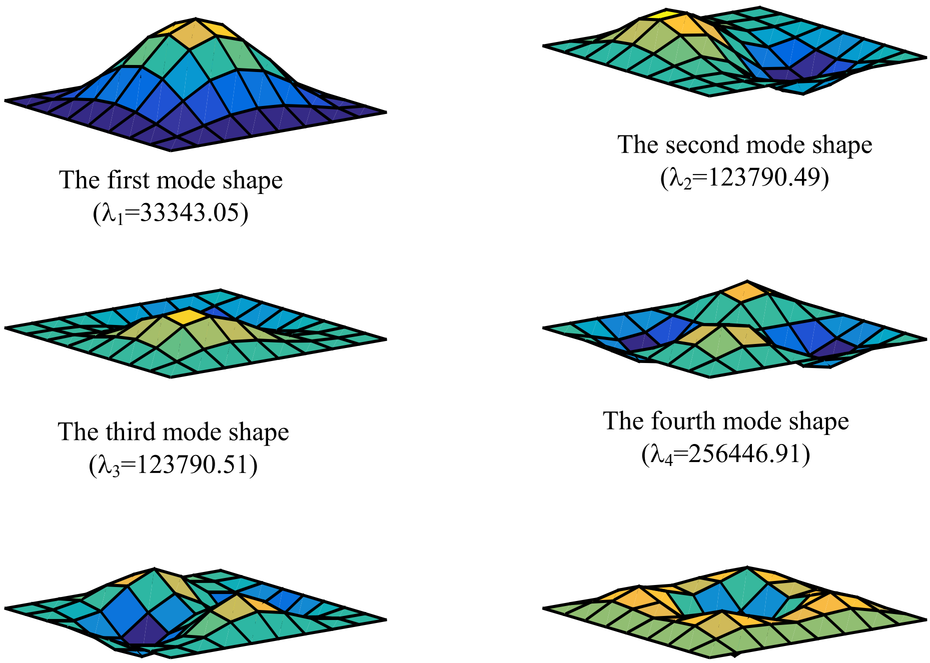


The sixth mode shape
(λ_6=16967.53)

Figure 5. The first six mode shapes of the thick free plates for b/a=1.0 and t/a=0.05 with subgrade reaction modulus k=5000.

It should be noted that the increase in the frequency parameters with increasing *t/a* ratios for a constant value of *b/a* ratio gets larger for larger values of the frequency parameters.

These observations indicate that the effects of the change in the *t/a* ratio on the frequency parameter of the thick plates clamped along all four edges are always larger than those of the change in the aspect ratio.



The first mode shape (λ_1=33343.05)

The second mode shape (λ_2=123790.49)

The third mode shape (λ_3=123790.51)

The fourth mode shape (λ_4=256446.91)

The fifth mode shape (λ_5=381026.62)

The sixth mode shape (λ_6=385539.73)

Figure 6. The first six mode shapes of the thick clamped plates for b/a=1.0 and t/a=0.05 with subgrade reaction modulus k=5000.

As also seen from Figure 4, the curves for a constant value of the aspect ratio, *b*/*a* are fairly getting closer to each other as the value of *t/a* decreases. This shows that the curves of the frequency parameters will almost coincide with each other when the value of the thickness/span ratio, *t/a*, decreases more. In other words, the decrease in the thickness/span ratio will not affect the frequency parameters after a determined value of *t/a*.

In this study, the mode shapes of the thick plates are also obtained for all parameters considered. Since presentation of all of these mode shapes would take up excessive space, only the mode shapes corresponding to the six lowest frequency parameters of the thick plate free, clamped along all four edges for *b/a* = 1 and *t/a* = 0.05 are presented. These mode shapes are given in Figures 5, and 6, respectively. In order to make the visibility better, the mode shapes are plotted with exaggerated amplitudes.

As seen from these figures, the number of half wave increases as the mode number increases. It should be noted that appearances of the mode shapes not given here for the thick plates clamped along all four edges are similar to those of the mode shapes presented here.

4. CONCLUSIONS

The purpose of this paper was to study parametric free vibration analysis of thick plates using higher order finite elements with Mindlin's theory and to determine the effects of the thickness/span ratio, the aspect ratio and the boundary conditions on the linear responses of thick plates subjected to vibration. As a result, free vibration analyze of the thick plates were done by using p version serendipity element, and the coded program on the purpose is effectively used. In addition, the following conclusions can also be drawn from the results obtained in this study.

