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Research Article

RITZ SOLUTION OF BUCKLING AND VIBRATION PROBLEM OF NANOPLATES EMBEDDED IN AN ELASTIC MEDIUM

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ABSTRACT

In this paper, free vibration and buckling of single-layered isotropic rectangular nanoplate is investigated based on classic plate theory (CPT). Nonlocal elasticity theory accounts for the small-nonlocal effects. Both Winkler-type and Pasternak-type foundation models are employed to simulate the surrounding elastic matrix. Governing differential weak form equations of the plate based on nonlocal elasticity theory are derived. The Ritz method is used to solve the problem of buckling and free vibration nanoplate for various boundary conditions. In order to confirm the accuracy of the results, data are compared with the other results published in literature. The effects of different parameters on the plate behavior, such as nonlocal parameter, aspect ratio, boundary conditions, Winkler and shear modulus are investigated.
Keywords: Buckling and vibration analysis, ritz method, nonlocal effects, elastic medium.

1. INTRODUCTION

Nano and micro-scale structures have attracted the attention of many researchers. Due to their superior electrical, mechanical and thermal properties, this structures are being used in micro- and nano electromechanical systems (MEMS/NEMS). Among the methods that can be used to analyze the mechanical nanostructures, involves experimental methods, molecular dynamics (MD) methods and continuous models. Due to the difficulty experiments and MD models, the continuum modeling of nanostructures has received the attention. But when the dimensions of the system is reduced to nanometer, the space between the atomic and intermolecular are considerable and system cannot be considered continuous. Also at the nanoscale, the effects of atomic and intermolecular forces on the static and dynamic behavior of structures is significant. Since continuum classical mechanics does not consider the size effects, so It is necessary to predict the behavior of micro and nanostructures, small scale effects to be considered, so modified classic continuum theories such as surface theory of elasticity[1], strain gradient theory [2], modified couple stress theory[3] and nonlocal elasticity theory[4], are reported. These modified continuum theories are being used for the analysis of nano/micro structure. The nonlocal elasticity theory which developed by Eringen[4], most common continuum theory is used to analyze small scale structures. To interfere the small scale effects in nonlocal elasticity theory it is

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assumed that stress at any point, as a function of the strains at all the other points in the domain. This contrasts with classic continuum theory which the stress at any point is only a function of strain in the same point. In this way, the internal size scale could be considered in the constitutive equations simply as a material parameter. Nanoplates are one of the most common nanostructures which due to low thickness in one direction, as are two-dimensional model and superior properties compared to other engineering materials. Understanding the characteristics of buckling and vibration of nanoplates is very important. Compared with the one-dimensional nano-structures such as nanobeam and nanowires, number of studies have been done is low about the mechanical behavior of nanoplates, In these studies focus more on the behavior of buckling and vibration. Pradhan and Phadikar[5] reported use of the Navier method for vibration nanoplates using nonlocal elasticity theory. Ansari et al.[6] investigated vibration analysis of single-layered graphene sheets(SLGSs) by using the nonlocal continuum plate model. They used the generalized differential quadrature method (DQM) to obtain the frequencies of free vibration of simply supported and clamped SLGS. Murmu et al.[7] introduced an analytical method to determine the natural frequencies of the non-local double-nanoplate system. They derived explicit closed-form expressions for natural frequencies for the case in which all four ends are simply supported. Pradhan and Phadikar[8] analyzed the Small scale effect on vibration of embedded multilayered graphene sheets based on nonlocal continuum models. The Navier-type solution method was used for simply supported nano-plates. Malekzadeh et al.[9] investigated the free vibration of orthotropic arbitrary straight-sided quadrilateral nano-plates by using the non-local elasticity theory. They used the DQM as an efficient and accurate numerical tool to solve the governing equation. Pouresmaeeli et al.[10] presented an analytical approach for free vibration analysis of double-orthotropic nano-plates with all edges simply-supported. Chakraverty and Behera[11] reported the Free vibration of rectangular nanoplates using Rayleigh–Ritz method. Mohammadi et al.[12] studied the free vibration behavior of circular and annular graphene sheet by using the non-local elasticity theory. Analytical frequency equations for circular and annular graphene sheets were obtained for different kinds of boundary condition. In other research[13] they investigated effect of shear in-plane load on the vibration analysis of graphene sheet embedded in an elastic medium. Levy type solution was applied to study the vibration and buckling behavior of nanoplates considering nonlocal theory by Aksencer and Aydogdu[14]. Pradhan and Murmu[15] analyzed the buckling of rectangular SLGSs under biaxial compression by use of the non-local elasticity. Hashemi and Samaei[16] proposed an analytical solution based on the non-local Mindlin plate theory and considering small-scale effects for analysis of the buckling of rectangular nano-plates. Murmu and Pradhan[17] studied the elastic buckling behavior of orthotropic small scale plates under biaxial compression. Farajpour et al.[18] investigated the buckling response of orthotropic SLGS by using non-local elasticity theory. They supposed two opposite edges of the plate were subjected to linearly varying normal stresses. Babaei and Shahidi[19] studied buckling of the quadrilateral nano-plates by use of non-local plate theory. Farajpour et al.[20] studied axisymmetric buckling of circular graphene sheets by using the non-local continuum plate model. Pradhan and Murmu[21] studied the influence of small scale effect on the buckling of single-layered graphene sheet embedded in an elastic medium. Murmu et al.[22] investigated biaxial and uniaxial buckling of bonded double-nanoplate systems. Behfar and Naghdabadi[23], investigated vibration characteristics of multi-layered nanoplates embedded into elastic medium with constant van der Walls force between nanoplates.

In this article, the free vibration and buckling behavior of single-layered isotropic rectangular plate is investigated when it is embedded in an elastic medium based on nonlocal elasticity theory. The governing differential equations for classical plate theory are achieved by using the principle of virtual work and Ritz method has been used to obtain natural frequency and buckling load of nanoplate. The accuracy of the results in special cases are compared with other results in literature. In addition, the effect of various parameters such as nonlocal parameter, aspect ratio, boundary conditions, Winkler and shear module parameter on the non-dimensional critical buckling and natural frequency of nanoplate are investigated.

