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Research Article

FREE AND FORCED WHIRLING ANALYSES OF A SINGLE-DISK ROTOR SUBJECT TO AXIAL FORCE

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ABSTRACT

Rotor vibrations and its control is an important subject in many industries such as power plants and gas stations. When lateral vibrations of rotors during operation exceed allowable level, a huge damage will be occurred. Surge and stall may be some common reasons of these vibrations. This paper aims to present a simple model for surge and stall and assumes that its effect as a distributed force acting on a rotor-disk system. This is a basic and conceptual model for future investigations in this area. Therefore, the effect of a distributed axial force exerted on an assembled disk on a rotating shaft is investigated theoretically. The set of equations for free and forced whirling analyses of a rotor- disc is derived. Also, transverse load composed of unbalanced mass and total weight of the system is considered. For forced whirling analysis, static deflection of the rotor is considered as the initial conditions and rotational speed of the rotor is considered as a time variable parameter which increases from zero to its nominal value in a limited period of time. For a simply supported rotor, the free whirling analysis is investigated using Galerkin method and using Galerkin and Newmark-beta methods, the forced whirling analysis is studied numerically. Forward and backward frequencies and Campbell diagrams are presented in free whirling analysis and variation of deflection, and shear force in any point of the rotor are depicted versus time in forced whirling analysis. The most advantages of the presented paper are consideration of time-dependency of rotating speed in forced whirling analysis. Results indicate that the axial load has a considerable effect on the forward and backward frequencies and lateral vibration amplitude of the rotor.
Keywords: Axial load, lateral vibration, whirling, campbell diagram.

1. INTRODUCTION

Rotor systems are widely used assemblies for power transmission in various kinds of machinery. The lateral vibrations of the rotors must be limited because excessive vibrations will affect the running status of it and may lead to catastrophic accidents. Due to several factors, which contribute to the energy transfer, the rotor rotations generate different modes of vibrations: lateral, torsional and axial modes. Among these modes, the lateral is of the greatest concern [1]. The simplest model that can be adopted to study the flexible behavior of rotors consists of a point mass attached to a massless shaft, often referred to as Jeffcott rotor. One of the most important vibration analysis parameter is a natural frequency. Equality of the system rotation speed and

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system frequency creates a critical state in the system. Also critical speed is an important parameter in the design of systems and their vibration analysis. The operation of rotating systems in the frequency range of the resonator can be very risky and cause serious damage to it. Parameters like mass and stiffness affect the natural frequency, and any change in the system that changes these two parameters will change the natural frequency of the system.

In general, axial forces are indeed small therefore are not considered. In aircraft engines, the thrust generates axial force in the rotor system. In compressors stall and surge phenomena can generate horizontal force on disks. These forces may be constant, while in rotating machinery is specific, they may be harmonic or random. This is the main motivation of the current research.

Grybos [2] studied the effect of shear deformations on the critical speeds of rotors. Jei and Lee [3] derived critical speeds of an un-symmetric rotor with rigid disks. Free and forced whirling analysis of viscoelastic rotors was investigated by Sturla and Argento [4]. Jun and Kim [5] studied the effect of torsional torque on whirling of rotors. Tiwari and Bhaduri [6] assessed the effect of boundary conditions on the natural frequencies of the rotor bearing system. Nelson et al. [7] studied the vibration analysis of the Timoshenko rotor with internal damping under axial load. Edney et al. [8] presented the dynamic analysis of the tapered Timoshenko rotor. Chen et al. [9] analyzed an exact and direct modeling for rotor bearing system subject to axial load. Choi et al. [10] presented the consistent derivation of a set of governing differential equations describing the vibration in two orthogonal planes and the torsional vibration of a straight rotor with dissimilar lateral principle moments of inertia, subjected to a constant compressive axial load. Huajian gouyang et al. [11] presented a dynamic model for the vibration of a rotating Timoshenko beam subjected to a three- directional load moving in the axial direction. Askarian et al. [12] assessed the effect of various parameters such as axial force, unbalance, and coupling misalignment on vibration of a rotor. Nawal H.Al et al. [13] derived the equation of motion that governs the transvers vibration of a beam loaded axially and compared the natural frequencies thereof. Motallebi et al. [14] adopted homotopy analysis based method to assess the vibration of a nonlinear beam subject to axial force. Torabi et al. [15] presented an analytical solution for whirling analysis of axial-loaded Timoshenko rotor and corresponding basic function.