The frequency parameters increases with increasing b/a ratio for a constant value of t/a up to the 3rd ferquency parameters, but after that the frequency parameters decrases with increasing b/a ratio for a constant value of t/a.

The frequency parameters decreases with increasing t/a ratio for a constant value of b/a up to the 3rd ferquency parameters, but after that the frequency parameters increases with increasing t/a ratio for a constant value of b/a.

The effects of the change in the t/a ratio on the frequency parameter of the thick plate are generally larger than those of the change in the b/a ratios considered in this study.

REFERENCES

[1] Ugural A.C. (1981), *Stresses in Plates and Shells*, McGraw-Hill., New York.

[2] Leissa A. W. The free vibration of rectangular plates. J Sound Vib 1973; 31 (3): 257-294.

[3] Leissa A.W. Recent research in plate vibrations, 1973–1976: classical theory. The Shock and Vibration, Digest 1977; 9 (10): 13–24.

[4] Leissa A.W. Recent research in plate vibrations, 1973–1976: complicatingeffects. The Shock and Vibration, Digest 1977; 9 (11): 21–35.

[5] Leissa A.W. Plate vibration research, 1976–1980: classical theory. The Shock and Vibration Digest 1981; 13 (9): 11–22.

[6] Leissa A.W. Plate vibration research, 1976–1980: complicatingeffects. The Shock and Vibration Digest 1981; 13 (10): 19–36.

[7] Leissa A.W. Plate vibration research,1981–1985—part I: classical theory. The Shock and Vibration Digest 1987; 19 (2): 11–18.

[8] Leissa A.W. Plate vibration research, 1981–1985—part II: complicatingeffects. The Shock and Vibration Digest 1987; 19 (3): 10–24.

[9] Providakis C. P., Beskos D. E. Free and forced vibrations of plates by boundary elements. Comput Meth Appl Mech Eng 1989;, 74: 231-250.

[10] Providakis C. P., Beskos D. E. Free and forced vibrations of plates by boundary and interior elements. Int J Numer Meth Eng 1989; 28: 1977-1994.

[11] Warburton G. B. The vibration of rectangular plates. Proceeding of the İnstitude of Mechanical Engineers 1954; 168: 371-384.

[12] Caldersmith G. W. Vibrations of orthotropic rectangular plates. ACUSTICA 1984; 56: 144-152.

[13] Grice R. M., Pinnington R. J. Analysis of the flexural vibration of a thin-plate box using a combination of finite element analysis and analytical impedances. J Sound Vib 2002; 249(3): 499-527.

[14] Sakata T., Hosokawa K. Vibrations of clamped orthotropic rectangular plates. J Sound Vib 1988; 125 (3): 429-439.

[15] Lok T. S., Cheng Q. H. Free and forced vibration of simply supported, orthotropic sandwich panel. Comput. Struct 2001; 79(3): 301-312.

[16] Si W.J., Lam K. Y., Gang S. W. Vibration analysis of rectangular plates with one or more guided edges via bicubic B-spline method. Shock Vib 2005; 12(5).

