Sigma J Eng & Nat Sci 35 (2), 2017, 187-200

Sigma Journal of Engineering and Natural Sciences
Sigma Mühendislik ve Fen Bilimleri Dergisi

Research Article

THE EARTHQUAKE RISK ANALYSIS BASED ON COPULA MODELS FOR TURKEY

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Received: 08.08.2016 Revised: 13.12.2016 Accepted: 08.02.2017

ABSTRACT

This study aims to explore the dependence structure between magnitude and frequency for Turkey earthquake data. In the literature, the Gutenberg Richter (GR) model based on lineer regression is often used to determine this dependence. The dependence structure is evaluated using copula models in this study. Copulas are useful statistical tool for modeling the dependence structure so it does not require assumptions such as linearity and normality. Therefore, as well as GR model, various copula functions are used to determine the magnitude-frequency relationship of earthquakes. An application is given to illustrate that the copulas can be used as alternatives to the GR model. The best copula models are selected by goodness of fit tests. Additionally, the probabilities of earthquake occurrence and the bivariate return periods are estimated for these selected copula models. It is seen that the probabilities of earthquakes occurrence for GR and copula models are almost identical, whereas the return periods based on copula models is more realistic than GR approach.
Keywords: Earthquake, magnitude-frequency, Gutenberg-Richter, dependence, copula, Turkey.

1. INTRODUCTION

Modeling of the relationship between the number of earthquakes and magnitudes is quite important in earthquake risk analysis. The Gutenberg-Richter (GR) model is often used to determine the magnitude-frequency relationship in the engineering literature [1]. In this model based on the linear regression, it is assumed that the dependent variable has normal distribution and the dependence structure of random variables is measured by the linear correlation coefficient. However, if marginal distributions are not normal, the linear correlation coefficient can not be used. In such cases, copula functions are useful tools to describe the dependence structure of random variables. Moreover, marginal distributions of random variables and their dependence function can be assessed separately by copulas. The concept of copula is first described by Nelsen [2] as a mathematical function with the copula function describing the dependence structure of random variables.

Gutenberg Richter (GR) approach is the existing approach to model the dependence between magnitude and frequency in earthquake engineering literature. GR model is based on assumptions about linear correlation and normal distributed marginals. It is not true to assume that magnitude-

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frequency variables are normally distributed. Thus, GR model based on linear regression might not be the best model to determine magnitude-frequency relationship in especially for earthquake data with high magnitude. Our approach is to use copulas as an alternative GR model so it does not require assumptions such as linearity and normality. For this purpose, Turkey earthquake data are used to determine dependence structure of magnitude- frequency.

A number of studies have been made using some statistical methods to determine the occurrence probability and return periods of earthquakes in different regions of Turkey [3-12]. For this purpose, the Poisson model [3, 7] has been widely used in the literture. In addition, earthquake risk analysis has been performed using models such as Gumbel extreme value distributions [3, 7], exponential distributions [5, 7], the Markov model [13-15].

There are the use of copulas in a number of scientific fields, such as engineering [16-18], multivariate flood frequency analysis [19], hydrology [20-22], insurance [23, 24] and actuarial [25, 26]. In addition, there are several earthquake studies that have focused on copula approach. For instance, Li et al. [27] demonstrate the use of copula for bivariate return periods and risk estimation. Goda and Ren [28] use copulas in evaluating the joint probability distribution of aggregate seismic losses. Nikoloulopoulos and Karlis [29] give an application to real data using copulas on the seismic gap hypothesis assumes that the intensity of an earthquake and the time elapsed from the previous one are positively related.

In the present paper, the purpose is to use copulas to examine the dependence between earthquake magnitude and frequency. Then, it is aimed to perform earthquake risk modeling based on copulas, which includes the estimation of the bivariate return periods and the probabilities of earthquake occurrence. So, the joint return periods based on copulas, given by Yue and Rasmussen [22], are used in this study.

Compared with the previous studies, it has been used a different approach to estimate earthquake risk in Turkey with copula models. This new approach, which has been shown to be useful for all Turkey earthquake data in this study, is thought to lead to earthquake risk studies, especially in high-risk earthquake zones those involving large numbers of high magnitude earthquakes.

Gaussian (Normal), Student's t, Clayton, Gumbel, and Frank copulas are the interested copula families in this study. Least Square Estimation (LSE) and Inference Function for Margins (IFM) methods are respectively used for estimating the parameters of GR and copula models. The best model selection is made using graphical tools, Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC).

The rest of the paper is organized as follows. Section 2 describes briefly GR model, copula models and bivariate return periods based on copulas. In addition, this section contains estimation methods of copula models and goodness of fit criteria for the model selection which will be used in this paper. In Section 3, a real earthquake data application is presented. Finally, conclusions are given in Section 4.

