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INVERSE KINEMATIC SOLUTION OF A 6 DOF SERIAL ROBOT MANIPULATOR WITH OFFSET WRIST BY USING ALO ALGORITHM

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ABSTRACT

The inverse kinematic problem is one of the most difficult problems to solve analytically for the serial robot manipulators. Its solution is highly related to the types and numbers of the joints of the manipulators. In this study the inverse kinematic problem of a six **D**egrees **O**f **F**reedoms (DOF) serial robot manipulator with offset wrist is solved by using **A**nt **L**ion **O**ptimization (ALO) algorithm. In order to evaluate the performance of the algorithm, five workspace points are randomly selected and the inverse kinematic problem of the manipulator for these points are solved by the ALO algorithm. The obtained results are assessed according to the positioning error and the solution time. It is shown that the ALO algorithm can be effectively used for the solution of the inverse kinematic problem of the serial robot manipulators in terms of the both of the evaluating criteria.
Keywords: Inverse kinematic problem, offset wrist, ALO algorithm, 6 DOF.

6 SD'Lİ VE EKLEM KAÇIKLIKLI BİLEKLİ BİR SERİ ROBOT MANİPÜLATORÜNÜN TERS KİNEMATİK DENKLEMLERİNİN ALO ALGORİTMASI İLE ÇÖZÜMÜ

ÖZ

Ters kinematik problem seri robotlar için analitik olarak çözülmesi en zor problemlerden biridir ve bu problemin çözümü robotun eklem yapısı ve türleriyle yakından ilişkilidir. Bu çalışmada altı serbestlik dereceli (DOF) ve eklem kaçıklıklı bileklikli bir seri robot manipülatörünün ters kinematik problemi Karınca Aslanı Optimizasyon algoritması (ALO) ile çözülmüştür. Algoritmanın performansını test etmek amacıyla rastgele seçilmiş beş çalışma uzayı noktası için ters kinematik çözüm gerçekleştirilmiştir. Elde edilen sonuçlar konumlanma hatası ve çözüm süresi açısından değerlendirilmiştir. Buna göre, ALO algoritmasının her iki değerlendirme kriterine göre de seri robotların ters kinematik probleminin çözümünde verimli bir şekilde kullanılabileceği gösterilmiştir.
Anahtar Sözcükler: Ters kinematik problem, eklem kaçıklıklı bile, ALO algoritması, 6 Serbestlik derecesi.

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1. INTRODUCTION

Serial robot manipulators constitute an important part of the robot manipulators used in the industry. These manipulators can be designed in different DOF and with different wrist types according to their usage purposes [1]. Sixteen three DOF serial robot manipulators were classified by Huang and Milenkovic [2] using a two-letter code. A six DOF robot manipulator can be constructed by adding a three DOF wrist to a manipulator type defined in [2]. In general, the manipulator wrists are designed in two different types, Euler and offset wrists. Three successive joint axes intersect at a common point in an Euler wrist while they do not intersect at a common point in an offset wrist [3]. The Euler wrists are used for the robot manipulators that are designed to reach regular shaped workspaces while the offset wrist are used with the robot manipulators that are needed to get long horizontal reach while maintaining the appropriate angle in their workspaces [1]. The solutions of the forward and inverse kinematic problems are the fundamental parts of the many analyses types for the serial robot manipulators such as workspace determination, dynamic analysis and trajectory generation, etc. The analytical solution of the forward kinematics problem can be easily obtained for all the serial robot manipulators. On the other hand, it can be very difficult to solve the inverse kinematic problem for some types of serial robot manipulators, especially for the robot manipulators that have offset wrist. Inverse kinematics is defined as finding the joint parameters by using kinematic equations of the robot for a pre-determined end-effector position. The analytical solution of the inverse kinematic problem of the six DOF serial robot manipulators with offset wrist is very difficult and cumbersome and it requires finding the roots of a sixteen degree polynomial [1]. In the literature, there are several methods have been proposed to solve the inverse kinematic problem of serial robot manipulators. Küçük and Bingül [4] obtained closed form solutions of the inverse kinematic problems of sixteen fundamental robot manipulators with Euler wrist. In [5] Köker et al. and in [6] Aggarwal, et al. proposed to use neural networks to solve the inverse kinematic problem of serial robot manipulators. In [7] Zhou et al. analytically solved inverse kinematic problem of a 7 DOF robot manipulator that has joint constraints. Sariyildiz et al. [8] compared three screw theory based inverse kinematic formulations and shown that the quaternion and dual quaternion based methods are more computationally efficient than the others. Qiao et al. [9] used dual quaternions to solve inverse kinematic problem of general type of 6R (R: revolute) serial robot manipulators.

