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AN ESTIMATION METHOD OF MEMBERSHIP FUNCTION FOR GIVEN FUZZY DATA

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ABSTRACT

In the present study for given fuzzy data an estimation method based on Max(F)Ent measure and generalized entropy optimization methods is suggested. A set of successive values of estimated membership function is defined as distribution which is closest to appropriate membership function and distribution which is furthest from mentioned membership function. In this study, fuzzy data analysis is fulfilled by applying GMax(F)EntM for fuzzy data. The performances of distributions $(\text{MinMaxEnt})_m$ and $(\text{MaxMaxEnt})_m$ are established by Chi-Square, Root Mean Square criterias and Max(F)Ent measure. It should be noted that the results are obtained by using MATLAB. It should be noted that the results are obtained by using MATLAB.

Keywords: Fuzzy set, membership function, generalized entropy optimization methods, fuzzy entropy.

1. INTRODUCTION

The entropy is very important concept for measurement of uncertain information. The fuzzy entropy is a basic concept in fuzzy set theory, many acquired researches are based on the definition of fuzzy entropy, especially in statistics, economics and engineering. Shannon [1] has introduced entropy as a measure of uncertainty of random variable X in the following form

$$H(X) = -\sum_{i=1}^{n} p_i log(p_i), \;\; i = 1,2,\dots,n$$

where $p_i = P\{X = x_i\}$, $\sum_{i=1}^{n} p_i = 1$.

This entropy has the following properties: H is non-neagative; $H = 0$ if and only if $p_i = 1$, $i = 1,2,,\dots,n$ and H gets its maximum value $H_{max} = logn$ when $p_1 = p_2 = \cdots = p_n = 1/n$.

Measure of fuzziness indicates the degree of fuzziness of a fuzzy set. In other words, the entropy of a fuzzy set is a measure of the fuzziness of a fuzzy set. The fuzzy entropy is defined by using the concept of membership function [2]. The fuzzy entropy proposed by De Luca and Termini is shown the following formula,

$$H(A) = -\sum_{i=0}^{n} [\mu_A(x_i) log\mu_A(x_i) + (1-\mu_A(x_i))\, log\big(1-\mu_A(x_i)\big)] \, ,$$

where A is a fuzzy set, $\mu_A(x)$ is membership function and $\mu_A(x_i)$ are the fuzzy values.

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This fuzzy entropy has following properties: $H(A) = 0$ if and only if A is crisp set and $H(A)$ is maximum if and only if $\mu_A(x_i) = 0.5, \forall x_i \in A$.

The fuzzy entropy is used to represent the degree of uncertainty. According to the Shannon entropy of random variables, Zadeh [3] firstly suggested that fuzzy entropy guantifies measurement of the uncertainty associated with each fuzzy value as a weighted Shannon entropy. Then, fuzzy entropy has been studied by lots of researchers. De Luca and Termini [4] first initialized the entropy of a fuzzy set by using Shannon probabilistic entropy measure of fuzzy entropy for a fuzzy set including finite number elements. Kauffman [5] proposed that the entropy of a fuzzy set can be measured with respect to the distance between the fuzzy set and its nearest set. After that, Knopfmacher [6] extended the definition of fuzzy entropy made by De Luca and Termini and Kauffman, respectively. Yager [7] defined another kind of fuzzy entropy measure by using distance of the fuzzy set and its complement. Kosko [8] introduced the fuzzy entropy based on the fuzzy set theory and distances between them. Also, there is a lot of research studies about fuzzy entropy and its applications such as Bhandari and Pal [9], Pal and Pal [10].

After the development of given by De Luca and Termini, a large number of measures of fuzzy entropy are discussed, characterized and generalized by various authors. Parkash, Sharma and Mahajan [11] introduced new measures of weigted fuzzy entropy including two moment conditions and acquired relationships among these measures according to their applications for the analysis of maximum weighted fuzzy entropy principle. Guo and Xin [12] have extended Zadeh's idea to improve some new generalized entropy formulas for fuzzy sets.