2. THEORY

The purpose of this part is to introduce nonlocal elasticity theory and its application in classical plate theory, As well as how to obtain the governing equations for single-layered isotropic rectangular plate, when it embedded in an elastic medium.

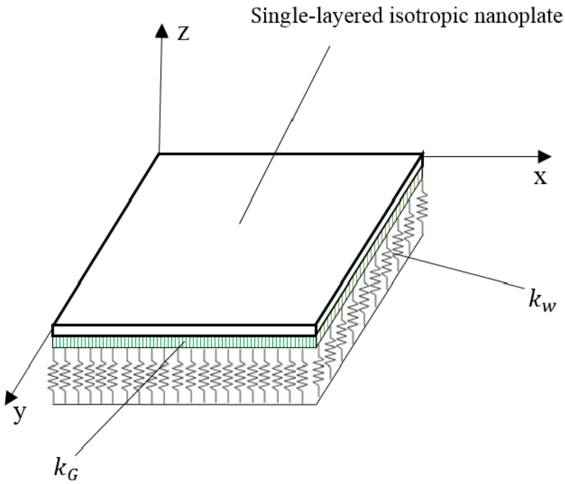

Figure 1. Definition of coordinate system for plate which is embedded in an elastic medium

2.1. Nonlocal elasticity theory

Based on nonlocal elasticity theory that has been proposed by the Eringen [4] the stress at reference point (x), not only a function of strain at that point, but also a function of strain at all other points of the body (x'). stress tensor at point x with considering effects nonlocal is defined by [4]:

$$t_{ij} = \int \alpha(|x' - x|, \tau)\, C_{ijkl}\varepsilon_{kl}(x')dv(x') \tag{1}$$

Where $t_{ij}(x)$ ، ε_{kl} و C_{ijkl} are the stress, strain, and fourth-order elasticity tensors, respectively. $\alpha(|x' - x|, \tau)$ is the non-local modulus, $|x' - x|$ is the Euclidean distance and τ is a material constant which depends on the internal (l_i) and external (l_e) characteristic lengths and is defined as $\tau = e_0(l_i/l_e)$. Since the solution of the integral Eq.(1) is very difficult, it can be transformed into a differential form which is more efficient. So the differential form of Eq.(1) can be written as [24]:

$$(1 - \mu\nabla^2)\, t_{ij} = C_{ijkl}\varepsilon_{kl} \tag{2}$$

Where μ is nonlocal parameter or small scale parameter.

2.2. Single-layered isotropic nanoplates

A single-layered rectangular nanoplates with length l_x and width l_y is considered. According Eq.(2), the two-dimensional nonlocal constitutive relations can be expressed as:

$$\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}-\mu\nabla^2\begin{Bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{xy}\end{Bmatrix}=\frac{E}{1-v^2}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix} \tag{3}$$

Where E and v are the modulus of elasticity and the Poisson's ratio of the nanoplate, respectively. And σ_{ij} are local stresses in the plate. According to the classical plate theory, the strains are expressed as:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix}-z\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix} \tag{4}$$

Here u ,v and w are components of the displacement vector in the mid-plane, along the x, y and z directions, respectively. Stress resultants can be written as:

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{xy}\end{Bmatrix}=\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix}-z\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix} \tag{5}$$

$$M_{ij}=\int_{-h/2}^{h/2}\sigma_{ij}z\,dz \quad ; \quad i,j\ are\ x,y \tag{6}$$

Where h is the thickness of the plate. By combing Eq.(6)–Eq.6 into Eq.(3) and integrating through of thickness, the final equations of the plate can be written in terms of displacement as follows:

$$\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix}-\mu\nabla^2\begin{Bmatrix}N_{xx}\\ N_{yy}\\ N_{xy}\end{Bmatrix}=\frac{Eh}{1-v^2}\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}\begin{Bmatrix}\frac{\partial u}{\partial x}\\ \frac{\partial v}{\partial y}\\ \frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\end{Bmatrix} \tag{7}$$

$$\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix}-\mu\nabla^2\begin{Bmatrix}M_{xx}\\ M_{yy}\\ M_{xy}\end{Bmatrix}=-D\begin{bmatrix}1 & v & 0\\ v & 1 & 0\\ 0 & 0 & \frac{1-v}{2}\end{bmatrix}\begin{Bmatrix}\frac{\partial^2 w}{\partial x^2}\\ \frac{\partial^2 w}{\partial y^2}\\ 2\frac{\partial^2 w}{\partial x\partial y}\end{Bmatrix} \tag{8}$$

Where $D=\frac{Eh^3}{12(1-v^2)}$ denotes the bending rigidity of nanoplate. Note that when small scale parameter is set to be zero, the relations given in Eq.(7) and Eq.(8) is reduced to the classical relations. Using the principle of virtual work, the equations of the motion of the nanoplate resting on Pasternak foundation can be obtained as follows:

$$\frac{\partial N_{xx}}{\partial x}+\frac{\partial N_{yx}}{\partial y}=m_0\frac{\partial^2 u}{\partial t^2} \tag{9}$$

$$\frac{\partial N_{yy}}{\partial y}+\frac{\partial N_{xy}}{\partial x}=m_0\frac{\partial^2 v}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 M_{xx}}{\partial x^2}+\frac{\partial^2 M_{yy}}{\partial y^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}-k_w w+k_G\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}\right)+\frac{\partial}{\partial x}\left(\bar{N}_{xx}\frac{\partial w}{\partial x}+\bar{N}_{xy}\frac{\partial w}{\partial y}\right)+\frac{\partial}{\partial y}\left(\bar{N}_{yy}\frac{\partial w}{\partial y}+\bar{N}_{xy}\frac{\partial w}{\partial x}\right)-m_0\frac{\partial^2 w}{\partial t^2}=0 \tag{11}$$

Where $m_0 = \rho h$ denotes the unit area mass and $k_w$and k_G denote the Winkler modulus and the shear modulus of the surrounding elastic medium respectively. Also $\bar{N}_{ij}$ shows the external in plane forces exerted on the plate.