ALO algorithm is a new swarm based optimization algorithm that proposed by Mirjalili [10] in 2015. It is based on hunting behavior of antlions while they in larval form. Although, it is a new algorithm ALO has been proposed to effectively solve several engineering problems such as optimum community detection, optimum load dispatch flexible process planning and optimum design of skeletal structures [11-14].

In this study the ALO algorithm is used to solve inverse kinematic problem of a 6-DOF serial robot manipulator with offset wrist. The performance of the algorithm is evaluated in terms of the positioning error and the solution time by using five random selected workspace points. The remainder of the paper is organized as follows; the 6-DOF serial robot manipulator is introduced and its forward kinematic equations are given in section 2, the ALO algorithm and its formulations are presented in section 3, problem definition, objective function and the performed experimental study are given in section 4 and finally, the study is concluded by the last section.

2. 6-DOF SERIAL ROBOT MANIPULATOR WITH OFFSET WRIST

In this study a 6-DOF serial robot manipulator with offset wrist is considered that is a NN type manipulator according to the two-letter code classification by Huang and Milenkovic [2]. The joint structure and the coordinate-system replacement of the manipulator is given in Figure 1 and the **D**enavit-**H**artenberg (DH) parameters [15] obtained from Figure 1 are given in Table 1.

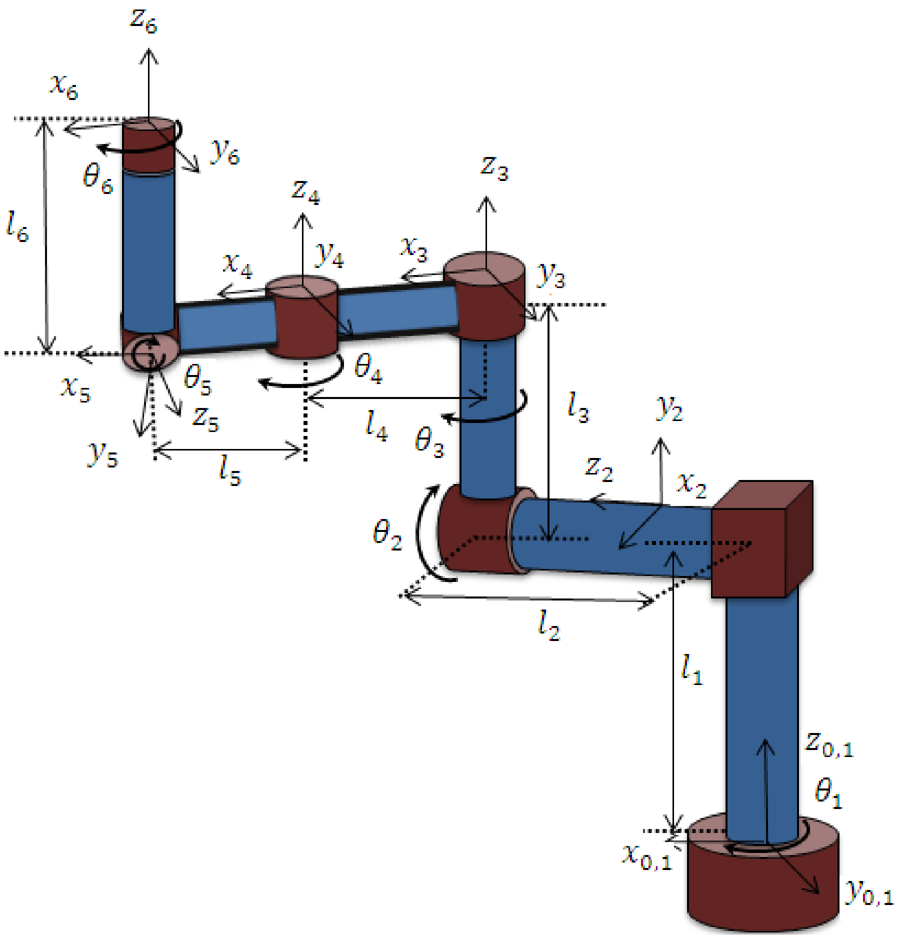


Figure 1. Joint structure and the coordinate-system replacement of the NN type manipulator with offset wrist

Table 1. DH parameters of the manipulator

Joint Number (i)	α_{i-1} (Radyan)	a_{i-1} (cm)	d_i (cm)	θ_i (variable)
1	0	0	l_1	θ_1
2	π/2	0	l_2	θ_2
3	-π/2	0	l_3	θ_3
4	0	l_4	0	θ_4
5	-π/2	l_5	0	θ_5
6	π/2	0	l_6	θ_6

The forward kinematic equations of the robot manipulator can be obtained by using the DH parameters and the general transformation matrix given in Equation 1 [1].

