

*This paper was recommended for publication in revised form by Regional Editor Bekir Sami Yilbaş*

## RECENT DEVELOPMENTS OF COMPUTATIONAL METHODS ON NATURAL CONVECTION IN CURVILINEAR SHAPED ENCLOSURES

**\*Hakan F. Öztop**

Dept. Mech. Eng. Firat University  
Elazig, Turkey

**Fatih Selimefendigil**

Department of Mechanical Eng.  
Celal Bayar University, Manisa, Turkey

**Eiyad Abu-Nada**

Department of Mechanical  
Engineering, Khalifa University  
of Science, Technology and  
Research, Abu Dhabi, United  
Arab Emirates

**Khaled Al-Salem**

Dept. Mech. Eng. King Saud Univ.  
Riyadh, Saudi Arabia

*Keywords: Curvilinear geometry, natural convection, nanofluid*

*\* Corresponding author: H.F. Öztop, Phone: 90 424 236 7064, Fax: 90 424 236 70 64  
E-mail address: hfzotop1@gmail.com*

### ABSTRACT

In this review work, thermal and flow fields due to natural convection of in curvilinear enclosures was conducted for different geometries using nnaofluids. Different computational techniques are applied to get results for this geometries. The main difficulties on this problem is to obtain of grid distribution. It was found that the geometry parameter is an important control parameter on heat and fluid flow in natural convection. In general, heat transfer increases with the addition of nanoparticle into the base fluid.

### INTRODUCTION

Many of the heat transfer and fluid flow problems occurs in a curvilinear shaped cavities in engineering applications such as building roofs, electronic equipments, dome shaped structures, heat exchangers or biomedical. Computational solution of this problem presents some difficulties when it compared with smooth surfaces due to complexity of the geometry. Thus, it needs special computational methods/techniques to get fast and correct results. Also, using of nanofluid is suggested new method to control heat transfer and fluid flow in recent years. Different types of nanoparticle are used with different base fluid.

The main purpose of this review paper is to clarify the effects of curvilinear boundaries on heat and fluid flow. Also, another aim of work is to discuss the main methods to discuss solution techniques.

### COMPUTATIONAL SOLUTIONS FOR CURVILINEAR GEOMETRIES

Computational solution for curvilinear geometries needs some modifications. Governing equations for general  $\phi$  variable in curvilinear coordinates can be rewritten as follows and

$$\frac{\partial}{\partial \zeta} (\rho_{\phi} U f) + \frac{\partial}{\partial \eta} (\rho_{\phi} V f) = \frac{\partial}{\partial \zeta} \left[ \frac{\Gamma_{\phi}}{J} \left( \alpha_{\phi} \frac{\partial f}{\partial \zeta} - \beta_{\phi} \frac{\partial f}{\partial \eta} \right) \right] + \frac{\partial}{\partial \eta} \left[ \frac{\Gamma_{\phi}}{J} \left( -\beta_{\phi} \frac{\partial f}{\partial \zeta} + \gamma_{\phi} \frac{\partial f}{\partial \eta} \right) \right] + JS_{\phi}$$

where U and V are defined below as contravariant velocities

$$U = \left( \frac{\partial y^*}{\partial \eta} U^* - \frac{\partial x^*}{\partial \eta} V^* \right)$$

$$V = \left( \frac{\partial x^*}{\partial \zeta} V^* - \frac{\partial y^*}{\partial \zeta} U^* \right)$$

where general variable,  $u$ , stands for any scalar component such as  $u$ ,  $v$ ,  $k$ ,  $T$ ,  $\epsilon$  and  $e$  which corresponds to  $x$ -momentum,  $y$ -momentum, energy, kinetic energy and energy dissipation, respectively. Coefficients of  $\alpha_\phi$ ,  $\beta_\phi$ ,  $\gamma_\phi$  and  $J$  are defined as follows Oztop (2005):

$$\alpha_\phi = \left[ \left( \frac{\partial x^*}{\partial \eta} \right)^2 + \left( \frac{\partial y^*}{\partial \eta} \right)^2 \right]$$

$$\beta_\phi = \left[ \left( \frac{\partial x^*}{\partial \zeta} \right) \left( \frac{\partial x^*}{\partial \eta} \right) + \left( \frac{\partial y^*}{\partial \zeta} \right) \left( \frac{\partial y^*}{\partial \eta} \right) \right]$$

$$\gamma_\phi = \left[ \left( \frac{\partial x^*}{\partial \zeta} \right)^2 + \left( \frac{\partial y^*}{\partial \zeta} \right)^2 \right]$$

$$J = \left[ \left( \frac{\partial x^*}{\partial \zeta} \right) \left( \frac{\partial y^*}{\partial \eta} \right) - \left( \frac{\partial x^*}{\partial \eta} \right) \left( \frac{\partial y^*}{\partial \zeta} \right) \right]$$

Then, grid must be generated by using different grid generation techniques.