For practical problems without sufficient information, the determination of uncertainty distributions of fuzzy values is an important problem in the fuzzy set theory and needs to be estimated with accessible information about fuzzy values. For a fuzzy set, in some circumstances, it isn't obtained the membership function clearly. At this point, Maximum Entropy Method (MaxEnt) proposed by Jaynes can successfully solve this problem by maximizing the Shannon entropy measure, subject to moment constraints, when the information is given moment functions. Following the Maximum Entropy Method, in fuzzy theory, it is introduced the Max(F)Ent measure which maximizes the value of fuzzy entropy subject to moment constraints. After Max(F)Ent measure is obtained by implicit function theorem [13] and Lagrange multipliers method [14], finding moment constraints is essential for the Max(F)Ent method. Shamilov [14-16] defined the MaxEnt functional for the first time by means of Shannon's entropy measure on the given compact and finite sets of moment vector functions. Furthermore, an approach to obtain $(\mathrm{MinMaxEnt})_{\mathrm{m}}$ and $(\mathrm{MaxMaxEnt})_{\mathrm{m}}$ distributions was formulated as a generalization of the entropy optimization principles in [17]. In Shamilov's studies, it was shown that the moment vector functions giving the least value and greatest value to the MaxEnt functional generate distributions in the form of $(\mathrm{MinMaxEnt})_{\mathrm{m}}$ and $(\mathrm{MaxMax(F)Ent})_{\mathrm{m}}$, respectively.

The aim of this paper is to realize an application of Maximum Fuzzy Entropy Method (Max(F)EntM) and Generalized Maximum Fuzzy Entropy Method (GMax(F)EntM) for Max(F)Ent measure subject to $m+1$ moment constraints. It should be noted that Max(F)EntM and GMax(F)EntM are developed in [18-20]. After the development of Generalized Maximum Entropy Methods, given by Shamilov [14,16], in fuzzy set theory, it is introduced the Generalized Maximum Fuzzy Entropy Methods and their solutions in the form of distributions $(\mathrm{MinMax(F)Ent})_{\mathrm{m}}$ which is closest to a given membership function and $(\mathrm{MaxMax(F)Ent})_{\mathrm{m}}$ which is furthest from a given membership function in the sense of Max(F)Ent measure [18-20].

This paper organized as follows. In Section 2, it is introduced the Max(F)Ent functional for fuzzy values, the process of determining $(\mathrm{MinMax(F)Ent})_{\mathrm{m}}$ and $(\mathrm{MaxMax(F)Ent})_{\mathrm{m}}$ distributions is given. In Section 3, an application on fuzzy data is fulfilled by using GMax(F)EntM in detail. Finally, the main results obtained in this study are summarized.

2. MAX (F) ENT FUNCTIONAL

The Maximum Fuzzy Entropy Method (Max(F)EntM) is a new approach to obtain a membership function for fuzzy data via Max(F)Ent characterizing moment vector functions which generate corresponding moment constraints to maximize Max(F)Ent function. According to Generalized Maximum Fuzzy Entropy Methods (GMax(F)Ent) [16], we have introduced a special functional $U(g)$ obtained on given compact set of Max(F)Ent characterizing moment vector functions g. By virtue of the Max(F)Ent characterizing moment vector functions which give respectively the least and the greatest values to the mentioned functional. This functional allows us to obtain Generalized Maximum Fuzzy Entropy (GMax(F)Ent) distributions in the form of $(\mathrm{MinMaxEnt})_\mathrm{m}$ and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ distributions for estimation of appropriate membership function. Vector function $g^{(0)}$ with m components giving minimum value to $U(g)$ obtains $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ distribution and vector function $g^{(1)}$ with m components giving maximum value to $U(g)$ obtains $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ distribution. It should be noted that $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ distribution is the closest to the appropriate membership function and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$ distribution is the furthest from the appropriate membership function in the sense of Max(F)Ent measure.