In this study the dynamics of a rotating shaft, modeled as a Timoshenko rotor, subject to distributed harmonic load acting on the surface of a disk on the rotor are investigated. For this, the dynamic equations of lateral vibrations of the rotor are derived using Timoshenko beam model and Newton's second law. Then, these governing equations are solved by Galerkin and Newmark method and effect of the symmetric axial force acting on the disk on the Campbell diagram and the rotor vibration response are analyzed.The most advantages of the presented paper are consideration of time-dependency of rotating speed in forced whirling analysis.

2. GOVERNING EQUATIONS AND SOLUTION PROCEDURE

A simply supported Timoshenko rotor subject to distributed axial force acted on disk is shown in Figure 1.

The rotational speed increases from zero to its nominal value (Ω_0) in time t_0 as:

$$\frac{\Omega}{\Omega_0} = \begin{cases} 2\frac{t}{t_0} - \left(\frac{t}{t_0}\right)^2 & t \le t_0 \\ 1 & t \ge t_0 \end{cases} \quad (1)$$

An element of rotor with forces and bending moment is shown in Figure 2.

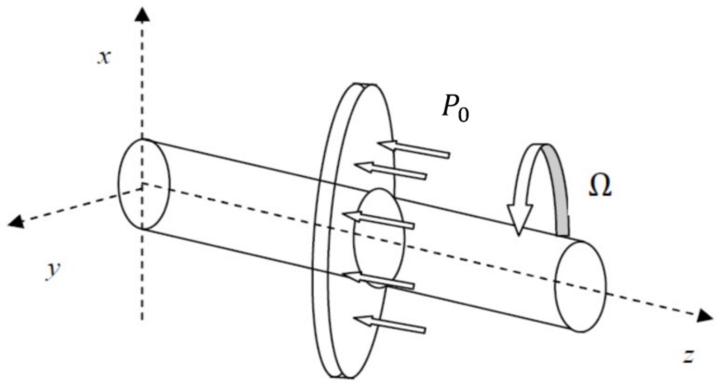

Figure 1. Rotating Timoshenko shaft with axial loaded disk

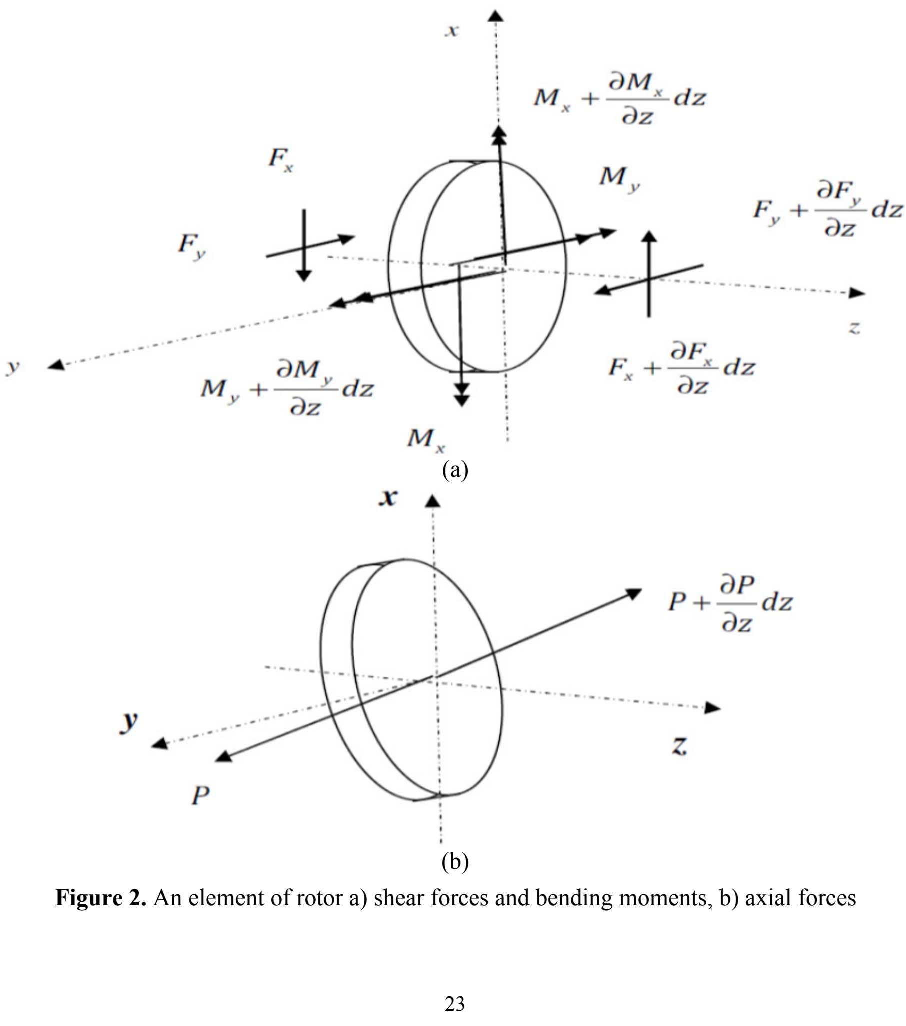

Figure 2. An element of rotor a) shear forces and bending moments, b) axial forces

By applying the equilibrium of forces and momentums, the set of equations of motion are extracted as follows:

$$F_x + \frac{\partial F_x}{\partial z}\,dz - F_x + \left(P + \frac{\partial P}{\partial z}\,dz\right)\left(\frac{\partial u_x}{\partial z} + \frac{\partial^2 u_x}{\partial z^2}dz\right) - P\frac{\partial u_x}{\partial z} + f_x(x\,,t\,) = [\rho A + M_0\delta(z - z_0)]dz\frac{\partial^2 u_x}{\partial t^2}$$