### STUDIES ON NATURAL CONVECTION IN NANOFLUID FILLED CURVILINEAR GEOMETRIES

A numerical study has been made to investigate the heat transfer performance and entropy generation of natural convection in a partially-heated wavy-wall square cavity filled with  $Al_2O_3$ -water nanofluid. They observed that the mean Nusselt number reduces and the total entropy generation increases as the amplitude and wavelength of the wavy-surface increase. Also, the Bejan number increases with an increasing amplitude and increasing wavelength of the wavy-surface.

Togun et al. (2014) made a revival work on various annular passage configurations have been used in the reviewed studies, namely circular, ellipse, rectangular, square, triangular, and rhombic annular channels with different fluid and boundary conditions for both nanofluid and pure fluid. They investigated the effects of heater length, as well as the Darcy, Prandtl, Reynolds, Grashof and Rayleigh numbers on heat transfer in concentric and eccentric annular passages are also investigated and found good relations between experimental and numerical works.

Rahman et al. (2014) made a study on natural convection heat transfer and entropy generation in a wavy-walled enclosure filled with nanofluid. They used finite element method to solve

governing equations. They observed that addition of the nanoparticle into base fluid (water) affects both heat transfer and fluid flow. Heat transfer increases with addition of nanoparticle and increasing of Rayleigh number. Conduction mode of heat transfer becomes dominant for the lower values of aspect ratio.

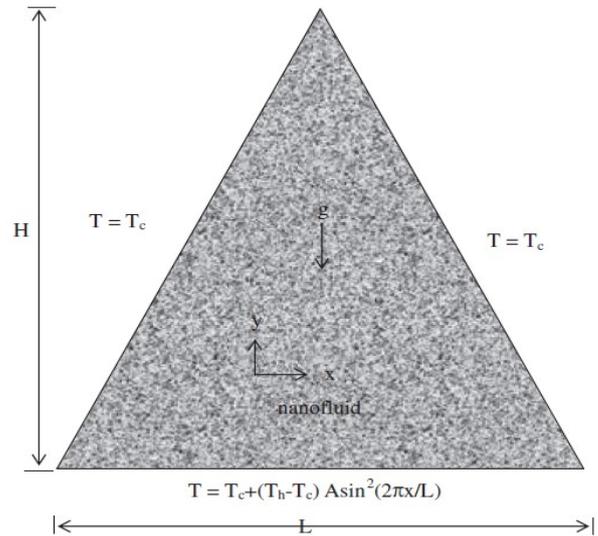


Fig. 1. Schematic view of the triangular shaped cavity with boundary conditions (Rahman, 2014)

Heat transfer in a two-dimensional inclined lid-driven triangular enclosure with nanofluids is investigated numerically for various pertinent parameters by Billah et al. (2013). They solved fluid mechanics and conjugate heat transfer in terms of continuity, linear momentum and energy equations and used the Galerkin finite element method. Results are obtained for a wide range of parameters such as the Grashof numbers ( $103 \leq Gr \leq 106$ ), and time step ( $0.01 \leq \tau \leq 1$ ). They found that heat transfer increased by 31.85% as volume fraction  $\delta$  increases from 0% to 25% at  $Gr = 10^5$  for  $\tau = 0.01$ .

Sheikholeslami et al. (2014) applied the control volume based finite element method (CVFEM) to investigate the heat transfer of  $CuO$ -water nanofluid in presence of magnetic field in an enclosure with sinusoidal wall under constant heat flux. The effective thermal conductivity and viscosity of nanofluid are calculated by KKL (Koo–Kleinstreuer–Li) correlation. Their results showed that Nusselt number is an increasing function of nanoparticles volume fraction, dimensionless amplitude of the sinusoidal wall and  $Ra$  number while it is a decreasing function of Hartmann number.