The problem of maximizing fuzzy entropy measure defined by De Luca and Termini by using Shannon probabilistic entropy measure, we shall use in the following form

$$H(A) = -\sum_{i=0}^{n}[\mu_A(x_i) ln\mu_A(x_i) + (1-\mu_A(x_i))\, ln(1-\mu_A(x_i))] \quad (1)$$

subject to constraints

$$\sum_{i=0}^{n}\mu_A(x_i)\, g_j(x_i) = \mu_j,\ j = 0,1,2,\dots,m \quad (2)$$

where $g_0(x) \equiv 1;\ \mu_j,\ j = 0,1,2,\dots,m$ are moment values of $\mu_A(x_i),\ i = 0,1,\dots,n$ with respect to moment functions $g_j(x),\ j = 0,1,2,\dots,m;\ m < n.$ It is possible to indicate that this problem has a solution

$$\mu_A(x_i) = \frac{1}{1+e^{\sum_{j=0}^{m}\lambda_j g_j(x_i)}},\ \ i = 0,1,\dots,n \quad (3)$$

where $\lambda_j,\ \ j = 0,1,\dots,m$ are Lagrange multipliers. We note that mentioned problem for entropy optimization measure is suggested and solved in [14-16].

If the moment vector value $\mu = (\mu_0,\mu_1,\dots,\mu_m)$ is obtained for each Max(F)Ent characterizing moment vector function $g(x) = (g_0(x), g_1(x),\dots,g_m(x))$ from the data, then distribution $\mu(x) = (\mu_A(x_0),\mu_A(x_1),\dots,\mu_A(x_n))$ is calculated by formula (3). If Equation (3) is substituted in Equation (1), the maximum value of Max(F)Ent measure (1) is obtained in the following form:

$$maxH_A = U(g) = -\sum_{i=0}^{n} ln\frac{e^{\sum_{j=0}^{m}\lambda_j g_j(x_i)}}{1+e^{\sum_{j=0}^{m}\lambda_j g_j(x_i)}} + \sum_{j=0}^{m}\lambda_j\mu_j. \quad (4)$$

In formula (4), $\mathrm{maxH_A}$ is considered as a functional of $g(x)$ and called Max(F)Ent functional. Thus, we use the notation $U(g)$ to denote the maximum value of $H(A)$ defined by (1) corresponding to $g(x) = (g_0(x), g_1(x),\dots,g_m(x))$.

A. MinMax(F)Ent and MaxMax(F)Ent Distributions

Generalized Maximum Fuzzy Entropy Distribution indicated as $(\mathrm{MinMax(F)Ent}\ )_\mathrm{m}$ is closest to the appropriate membership function and $(\mathrm{MaxMax(F)Ent}\ )_\mathrm{m}$ is furthest from the appropriate membership function in the sense of Max(F)Ent measure. Problems suggested to estimate membership functions in the form of $(\mathrm{MinMax(F)Ent})_\mathrm{m}$ and $(\mathrm{MaxMax(F)Ent})_\mathrm{m}$distributions are called MinMax(F)Ent and MaxMax(F)Ent problems, respectively.

Let $K_0 = \{g_0, g_1, \ldots, g_r\}$ be the set of characterizing moment vector functions and all combinations of r elements of K_0 taken m elements at a time be $K_{(0,m)}$. We note that each element of $K_{(0,m)}$ is vector g with m components. Note that the number of vectors g is equal to $\binom{r}{m}$.

