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Wind Turbine Blade Flapwise and Edgewise Bending Vibration Analyses Using Energy Methods

***A. Celik**
Faculty of Mechanical Engineering,
Yildiz Technical University (YTU)
Istanbul, Turkey

N. Javani
Faculty of Mechanical Engineering,
Yildiz Technical University (YTU)
Istanbul, Turkey

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** Corresponding author: Phone: +90 (212) 383 2906: Fax: +90 (212) 383 3025*
E-mail address: acelik@yildiz.edu.tr

ABSTRACT
Renewable based electricity, generated from wind turbines is considered as a clean alternative energy resource. In a wind power plant, it is of vital importance to avoid resonance of the blade when a frequency of an exciting force exists, especially when it coincides with one of the natural frequencies of the system. The present study deals with the vibrational behaviour of a typical wind turbine blade, NACA 4415. Once the blade is under static load, its natural frequencies are analysed. Two energy methods of Rayleigh-Ritz and optimized Rayleigh-Ritz are employed to carry out the analyses. For the applied loads, the natural frequencies have also been specified. In case of bending is applied in vertical and horizontal directions has been investigated. There is a good agreement between the results of both methods. The results show that for the given blade profile, rotational speeds corresponding to the specific dimensional natural frequency should be avoided during the blade operation to maximize the output power of the wind turbine and extend its lifespan.

INTRODUCTION
Technology and finance play a main role to bring new solutions for ever-increasing energy demand and reducing greenhouse gas emissions. Such a scenario needs more energy demand to continue its trend and a noticeable portion of this energy is supplied from fossil fuels which have their known drawbacks. Renewable energy is a promising solution for a sustainable energy resource in the upcoming years. Among various types of renewable energies, the wind energy is considered as one of the most attractive solutions and many countries are investing in this field. In terms of large-scale development and also the feasibility to invest and make it a commercial source, the wind energy is considered as a very suitable candidate [1]. The USA has recently revealed that by 2030, about 20% of the country's electricity is going to be generated by wind power [2]. Researchers are collecting data for different provinces in USA to supply a roadmap for the future of the wind energy [3]. Also, China is planning to provide 100 GW of wind capacity by 2020, though by the time being, it has the most installed wind-capacity in the world [4].

In order to have a more clear idea about the globe intention toward the wind energy, the annually installed wind energy capacity is shown in Figure 1. for a time span between 1996 to 2013 [5].

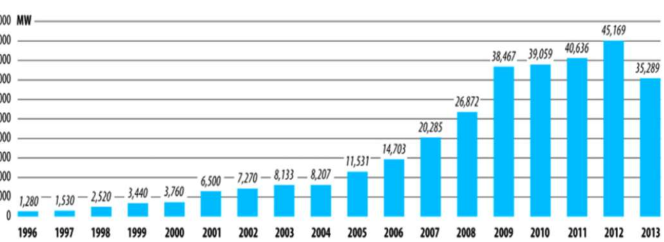

Figure 1. Global annual installed wind energy capacity 1996–2013 (Annual Market Update 2013) [5].

Among different methods to achieve a successful design is to decreases the vibrational forces and avoid potential side effects of resonance in blades is deterministic. One of the possible methods is to separate the natural frequencies of the structure from the harmonies of rotor speed to avoid the high order vibrational amplitudes [6]. Ddynamics stall can cause more violent fluctuation for the blade aerodynamic loads compared with the steady aerodynamic model, which can considerably affect the blade fatigue load spectrum analysis and the fatigue life design.
Aerodynamic load is influenced by blade vibration and its deformation. In case of the aerodynamic loads of the blade, more fluctuations can be imposed due to dynamic stall which will cause

blade fatigue load and its lifetime [7]. Dynamic analyses of blades is important because an unsteady loading such as different wind loads in variable conditions will affect the performance of the wind turbine, specially a small-scale horizontal wind turbine [8]. Instability of a wind turbine blade also depends on the material of the wing and therefore can be delayed [9].