Cho et al. (2013) performed a work on natural convection heat transfer in a water-based nanofluid filled enclosure bounded by wavy walls and flat upper and lower surfaces. They used three different nanofluids such as  $Cu$ -water,  $Al_2O_3$ -water and  $TiO_2$ -water. The finite volume method is used as numerical method. They observed that the total entropy generation minimized via

an appropriate tuning of the wavy surface geometry parameters. A generalized coordinate transform technique was used to transform to governing equations. Then, they discretized and solved with SIMPLEX algorithm. They made another work on their study by Cho et al. (2012) they investigated the natural convection heat transfer in nanofluid filled medium.

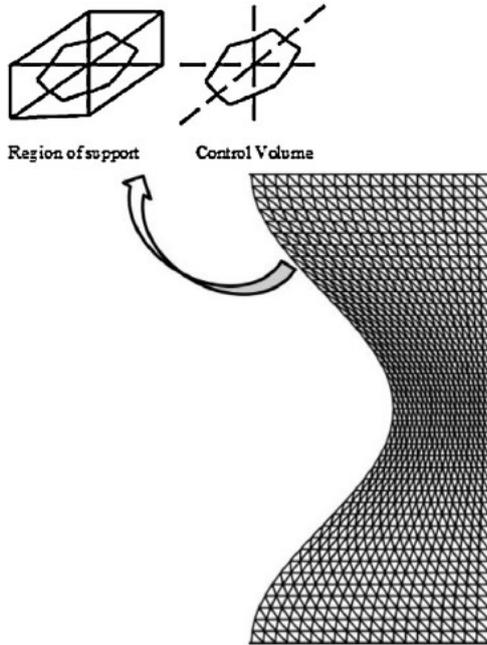


Fig. 2. A sample grid for sinusoidal enclosure (Sheikholeslami et al. 2014)

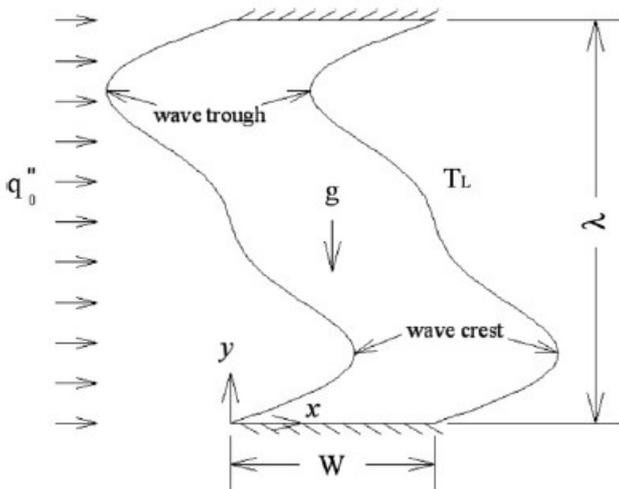


Fig. 3. Configuration of Cho et al. (2012).

Kashani (2012) studied on the effects of surface waviness and nanoparticle dispersion on solidification of Cu-water nanofluid in an enclosure by using enthalpy porosity technique to trace the solid and liquid interface. They defined an equation to define the boundaries as

$$\frac{x}{H} = \begin{cases} \lambda \left( 1 - \cos\left(\frac{2\pi y}{H}\right) \right) \\ 1 + \lambda \left( 1 - \cos\left(\frac{2\pi y}{H}\right) \right) \end{cases}$$

A numerical study is performed by Abdollahzadeh and Esmailpour (2015) to study on the effects of surface waviness and nanoparticle dispersion on solidification of Cu-water nanofluid filled vertical enclosure. In their case, a geometry with sinusoidally curved wavy surface which is given in Fig. 5 can be used to enhance the heat transfer performance. They showed that surface waviness of 0.25 and 0.4, a maximum of 60% decrease in solidification time for Gr = 106 is observed in comparison with Gr = 105 which indicates the increasing effects of natural convection on solidification due to distortion on surface. Therefore, surface waviness can be used to control the solidification time based on enhancing different mechanism of solidification.