2. METHODS

In this section, the methodologies used for estimating the earthquake risk are explained.

2.1. Gutenberg Richter Model

Gutenberg–Richter Model is often used to determine magnitude-frequency relationship in earthquake risk analysis,

$$y = logN = a - bM \qquad (1)$$

where M is the earthquake magnitude and N is the number of cumulative earthquakes (cumulative frequency). The cumulative frequency indicates the number of earthquakes for magnitude earthquakes equal to or greater than M. Parameters a and b in the model are described as regression parameters and they are calculated using the Least Squares (LS) as follows [30];

$$a = \bar{y} + b\bar{M} \quad \text{and} \quad b = -\frac{\sum_{i=1}^{k} M_i y_i - k\bar{M}\bar{y}}{\sum_{i=1}^{k} M_i^2 - k\bar{M}^2} \tag{2}$$

Here, k is the number of groups or class, $\bar{y}$ and $\bar{M}$ are respectively the mean of frequency and magnitude. For the frequency values in this study, cumulative frequency values (N) were divided by time period t (year) and then annual logarithmic frequency values $y = \log(N/t)$ were obtained by taking their logarithms.

Calculations such as earthquake occurrence probabilities and return periods for the determination of earthquake hazard can be made by probability methods. The Poisson method, which is assumption that earthquakes are independent from the times and places that they occur, is the most widely used method for these calculations [31]. In this study, it is assumed that the earthquake occurrences of having a certain magnitude within a certain period follow a Poisson process.

The earthquake risk parameters are defined using Poisson method as follows by Ünal et al. [12] and Gençoğlu [32];

$$n(M) = 10^{a-bM} \tag{3}$$

$$R(M) = 1 - e^{-n(M)T} \tag{4}$$

$$Q = \frac{1}{n(M)} \tag{5}$$

where, $n(M)$ is the annual average number of earthquakes; $R(M)$ is the risk of occurrence of an earthquake with magnitude M within T years, in any given region for a t-year observation interval; Q is the recurrence (return) period.

2.2. Copula Models

Copulas are used to describe the dependence between continuous random variables, X and Y. Sklar's Theorem states that any joint distribution can be written in terms of univariate marginal distribution functions and a copula [33]. A copula for any continuous random vector (X, Y) is defined such as $C: [0,1]^2 \to [0,1]$ uniquely determines

$$H(x, y) = C(F(x), G(y)) \tag{6}$$

where $H(x, y) = P(X \le x, Y \le y)$ is the joint distribution function of X and Y random variables, F and G are the marginal distribution of X and Y. The joint survival function $\bar{H}(x, y) = P(X > x, Y > y)$ is defined as follows:

$$\bar{H}(x, y) = \bar{F}(x) + \bar{G}(y) - 1 + C(1 - \bar{F}(x), 1 - \bar{G}(y)). \tag{7}$$

In case of the marginal distributions with $U(0,1)$-uniform random variables can be done properly similar descriptions. $C: [0,1]^2 \to [0,1]$ copula and $\hat{C}: [0,1]^2 \to [0,1]$ survival copula are defined respectively,

$$C(u, v) = H(F^{-1}(u), G^{-1}(v)) \tag{8}$$

$$\hat{C}(u, v) = u + v - 1 + C(1 - u, 1 - v). \tag{9}$$

The joint survival function based on uniform variables $\bar{C}(u, v) = P(U > u, V > v)$ is defined as follows:

$$\bar{C}(u, v) = 1 - u - v + C(u, v) = \hat{C}(1 - u, 1 - v). \tag{10}$$

In addition, the definitions of dual copula and co-copula are respectively given also with probabilities $\tilde{C}(u, v) = P(U \le u \text{ or } V \le v)$ and $C^*(u, v) = P(U \ge u \text{ or } V \ge v)$ as below:

$$\tilde{C}(u, v) = u + v - C(u, v) \tag{11}$$

$$C^*(u,v) = 1 - C(1-u, 1-v). \tag{12}$$

$\tilde{C}$ and C^* are not copula functions, but they can be used only to calculate the specified probability whereas $\hat{C}$ and $\bar{C}$ are copula [2].

More details on theoretical background and properties of various copula families can be found in [2, 34, 35]. There are a number of copula functions that have been widely used in practice. In this study, it is concentrated on the elliptical copula family (Gaussian and Student's t) and the Archimedean copula family (Clayton, Frank, Gumbel) which are often used in the statistics and other areas. $C(u,v)$, $\hat{C}\,(u,v)$ and $\bar{C}(u,v)$ and their parameter spaces are given in Table 1 for copulas used in this study.

Table 1. $C(u,v)$, $\hat{C}\,(u,v)$ and $\bar{C}(u,v)$ and their parameter spaces for Gaussian, Student's t, Clayton, Gumbel and Frank copulas.