$$F_y + \frac{\partial F_y}{\partial z}\,dz - F_y + \left(P + \frac{\partial P}{\partial z}\,dz\right)\left(\frac{\partial u_y}{\partial z} + \frac{\partial^2 u_y}{\partial z^2}dz\right) - P\frac{\partial u_y}{\partial z} + f_y(x\,,t\,) = [\rho A + M_0\delta(z - z_0)]dz\frac{\partial^2 u_y}{\partial t^2}$$

$$M_x + \frac{\partial M_x}{\partial z}dz - M_x - \left(F_y + \frac{\partial F_y}{\partial z}dz\right)dz = [\rho I_x + I_x\delta(z - z_0)]\frac{\partial^2\varphi_x}{\partial t^2}dz + [\rho I_p + I_p\delta(z - z_0)]\Omega dz\frac{\partial\varphi_y}{\partial t} \tag{2}$$

$$M_y + \frac{\partial M_y}{\partial z}dz - M_y - \left(F_x + \frac{\partial F_x}{\partial z}dz\right)dz = [\rho I_y + I_y\delta(z - z_0)]\frac{\partial^2\varphi_y}{\partial t^2}dz + [\rho I_p + I_p\delta(z - z_0)]\Omega dz\frac{\partial\varphi_x}{\partial t}$$

where, u_x , u_y, φ_x *and* φ_y are displacement and rotation components in x and y directions, respectively; and ρ is the mass density. A, I_x , I_y *and* I_p are the cross- sectional area, moment of inertia around the *x* and y axes and polar moment of inertia, respectively, and f_x *and* f_y *and P* and z_0 are the forces per unit length in *x* and y directions and axial force and the disk position.

According to Timoshenko beam theory, components of bending moment (*M*) and shear force (*F*) in *x* and y directions are presented as follows [16]:

$$F_x = kGA\left(\frac{\partial u_x}{\partial z} - \varphi_y\right) \qquad M_x = \mathrm{EI}_x\frac{\partial\varphi_x}{\partial \mathrm{z}}$$

$$F_y = kGA\left(\frac{\partial u_y}{\partial z} - \varphi_x\right) \qquad M_\mathrm{y} = \mathrm{EI}_\mathrm{y}\frac{\partial\varphi_\mathrm{y}}{\partial \mathrm{z}} \tag{3}$$

where *E* and *G* are the modulus of elasticity and shear modulus, respectively; *k* is the shear correction factor depending on the shape of the section and Poisson's material ratio [17].