There are different reasons for blade failure. In order to design and have more information about the blade stability, it is necessary to calculate the natural frequencies of the wind turbine blade. In such a situation, the blade will resonate and sudden failure can occur in the system. It is worth noting that in low frequencies, vibrational amplitude and its imposed energy in the blades will be more pronounced compared to the higher frequencies and bending and torsion vibrations can represent the situation [10]. Furthermore, by increasing the size of the wind turbines, due to their capacity increase, there is a need to control the blade load in order to prevent the unexpected applied forces and resonations. These methods include different aspects from smart material for the blade [11] to controlling the vibrational behavior of the blades.

Due to the semi-elastic structure of materials used in wind turbine blades, the vibrating results are expected especially in unsteady environments. Once the forcing frequency (exciting) gets close to one of the natural frequencies in a structure, the resonance occurs which will bring a distinguishable displacement and internal loads [12]. Therefore, in order to avoid the resonance region in wind turbine elements, blades should be designed accordingly. The frequency in which the blade is sensible to vibration is revealed from modal and this makes it possible to avoid the phenomena. Reliability and blade life time of the turbine then mainly depends on the resonating operation [13]. Wind turbines can be located in different places for a considered layout which leads to alternative configurations. Specific criteria such as wind turbine blade vibration and resonance can have negative effects and they should be considered in these selection methods such as aggregation technique [14].

The blades should have specific characteristics for proper operation. Therefore, various parameters such as stability, strength, cost and also vibration can affect it. In a system where bending free vibration and torsional vibrations are present, it has been shown that in the first modes, bending vibration values are more prevalent than the torsional vibrations and are more effective [10]. Comparing the bending and torsion vibrations, one can notice that the first one is more dominant. At the same time, due to lower rotational speed of wind turbines, only the first few vibration modes of the blade will be dominate in their vibrational behavior [15].

When the order increase, the blade frequency increases rapidly and therefore, at high-speed rotational situations, the blade stiffness will increase and this leads to more vigorous vibrations on top of the blade. In large-amplitude vibrations, new effects will arise such as nonlinear shear deformation in the blade which has been studied elsewhere [16].

It is known that in rotational motion and by increasing the speed, the blade shows more rigid behavior and will have higher frequencies, but regarding the first modes, the bending vibration has more importance and that is why this case has been noticed in the current study. Fang et. al used Chebyshev polynomials with respect to boundary conditions as admissible functions in the Ritz minimization method. They studied the modal analysis with various boundary conditions. They also investigated the effect of dimensionless rotational speed ratio and other parameters on natural frequencies [17].

Table 1. The first 10 orders modal frequency [10]

Model Order	Test Frequency		Calculated Frequency	Relative Error (%)	Vibration Types
	Before Elimination	After Elimination			
1	62.42	63.25	60.89	1.33	Bending
2	261.39	262.34	254.14	0.36	Bending
3	492.12	496.14	498.48	0.82	Torsion
4	705.93	711.36	716.98	0.77	Bending
5	1046.66	1052.35	1050.62	0.54	Composed
6	1172.01	1193.21	1201.75	1.80	Composed
7	1499.3	1548.59	1536.36	3.29	Torsion
8	1946.79	2003.62	1984.23	2.92	Composed
9	2211.18	2315.64	2278.36	4.72	Composed
10	2306.08	2406.58	2352.13	4.32	Bending

Hamilton principle can be used to obtain the governing partial differential equations in the blades with large bending deflection for the case of longitudinal and also transverse vibrations in coupled format. Numerical integration of Green's function can be employed to solve the modal problem [18]. - Natural frequencies of rotor blades of NACA 4415 for an average family wind turbine have been studied [19]. They have employed Rayleigh's method and finite element method and natural frequencies have been found. It has been analyzed using analytical approximation techniques and satisfactory agreements have been observed with the experimental data [20].

The purpose of this study is to analysis the free vibrations of a wind turbine blade with a complex geometry. When the governing equation for the blade geometry is difficult to obtain, cross- sections and second moment of inertia variation functions have been calculated numerically and frequencies of the blades are obtained. Along the blade longitudinal direction, the cross sections has been discretized in 10 sections and in each cross-section, the corresponding area and second moment of inertia have been calculated by using a suitable software. After obtaining data for cross-section area and second moment of inertia, curve fitting by employing the MATHEMATICA software are obtained which represents the NACA 4415 blade accurately.