With assumption $w(x,y,t) = w(x,y)e^{i\omega t}$ and using Eq.(7)-(8) and Eq.(11) one can obtain the nonlocal governing differential equation for the vibration and buckling of single-layered nanoplates based on displacement as below.

$$-D\frac{\partial^4 w}{\partial x^4} - D\frac{\partial^4 w}{\partial y^4} - 2D\frac{\partial^4 w}{\partial x^2 \partial y^2} + \mu\nabla^2\left[-\frac{\partial}{\partial x}\left(\bar{N}_{xx}\frac{\partial w}{\partial x}\right) - \frac{\partial}{\partial y}\left(\bar{N}_{yy}\frac{\partial w}{\partial y}\right) - \frac{\partial}{\partial x}\left(\bar{N}_{xy}\frac{\partial w}{\partial y}\right) - \frac{\partial}{\partial y}\left(\bar{N}_{xy}\frac{\partial w}{\partial x}\right) - m_0 w\omega^2 - k_G\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + k_w w\right] + \frac{\partial}{\partial x}\left(\bar{N}_{xx}\frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial y}\left(\bar{N}_{yy}\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial x}\left(\bar{N}_{xy}\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial y}\left(\bar{N}_{xy}\frac{\partial w}{\partial x}\right) - k_w w + k_G\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) + m_0 w\omega^2 = 0 \quad (12)$$

As part of this study, we use the Ritz method to solve the Eq.(12) so that we can investigate buckling and vibration behavior of single-layer isotropic plate.

3. RITZ SOLUTION

It is obvious that to use Ritz solution for Eq.(12), the weak form of the equation should be obtained. To do this, by using the part by part integral, the final form of the weak form of the problem can be written as:

$$\oint_A \varphi(Eq.12)dA=0 \quad (13)$$

Here φ is a trial function so that satisfies the essential boundary conditions. After performing integration by parts on Eq.(13) and Assuming $\bar{N}_{yy} = k\bar{N}_{xx}$ and $\bar{N}_{xy} = 0$, the weak statement can be written as follows:

$$\int_0^{l_y}\int_0^{l_x}\Big(-Dw_{,xx}\varphi_{,xx} - Dw_{,yy}\varphi_{,yy} - 2Dw_{,xy}\varphi_{,xy} - k_w w\varphi - \mu k_w\left(w_{,x}\varphi_{,x} + w_{,y}\delta w_{,y}\right) - k_G\left(w_{,x}\varphi_{,x} + w_{,x}\varphi_{,y}\right) - \mu k_G\left(w_{,xx}\varphi_{,xx} + 2w_{,xy}\varphi_{,xy} + w_{,yy}\varphi_{,yy}\right) - \bar{N}_{xx}\left[w_{,x}\varphi_{,x} + \mu\left(w_{,xx}\varphi_{,xx} + w_{,xy}\varphi_{,xy}\right)\right] - k\bar{N}_{xx}\left[w_{,y}\varphi_{,y} + \mu\left(w_{,yy}\varphi_{,yy} + w_{,xy}\varphi_{,xy}\right)\right] + \rho h\omega^2\left(w\varphi + \mu\left(w_{,x}\varphi_{,x} + w_{,y}\varphi_{,y}\right)\right)\Big)dxdy = 0 \quad (14)$$

Where l_x and l_y denotes the plate dimensions in the x and y directions. To solve the Eq.(14) by using Ritz method, the deflection of the plate has been written as follow:

$$w(x,y) = \sum_{i=1}^{n} c_i \Gamma \Delta_i \quad (15)$$

The c_i is the unknown coefficients and Δ_i and Γ are two-dimensional simple polynomials and boundary polynomial respectively, and can be written as follows:

$$\Delta_i = \{1, x, y, x^2, xy, y^2, x^3, x^2y, xy^2, y^3 \ldots\} \quad (16)$$

$$\Gamma = x^i(l_x - x)^j y^k (l_y - y)^l \quad (17)$$

Γ is taken such that, the boundary condition of the plate is satisfied. Substituting Eq.(15) into Eq.(14), the following equations of the nanoplate are obtained:

$$\sum_{j=1}^{n}\sum_{i=1}^{n}\left([K_{ij}] - \bar{N}_{xx}[\beta_{ij}] - \omega^2[M_{ij}]\right)c_i = 0 \quad (18)$$

Where K_{ij}, β_{ij} and M_{ij} are stiffness, buckling and mass matrices respectively.

4. RESULTS AND DISCUSSION

In this section the results of the nanoplate embedded in an elastic medium is presented. The properties of isotropic nanoplate in all examples of this article (except from those mentioned) are assumed as follows[7]. The thickness $h = 0.34nm$, Young's modulus $E = 1.06TPa$, Poisson's ratio $v = 0.25$, and mass density $\rho = 2250\frac{kg}{m^3}$. The length and width of the plate vary in different examples. In addition to the vibration analysis, the buckling of nanoplate under uniform uniaxial and biaxial loadings is considered in this part.

The non-dimensional buckling load, frequency vibration, Winkler and shear modulus parameter are defined as:

$$\bar{N} = \frac{\bar{N}_{xx} l_x^2}{D}, \qquad \bar{\omega} = l_x^2 \omega \sqrt{\frac{\rho h}{D}}, \qquad KW = \frac{k_w l_x^4}{D}, \qquad KG = \frac{k_G l_x^2}{D}$$

The letters S and C denote the simply supported and clamped boundary conditions of the plate, respectively. The boundary conditions are taken in clockwise direction starting at the edge $x = 0$.

4.1. Verification

In this part, the proposed Ritz method was evaluated and results compared with other data reported in the literature. Aksencer and Aydogdu[14] applied Navier method for the uniaxial buckling analysis of rectangular nanoplates. In order to validate the numerical results, the comparison of solution Ritz, with those of exact ones[14] is presented. In this comparison it is assumed that $l_x/l_y = 0.5$ and nonlocal parameter is set to 1nm^2. The nonlocal buckling Load to local buckling ratio (N^{NL}/N^{L}) are calculated for SSSS boundary conditions and is shown in Figure 2. It is seen that the present results are in a good agreement with the results of Aksencer and Aydogdu[14].