By neglecting the term of $(dz)^2$ and applying the following equation for a circular section:

$$I_\mathrm{p} = 2I_x = 2I_y = 2I \tag{4}$$

Eq.1 can be rewritten as:

$$kAG\left(\frac{\partial^2 u_x}{\partial \mathrm{z}^2} - \frac{\partial\varphi_y}{\partial \mathrm{z}}\right) + \frac{\partial}{\partial \mathrm{z}}\left(P\frac{\partial u_x}{\partial z}\right) + \mathrm{f}_x(\mathrm{x},\mathrm{t}) = [\rho A + M_0\delta(\mathrm{z}-\mathrm{z}_0)]\frac{\partial^2 u_x}{\partial \mathrm{t}^2}$$

$$kAG\left(\frac{\partial^2 u_y}{\partial \mathrm{z}^2} - \frac{\partial\varphi_x}{\partial \mathrm{z}}\right) + \frac{\partial}{\partial \mathrm{z}}\left(P\frac{\partial u_y}{\partial z}\right) + \mathrm{f}_\mathrm{y}(\mathrm{x},\mathrm{t}) = [\rho A + M_0\delta(\mathrm{z}-\mathrm{z}_0)]\frac{\partial^2 u_y}{\partial \mathrm{t}^2}$$

$$\mathrm{EI}\frac{\partial^2\varphi_x}{\partial \mathrm{z}^2} - \mathrm{kGA}\left(\frac{\partial u_y}{\partial z} + \varphi_x\right) = [\rho I + I\delta(\mathrm{z}-\mathrm{z}_0)]\frac{\partial^2\varphi_\mathrm{x}}{\partial \mathrm{t}^2} + 2[\rho I + I_0\delta(\mathrm{z} - \mathrm{z}_0)]\Omega\frac{\partial\varphi_y}{\partial \mathrm{t}} \tag{5}$$

$$\mathrm{EI}\frac{\partial^2\varphi_y}{\partial \mathrm{z}^2} - \mathrm{kGA}\left(\frac{\partial u_x}{\partial \mathrm{z}} + \varphi_y\right) = [\rho I + I_0\delta(\mathrm{z}-\mathrm{z}_0)]\frac{\partial^2\varphi_\mathrm{y}}{\partial \mathrm{t}^2} + 2[\rho I + I_0\delta(\mathrm{z}-\mathrm{z}_0]\Omega\frac{\partial\varphi_x}{\partial \mathrm{t}}$$

The force at the cross section of rotor is written as:

$$P(\mathrm{z},t) = -R_0 - P_0(t)H(\mathrm{z}-\mathrm{z}_0) \tag{6}$$

where, R_0 and P_0 are the reaction of force in the support and axial force exerted on disk. By re-arranging the equations of motion Eq.5 is yield:

$$\begin{aligned}
&kGA\left(\frac{\partial^2 u_x}{\partial z^2}+\frac{\partial \varphi_x}{\partial z}\right)-[R_0+P_0(t)H(z-z_0)]\frac{\partial^2 u_x}{\partial z^2}-P_0(t)H(z-z_0)\frac{\partial u_x}{\partial z}+f_x(x,t)\\
&\qquad=\left[\rho A+M_0\delta(z-z_0)\right]\frac{\partial^2 u_x}{\partial t^2}\\
&kGA\left(\frac{\partial^2 u_y}{\partial z^2}+\frac{\partial \varphi_y}{\partial z}\right)-[R_0+P_0(t)H(z-z_0)]\frac{\partial^2 u_y}{\partial z^2}-P_0(t)\delta(z-z_0)\frac{\partial u_y}{\partial z}+f_y(x,t)\\
&\qquad=[\rho A+M_0\delta(\mathrm{z}-\mathrm{z}_0)]\frac{\partial^2 u_y}{\partial t^2}\\
&EI\frac{\partial^2 \varphi_x}{\partial z^2}-kGA\left(\frac{\partial u_y}{\partial z}+\varphi_x\right)=[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)]\frac{\partial^2 \varphi_x}{\partial t^2}+2[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)]\Omega\frac{\partial \varphi_y}{\partial t}\\
&EI\frac{\partial^2 \varphi_y}{\partial z^2}+kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)=[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0]\frac{\partial^2 \varphi_y}{\partial t^2}+2[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0]\Omega\frac{\partial \varphi_x}{\partial t}
\end{aligned} \tag{7}$$