$$ {}^{i-1}_{\ i}D = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos(a_{i-1}) & \cos\theta_i\cos(a_{i-1}) & -\sin(a_{i-1}) & -\sin(a_{i-1})d_i \\ \sin\theta_i\sin(a_{i-1}) & \cos\theta_i\sin(a_{i-1}) & \cos(a_{i-1}) & \cos(a_{i-1})d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1} $$

Where ${}^{i-1}_{\ i}D$ $(i = 1,2,\dots,6)$ is the transformation matrix between the (i-1)'th and i'th joints. A transformation matrix between two consecutive joints can be obtained by substituting the related row of the DH parameters table in the Equation 1.

$$ {}^{0}_{1}D = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & l_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}; {}^{1}_{2}D = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & -1 & -l_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}^{2}_{3}D = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 0 \\ 0 & 0 & 1 & l_3 \\ -\sin\theta_3 & -\cos\theta_3 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; {}^{3}_{4}D = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & l_4 \\ \sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2} $$

$$ {}^{4}_{5}D = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & l_5 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_5 & -\cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; {}^{5}_{6}D = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & -1 & -l_6 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

Finally, the transformation matrix between the base and the tool frame can be find by multiplying all the transformation matrices in the given order as follows;

$$ {}^{0}_{6}D = {}^{0}_{1}D\,{}^{1}_{2}D\,{}^{2}_{3}D\,{}^{3}_{4}D\,{}^{4}_{5}D\,{}^{5}_{6}D \tag{3} $$

The forward kinematic equations of the robot manipulator are the elements of the ${}^{0}_{6}D$ matrix as follows;

$$ {}^{0}_{6}D = \begin{bmatrix} & & & P_x \\ & {}^{0}_{6}R & & P_y \\ & & & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4} $$

Where ${}^{0}_{6}R$ is the 3x3 rotation matrix between base and tool frame while P_x, P_y and P_z represent the position of the end effector according to the base frame in 3D Cartesian space.