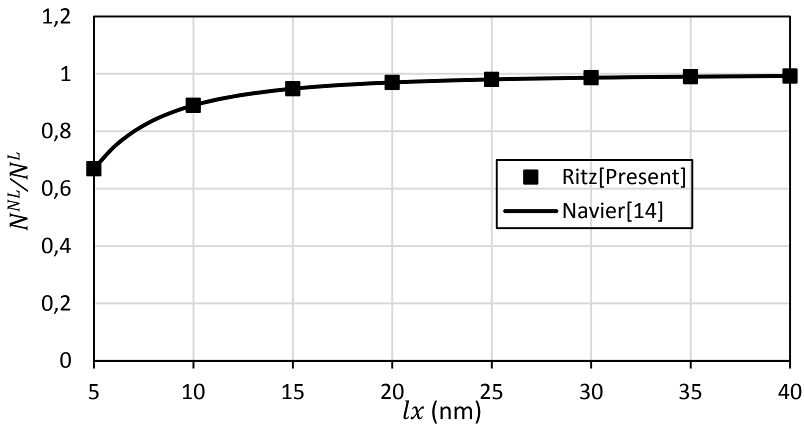

Figure 2. Comparison of Ritz method (present) with Navier method[14] for an isotropic nanoplate under uniaxial loading.

In another example, the present results are compared with the natural frequencies of square single layer nanoplate reported by Pradhan and Phadikar [5] as is shown in Figure 3. The nonlocal parameter of the plate is $\mu = 1nm^2$ and plate has SSSS boundary conditions. From this figure, it is clearly seen that the results are in good agreement with the results of Navier methid.

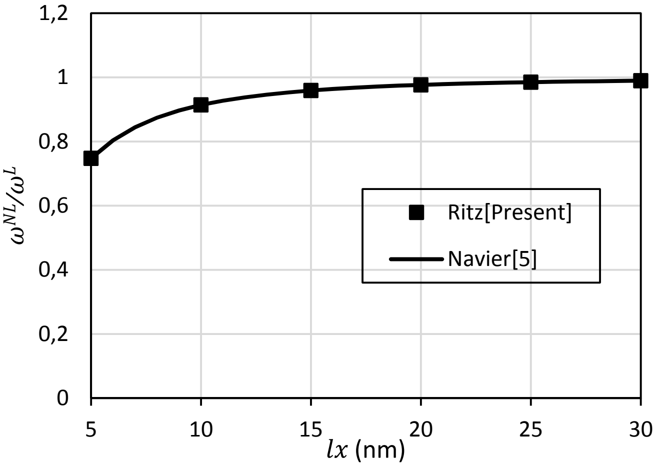

Figure 3. Comparison Ritz method (present) with Navier method [5] for an isotropic nanoplate under free vibration

4.2. Buckling analysis of nanoplate

In this section, the buckling load of the nanoplate versus nonlocal parameter is investigated. Table 1 shows a convergence study for buckling load a square nanoplate with all edges simply supported. The length of the plate is $l_x = 10nm$. As shown in Table 1 it is seen that by selecting the 18 terms of polynomials, the error is under 0.01 percent. The nanoplate under uniform uni-axial and biaxial loadings is considered in this section. Figure 4 and Figure 5 shows the non-dimensional buckling load for square single-layered nanoplate ($l_x = 10nm$) subjected to biaxial uniform compression loading for SSSS and CCCC boundary conditions. The figures are plotted versus the nonlocal parameter for different values of compression ratio ($k = 1, k = 1.2,\ k = 1.4, k = 1.6, k = 1.8$ و $k = 2$). In all cases, by increasing of compression ratio, the non-dimensional buckling load decreases. Moreover it can be concluded from this figures that increasing the nonlocal parameter, leads to decrease in non-dimensional critical buckling load. It means that the stiffness of system, reduced with the increasing of nonlocal parameter.

Table 1. Convergence study for the Ritz method

Number terms(n)	$\mu = 0nm^2$	$\mu = 1nm^2$	$\mu = 2nm^2$	$\mu = 4nm^2$
	Buckling load	Buckling load	Buckling load	Buckling load
9	19.7495	16.4923	14.1574	11.0334
12	19.7442	16.4886	14.1547	11.0317
15	19.7392	16.4852	14.1522	11.0302
18	19.7392	16.4852	14.1522	11.0302

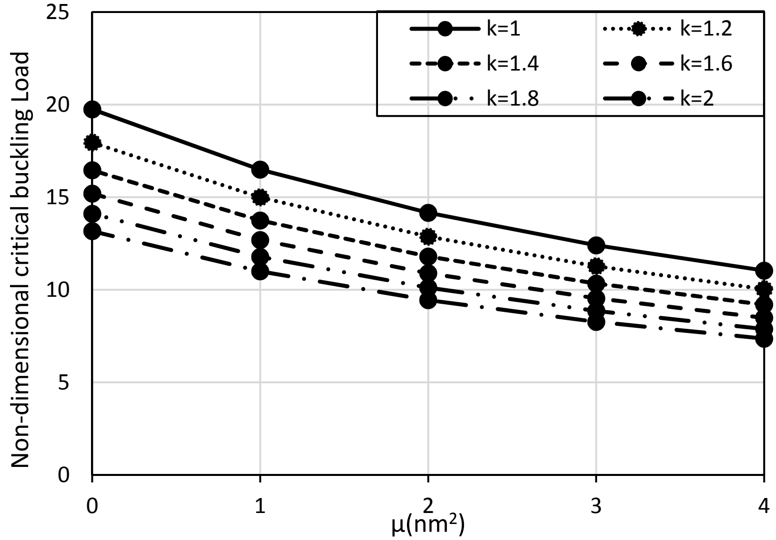

Figure 4. Variation of non-dimensional buckling load of nanoplate versus nonlocal parameter for different compression ratio (SSSS boundary conditions)