MATHEMATICAL MODEL

In the current study, standard NACA 4415 wind blade has been considered and cross section of the blade has been investigated (Figure 2).

To find the stresses in the blade structure, the governing partial differential equation due to rotational effect is derived. Thereafter, the free vibrations have been found without taking into account the rotational system and only bending is considered in this study. In Figure 3 stress is starting from the blade tip (initial point) as the reference point. Force balance equation along the z direction is applied for an element of dz.

Figure 2. The geometry of considered NACA 4415 blade

In order to analyses the blade vibration, two energy methods are employed to obtain natural frequencies.

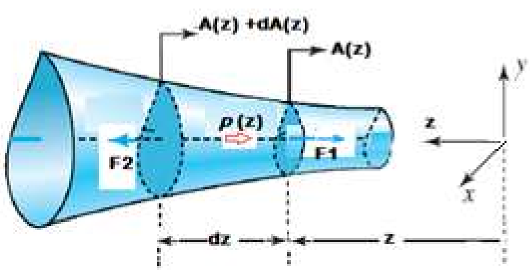

Figure 3. Differentials cross-section for blade equilibrium

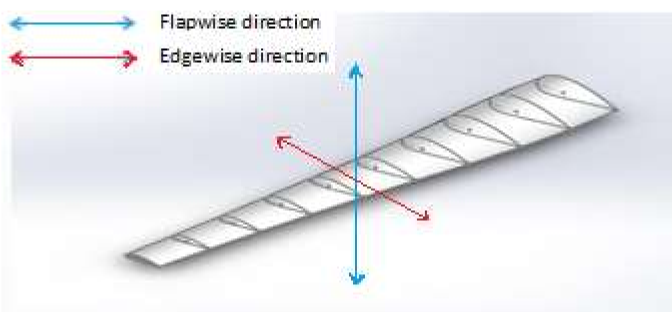

Figure 4. Flapwise and edgewise directions of wind turbine blade

The governing equation for the element is

$$-\sigma(z).A(z) - \rho\Omega^2 A(z) z\, dz + [\sigma(z) + d\sigma(z)][A(z) + dA(z)] = 0 \quad (1)$$

At the blade base (rotation axis) and along the z-direction, force equilibrium for the blade element leads to the following equation

$$\frac{\sigma(z)}{A(z)}\frac{d\,A(z)}{dz} + \frac{d\ \sigma(z)}{dz} = \rho\Omega^2 z \quad (2)$$

In this equation, the cross-section area is a function of blade longitudinal direction (z)

$$A(z) = a_0 + a_1 z^1 + a_2 z^2 + a_3 z^3 + \cdots + a_9 z^9 \quad (3)$$

Here, the coefficients are as follows:

a_0 = 158.8
a_1 =1.90304
a_2 = -0.051490
a_3 = 0.0008409
a_4 = -7.00445E-06
a_5 = 3.14179E-08
a_6 = -1.005757 E-10
a_7 =1.7574E-13
a_8 = -1.67472E-16
a_9 = 6.67511E-20

For calculating purposes, the area function can be written as equation 4.

$$A(z) = A_0[b_0 + b_1 z^1 + b_2 z^2 + b_3 z^3 + \cdots + b_9 z^9] \quad (4)$$

where the coefficients are

b_0 = 1
b_1 = 0.0119839
b_2 = -0.0003237
b_3 = 5.2958E-06
b_4 = -4.41086E-08
b_5 = 2.1523E-10
b_6 = -6.333348E13
b_7 = 1.106717E-15
b_8 = -1.054611E-18
b_9 = 4.20347E-22

For the second moment of inertia with respect to *x*-axis, the used equation is

$$I_x(z) = c_0 + c_1 z^1 + c_2 z^2 + \cdots + c_9 z^9 \quad (5)$$

Here we have for the "*c*" coefficients as:

c_0 = 281.45
c_1 = 78.214
c_2 = -3.0554
c_3 = 0.04926
c_4 = -0.0004159
c_5 = 2.04898e-6
c_6 = -6.0723 e-9
c_7 =1.06469 e-11
c_8 = -1.01298 e-14
c_9 = 4.01327 e-18