Copula	$C(u,v)$	$\hat{C}\,(u,v)$	$\bar{C}(u,v) = \hat{C}(1-u,1-v)$	Parameter space
Gaussian	$\Phi_\rho(\Phi^{-1}(u),\Phi^{-1}(v))$	$u+v-1+\Phi_\rho(\Phi^{-1}(1-u),\Phi^{-1}(1-v))$	$1-u-v+\Phi_\rho(\Phi^{-1}(u),\Phi^{-1}(v))$	$\rho \in (-1,1)$
Student's t	$t_{d,\rho}(\,t_d^{-1}(u),t_d^{-1}(v))$	$u+v-1+t_{d,\rho}(\,t_d^{-1}(1-u),t_d^{-1}(1-v))$	$1-u-v+t_{d,\rho}(\,t_d^{-1}(u),t_d^{-1}(v))$	$\rho \in (-1,1), d \in (0,\infty)$
Clayton	$(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}$	$u+v$ $-1+((1-u)^{-\theta}+(1-v)^{-\theta}-1)^{-1/\theta}$	$1-u-v+(u^{-\theta}+v^{-\theta}-1)^{-1/\theta}$	$\theta \in (0,\infty)$
Gumbel	$exp[-(u^{-\theta}+v^{-\theta})^{-1/\theta}]$	$u+v-1$ $+exp[-((1-u)^{-\theta}+(1-v)^{-\theta})^{-1/\theta}]$	$1-u-v+exp[-(u^{-\theta}+v^{-\theta})^{-1/\theta}]$	$\theta \in [1,\infty)$
Frank	$-\frac{1}{\theta}ln\left[1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right]$	$u+v-1$ $-\frac{1}{\theta}ln\left[1+\frac{(e^{-\theta(1-u)}-1)(e^{-\theta(1-v)}-1)}{(e^{-\theta}-1)}\right]$	$1-u-v$ $-\frac{1}{\theta}ln\left[1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right]$	$\theta \in (-\infty,\infty)\backslash\{0\}$

Here, Φ_ρ is the bivariate standart normal distribution function with parameter ρ, and Φ^{-1} is the functional inverse of the univariate standart normal, Φ cdf. $t_{(d,\rho)}$ is the bivariate Student's t distribution, and t_d^{-1} is the functional inverse of Student's t, t_d cdf with d degrees of freedom.

In the literature, Inference Function of Margins (IFM) method is often used for parameter estimation of copulas. This method consists of two steps [34]. First, the parameters of the marginals are estimated by MLE

$$\hat{\theta}_1 = argmax_{\theta_1} \sum_{t=1}^{T} \sum_{i=1}^{2} log\ f_i(x_{i,t};\theta_1) \tag{13}$$

and then, given $\hat{\theta}_1$, the parameters of the copula model are estimated as

$$\hat{\theta}_2 = arg\,max_{\theta_2} \sum_{t=1}^{T} log\, c(F_1(x_{1,t}), F_2(x_{2,t}); \theta_2, \hat{\theta}_1) \tag{14}$$

The IFM estimator is defined as $\hat{\theta} = (\hat{\theta}_1, \hat{\theta}_2)'$.

The best fit copula to the observed data must be selected using some methods. The graphical tools can be used for this purpose. The other most used methods are AIC: Akaike's Information Criterion [36] and BIC: Bayesian Information Criterion [37] and are described as follows:

$$AIC = -2 \cdot LL + 2 \cdot l \tag{15}$$

$$BIC = -2 \cdot LL + ln\,(n) \cdot l \tag{16}$$

where LL is loglikelihood value, l is the number of parameters of the copula model, n is the number of observations. The best fits model to data according to these criteria is the model with the smallest AIC or BIC value.

2.3. The Bivariate Return Periods and Risks Based on Copula Models

The occurrence probabilities and return periods of earthquakes based on copula models can be performed. A return period is defined as an estimate of the interval of time between events. This statistical measurement denotes the average recurrence interval over an extended period of time [27]. If appropriate copula functions are selected to model dependence among earthquake magnitude and frequency, the bivariate return periods can be obtained using the approach given by Yue and Rasmussen [22]. Joint return periods in case OR and AND are defined as follows by Fan [20] and Salvadori [38].

$$P^{OR} = P(U > u \; or \; V > v) = 1 - C_{U,V}(u,v) \tag{17}$$

$$P^{AND} = P(U > u \; and \; V > v) = \hat{C}_{U,V}(1-u, 1-v) \tag{18}$$

$$Q^{OR} = \frac{\mu}{P^{OR}} = \frac{\mu}{1 - C_{U,V}(u,v)} \tag{19}$$

$$Q^{AND} = \frac{\mu}{P^{AND}} = \frac{\mu}{1 - u - v + C_{U,V}(u,v)} \tag{20}$$

where μ can be considered as the mean inter arrival time of the two earthquake events. In this study, $\mu = t/N$ will be calculated for each magnitude values.