3. ANT LION OPTIMIZATION ALGORITHM

ALO algorithm proposed by Mirjalili [10] in 2015. It mimics the hunting behavior of the antlions in the larval form. An antlion prepares a cone-shaped trap in the sand and it hides underneath of the bottom of the trap to hunt [10]. ALO is a swarm based optimization algorithm and it uses two swarms to find the optimum solution of the problem; one for the antlions and the other for the prey, the ants [10].

$$ K = \begin{bmatrix} K_{1,1} & \cdots & K_{1,d} \\ \vdots & \ddots & \vdots \\ K_{n,1} & \cdots & K_{n,d} \end{bmatrix}, KA = \begin{bmatrix} KA_{1,1} & \cdots & KA_{1,d} \\ \vdots & \ddots & \vdots \\ KA_{n,1} & \cdots & KA_{n,d} \end{bmatrix} \tag{5} $$

Where, $K_{i,j}$ and $KA_{i,j}$ $(i = 1,2,\dots,n\,; j = 1,2,\dots,d)$ are the individuals of the swarms of the ants and antlions, while d and n represents the dimension of the problem and the size of the populations, respectively. The ants randomly walk around in the search space of the solution according to the following random walk matrix [10];

$$ RY(t) = [0, C(2r(t_i) - 1), \dots, C(2r(t_n) - 1)] \tag{6} $$

Where, $RY(t)$ is the random walk matrix, t $(i = 1,2,..n)$ is the iteration number and C is a cumulative sum function. Finally $r(t)$ is defined as follows [10];

$$r(t) = \begin{cases} 1 & if\ rand > 0.5 \\ 0 & if\ rand \leq 0.5 \end{cases} \tag{7}$$

Where, $rand$ is a random number in [0,1]. The random walk of the ants are normalized according to the following equations in order to keep them in the search space. [10].

$$N_i^t = \frac{(N_i^t - a_i)(d_i^t - c_i^t)}{(b_i - a_i)} + c_i^t \tag{8}$$

Where, N_i^t, c_i^t and d_i^t are, random walk matrix, the maximum and the minimum values of the search space boundaries for i'th variable at t'th iteration , respectively. Finally, a_i and b_i are the maximum and the minimum of the random walk values of the i'th variable. Since the ALO algorithm has two populations the objective function values are determined for both of them.

$$F_K = f(K) \text{ and } F_{KA} = f(KA) \tag{9}$$

Where F_K and F_{KA} are the vectors of the fitness values for the K and KA matrices and f is the objective function. In each iteration, it is assumed that one ant is only in one antlion's trap. This assumption is realized by means of a roulette wheel selection method and the random walk matrix of each ant is affected by the selected antlion as follows [10].

$$c_i^t = c^t + KA_j^t \ ; \ d_i^t = d^t + KA_j^t \tag{10}$$

Where, KA_j^t $(j = 1,2,\ldots,n)$ is the position of the j 'th antlion at t'th iteration while c^t and d^t are the minimum and the maximum values of the i' th ant among all the variables [10]. c^t and d^t are updated according to the following equation;

$$c^t = \frac{c^t}{I} \ ; \ d^t = \frac{d^t}{I} \tag{11}$$

Where, I is defined as follows [10].

$$I = 10^w \frac{t}{T} \tag{12}$$

Where T is the maximum iteration number and w is determined according to the Equation 13 [10].

$$w = \begin{cases} 2 \text{ if } t > 0.1T \text{ ; } 3 \text{ if } t > 0.5T \text{ ;} \\ 4 \text{ if } t > 0.75T \text{ ; } 5 \text{ if } t > 0.9T \\ \quad 9 \text{ if } t > 0.95T \end{cases} \tag{13}$$

In the ALO algorithm an ant is assumed to be catch by an antlion if its fitness value is greater than the fitness value of the antlion [10].

$$KA_j^t = K_i^t \text{ If } f(K_i^t) > f\left(KA_j^t\right) \tag{14}$$

Finally, the elitism is performed by the following equation [10].

$$K_i^t = \frac{R_{KA}^t + R_E^t}{2} \tag{15}$$

Where R_{KA}^t and R_E^t are the random walks of the i'th ant towards the antlion that selected by the roulette wheel and the elitist antlion, respectively [10].