Solving the MinMax(F)Ent and MaxMax(F)Ent problems require to find vector functions $\left(g_0, g^{(1)}(x)\right)$, $\left(g_0, g^{(2)}(x)\right)$ where $g_0(x) \equiv 1$, $g^{(1)} \in K_{(0,m)}$, $g^{(2)} \in K_{(0,m)}$ gives minimum and maximum values to functional $U(g)$ defined by (4), respectively. It must be noted that $U(g)$ reaches its minimum (maximum) value subject to constraints (2) generated by function $g_0(x)$ and all m- dimensional vector functions $g(x)$, $g \in K_{(0,m)}$. In other words, minimum (maximum) value of $U(g)$ is least (greatest) value of $U(g)$ corresponding to $(g_0(x), g)$, $g \in K_{0,m}$. In other words, $(\mathrm{MinMax(F)Ent})_m((\mathrm{MaxMax(F)Ent})_m)$ is distribution giving minimum (maximum) value to functional $U(g)$ along of all distributions generated by $\binom{r}{m}$ number of moment vector functions $(g_0(x), g)$, $g \in K_{0,m}$. Therefore, we denote mentioned distributions in the form of $(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$.

The existence and method of evalution of distributions $(\mathrm{MinMax(F)Ent})_m$, $(\mathrm{MaxMax(F)Ent})_m$ is proved by the following theorem.

Existence theorem. Let us the following conditions are satisfied:

1.Max(F)Ent characterizing moment functions $g_j(x)$, $j = 0,1,2, \ldots, m$ a are linearly independent;

2. The inequality $n > m$ is satisfied;

3. Moment values $\tilde{\mu}_j$, $j = 0,1, \ldots, m$ are obtained by virtue of given fuzzy values $\tilde{\mu}_A(x_i)$, $i = 0,1, \ldots, n$ and $g_j(x)$, $j = 0,1, \ldots, m$ in the form of equalities

$$\sum_{i=0}^{n} g_j(x_i)\, \tilde{\mu}_A(x_i) = \tilde{\mu}_j, \; j = 0,1, \ldots, m.$$

Then, Maximum Fuzzy Entropy Problem (Max(F)EntP) which consists of maximizing fuzzy entropy measure (1) with respect to membership functions $\mu_A(x)$ with finite number of the fuzzy values $\mu_A(x_i)$, $i = 0,1,\ldots,n$ subject to constraints (2) has a solution $\mu(x) = \left(\mu_A(x_0), \mu_A(x_1), \ldots, \mu_A(x_n)\right)$.

3. APPLICATION ON FUZZY DATA FOR GENERALIZED MAXIMUM FUZZY ENTROPY DISTRIBUTIONS

In this section, $(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$ distributions for membership function values corresponding to fuzzy data given by Table 1 are obtained. It should be noted that mentioned distributions are calculated by using MATLAB.

Table 1. Calculated membership function values

x_i	$X(m/sn)$
0.1000	0.0001
0.6000	0.0002
1.1000	0.0004
1.6000	0.0011
2.1000	0.0030
2.6000	0.0082
3.1000	0.0219
3.6000	0.0573
4.1000	0.1419
4.6000	0.3100
5.1000	0.5498
5.6000	0.7685
6.1000	0.9002
6.6000	0.9608
7.1000	0.9852
7.6000	0.9945
8.1000	0.9980
8.6000	0.9993
9.1000	0.9997
9.6000	0.9999

In order to calculate distributions in the form of $(\mathrm{MinMax(F)Ent})_{\mathrm{m}}$ and $(\mathrm{MaxMax(F)Ent}\,)_{\mathrm{m}}$, the following steps are realized:

1. According to fuzzy data, determine moment functions generating moment conditions in other words Max(F)Ent characterizing moments in the following form:

$E\{\sqrt{x}\},\ E\{lnx\},\ E\{ln(1+x)\},\ E\{ln(1+x^2)\}.$

2. Calculate Max(F)Ent distributions subject to each of Max(F)Ent characterizing moments.
3. Calculate the fuzzy entropy measures of the Max(F)Ent distribution.
4. Determine $(\mathrm{MinMax(F)Ent})_{\mathrm{m}}$ and $(\mathrm{MaxMax(F)Ent}\,)_{\mathrm{m}}$ distributions corresponding to selected Max(F)Ent characterizing moments which generate corresponding moment constraints.