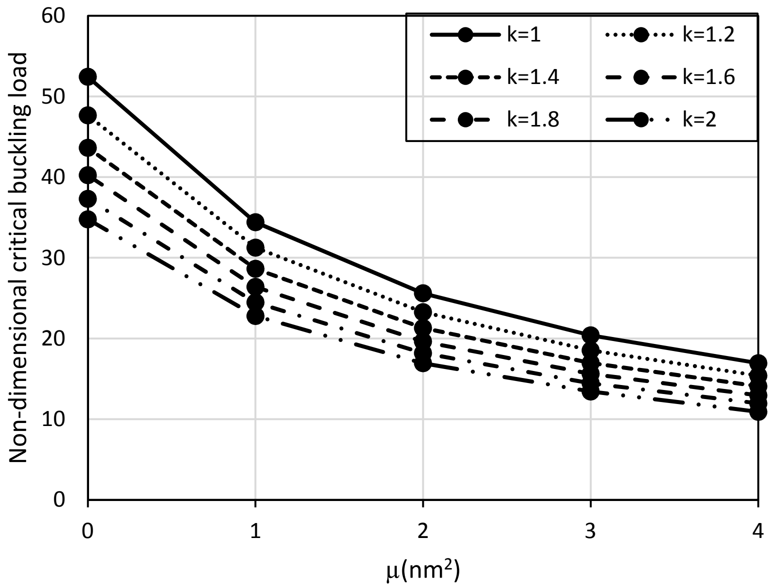

Figure 5. Variation of non-dimensional buckling load of nanoplate versus nonlocal parameter for different compression ratio (CCCC boundary conditions)

The effect of aspect ratio on the buckling load of single-layered nanoplate with different nonlocal parameter ($\mu = 0nm^2, 1nm^2, 2nm^2, 3nm^2, 4nm^2$) for CCCC and SSSS boundary conditions is also studied as shown in Figure 6 and Figure 7 respectively. The plate is assumed to have $l_x = 10nm$ and under uniaxial loading in the *x* direction. Figure 6 reveals that increasing in aspect ratio, reduces the small scale effects and the nonlocal curves converges to the classical theory results (μ=0). Also it can be concluded from figures, as the aspect ratio of the plate increases, the non-dimensional critical buckling load increases.

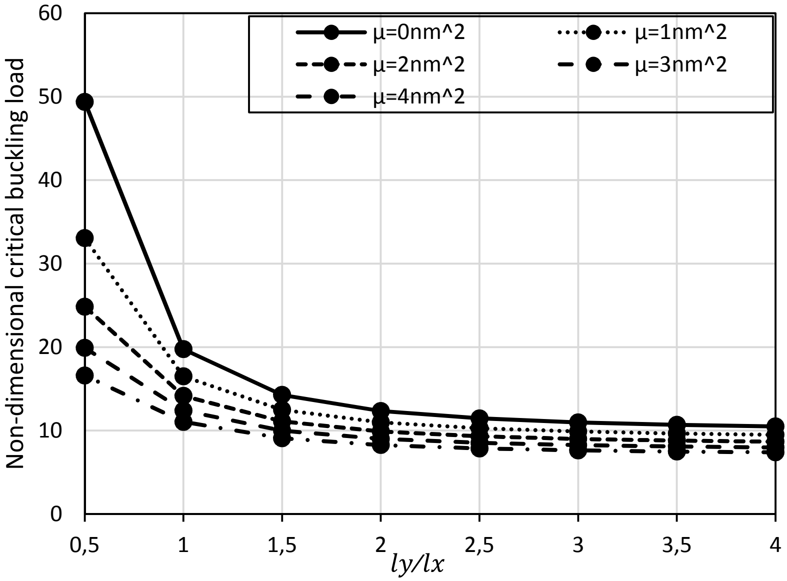

Figure 6. Variation of non-dimensional buckling load of nanoplate versus aspect ratio for different nonlocal parameter under uniaxial load (SSSS boundary conditions)

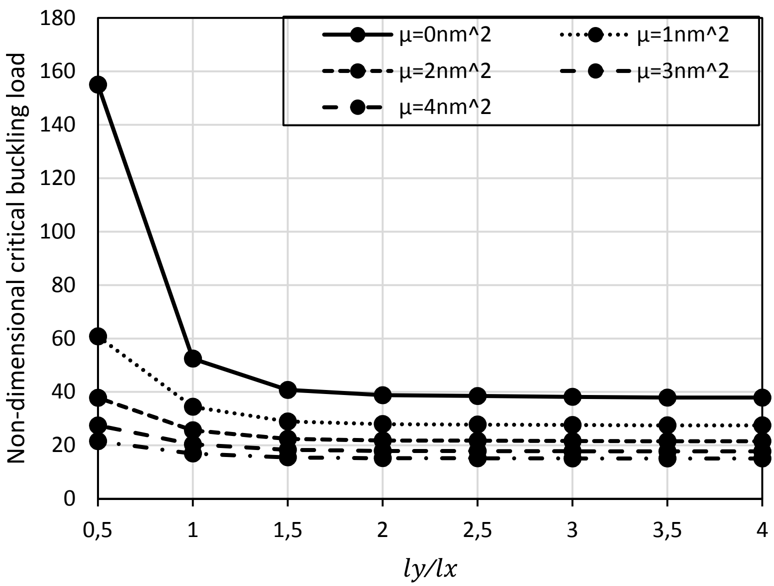

Figure 7. Variation of non-dimensional buckling load of single-layered nanoplate with apect ratio for different nonlocal parameter under uniaxial load (CCCC boundary conditions)

Figure 8 depicts the variation of Non-dimensional critical buckling load with the nonlocal parameter for different boundary conditions (SSSS, CCCC, SCSC, CSCS). A value of is assumed for the square nanoplate under uniaxial loading.