Governing equations for the second moment of inertia with respect to u-axis is as follows:

$$I_y\,(z) = h_0 + h_1 z^1 + h_2 z^2 + \cdots + h_9 z^9 \qquad (6)$$

where the "h" coefficients are

h_0 = 15538.2
h_1 = 1444.95
h_2 = -53.959
h_3 = 0.870669
h_4 = -0.00733075
h_5 = 0.0000361152
h_6 = -1.07281 e-7
h_7 =1.88991 e-10
h_8 = -1.08104 e-13
h_9 = 7.22992 e-17

ENERGY EQUATIONS

There are couple of methods to determine the blade natural frequencies such as Frobenius method [6] and energy methods. In the current study, two different energy methods are employed to obtain the free vibration in the blade. These two methods are described below.

RAYLEIGH-RITZ METHOD

Potential Energy of the blade

In order to obtain transverse displacement of the blade under harmonic vibrations, the following function can be used.

$$w = W(z) cos \omega_n t \qquad (7)$$

where W(z) is the displacement function which satisfies the blade boundary conditions. Maximum potential energy due to the bending in the blade is formulated as following:

$$V_{b_{max}} = \frac{E}{2} \int_0^L I_x(z) \left(\frac{\partial^2 W(z)}{\partial z^2}\right)^2 dz \qquad (8)$$

Here $I_x(z) = I_0\, \emptyset\,(z)$ $\qquad 0 \le z \le L$
E is the elasticity modulus as a function of blade material and

$$\emptyset\,(z) = d_0 + d_1 z + d_2 z^2 + d_3 z^3 \ldots + d_9 z^9 \qquad (9)$$

Kinetic Energy for the Blade Unit Length

The kinetic energy of the blade along its longitudinal direction can be written as

$$dT = \frac{1}{2} \left(\frac{dw}{dt}\right)^2 \rho A\, dz \qquad (10)$$

Maximum kinetic energy is given by the following equation as follows:

$$T_{max} = \frac{1}{2} \rho\, \omega_n{}^2 \int_0^L \mathrm{W(z)}^2 A(z) dz \qquad (11)$$

$W(z)$ should satisfy the blade boundary conditions, therefore, it will have the following formulation:

$$W(z) = k_1 [\mathrm{L-z}]^2 + k_2 [\mathrm{L-z}]^3 + \ldots + k_\mathrm{n} [\mathrm{L-z}]^{\mathrm{n+1}} \qquad (12)$$

RAYLEIGH-RITZ METHOD APPLICATION

In order to obtain the vibrational frequencies through the Rayleigh-Ritz Method, derivative of potential and kinetic energy differences with respect to coefficients in transverse displacement function (k_i) is set to zero to minimize its value. This can be shown as the following equation

$$\frac{\partial}{k_i}(V - T) = 0 \qquad i = 1,2,3, \ldots\ , n \qquad (13)$$

This equation can be re-written in matrix format as follows:

$$[A] - \omega_n{}^2 [B] \begin{pmatrix} k_1 \\ k_2 \\ \vdots \\ k_n \end{pmatrix} = 0 \qquad (14)$$

Now, dimensional frequencies can be extracted from equation (14) as follows:

$$\omega_n = \bar{\omega}_n \sqrt{\frac{EI_0}{\rho A_0 L^4}} \qquad (15)$$

OPTIMIZED RAYLEIGH METHOD

The Rayleigh's quotient has been extensively in use for many structures to find approximate values of natural frequencies. This can include beams with varying cross-section [21, 22] . The first natural frequency of any structure can be calculated using the trial function (Eq. 16) by Rayleigh method.

$$W(z) = k_1 [\mathrm{L-z}]^\mathrm{n} \qquad (16)$$

The optimum value of "n" and natural frequencies to minimize the Rayleigh quotient can be obtained by following equation.

$$\omega_n = \frac{\frac{1}{2} E \int_0^L I_x(z) \left(\frac{\partial^2 W(z)}{\partial z^2}\right)^2 dz}{\frac{1}{2} \rho \int_0^L W(z)^2 A(z) dz} \qquad (17)$$