4. PROBLEM DEFINITION AND EXPERIMENTAL STUDY

In this study inverse kinematic problem of an NN type serial robot manipulator with offset wrist is solved by the ALO algorithm. The problem definition an the proposed objective function can be defined as in the following sub-sections.

4.1. Problem Definition and the Objective Function

Inverse kinematics is finding the joint parameters of the robot manipulator for a pre-determined end-effector position in the 3D Cartesian coordinate system [1]. It should be noted that, there can be multiple solutions for one end effector position because of the structural design of the manipulator. Aim of the solution of the inverse kinematics is finding a proper solution to the problem instead of finding all the solutions for one end effector position. In order to define the inverse kinematic problem, let the end-effector position of the robot manipulator in the 3D Cartesian coordinate system is,

$$P = [P_x \; P_y \; P_z]^T \tag{16}$$

where, P_x, P_y and P_z are the displacement of the end-effector in the coordinate system according to the x, y and z axes, respectively. Accordingly, the joint variables required for the robot manipulator to reach the P point can be defined as follows;

$$H = [\theta_1, \theta_2, \theta_{3,}, \ldots., \theta_6] \tag{17}$$

Where, θ_j $(j = 1,2,3,..,6)$ are the j'th joint variable and the forward kinematic of the robot manipulator can be easily obtained by substituting the H vector in the Equation 4. The aim of the inverse kinematic solution is to find a proper H vector that gives the pre-determined P vector through the Equation 4. Therefore, the H vector can be used as an individual of the ALO algorithm and the K and KA populations can be defined as follows;

$$K = \begin{bmatrix} \theta_{11} & \cdots & \theta_{16} \\ \vdots & \ddots & \vdots \\ \theta_{n1} & \cdots & \theta_{n6} \end{bmatrix}, KA = \begin{bmatrix} \theta_{11} & \cdots & \theta_{16} \\ \vdots & \ddots & \vdots \\ \theta_{n1} & \cdots & \theta_{n6} \end{bmatrix} \tag{18}$$

where n is the number of the individuals of the populations.

The objective function is one of the most important parts of the solution of an optimization problem. Since a proposed solution of the inverse kinematic problem needs to satisfy the aimed position for the end-effector, the objective function can be defined as the positioning error between the aimed position (P) and the position that calculated by a candidate solution ($P' = [P_x', P_y', P_z']$) in the 3D Cartesian coordinate system by using the Euler distance formula.

$$E_r = \sqrt{(P_x' - P_x)^2 + (P_y' - P_y)^2 + (P_z' - P_z)^2} \tag{19}$$

where E_r is the positioning error between the points P and P'. Minimization of E_r is the main aim of this study.

4.2. Experimental Studies

In order to test the performance of the ALO algorithm in solving the inverse kinematic problem of a NN type serial robot manipulator, in this study, five end-effector positions are randomly selected from the workspace of the manipulator. The coordinates of these points and the structural parameters of the manipulator are given in the Table 2.

Table 2. Structural parameters of the manipulator and the coordinates of five randomly selected end-effector positions.