[17] Ayvaz Y., Durmuş A. Earhquake analysis of simply supported reinforced concrete slabs. J Sound Vib 1995; 187(3): 531-539.
[18] Reissner E. The Effect of Transverse Shear Deformation on the Bending of Elastic Plates. J Appl Mech (ASME) 1945; 12: A69-A77.
[19] Reissner E. On Bending of Elastic Plates. Quarterly Appl Math 1947; 5(1): 55-68.
[20] Reissner,E. On a Variational Theorem in Elasticity. J Math Physics 1950; 29: 90-95.
[21] Mindlin RD. Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates. J Appl Mech 1951; 18(1): 31-38.
[22] Hinton E., Huang HC. A family of quadrilateral Mindlin plate element with substitute shear strain fields. Comput Struct 1986; 23(3): 409-431.
[23] Zienkiewich OC. Taylor RL., Too JM., "Reduced integration technique in general analysis of plates and shells. Int J Numer Meth Eng 1971; 3: 275-290.
[24] Bergan PG., Wang X. Quadrilateral Plate Bending Elements with Shear Deformations. Comput Struct 1984; 19(1-2): 25-34.
[25] Özkul T. A., Ture U. The transition from thin plates to moderately thick plates by using finite element analysis and the shear locking problem. Thin-Walled Struct 2004; 42: 1405-1430.
[26] Hughes T. J. R., Taylor R. L., Kalcjai W. Simple and efficient element for plate bending. Int J Numer Meth Eng 1977; 11: 1529-1543.
[27] Özdemir Y. I., Bekiroğlu S., Ayvaz Y. Shear locking-free analysis of thick plates using Mindlin's theory. Struct Eng Mech 2007; 27(3): 311-331.
[28] Özdemir Y. I. Development of a higher order finite element on a Winkler foundation. Finite Elem Anal Des 2012; 48: 1400-1408.
[29] Wanji C., Cheung Y. K. Refined quadrilateral element based on Mindlin/Reissner plate theory. Int J Numer Meth Eng 2000; 47: 605-627.
[30] Soh A. K., Cen S., Long Y., Long Z. A new twelve DOF quadrilateral element for analysis of thick and thin plates. Eur J Mech ; A/Solids 2001; 20: 299-326.
[31] Brezzi F., Marini L. D. A nonconforming element for the Reissner-Mindlin plate. Comput Struct 2003; 81: 515-522.
[32] Belounar L., Guenfound M. A new rectangular finite element based on the strain approach for plate bending. Thin-Wall Struct 2005; 43: 47-63.
[33] Senjanovic I., Tomic M., Hadzic N., Vladimir N. Dynamic finite element formulations for moderately thick plate vibrations based on the modified Mindlin theory. Eng. Struct. 2017; 136:100-113.
[34] Cen S., Long Y.Q., Yao Z.H., Chiew S.P. Application of the quadrilateral area co-ordinate method: Anew element for Mindlin-Reissner plate. Int J Numer Meth Eng 2006; 66: 1-45.
[35] Shen H. S., Yang J., Zhang L. Free and forced vibration of Reissner-Mindlin plates with free edges resting on elastic foundation. J Sound Vib 2001; 244(2): 299-320.
[36] Woo K.S., Hong C.H., Basu P.K. and Seo C.G. Free vibration of skew Mindlin plates by p-version of F.E.M. J Sound Vib 2003; 268: 637-656.
[37] Qian, R.C. Batra, L.M. Chen. Free and forced vibration of thick rectangular plates using higher-orde shear and normal deformable plate theory and meshless Petrov-Galerkin (MLPG) method. Comput Modeling Eng & Sciences 2003; 4(5): 519-534.
[38] Özdemir Y. I., Ayvaz Y. Shear locking-free earthquake analysis of thick and thin plates using Mindlin's theory. Struct Eng Mech 2009; 33(3): 373-385.
[39] Gunagpeng Z., Tianxia Z., Yaohui S. Free vibration analysis of plates on Winkler elastic foundation by boundary element method. Opt Elect Materials Applicat II 2012; 529: 246-251.

[40] Fallah A., Aghdam M. M. and Kargarnovin M.H. Free vibration analysis of moderately thick functionally graded plates on elastic foundation using the extended Kantorovich method. Arch Appl Mech 2013; 83(2): 177-191.

[41] Jahromi H. N., Aghdam M. M., Fallah A. Free vibration analysis of Mindlin plates partially resting on Pasternak foundation. Int J Mech Sciences 2013;75: 1-7.

[42] Özgan K., Daloglu A. T. Free vibration analysis of thick plates on elastic foundations using modified Vlasov model with higher order finite elements. Int J Eng Materials Sciences 2012; 19: 279-291.

[43] Özgan K., Daloglu A. T. Free vibration analysis of thick plates resting on Winkler elastic foundation. Challenge J Struct Mech 2015; 1(2): 78-83.

[44] Cook R.D., Malkus D. S., Michael E. P. Concepts and Applications of Finite Element Analysis. John Wiley & Sons, Inc., Canada; 1989.

[45] Bathe KJ. Finite ElementProcedures. Prentice Hall, Upper Saddle River, New Jersey; 1996.

[46] Tedesco J. W., McDougal W. G., Ross C.A. Structural Dynamics. Addison Wesley Longman Inc., California; 1999.

[47] Weaver W., Johnston PR. Finite Elements for Structural Analysis. Prentice Hall, Inc., Englewood Cliffs, New Jersey; 1984.