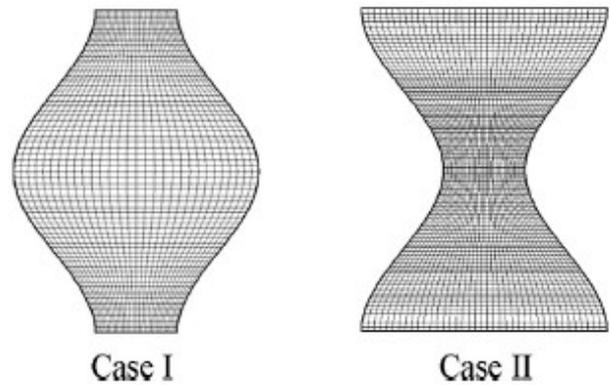


Fig. 5. Curvilinear geometries of Abdollahzadeh and Esmailpour (2015)

Mahmoodi and Hashemi [ ] worked on natural convection fluid flow and heat transfer in C-shaped enclosures filled with Cu-Water nanofluid has been investigated numerically using finite volume method and SIMPLER algorithm. They observed that the obtained results showed that the rate of heat transfer

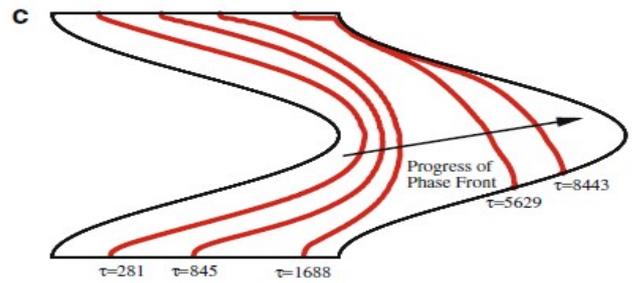


Fig.4. Phase front displacement for three waviness for Gr = 10<sup>5</sup> and phi = 0.5, lambda = 0.125 (Kashani et al., 2012)

increased with decreasing the aspect ratio of the cavity. Also, it was found that the rate of heat transfer increased with increase in nanoparticles volume fraction.

Kasaeipour et al. (2015) made a numerical study on the mixed convection of Cu-water nanofluid in a T-shaped cavity in the presence of a uniform magnetic field. The governing equations are solved numerically with a finite volume approach using the SIMPLE algorithm. They observed that the influence of nanofluid on the heat transfer enhancement increases as AR increases.

Natural convection of Cu-water nanofluid in a cold outer circular enclosure containing a hot inner sinusoidal circular cylinder in the presence of horizontal magnetic field is investigated numerically using the Control Volume based Finite Element Method (CVFEM) by Sheikholeslami et al. (2014). They presented that the average Nusselt number is an increasing function of nanoparticle volume fraction, the number of undulations and Rayleigh numbers while it is a decreasing function of Ha number.

Siddiqi et al. (2006) made a numerical study of natural convection flow along an irregular semi-infinite triangular wavy horizontal surface. They used coordinate transformation to transform the boundary layer equations into a form which is more suitable for computational study. They observed that heat liberation is more effective in sinusoidal wavy surface than in triangular surface.

A numerical study is made on the mixed convection of copper-water nanofluid inside a differentially heated skew enclosure by Nayak et al. (2015). They used co-ordinate transformations are used to transform the physical domain to the computational domain in an orthogonal co-ordinate. The finite volume based SIMPLEC algorithm is used to solve the transformed equations for fluid flow and heat transfer equations in the computational domain. They found that the heat transfer rate increases remarkably by the addition of nanoparticles and the flow field is sensible to the skew angle variation.

El Abdallaoui et al. (2015) studied the natural convection around a decentered triangular cylinder placed in a square cylinder numerically using the lattice-Boltzmann method. They used for pure water and water-silver nanofluid. Their results showed that fluid flow and heat transfer characteristics are highly affected by the heating cylinder position. The increase of nanoparticles volume fraction has a positive impact on the average Nusselt number for all considered positions of the heating block.

Moraveji and Hazajian (2013) performed a study of natural convection heat transfer in rectangular cavities with an inside oval-shaped heat source filled with Fe<sub>3</sub>O<sub>4</sub>/water nanofluid. They used finite element method to solve the governing equations for this problem. They showed the concentration of

the nanoparticle, geometry of the heat source, and the value of Rayleigh number different behaviors are monitored for average Nusselt numbers. Their interesting result showed that addition of the nanoparticles has a negative effect as given in Fig. 6 .on the magnitude of Nusselt number for this problem.