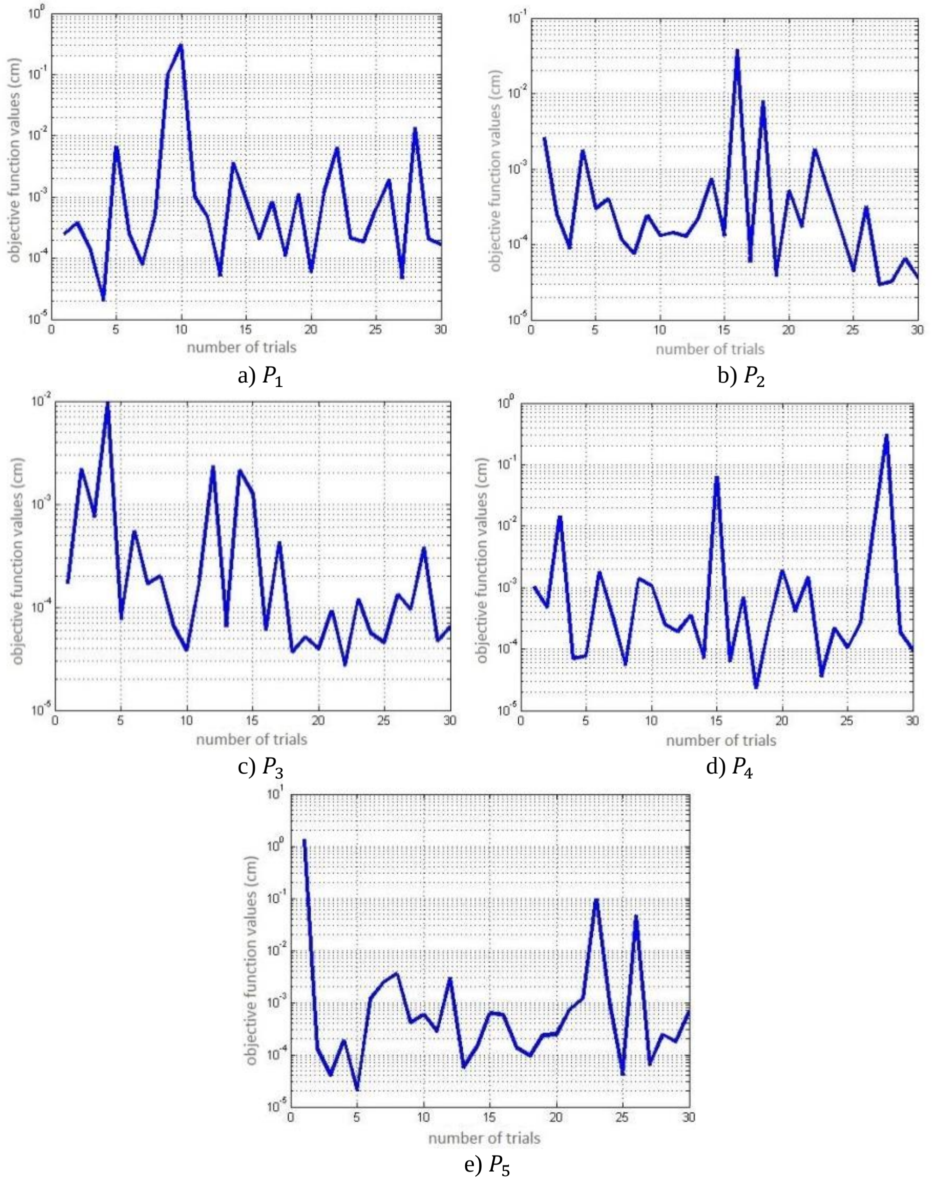
Coordinates (cm)	P_1	P_2	P_3	P_4	P_5
P_x	21.1736	1.0086	-48.0754	8.8484	40.1814
P_y	-57.2067	-63.1067	-119.2732	-178.875	21.4357
P_z	276.4353	303.8828	229.7840	264.7266	299.5422
Structural Parameters (cm)		$l_1 = 200, l_2 = 120, l_3 = 50,$ $l_4 = 50, l_5 = 50, l_6 = 40$			

The optimization parameters of the ALO algorithm are determined as 40 for the number of the individuals of the populations and 80 for the maximum iteration number. The stopping criterion is determined as the maximum number of the iterations. The algorithm is run 30 times for each of the end-effector position. All the study is performed by using a PC that equipped with an Intel Core i5 3.2 GHz microprocessor and 4 GB ram. Some statistical data (minimum, maximum, mean and standard derivation) are calculated from the results of the objective function values of all the runs for the five end-effector positions and given in Table 3.