By introducing the following complex variables:

$$\begin{aligned}
u &= u_x + ju_y\\
\varphi &= \varphi_x + j\varphi_y\\
f &= f_x + jf_y
\end{aligned} \tag{8}$$

and The external forces that consisting weight and unbalanced force written as:

$$f(z,t)=f_1(z,t)+f_2(z,t) = -\rho Ag - M_0\, g\delta(z-z_0) + \Omega^2 m_0^p e_0[\cos(\Omega \mathrm{t}+\theta_0) + j\sin(\Omega \mathrm{t}+\theta_0)]\delta(z-z_0) \tag{9}$$

where, M_0 is the disk mass, m_0^p is the unbalanced mass, e_0 is the unbalanced radius and θ_0 is the angle of unbalanced mass with respect to horizontal axis, the following set of equations of motion is yield:

$$\begin{aligned}
&-\left[\rho A+M_0\delta(z-z_0)\right]\frac{\partial^2 u}{\partial t^2}+kGA\left(\frac{\partial^2 u}{\partial z^2}+j\frac{\partial \varphi}{\partial z}\right)-[R_0+P_0(t)H(z-z_0)]\frac{\partial^2 u}{\partial z^2}\\
&\qquad-P_0(t)\delta(z-z_0)\frac{\partial u}{\partial z}\\
&\quad=\ \rho Ag+\left[M_0 g-m_0^p\Omega^2 e_0\cos(\Omega t+\theta_0)\right]\delta(z-z_0)-j\Omega^2 m_0^p e_0\sin(\Omega t\\
&\qquad+\theta_0)\delta(z-z_0)\\
&-[\rho I+I_0\delta(z-z_0)]\frac{\partial^2 \varphi}{\partial t^2}+2j[\rho I+I_0\delta(z-z_0)]\Omega\frac{\partial \varphi}{\partial t}+EI\frac{\partial^2 \varphi}{\partial z^2}-kGA\left(\varphi-j\frac{\partial u}{\partial z}\right)\\
&\qquad=0
\end{aligned} \tag{10}$$

For a rotor with simple support the following equation must be met:

$$\begin{aligned}
z=0:\ u=0,\ \frac{\partial \varphi}{\partial z}=0\\
z=L:\ u=0,\ \frac{\partial \varphi}{\partial z}=0
\end{aligned} \tag{11}$$

The solution of Eq.10 is assumed as:

$$u(z,t)=\sum_{n=1}^{\infty} a_n(t)\sin(\frac{n\pi z}{L}), \qquad \varphi(z,t)=\sum_{n=1}^{\infty} b_n(t)\cos[\frac{(n-1)\pi z}{L}] \tag{12}$$

Inserting Eq.12 into Eq.10 and then running the required simplifications the following equations are obtained:

$$\sum_{n=1}^{\infty}\left\{\begin{matrix}[\rho A+M_0\delta(z-z_0)]sin\left(\frac{n\pi z}{L}\right)\ddot{a}_n(t)\\ +\left\{\begin{matrix}\left(\frac{n\pi}{L}\right)^2[kGA-R_0-P_0(t)H(\mathrm{z}-\mathrm{z}_0)]sin\left(\frac{n\pi z}{L}\right)\\ +[P_0(t)\delta(\mathrm{z}-\mathrm{z}_0)]\frac{n\pi}{L}cos\left(\frac{n\pi z}{L}\right)\end{matrix}\right\}a_n(t)\\ +jkGA[\frac{(n-1)\pi z}{L}]sin[\frac{(n-1)\pi z}{L}]b_n(t)\end{matrix}\right\} \tag{13}$$

$$=-\rho Ag+\{m_0^pe_0\Omega^2[\cos(\Omega\mathrm{t}+\theta_0)+jsin(\Omega\mathrm{t}+\theta_0)]-M_0g\}\delta(z-z_0)$$

$$\sum_{n=1}^{\infty}\left\{\begin{matrix}[\rho I+I_0\delta(z-z_0)]\ddot{b}(t)-2j\Omega[\rho I+I_0\delta(\mathrm{z}-\mathrm{z}_0)]\dot{b_n}(t)\\ +El\left(\frac{n\pi}{L}\right)^2b_n(t)+kGA\left[b_n(t)-j\frac{n\pi}{L}a_n(t)\right]\end{matrix}\right\}\cos\left[\frac{(n-1)\pi z}{L}\right]=0$$