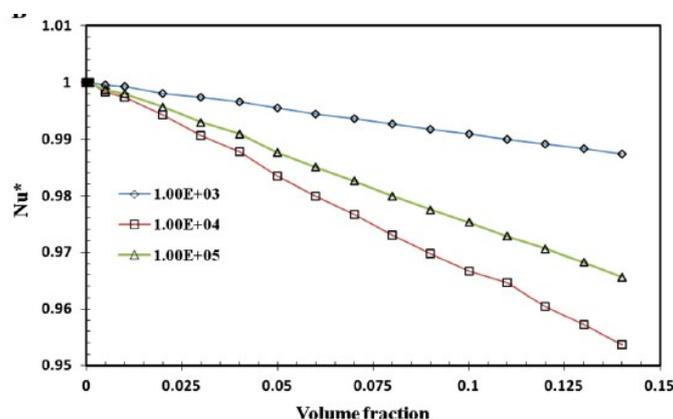


Fig. 6. Variation of mean Nusselt number with volume fraction [Moraveji and Hazajian (2013)]

Parvin and Chamkha (2014) solved a computational problem in an odd-shaped nanofluid filled geometry (It is a combination of the horizontal and vertical enclosure) to obtain heat and fluid flow due to natural convection. They showed that the proper choice of Rayleigh number could be able to maximize heat transfer rate simultaneously minimizing entropy generation.

Selimefendigil et al. (2014a) numerically studied the natural convection of ferrofluid in a partially heated cavity by using the Galerkin weighted residual finite element method. They showed that the magnetic dipole source strength and position have a profound effect on the local and averaged heat transfer and averaged heat transfer increases with decreasing values of horizontal position of the magnetic dipole source.

Selimefendigil and Oztop (2015a) investigated the natural convection with ferrofluids in a partially heated triangular cavity with finite element method. They also proposed a prediction method based on Proper Orthogonal Decomposition and Generalized Neural Networks. Their results showed that the external magnetic field can be used as a control parameter for fluid flow and heat transfer and the proposed prediction method gives satisfactory results in evaluating the thermal performance of the system.

Selimefendigil and Oztop (2014b) made a numerical study of natural convection of nanofluid filled trapezoidal cavity with a stationary adiabatic circular cylinder under the influence of magnetic field using the Galerkin finite element method. They found that the averaged Nusselt number decreases with increasing Hartmann number, decreasing Grashof numbers and increasing inclination angles of the side walls. The averaged heat transfer is deteriorated with presence of the cylinder and

enhanced as the solid volume fraction of nanoparticles increases.

Selimefendigil and Oztop (2015b) numerically investigated the natural convection and entropy generation in a nanofluid filled cavity having different shaped obstacles with magnetic field and internal heat generation using the finite element method. They showed that the presence of the obstacles deteriorates the heat transfer process and this is more pronounced with higher values of external Rayleigh number. The local and averaged heat transfer reduce with increasing values of internal Rayleigh number and Hartmann number.

### CONCLUSION

The purpose of this review paper is to clarify the energetic value of achieving highly efficient energy transport at reduced cost via different approaches, including change in the structural configuration of thermal systems and employing high thermal conductivity fluids. It is observed that curvilinear surface is the main effective parameter on heat and fluid flow in a nanofluid filled enclosure due to natural convection. But there is a discussion on augmentation or decreasing of heat transfer in the system with addition of nanoparticle.

### NOMENCLATURE

AR	aspect ratio
e	energy dissipation
g	gravitational acceleration
Gr	Grashof number
k	kinetic energy
Nu	Nusselt number
Ra	Rayleigh number
T	temperature
U, V	contravariant velocity
u,v	velocities
x,y	cartesian coordinates

### Greek characters

$\lambda$	wave frequency
$\delta$	solid volume fraction
$\tau$	time step

### Subscripts

c	cold
h	hot

### REFERENCES

Abdollahzadeh, M., Esmaelpour, M., Enhancement of phase change material (PCM) based latent heat storage system with nanofluid and wavy surface, *Int. J. Heat Mass Transfer*, 80 (2015) 376-385.

Billah, M.M., Rahman, M.M., Razzak, M.A., Saidur, R., Mekhilef, S., Unsteady buoyancy-driven heat transfer

enhancement of nanofluids in an inclined triangular enclosure, *International Communications in Heat and Mass Transfer* 49 (2013) 115–127.

Cho, C. C., Chen, C. L., & Chen, C. K. (2013). Natural convection heat transfer and entropy generation in wavy-wall enclosure containing water-based nanofluid. *International Journal of Heat and Mass Transfer* 61, 749-758.

Cho, C.C., Chen, C.L., Chen, C.K., (2012) Natural convection heat transfer performance in complex-wavy-wall enclosed cavity filled with nanofluid, *International Journal of Thermal Sciences* 60, 255-263.

Cho, C.C., (2014) Heat transfer and entropy generation of natural convection in nanofluid-filled square cavity with partially-heated wavy surface, *Int. J. Heat Mass Transfer*, 77, 818-827.