The bivariate risk value (R^{OR}) associated with the joint return period (Q^{OR}) is defined by Fan [20] and Yen [39] as follows;

$$R^{OR} = 1 - \left(1 - \frac{1}{Q^{OR}}\right)^t \tag{21}$$

here *t* is the number of observed year. The other risk formulation is given by Fan [20] as below;

$$R = 1 - (1-p)^t \tag{22}$$

where p is the exceedance probabilities. The bivariate risk values can be rewritten according to P^{OR} copula probabilites as follows;

$$R = 1 - C_{U,V}(u,v) \quad . \tag{23}$$

3. AN APPLICATION FOR TURKEY EARTHQUAKE DATA

In this study, in order to perform the earthquake risk analysis of Turkey, the earthquake data of 4863, whose magnitudes are $4.0 \leq M \leq 7.9$ occurred in Turkey between 1900-2014 years, which is within the coordinates of $(36 - 42^{\circ}N)$ latitude and $(26 - 45^{\circ}E)$ longitude, was used from the database contained in Bogazici University Kandilli Observatory and Earthquake Research Institute National Earthquake Observation Centre [40].

This section consists of three phases. First, GR model is applied to data. Second, the best copula models are selected that describe the dependence between magnitude and frequency the data. Finally, the bivariate return periods and earthquake risks are estimated according the selected copula models. The results based on copula models are compared with those found by using GR model.

The number of earthquakes occurred between 1900 and 2014 years was taken as the dependent variable and magnitude values were taken as the independent variable. Regression analysis for GR, and parameter estimations, probability calculations and the goodness of fit tests for copula models were made by using SPSS 18.0, Matlab R2013a and Excel package software.

In order to determine the magnitude-frequency relationship, the earthquake risk analysis was performed by using Gutenberg-Richter (GR) model and copula models (Elliptical copulas: Gaussian, Student's t, Archimedean copulas: Clayton, Frank, Gumbel). Magnitude values were taken with 0.1 unit intervals, and the number of earthquakes (frequency) (n) occured within $t = 115$ years, whose magnitudes were $M \geq 4.0$, cumulative frequency (N), (N/t), $y = log(N/t)$ and descriptive statistics related to these are given in Table 2 and Table 3. Accordingly, mean and standard deviation values for magnitude were found to be 5.95 and 1.169 respectively. Values of $y = log(N/t)$ as the dependent variable were used in this study to examine the magnitude-frequency relationship. Mean and standard deviation values of y were found as 0.0035 and 1.0893, respectively. The Pearson (-0.9947), Spearman's ρ (-1.0000) and Kendal's τ (-0.9994) coefficients are utilized for the assessment of dependence. There is the presence of a relatively high negative dependence between the two earthquake variables. The magnitude and frequency can be modelled by the Gaussian (Normal) distribution according to Jarque-Bera test results ($p > 0.05$). Figure 1 depicts QQ-plots showing the fit of marginal models for the magnitude and frequency.

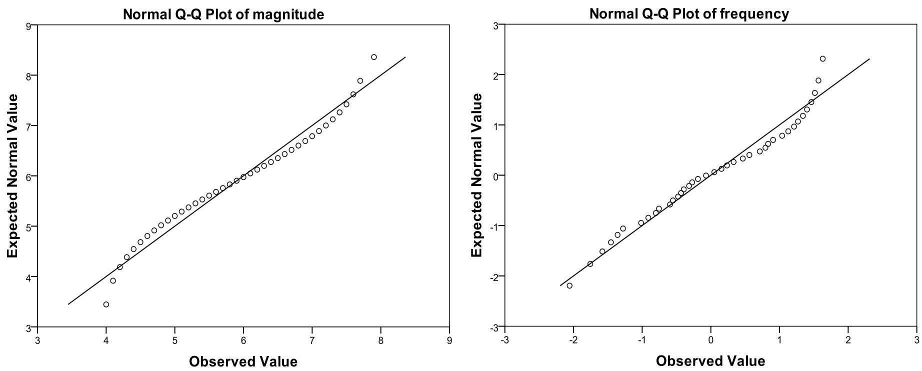
Table 2. The distribution of the magnitude of the earthquakes in Turkey between 1900-2014 years (Magnitude interval=0.1)