By applied Galerkin method Eq.13 is rewritten as:

$$\sum_{n=1}^{\infty}\left\{\begin{matrix}\left[\frac{\rho AL}{2}\delta_{mn}+M_0sin\left(\frac{n\pi z_0}{L}\right)sin\left(\frac{m\pi z_0}{L}\right)\right]\ddot{a}_n(t)\\ +\frac{n\pi}{L}\left\{\begin{matrix}\frac{n\pi}{2}\left[kGA-R_0-\left(1-\frac{\mathrm{z}_0}{L}\right)P_0(t)\right]\delta_{mn}\\ P_0(t)cos\left(\frac{n\pi z_0}{L}\right)sin\left(\frac{m\pi z_0}{L}\right)\end{matrix}\right\}a_n(t)+j\frac{n\pi kGA}{2}\delta_{mn}b_n\end{matrix}\right\} \tag{14}$$

$$=\frac{-\rho AL}{m\pi}[1-\cos(m\pi)]$$

$$+\{m_0^pe_0\Omega^2[\cos(\Omega\mathrm{t}+\theta_0)+jsin(\Omega\mathrm{t}+\theta_0)]-M_0g\}\sin\left(\frac{m\pi z}{L}\right)$$

$$\sum_{n=1}^{\infty}\left\{\begin{matrix}\left[\frac{\rho AL}{2}\delta_{mn}+I_0cos\left(\frac{n\pi z_0}{L}\right)cos\left(\frac{m\pi z_0}{L}\right)\right]\ddot{b}_n(t)\\ -j\Omega\left[\rho IL\delta_{mn}+I_0cos\left(\frac{n\pi z_0}{L}\right)cos\left(\frac{m\pi z_0}{L}\right)\right]\dot{b}_n(t)\\ +\frac{1}{2}\left[\frac{EI}{L}(n\pi)^2+kGAL\right]\delta_{mn}b_n(t)-j\frac{kGA}{2}n\pi\delta_{mn}a_n(t)\end{matrix}\right\}=0$$

The simplified form is briefed as:

$$[M]\{\ddot{X}(t)\}+[C]\{\dot{X}(t)\}+[K(t)]\{X(t)\}=[F(t)] \tag{15}$$

3. NUMERICAL RESULTS AND DISCUSSION

In order to present and analyze the results obtained in this study, some specifications of the subject rotor are selected from Table 1.

Table 1. Considered rotor characteristics

characteristics	Value	Symbols& dimensions
Rotor diameter	50	$d\ (mm)$
Rotor length	4	$L\ (m)$
Modulus of elasticity	200	$E\ (GPa)$
Poisson's ratio	0.3	υ
Density	7860	$\rho\ \left(\frac{Kg}{m^3}\right)$
Location of disk	L/4	z_0
Disk diameter	90	*D(mm)*
Disk thickness	20	*t(mm)*
Unbalance mass	50	$m_0^p\ (gr)$
Unbalance radius	70	$e_0\ (mm)$
Unbalance angle with x direction	45	$\theta_0\ (\text{degree})$
Rotational speed	50	$\Omega\ (rpm)$

3.1. Free whirling

In order to determine the forward and backward rotor frequencies for free vibration, we have the following definitions:

$$u(z,t) = \sum_{n=1}^{\infty} a_n e^{j\omega t} \sin\frac{n\pi z}{L} \tag{16}$$

$$\varphi(z,t) = \sum_{n=1}^{\infty} b_n e^{j\omega t} \cos[\frac{(n-1)\pi z}{L}]$$

In this regard, ω is the natural frequency of the whirling. By applied Galerkin method can be written:

$$(\omega^2[M] + \omega[C] + [k])\{X\} = \{0\} \tag{17}$$

In order to obtain an unbiased zero response in (17), the determinants of the coefficients must be zero; in other words:

$$|\omega^2[M] + \omega[C] + [k]| = 0 \tag{18}$$

By using equation (18), the natural frequencies will be obtained for the motion of the whirling. Given the coherent coefficients in the stiffness matrix, all the roots of the equation will be purely imaginary, and they will result in forward and backward frequencies.

Initially, a rotor without axial force was investigated, the results of which are shown in the form of Campbell diagram for the first and second modes in Figures 3 and 4. As observed in Figures, when the rotor is stationary, the forward and backward frequencies are equal and, with increasing rotor speed, the forward frequencies increase and the backward frequencies decrease. The reason for this is the effect of the gyroscopic phenomenon.