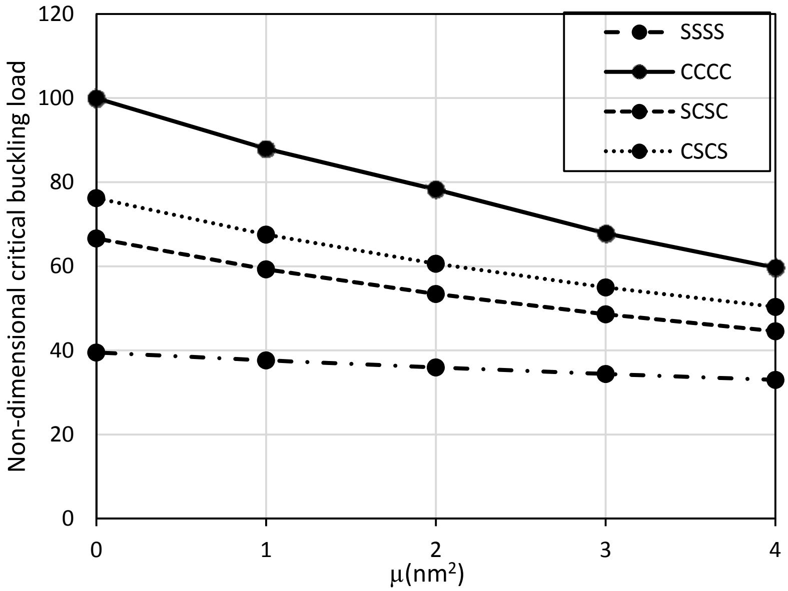

Figure 8. Variation of non-dimensional critical buckling load of single-layered nanoplate with nonlocal parameter for different boundary conditions under uniaxial load

To see the effects the elastic medium of Winkler-type foundation on the buckling behavior of square nanoplate subjected to uniaxial compression loading, the non-dimensional critical buckling load for various values of nonlocal parameter and Winkler modulus parameter for SSSS and CCCC are plotted in Figure 9 and Figure 10 respectively. It should be noticed , it signifies that the plate is free and not embedded in an elastic medium. It can be seen that by increasing Winkler modulus, the critical buckling load for all values of nonlocal parameter is increases.

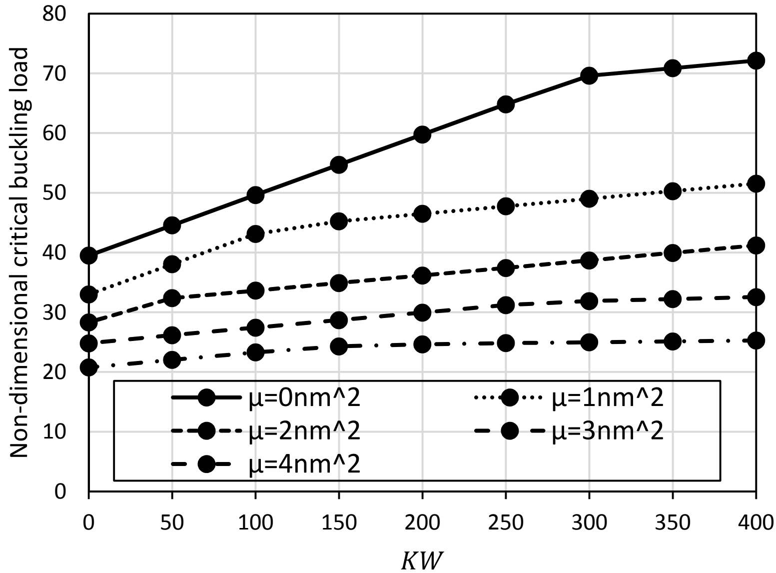

Figure 9. Variation of non-dimensional critical buckling load of single-layered nanoplate with Winkler modulus parameter for different nonlocal parameter under uniaxial load(SSSS boundary conditions)

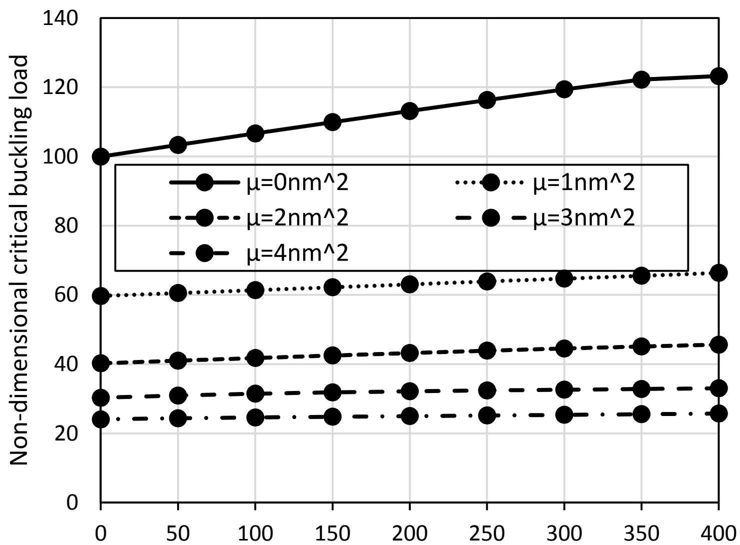

Figure 10. Variation of non-dimensional critical buckling load of single-layered nanoplate with Winkler modulus parameter for different nonlocal parameter under uniaxial load (CCCC boundary conditions)

Figure 11 and Figure 12 illustrate the effect of shear modulus parameter on the buckling load of a nanoplate with elastic medium modeled as Pasternak foundation for SSSS and CCCC boundary conditions. Length of plate is taken as 10nm and value of Winkler modulus parameter is $KW = 250$ and the plate is under uniaxial loading. As can be seen for all cases, with the increases of shear modulus parameter, the non-dimensional critical buckling load increases.

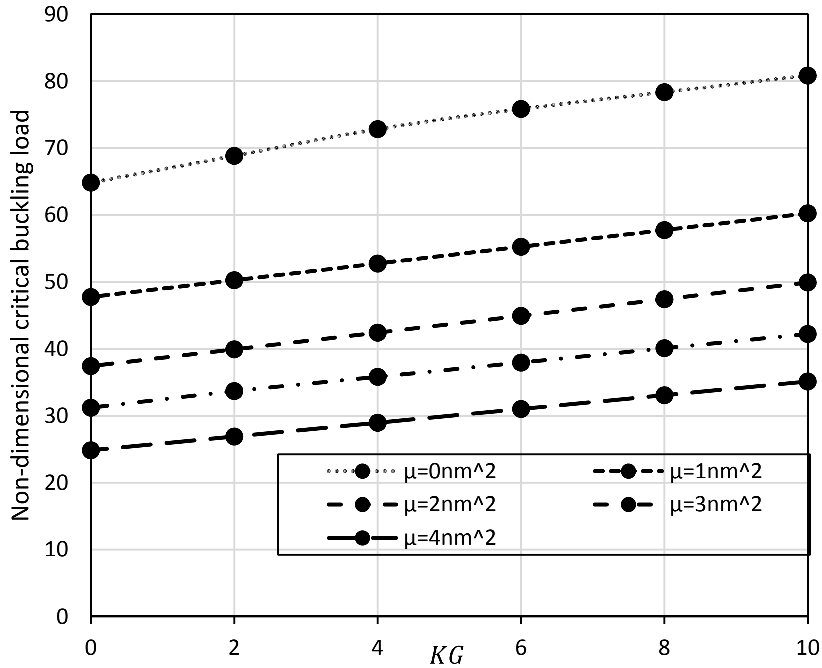

Figure 11. Variation of non-dimensional critical buckling load of single-layered nanoplate with shear modulus parameter for different nonlocal parameter under uniaxial load (SSSS boundary conditions)