k	M	n	N	N/t	y	k	M	n	N	N/t	y
1	4.0	651	4863	42.287	1.626	21	6.0	23	97	0.843	-0.074
2	4.1	478	4212	36.626	1.564	22	6.1	13	74	0.643	-0.191
3	4.2	412	3734	32.470	1.511	23	6.2	6	61	0.530	-0.275
4	4.3	456	3322	28.887	1.461	24	6.3	9	55	0.478	-0.320
5	4.4	366	2866	24.922	1.397	25	6.4	4	46	0.400	-0.398
6	4.5	380	2500	21.739	1.337	26	6.5	4	42	0.365	-0.437
7	4.6	264	2120	18.435	1.266	27	6.6	6	38	0.330	-0.481
8	4.7	328	1856	16.139	1.208	28	6.7	3	32	0.278	-0.556
9	4.8	282	1528	13.287	1.123	29	6.8	9	29	0.252	-0.598
10	4.9	327	1246	10.835	1.035	30	6.9	2	20	0.174	-0.760
11	5.0	141	919	7.991	0.903	31	7.0	4	18	0.157	-0.805
12	5.1	67	778	6.765	0.830	32	7.1	3	14	0.122	-0.915
13	5.2	121	711	6.183	0.791	33	7.2	5	11	0.096	-1.019
14	5.3	175	590	5.130	0.710	34	7.3	1	6	0.052	-1.283
15	5.4	82	415	3.609	0.557	35	7.4	1	5	0.043	-1.362
16	5.5	88	333	2.896	0.462	36	7.5	1	4	0.035	-1.459
17	5.6	49	245	2.130	0.328	37	7.6	1	3	0.026	-1.584
18	5.7	33	196	1.704	0.232	38	7.7	1	2	0.017	-1.760
19	5.8	35	163	1.417	0.151	39	7.8	0	1	0.009	-2.061
20	5.9	31	128	1.113	0.047	40	7.9	1	1	0.009	-2.061

Table 3. Descriptive statistics, normality and correlations tests

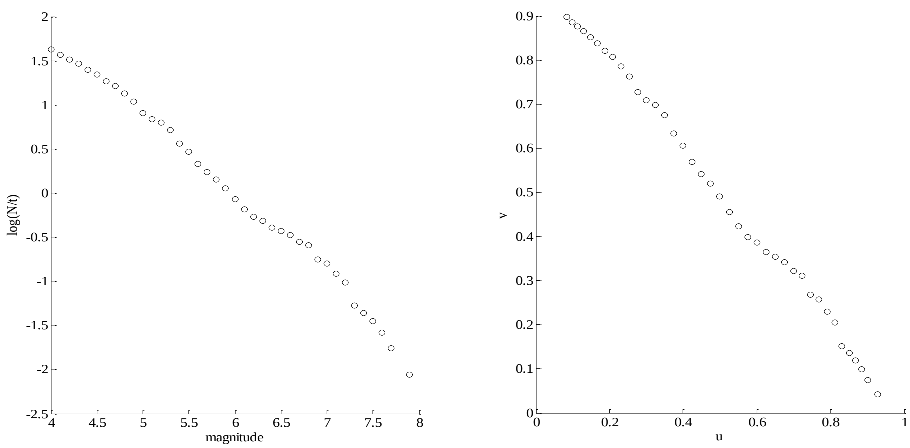
Descriptive statistics	**Magnitude (M)**		$log(N/t) = y$
Mean	5.9500		0.0035
Standart Deviation	1.1690		1.0893
Kurtosis	-1.2000		-1.0201
Skewness	0.0000		-0.2094
Class number	40		40
Jarque-Bera (χ^2stat.)	2.4060 (0.1443*)		2.0849 (0.1856*)
Correlation Tests	Kendall's τ	Spearman ρ	Pearson
	-0.9994 (0.0000**)	-1.000 (0.0000**)	-0.9947 (0.0000**)

Here, the values in brackets denote p- values. If the p-value is below the default significance level of 5%, (*) the test rejects the null hypothesis that the distribution is normal and (**) the correlation is significantly different from zero.



(a) (b)

Figure 1. QQ-plots showing the fit of marginal models for magnitude (a) and frequency (b) data in Turkey.



(a) (b)

Figure 2. The scatter plots of observation pairs (a) and the transformed observation pairs to uniform (b)

3.1. Earthquake Risk Analysis Based on Gutenberg Richter (GR) Model

In this section, in order to determine the probabilities of earthquake occurrence and the return periods, the magnitude-frequency relationship was examined with GR model. According to the results obtained with GR model, the model was found as $\hat{y} = 5.5187 - 0.9269M$ for the magnitude-frequency relationship and it has $R^2 = 0.9895$ coefficient of determination. It indicates that earthquake data can be quite well described with this model. In addition, it is seen that model and model parameters is statistically significant at 0.05 significance level ($p < 0.05$).