Table 3. Some statistical data on the objective function values of all the runs for the five end-effector positions

Position/Data Type	P_1	P_2	P_3	P_4	P_5
minimum	2.01E-05	2.93E-05	2.70E-05	2.29E-05	2.02E-05
maximum	0.310788	0.038946	0.009727	0.318156	1.363192
mean	0.015185	0.001939	0.000722	0.013963	0.05087
standard derivation	0.058919	0.007151	0.001829	0.058664	0.248615

According to the table the maximum value of the positioning error for the P_1, P_2, P_3 and P_4 are obtained approximately 0.3 cm while the minimum error is obtained as 2.01E-05 cm. By assessing the results for these four points it can be seen that the mean value of the all runs is smaller than 0.016 cm. Although the maximum error for the P_5 point is 1.4 cm, it can be seen that the mean error is approximately 0.05 cm and the standard derivation is 0.248615 cm. Therefore, it can be said that the ALO algorithm is solved the inverse kinematic problem for all the five points, successfully. Moreover, in order to make a detailed analysis, the minimum objective function values obtained for the five end-effector positions for 30 trials are given in Figure 2. According to the figure, it can be seen that the positioning error for P_1, P_2, P_3 and P_4 points are less than 0.4 cm for all the trials. For P_5 point, at only one trial the positioning error occurs near 1.4 cm and at all the other trials the error is less than 0.5 cm.



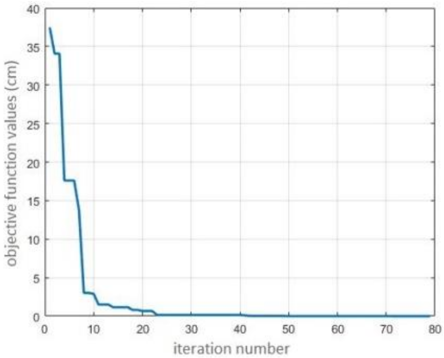
a) P_1 b) P_2

c) P_3 d) P_4

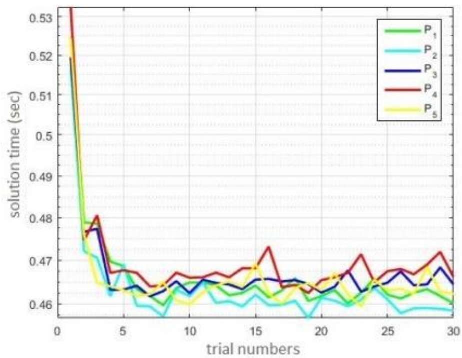
e) P_5

Figure 2. Minimum objective function values obtained for the five end-effector positions for 30 trials.

In order to show the convergence speed of the ALO algorithm, the objective function values of the best solution of P_1 point are drawn on Figure 3a. And also, the algorithm execution time values for all the trials of the five points are shown in Figure 3b so as to indicate the performance of the algorithm in solving the inverse kinematic problem in terms of the CPU time.



a)



b)

Figure 3. a) Objective function values of the best solution of P_1 point b) Algorithm execution time values for all the trials

As can be seen in the Figure 3.a, the ALO algorithm reaches the best solution at the end of 40 iterations and the algorithm execution times for all the trials are approximately 0.5 seconds according to Figure 3b. As a result of the both of the figures it can be indicated that the ALO algorithm has a fast converge characteristic and also it is very efficient in terms of the computational cost.

As a final assessment by considering the overall results of the study it can be said that, the ALO algorithm can successfully solve the inverse kinematic problem that is very difficult to solve analytically, for a 6-DOF serial robot manipulator with offset wrist.

5. CONCLUSION

The inverse kinematic problem of a NN type, 6-DOF serial robot manipulator with offset wrist is solved by the ALO algorithm. In order to test the performance of the algorithm, five workspace points are randomly selected and the inverse kinematic problem for these points is solved. According to the obtained results it is shown that, the ALO algorithm can successfully solve the inverse kinematic problem by a less than 0.4 cm positioning error in nearly 0.5 seconds. Since the selected mechanism has six revolute joints and an offset wrist, it's inverse kinematic problem is very hard to solve analytically. Therefore, it can be concluded that the ALO algorithm can be efficiently used to solve the inverse kinematics problem of the other serial robot manipulators.

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