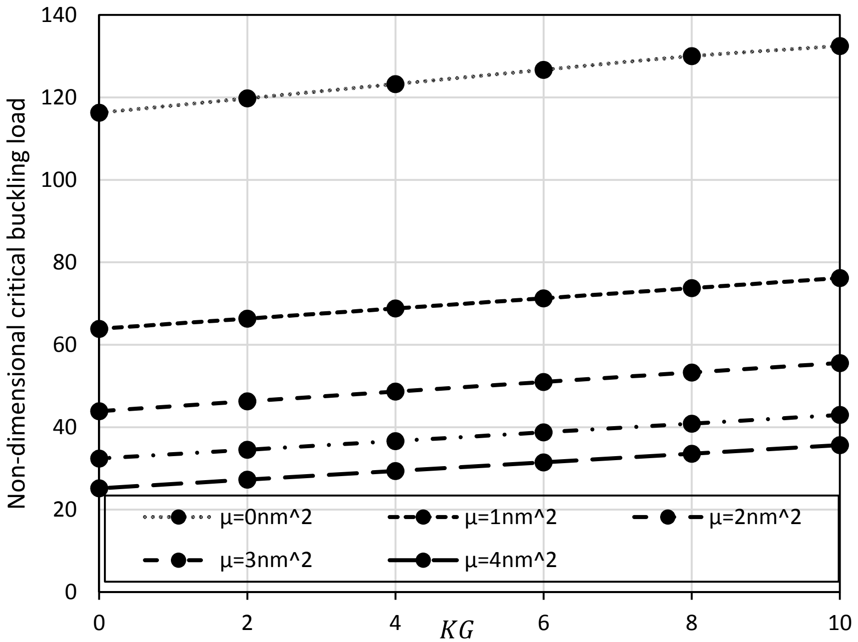

Figure 12. Variation of non-dimensional critical buckling load of single-layered nanoplate with shear modulus parameter for different nonlocal parameter under uniaxial load (CCCC boundary conditions)

4.3. Vibration analysis of nanoplate

In this section, the vibration analysis of the nanoplate is studied. In Figure 13 and Figure 14, the non-dimensional natural frequency of nanoplate, have been depicted for SSSS and CCCC boundary conditions respectively. The figures are plotted based on variation of aspect ratio and for different nonlocal parameter, for a square plate with $l_x = 10nm$. As it is seen, in constant aspect ratio, by increasing nonlocal parameter, the non-dimensional natural frequency reduced. The reason for this phenomenon is the decreasing of stiffness nanoplate. Also, it is seen that in all boundary condition, by increasing the nonlocal parameter, the natural frequency is decreasing.

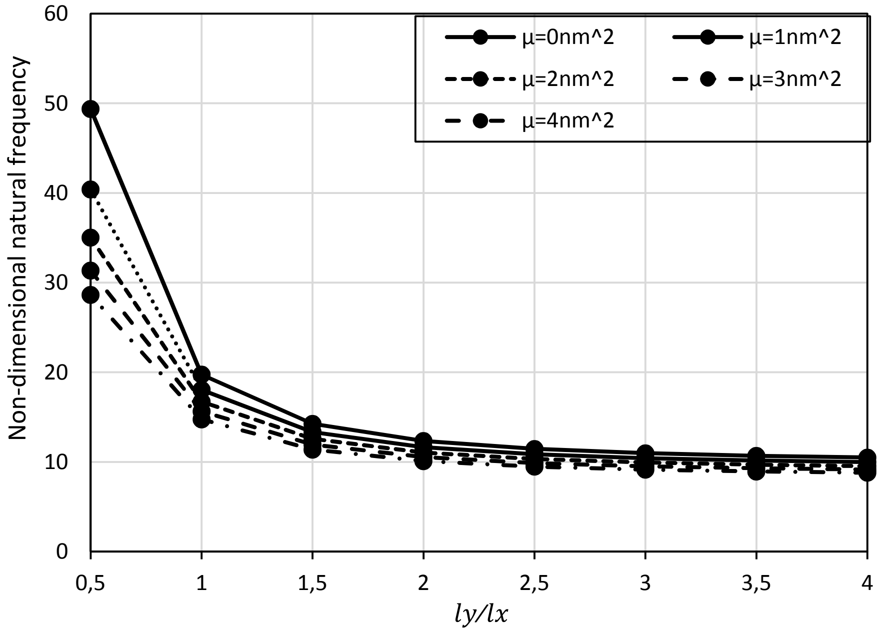

Figure 13. Variation of non-dimensional natural frequency of nanoplate versus aspect ratio for different nonlocal parameter (SSSS boundary conditions)