Results obtained by calculating risk parameter estimations, seismic risk and return period values for GR model are shown in Table 4. Possibilities of exceeding the earthquake magnitudes in the ($t = 115$ years) periods for these models are shown in Figure 3. 1-year, 5-years, 10-years, 20-years, 30-years, 50 years, 75-years and 100-years periods were used in these calculations. A period of 115 years data is used to estimate the risk values and return periods from 1- to 100- year periods. For only T=115 years, the risk values and the return periods are given in Table 7 to compare with copula results. According to the results obtained, it was found that the occurrence possibility of an earthquake with $M \geq 7.5$ within 10 years is 0.3085, return period is 27.1131 years. Probabilites of exceeding earthquake magnitudes in given periods for GR model are shown in Figure 3. Here, while magnitude value is increasing, corresponding earthquake possibilities are decreasing.

Table 4. Earthquake risk analysis results obtained by using GR model for Turkey.

Model	GR	$logN = 5.5187 - 0.9269M$								
					Period (115 years)					
M	$n(M)$	1 year	5 years	10 years	20 years	30 years	50 years	75 years	100 years	Q
4	64.7159	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0155
4.5	22.2615	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.0449
5	7.6577	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.1306
5.5	2.6342	0.9282	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.3796
6	0.9061	0.5959	0.9892	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.1036
6.5	0.3117	0.2678	0.7895	0.9557	0.9980	0.9999	1.0000	1.0000	1.0000	3.2082
7	0.1072	0.1017	0.4150	0.6577	0.8829	0.9599	0.9953	0.9997	1.0000	9.3266
7.5	0.0369	0.0362	0.1684	0.3085	0.5218	0.6693	0.8418	0.9371	0.9750	27.1131

3.2. The Earthquake Risk Analysis Based on Copula models

The study offers an alternative method to estimate the number of earthquakes as a function of magnitude compared to Gutenberg-Richter method. Copula approach is implemented to determine the dependence for magnitude–frequency relationship. The earthquake probabilities and return periods are estimated by fitting certain copula families to magnitude and number of earthquakes.

The parameter estimations and model selection criteria are given in Table 5. According to these results, it is seen that the data fit better to Student's t with at the smallest AIC ($= -231.22$) value among the elliptical copula models and Frank with at the smallest AIC ($= -172.10$) value among the Archimedean copula models which it has suggested as alternative to GR model. These results can also be supported by looking at Figure 4 and 5. Random sample of size 100 are obtained by simulating from the five selected copulas. The copula parameters are estimated by the method of IFM using the magnitude-frequency data. Figure 4 shows the scatter plots of the simulated data and pairs of observations. Figure 5 shows uniform transformation (u, v) of the marginal distributions for the selected models to the magnitude-frequency data.

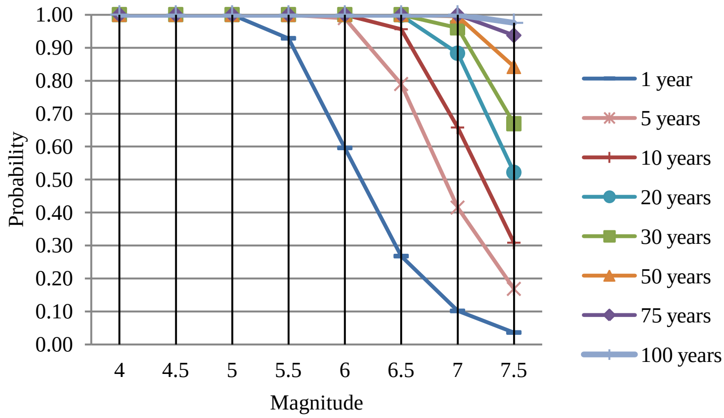


Figure 3. Probabilites of exceeding earthquake magnitudes in periods given for GR model. (t = 115 years)

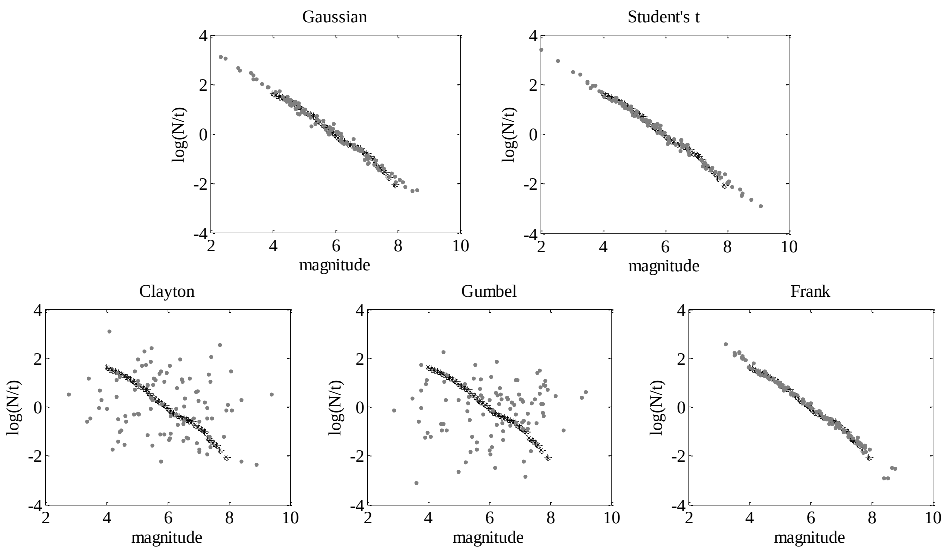


Figure 4. The scatter plots of the simulated and actual observations for selected copulas