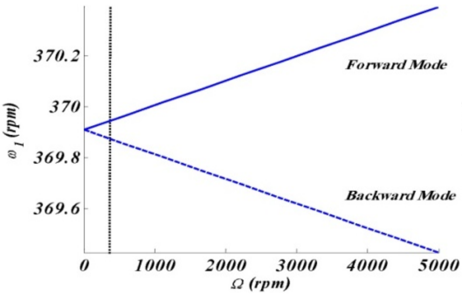

Figure 3. The Campbell diagram of the first mode of rotor without axial force

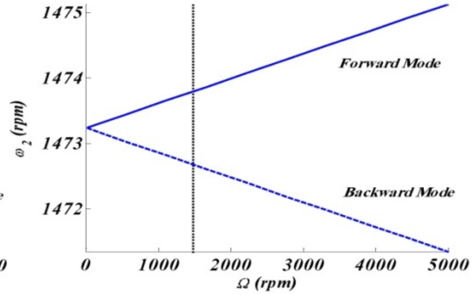

Figure 4. The Campbell diagram of the second mode of rotor without axial force

Then the force of 10000N is applied to the disk. The Campbell diagram for the first and second modes is shown in Figures 5 and 6. The blue charts are related to the rotor without axial force, which is presented for comparison.

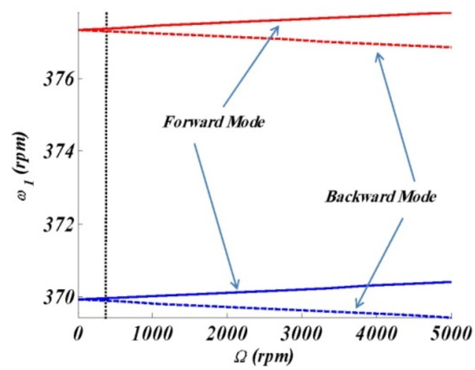

Figure 5. The Campbell diagram of the first mode of rotor with a force of 10,000 N

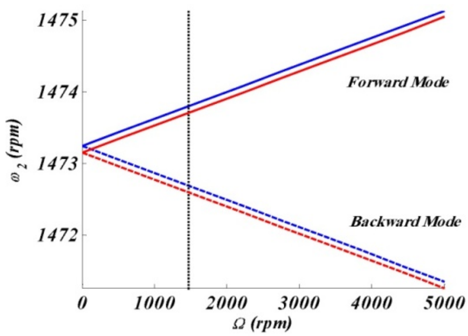

Figure 6. The Campbell diagram of the second mode of rotor with a force of 10,000 N

Applying this force will increase the frequency of the system in the first mode. Also, with increasing force, in the second mode the system frequencies begin to decrease. These trends depend on the position of the disk. There exists an appropriate agreement between results of this study and [12].

3.2. Forced Whirling

In order to examine the accuracy of the drawn up codes, the rotor is first modeled as a simple beam without rotation, and the results are compared with the accurate solution. These results are then obtained by assuming $\alpha=1/2$, $\beta=1/4$ and $\Delta t=0.01$ in Newmark-beta solution and by considering the first five sentences of Galerkin solution ($N_i=5$) by coding them in the MATLAB software.

Given that there exist no reference to verify the results obtained here, the modeling method, drawn up codes, the axial force and the rotor's rotation are removed, from this introduced system, leaving a beam with a simple support was taken into account. Therefore, the extent of motion, and the shear force along *x* axis in the middle and left support of the rotor are calculated and the results are shown in Figures 7 -8.

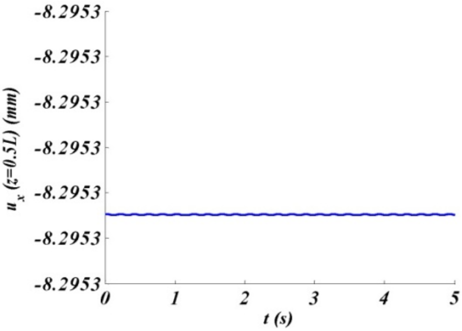

Figure 7. Displacement (x direction)

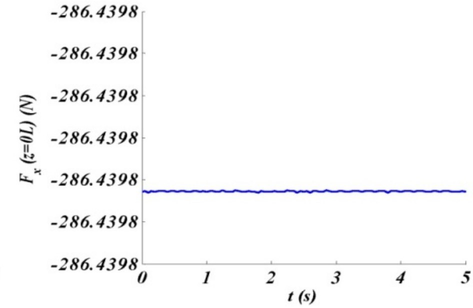

Figure 8. Shear force (x direction)

The results have reveals that a constant force of approximately 286 N is exerted on the left support. The present solution and analytical solution have been compared in Table 2. Analytical solution in Table 2 has been derived using strength of material formula. There exists an appropriate agreement between results and the drawn up codes of this study and [18].