If a non-integer multiplier Rayleigh approach is used, then we can employ trial function as follows:

$$W(z) = [\mathrm{L-z}]^2 + k_1 [\mathrm{L-z}]^3 + \ldots + k_\mathrm{n} [\mathrm{L-z}]^{\mathrm{n+2}} \qquad (18)$$

where the both boundary conditions of $W(L) = 0$ and $W'(L) = 0$ are satisfied. In optimized Rayleigh method, to find the k_i coefficients which can minimize the Rayleigh method, the derivative of this expression is set to be zero as follows:

$$\frac{\partial}{k_i}\left(\frac{\frac{1}{2}E\int_0^L I_x(z)\left(\frac{\partial^2 W(z)}{\partial z^2}\right)^2 dz}{\frac{1}{2}\rho\int_0^L W(z)^2 A(z)dz}\right)=0 \quad (19)$$

As a result of this method, the natural frequencies and k_i values are obtained. The wind blade has a complex geometry. In order to take into account cross-sectional area and second moments of inertia changes, their mathematical expressions are difficult to find. These characteristics have to be used in the calculations and therefore, by geometrical methods, they are obtained. The blade has been discrete into 10 sections.

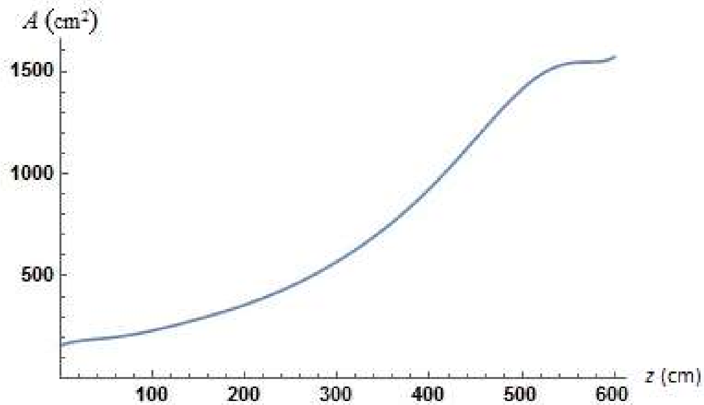

Figure 5. Cross-sectional variations along the z-direction of the blade

Figure 5 shows the variations of cross sectional area along the blade longitudinal direction (z-direction). Approaching to the wind turbine hub, the area is increased as expected. It is important to have the second moment of inertia variations in the blade at different locations. Figure 6 shows how $I_x(z)$ changes along the total length (600cm) of the considered NACA blade.

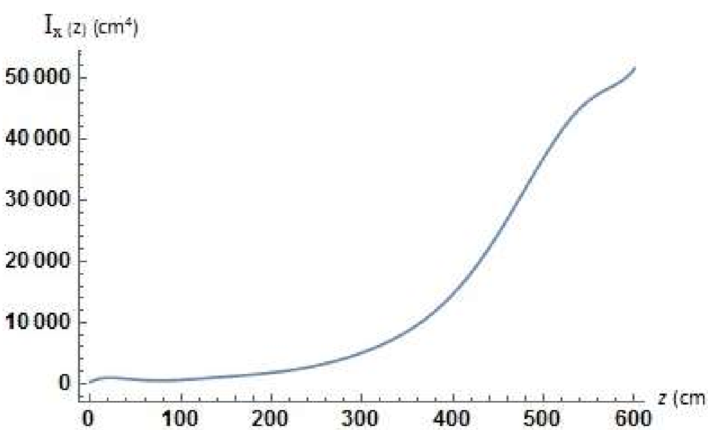

Figure 6. Second moment variation of Inertia vs. x-axis along the z-direction of the blade

RESULTS AND DISCUSSIONS

The main parameter in the study, as it was mentioned before, is the natural frequencies of the blade. For the considered blade, these values are presented in dimensionless form as can be seen in Table 3.