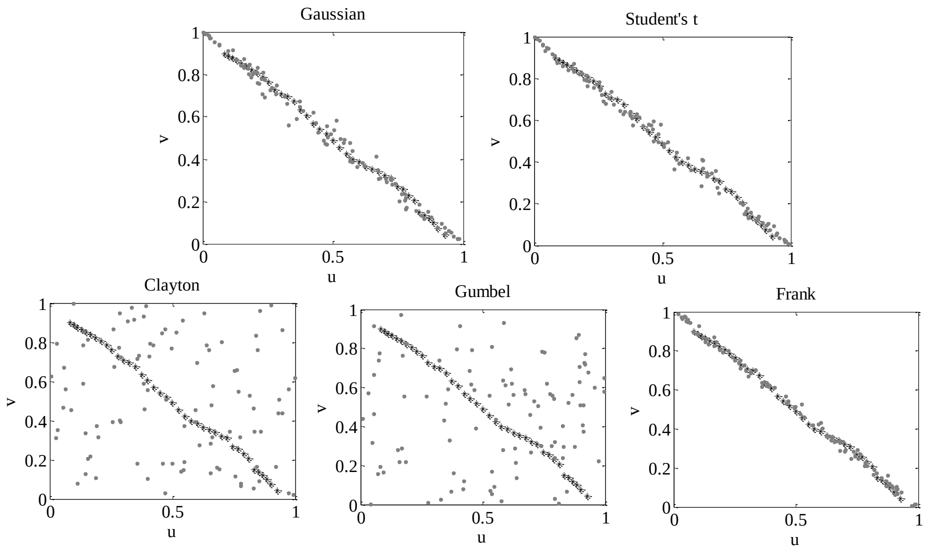


Figure 5. The scatter pilots of the simulated and actual observations transformed into uniform (u, v) for selected copulas

Table 5. Parameter estimations of the models and model selection

		GR	Student's t	Gaussian	Frank
Parameter Estimations		$a = 5.5187$	$\rho = -0.9956$	$\rho = -0.9957$	$\theta = -96.7252$
		$b = -0.9269$	$d = 3.1857$		
Model Selection	AIC	-266.082	-231.22	-212.58	-172.10
	BIC	-271.073	-228.17	-211.00	-169.05

For Gaussian, Student's t and Frank copula models, the probabilities for copula functions, survival copulas and survival functions, given respectively with equalities (8), (9) and (10) are numerically calculated for different magnitudes and the results are presented in Table 6.

Table 6. Earthquake risk analysis results by using copula models

				Gaussian Copula ($\rho = -0.9957$)			Student's t Copula ($\rho = -0.9956, d = 3.1857$)			Frank Copula ($\theta = -96.7252$)		
M	y	u	v	$C(u,v)$	$\hat{C}(u,v)$	$\bar{C}(u,v)$	$C(u,v)$	$\hat{C}(u,v)$	$\bar{C}(u,v)$	$C(u,v)$	$\hat{C}(u,v)$	$\bar{C}(u,v)$
4.0	1.6262	0.0813	0.8991	0.0007	0.0007	0.0203	0.0007	0.0007	0.0204	0.0014	0.0014	0.0211
4.5	1.3372	0.1628	0.8408	0.0110	0.0110	0.0074	0.0105	0.0105	0.0068	0.0092	0.0092	0.0055
5.0	0.9026	0.2694	0.7343	0.0142	0.0142	0.0105	0.0139	0.0139	0.0102	0.0092	0.0092	0.0055
5.5	0.4618	0.3888	0.6163	0.0169	0.0169	0.0118	0.0170	0.0170	0.0119	0.0100	0.0100	0.0049
6.0	-0.0740	0.5124	0.4695	0.0075	0.0075	0.0256	0.0079	0.0079	0.0260	0.0017	0.0017	0.0197
6.5	-0.4374	0.6356	0.3703	0.0172	0.0172	0.0112	0.0172	0.0172	0.0113	0.0105	0.0105	0.0046
7.0	-0.8054	0.7533	0.2754	0.0315	0.0315	0.0028	0.0319	0.0319	0.0031	0.0293	0.0293	0.0006
7.5	-1.4587	0.8558	0.1374	0.0054	0.0054	0.0121	0.0048	0.0048	0.0116	0.0043	0.0043	0.0111