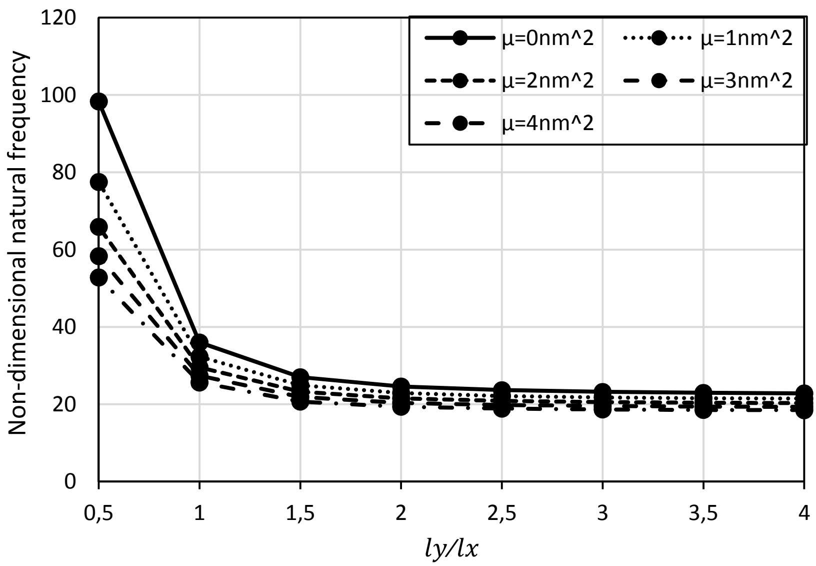

Figure 14. Variation of non-dimensional natural frequency of nanoplate with aspect ratio for different nonlocal parameter (CCCC boundary conditions)

To study the small-scale effects on the natural frequencies of nanoplate with different boundary conditions, effects of nonlocal parameter on the non-dimensional natural frequency of nanoplate with $l_x = 10nm$ and aspect ratio is 1 for different boundary conditions (SSSS, CCCC, SCSC, CSSC) is plotted in Figure 15. It is implied that the difference between curves decreases when the nonlocal parameter takes greater values.

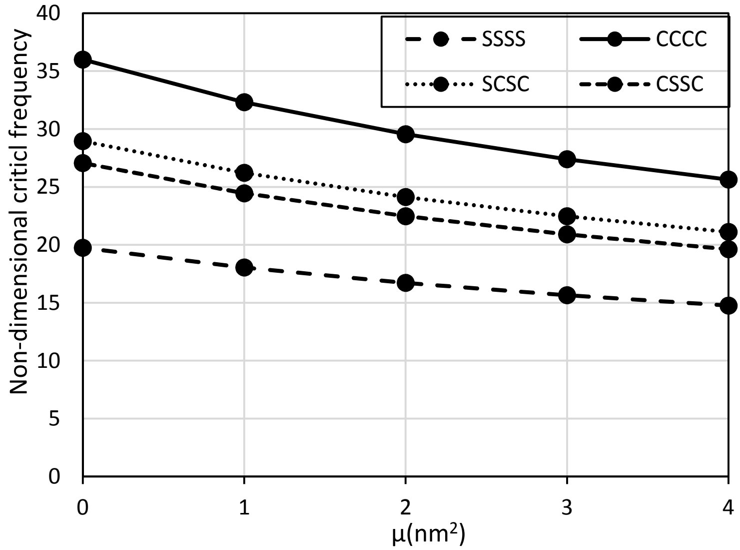

Figure 15. Variation of non-dimensional natural frequency of single-layered nanoplate with nonlocal parameter for different boundary conditions

Figure 16 shows the Winkler modulus parameter effect on non- dimensional natural frequency of square nanoplate with different nonlocal parameter. The length of plate is considered as $l_x = 10nm$. In these figures it is obvious that with increasing Winkler modulus, the non-dimensional natural frequency for all values of nonlocal parameter is increases.

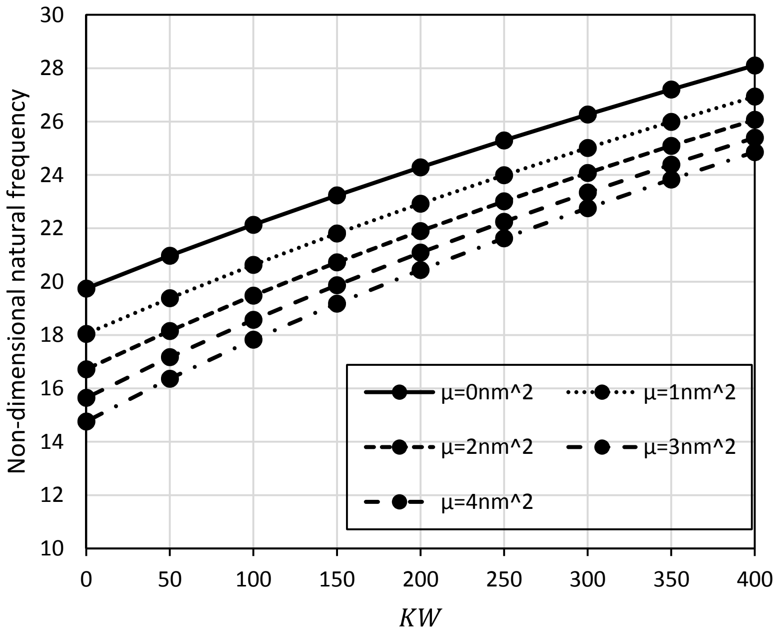

Figure 16. Variation of non-dimensional natural frequency of nanoplate with Winkler modulus parameter for different nonlocal parameter (SSSS boundary conditions)

5. CONCLUSIONS

In this paper, we studied the small scale effects on the buckling and vibration analysis of single-layered rectangular nanoplate, when it is embedded in an elastic medium. The governing equations of nanoplate obtained by considering the Eringen nonlocal theory. The Ritz method was used to solve the equation. Both Winkler-type and Pasternak-type foundation models are employed to simulate the surrounding elastic matrix. Effects of various parameters such as nonlocal parameters, aspect ratio, boundary conditions, Winkler and shear module parameters on the buckling load and natural frequency of nanoplate are investigated. It is shown that the small scale, has a significant influence on the buckling load and natural frequency of nanoplate. Also it is seen that by increasing aspect ratio and Winkler and shear modulus parameters, the difference between the local and nonlocal theories decreases. By increasing the Winkler and shear module parameters, the nanoplate shows stiffer behavior and has greater values of natural frequency and buckling load. It is seen that in higher values of small scale parameter, the Winkler and shear module has lower effect on the natural frequency and buckling load of the nanoplate. Also it is seen that the shear module has more effect on the natural frequency and buckling load of the nanoplate than Winkler module. It is worth to note that, if we want to have a naoplate with greater natural frequency, we should use Winkler or shear foundation.

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