El Abdallaoui, M., Hasnaoui, M., Amahmid, A., (2015) Numerical simulation of natural convection between a decentered triangular heating cylinder and a square outer cylinder filled with a pure fluid or a nanofluid using the lattice Boltzmann method, *Powder Technology* 277, 193–205

Kashani, S., Ranjbar, A.A., Abdollahzadeh, M., Sebt, S., (2012) Solification of nano-enhanced phase change material (NEPCM) in a wavy cavity, *Heat Mass Transfer*, 48, 1155-1166.

Kasaeipour, A., Ghasemi, B., Aminossadati, S.M., (2015) Convection of Cu-water nanofluid in a vented T-shaped cavity in the presence of magnetic field, *Int. J. Thermal Sci.*, 94, 50 - 60

Mahmoodi, M., & Hashemi, S. M. (2012). Numerical study of natural convection of a nanofluid in C-shaped enclosures. *International Journal of Thermal Sciences*. 55, 76-89.

Moraveji, M. K., & Hejazian, M. (2013). Natural convection in a rectangular enclosure containing an oval-shaped heat source and filled with Fe<sub>3</sub>O<sub>4</sub>/water nanofluid. *International Communications in Heat and Mass Transfer*. 44, 135-146.

Nayak, R.K., Bhattacharyya, S., Pop, I. (2015) Numerical study on mixed convection and entropy generation of Cu–water nanofluid in a differentially heated skewed enclosure, *Int. J. Heat Mass Transfer*, 85, 620–634

Öztop, H.F., (2005) Numerical study of flow and heat transfer in curvilinear ducts: applications of elliptic grid generation, *Applied Mathematics and Computation* 168, 1449–1460.

Parvin, S. Chamkha, A.J., (2014) An analysis on free convection flow, heat transfer and entropy generation in an odd-shaped cavity filled with nanofluid, *International Communications in Heat and Mass Transfer* 54, 8–17.

Rahman, M.M., Öztop, H.F., Mekhilef, S., Saidur, R., Al-Salem, K., Unsteady natural convection in Al<sub>2</sub>O<sub>3</sub>-water nanofluid filled in isosceles triangular enclosure with sinusoidal thermal boundary condition on bottom wall, Superlattices and Microstructures, 67 (2014) 181-196.

Sheikholeslami, M. Gorji-Bandpy, M., Ganji, D.D., Soleimani, S. (2014) Natural convection heat transfer in a cavity with sinusoidal wall filled with CuO–water nanofluid in presence of magnetic field, J. Taiwan Institute Chemical Eng. 45, 40–49.

Sheikholeslami, M., Gorji-Bandpy, M., Ganji, D.D., Soheil Soleimani, S.M. Seyyedi, (2012) Natural convection of nanofluids in an enclosure between a circular and a sinusoidal cylinder in the presence of magnetic field, International Communications in Heat and Mass Transfer 39, 1435–1443.

Siddiq, S., Anwar Hossain, M., Gorla, R. S. R. (2006) Natural convection flow of viscous fluid over triangular wavy horizontal surface, Computers Fluids, 106, 130-134.

Togun, H., Abdulrazzaq, T. , S.N. Kazi A. Badarudin, A.A.H. Kadhum, Sadeghinezhad, E., (2014) A review of studies on forced, natural and mixed heat transfer to fluid and nanofluid flow in an annular passage, Ren. Sust. En. Rev. 39, 835–856.

Selimefendigil, F., Öztop, HF, Al-Salem K., (2014a) Natural convection of ferrofluids in partially heated square enclosures, Journal of Magnetism and Magnetic Materials 372, 122–133.

Selimefendigil, F., Öztop, HF.(2015a) Numerical Study and POD-Based Prediction of Natural Convection in a Ferrofluids–Filled Triangular Cavity with Generalized Neural Networks, Numerical Heat Transfer, Part A: Applications 67, 1136-1161

Selimefendigil, F., Öztop, HF.(2014b) MHD natural convection of nanofluid filled trapezoidal enclosure with a stationary adiabatic cylinder, Proc. of the Second Intl. Conf. on Advances In Mechanical and Robotics Engineering- AMRE 2014, Switzerland.

Selimefendigil, F., Öztop, HF.(2015b) Natural convection and entropy generation of nanofluid filled cavity having different shaped obstacles under the influence of magnetic field and internal heat generation, J. Taiwan Institute Chemical Eng. (in press).