Table 2. The natural frequency values correspond to bending vibration of blade around horizontal x-axis (Hz)

Mode	Rayleigh - Ritz Method	Optimized Rayleigh Method
1	6.8696	6.8696
2	20.4633	20.4633

Table 3. The natural frequency values corresponding to bending vibration of blade around vertical y-axis (Hz)

Mode	Rayleigh - Ritz Method	Optimized Rayleigh Method
1	34.0161	34.0161
2	107.1488	107.1488

Depending on the properties of the blade material, the corresponding parameters can be calculated. In this way, the critical operating conditions for which the resonance can occur, will be specified and by avoiding these situations, the life-span of the blade will increase and a successful blade design will be possible.

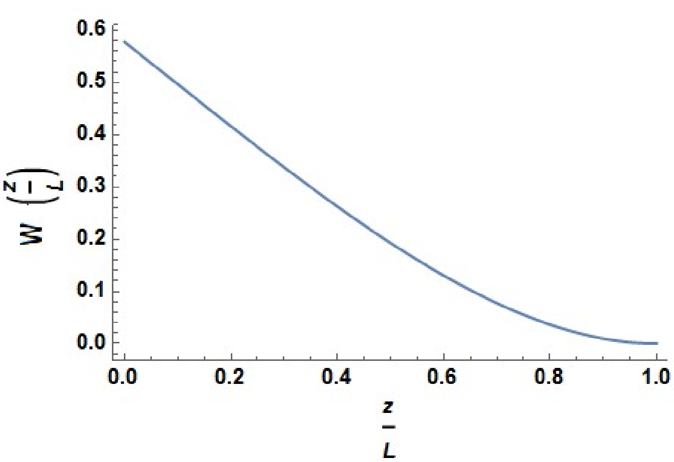

Figure 7. Mode shape of blade for first mode (n =1)

Figures 7 and 8 show the modal shapes of the blade for the first and second modes. The dimensionless natural frequency is already presented in Table 3. These two Figures give the blade physical displacement behaviour when it is exposed to first and second modes and has practical importance in designing of the blade.

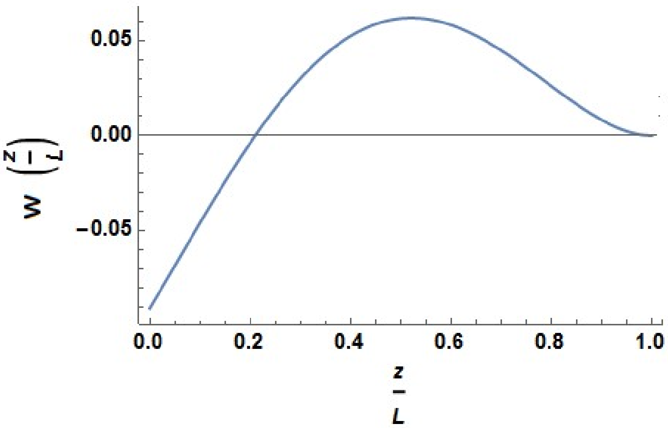

Figure 8. Second mode shape of the blade (n =2)

CONCLUSION

One of the main criteria for a successful blade design is to avoid critical operating conditions in which the imposed dynamic forces can lead to the blade failure. The natural frequency of a blade, therefore, has a great importance. The exciting frequencies due to operating conditions can lead to the resonance in the blade if they are approaching to the natural modes of the blade. In this study, two energy methods have been employed to find the natural frequencies in the blade. Rayleigh-Ritz method and optimized Rayleigh method are used for a NACA 4415 wind turbine blades. Results show that both methods give accurate and identical natural frequencies for the considered blade. Furthermore, due to complex geometry of the blade, it is hard to find a mathematical expression for the cross-sectional and second moment of inertia variations for the blade. In this study, blade has been discretized to a number of sections and hence, the required formulations are obtained with a satisfactory accuracy. The results of current study also verifies that using both methods, there will be higher frequencies in edge-wise which is because of higher blade stiffness.

NOMENCLATURE

A Blade cross-sectional area

E Module of elasticity

I Second moment of inertia

k Transverse displacement coefficient

L Blade length

T Kinetic energy

V Potential energy

W Transverse displacement

Greek abbreviation

σ Stress

ρ Density

Ω Rotational speed of the blade

ω Frequency

Subscripts

0 Blade tip conditions

n Mode number

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