The steps mentioned above are repeated for two moment constraints. In step 3, the Lagrange multipliers for distributions are found by using the Newton method on the MATLAB program. As an example for this process, the corresponding moment constraints to maximum information are listed in Tables 2–3 using the fuzzy data.

Table 2. Entropy of calculated Max(F)Ents subject to two moment functions

Moment functions	Fuzzy Entropy
$(1,\sqrt{x})$	0.0371
$(1,lnx)$	**0.0600**
$(1,ln(1+x))$	**0.0368**
$(1,ln(1+x^2))$	0.0422

Table 3. Entropy of calculated Max(F)Ents subject to three moment functions

Moment functions	Fuzzy Entropy
$(1, \sqrt{x}, lnx)$	0.0241
$(1, \sqrt{x}, ln(1+x))$	**0.0219**
$(1, \sqrt{x}, ln(1+x^2))$	**0.0300**
$(1, lnx, ln(1+x))$	0.0289
$(1, lnx, ln(1+x^2))$	0.0296
$(1, ln(1+x), ln(1+x^2))$	0.0298

For $m = 1$, $K_{(0,1)} = \{(1, \sqrt{x}), (1, lnx), (1, ln(1+x)), (1, ln(1+x^2))\}$

From Table 2, it is shown that $(g_0, g^{(1)}) = (1, ln(1+x))$, $g^{(1)} \in K_{(0,1)}$ gives to least value to $U(g)$, consequently corresponding distribution is $(\text{MinMax(F)Ent})_1$ and $(g_0, g^{(2)}) = (1, lnx)$, $g^{(2)} \in K_{0,1}$ gives to greatest value to $U(g)$, consequently corresponding distribution is $(\text{MaxMax(F)Ent})_1$. In a similar way,

For $m = 2$, $K_{(0,2)} = \{(1, \sqrt{x}, lnx), (1, \sqrt{x}, ln(1+x)), (1, \sqrt{x}, ln(1+x^2)), (1, lnx, ln(1+x)), (1, lnx, ln(1+x^2)\}$.

From Table 3, it is shown that $(g_0, g^{(1)}) = (1, \sqrt{x}, ln(1+x))$, $g^{(1)} \in K_{0,2}$ gives to least value to $U(g)$, consequently corresponding distribution is $(\text{MinMax(F)Ent})_2$ and $(g_0, g^{(2)}) = (1, \sqrt{x}, lnx)$, $g^{(2)} \in K_{0,2}$ gives to greatest value to $U(g)$, consequently corresponding distribution is $(\text{MaxMax(F)Ent})_2$.

According to this moment conditions, the frequency distributions calculated by the $(\text{MinMax(F)Ent})_m$ and $(\text{MaxMax(F)Ent})_m$ distributions are given in Table 4.

In the Table 4-6, corresponding to $K_{(0,m)}$ $(m = 1,2)$ distributions are indicated in the following form. Let $(\text{MinMax(F)Ent})_1$ be $(\text{MinMFE})_1$ and $(\text{MinMax(F)Ent})_2$ be $(\text{MinMFE})_2$ subject to two constraints. Then, $(\text{MaxMax(F)Ent})_1$ is the $(\text{MaxMFE})_1$ and $(\text{MaxMax(F)Ent})_2$ is the $(\text{MaxMFE})_2$ subject to three constraints.

Table 4. Distributions of $(\text{MinMax(F)Ent})_M$, $(\text{MaxMax(F)Ent})_M$

$(\text{MinMFE})_1$	$(\text{MinMFE})_2$	$(\text{MaxMFE})_1$	$(\text{MaxMFE})_2$
0.9998	0.9999	0.9987	0.9999
0.9998	0.9999	0.9995	0.9999
0.9998	0.9999	0.9997	0.9999
0.9998	0.9999	0.9997	0.9999
0.9999	0.9999	0.9998	0.9999
0.9999	0.9999	0.9998	0.9999
0.9999	0.9999	0.9998	0.9999
0.9999	0.9999	0.9998	0.9999
0.9999	0.9999	0.9998	0.9999
0.9999	0.9999	0.9998	0.9999
0.9999	0.9999	0.9999	0.9999
0.9999	0.9999	0.9999	0.9999
0.9999	0.9999	0.9999	0.9999
0.9999	0.9999	0.9999	0.9999
0.9999	1.0000	0.9999	0.9999
0.9999	1.0000	0.9999	0.9999
0.9999	1.0000	0.9999	0.9999
0.9999	1.0000	0.9999	0.9999