Table 2. Comparison between present method and the analytical results

Parameter	Analytical solution	Present solution	Error percentage
F_x (N)	308	286	7.1

In order to study the effects of axial load on the responses¸ a harmonic axial load on the disk is considered with an amplitude and frequency of 30000 N and 5 rad/s, respectively. The shear force at the left support for the unbalanced rotor with this axial harmonic force is shown in Figures 9-10.

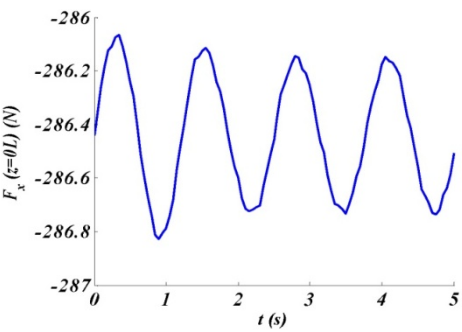

Figure 9. Shear force (x direction)

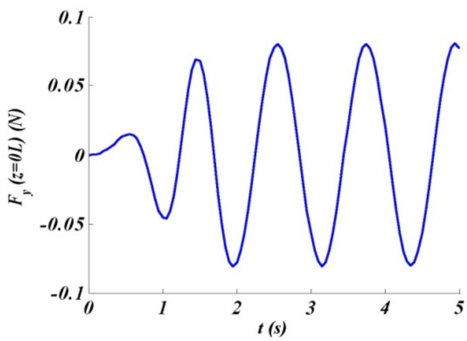

Figure 10. Shear force (y direction)

As observed in this set up the value of the shear force in the support is oscillated due to both the unbalanced and horizontal force.

As to the unbalanced rotor with axial harmonic force of p=300000sin (5t) the obtained results are shown in Figures 11-12.

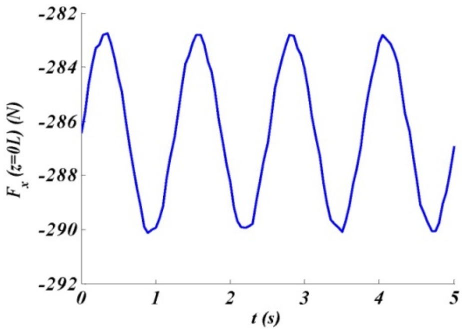

Figure 11. Shear force (x direction)

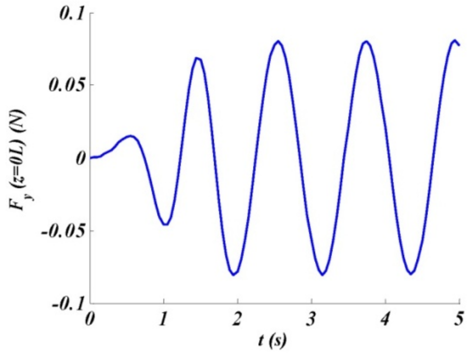

Figure 12. Shear force (y direction)

As observed there exists a direct relation between horizontal force and x direction shear force. Amplitude increasing of x direction shear force can be easily seen in these figures. In general, these results indicate which forces are created horizontally the effects of stall and surge can increase the amplitude of rotor lateral vibrations. Consequently, by measuring the vibrations in the bearing of the system, it is possible to detect the occurrence of these phenomena because they can lead to system failure.

4. CONCLUSIONS

In this study, the effect of applied horizontal distributed force on a disk of a single disk rotor was studied using Campbell's diagram. The rotor was assumed to be uniform and Timoshenko theory was used. Partial differential equations of motion were derived by considering equilibrium equations for an element of the rotor. The rotor was considered simply supported and Galerkin-Newmark method was applied directly to the partial differential equations of motion.The results show that the application of horizontal force in a rotor causes a change in the critical speed of the system. This is due to the effect of horizontal force on the stiffness of the system. The greatest effect is seen in the first mode of the system and increases with the increase of the applied horizontal force. However, with the increase of the force of the second mode, the second mode also changes the critical velocities, which is less than the first mode and decreases. The results of this study show that the effect of this force should be considered in designing rotary systems that are observing the presence of a horizontal force. Also¸ the effects of axial symmetrically force acted on disk on lateral vibration of a single disk rotor was analysed. Responses of lateral vibration for without and with axial symmetrically force were a analysed. From the results it is observed that by increasing the amplitude of force the amplitude of x direction shear force increases.

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