In this study, the return periods (Q) and the probabilities of the earthquake occurrence (R) based on GR model are given for $T = 115$ years in Table 7. Similarly, the bivariate joint return periods and risk values are given for copula models (Gaussian, Student's t and Frank) in Table 8. In here, Q, R^{OR} and R values calculated by using (19), (21) and (23) equations for each copulas, respectively. Compared to the GR and the copula models, it is seen that the estimated return period values for the selected copula models give more realistic results than the GR model, whereas the risk values were not significantly different. That is, where μ, defined as the mean inter occurrence time of the two earthquake events such as given in Subsection 2.3, is calculated using the actual observation values. Q is estimated from used models for GR and copula models using (5) and (19) equations, respectively. Note that the Q and μ values in Tables 7 and 8 show that the return estimates based on the copula models yield more accurate results. For this reason, it can be said that copula models can be used in earthquake models.

Table 7. The return periods and risk values for GR model ($T = 115$ years)

M	$n(M)$	Q	R
4.0	64.7159	0.0155	1.0000
4.5	22.2615	0.0449	1.0000
5.0	7.6577	0.1306	1.0000
5.5	2.6342	0.3796	1.0000
6.0	0.9061	1.1036	1.0000
6.5	0.3117	3.2082	1.0000
7.0	0.1072	9.3266	1.0000
7.5	0.0369	27.1131	0.9856

Table 8. The bivariate return periods and bivariate risk values for copula models

				Q-Bivariate Return Periods			R- Bivariate Risk values					
							R^{OR}			R		
M	y	N	μ	Gaussian	Student's t	Frank	Gaussian	Student's t	Frank	Gaussian	Student's t	Frank
4	1.6262	4863	0.0237	0.0237	0.0237	0.0237	1.0000	1.0000	1.0000	0.9993	0.9993	0.9986
4.5	1.3372	2500	0.0460	0.0465	0.0465	0.0464	1.0000	1.0000	1.0000	0.9890	0.9895	0.9908
5	0.9026	919	0.1251	0.1269	0.1269	0.1263	1.0000	1.0000	1.0000	0.9858	0.9861	0.9908
5.5	0.4618	333	0.3453	0.3513	0.3513	0.3488	1.0000	1.0000	1.0000	0.9831	0.9830	0.9900
6	-0.0740	97	1.1856	1.1945	1.1950	1.1876	1.0000	1.0000	1.0000	0.9925	0.9921	0.9983
6.5	-0.4374	42	2.7381	2.7860	2.7860	2.767	1.0000	1.0000	1.0000	0.9828	0.9828	0.9895
7	-0.8054	18	6.3889	6.5967	6.5994	6.5817	1.0000	1.0000	1.0000	0.9685	0.9681	0.9707
7.5	-1.4587	4	28.7500	28.9061	28.8887	28.8741	1.0000	1.0000	1.0000	0.9946	0.9952	0.9957

4. CONCLUSIONS

In this study, it was tried to be shown that magnitude frequency relation for Turkey earthquake data could be modeled with a statistical approach, copula. For this purpose, firstly earthquake magnitude and frequency relation of Turkey was determined by GR model based on linear regression. By using these model parameters and the Poisson method, the occurrence probabilities and return periods of earthquakes for different magnitudes were estimated. However, the assumptions of linearity and normality, which must be provided in linear regression, can not be achieved especially for earthquake data with high magnitude. Therefore, in this study, the use of copula models as an alternative to the GR model was proposed to determine the earthquake magnitude and frequency relationship. The copula is a useful statistical tool with which to provide flexibility about linearity and marginal distributions. Also, marginal properties and dependence structure can be separated by copulas. For this reason, the occurrence probabilities and return periods of earthquakes were estimated using copula functions for different magnitudes in Turkey. Compared with the GR model, copula models gave the more realistic estimations for return periods, while the probabilities of earthquake occurrence were not substantially different.

This study presented a different approach showing that the earthquakes occurring in Turkey can be modeled successfully by copula model. On the other hand, earthquake occurrence times other than magnitude and frequency variables also play an important role in earthquake models. For this reason, in the next study that it is considered to make a significant contribution to the literature of earthquake engineering, by adding the time factor to the proposed copula model, that it is aimed to perform earthquake risk analysis separately especially for earthquake risk zones with large magnitude in Turkey. Thus, more accuracy and necessary precautions can be taken by using the multivariate return periods of earthquakes based on copulas.

Acknowledgements

The author would like to thank the referees for their criticism that helped improving the paper.

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