Now, chosing the best distribution function for fuzzy data can be determined according to the lowest values RMSE, χ^2 and Max(F)Ent measure. Therefore, in order to obtain the performance of the mentioned distributions, it has been used various criterias as Chi-Square, Root Mean Square Error (RMSE) and maximum fuzzy entropy values of distributions. The obtained results are demonstrated in Table 5-6.

Table 5. The obtained results for $(\mathrm{MaxMax(F)Ent})_M$, $M = 1,2$

Distributions of $(\mathrm{MaxMax(F)Ent})_m$	Moment Constraints	H	χ^2	RMSE
$(\mathrm{MaxMFE})_1$	$(1, lnx)$	0.0600	0.5818	0.6816
$(\mathrm{MaxMFE})_2$	$(1, \sqrt{x}, ln(1+x^2))$	0.0300	0.5811	0.6818

Table 6. The obtained results for $(\mathrm{MinMax(F)Ent})_M$, $M = 1,2$

Distributions of $(\mathrm{MinMax(F)Ent})_m$	Moment Constraints	H	χ^2	RMSE
$(\mathrm{MinMFE})_1$	$(1, ln(1+x))$	0.0368	0.5811	0.6818
$(\mathrm{MinMFE})_2$	$(1, \sqrt{x}, ln(1+x))$	0.0300	0.5810	0.6819

Tables 5-6 show that in the sense of χ^2 criteria each of $(\mathrm{MinMax(F)Ent})_m$ $(m = 1,2)$ distribution is better than corresponding $(\mathrm{MaxMax(F)Ent})_m$ $(m = 1,2)$ distribution. In particular, $(\mathrm{MinMax(F)Ent})_2$ distribution is more suitable for fuzzy data than $(\mathrm{MinMax(F)Ent})_1$ distribution in the sense of RMSE, χ^2 criterias and Max(F)Ent measure. Therefore, it can be explained that $(\mathrm{MinMax(F)Ent})_m$, $m = 1,2$ distributions, which are regarded as the closest distributions, show better performance, since the moment functions sets chosen in this application can be suitable. Thus, it is noted that selection of moment functions is important in the application of the Max(F)Ent method just as MaxEnt method [21].

4. CONCLUSION

The fuzzy entropy provides a quantitative measurement of the degree of uncertainty of fuzzy values. Generalization of fuzzy entropy measure is a new approach in the fuzzy set theory. In this paper, we introduce the concept optimization of maximum fuzzy entropy for fuzzy values and its mathematical properties. Then, we studied fuzzy entropy in terms of optimization measure and presented a framework of Max(F)Ent measure for finite fuzzy values when some Max(F)Ent characterizing moments are given. It should be noted that the suggested distributions $(\mathrm{MinMax(F)Ent})_m$, $(\mathrm{MaxMax(F)Ent})_m$ are applied for the first time to the fuzzy data.

$(\mathrm{MinMax(F)Ent})_m$ and $(\mathrm{MaxMax(F)Ent})_m$ distributions according to estimated fuzzy data are compared using different criterias in terms of modelling data. It is shown that the obtained $(\mathrm{MinMax(F)Ent})_m$ distributions are more suitable in modelling fuzzy data than the $(\mathrm{MinMax(F)Ent})_m$ distributions in the sense of χ^2 and RMSE criterias. Moreover, the present study gives different and useful results for fuzzy data analysis.

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