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A GENERALIZATION OF LUCKY GUESS LIE GROUP $LG(3n)$ AND ITS LIE ALGEBRA $\mathfrak{lg}(3n)$

Ayşe KARA HANSEN*, Mahmut KUDEYT

Yıldız Technical University, Department of Mathematics, Esenler-ISTANBUL

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ABSTRACT

In this work, we generalize the Lucky Guess Lie group of dimension three [1], to the dimension $3n$ which is a solvable and non-nilpotent Lie group. We calculate general forms of the elements of both the Generalized Lucky Guess Lie group of dimension $3n$ and its Lie algebra, and study some algebraic and topological properties [4].
Keywords: Lucky guess lie group $LG(3)$, lie algebra $\mathfrak{lg}(3)$, generalized lucky guess lie group $LG(3n)$, generalized lie algebra $\mathfrak{lg}(3n)$.

$LG(3n)$ LUCKY GUESS LIE GRUBU VE ONUN LIE CEBİRİ $\mathfrak{lg}(3n)$

ÖZ

Bu çalışmada, üç boyutlu Lucky Guess Lie grup [1], çözülebilir ve nilpotent olmayan $3n$ boyutlu olacak şekilde genelleştirdik. Hem $3n$ boyutlu Genel Lucky Guess Lie Grubu ve onun Lie cebirinin genel formları hesaplanmış ve bazı cebirsel ve topolojik özellikleri incelenmiştir.
Anahtar Sözcükler: Lucky guess lie grup $LG(3)$, lie cebiri $\mathfrak{lg}(3)$, genelleştirilmiş lucky guess lie grup $LG(3n)$, genelleştirilmiş lie cebiri $\mathfrak{lg}(3n)$.

1. INTRODUCTION

In this work, we study Lucky Guess Lie Group which has been introduced by Bowers , [1], in the three dimensional case. In [1], Bowers gives the Lie algebra of Lucky Guess in three dimension.

In section 2, we calculate $LG(3n)$ and $LG(6)$ the Lucky Guess Lie groups of dimensions three and six, respectively. We calculate derivations of the Lucky Guess Lie algebra of dimension three.

In section 3, we generalize Lucky Guess to dimension $3n$ and calculate the generators of Lucky Guess Lie algebra $\mathfrak{lg}(3n)$

* Corresponding Author/Sorumlu Yazar: e-mail/e-ileti: aysekara66@gmail.com, tel: (+45) 60 41 72 31

And in section 4, we study algebraic and topological properties of the generalized Lucky Guess Lie group. We use some of the techniques given in [4].

2. LUCKY GUESS LIE GROUP $LG(3)$ AND LIE ALGEBRA $\mathfrak{lg}(3)$

In [1], Bowers defines the Lucky Guess Lie algebra of dimension three, and in this section, we calculate Lie group of the Lucky Guess Lie algebra of dimension three and six.

The Lucky Guess Lie algebra of dimension three has basis elements

$$e_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix}$$

where $[e_1, e_2] = 0,\ [e_1, e_3] = e_1,\ [e_2, e_3] = e_1 + e_2$.

Since

$$exp: \mathfrak{lg}(3) \to LG(3)$$
$$A \to \exp(A)$$

$$\exp(x.e_1) = \begin{pmatrix} 1 & 0 & x \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \exp(y.e_2) = \begin{pmatrix} 1 & 0 & y \\ 0 & 1 & -y \\ 0 & 0 & 1 \end{pmatrix}, \exp(z.e_3) = \begin{pmatrix} 1 & z & -1 - z + e^z \\ 0 & 1 & -1 + e^z \\ 0 & 0 & e^z \end{pmatrix}.$$

Then, we obtain the Lucky Guess group $LG(3)$:

$$LG(3) = \left\{ \begin{pmatrix} 1 & z & x + y - 1 - z + e^z \\ 0 & 1 & -y - 1 + e^z \\ 0 & 0 & e^z \end{pmatrix} \middle| x, y, z \in \mathbb{R} \right\}.$$

2.1. Derivation Algebra of $\mathfrak{lg}(3)$

In the following, we calculate all derivations by [5] of the Lucky Guess Lie algebra of dimension three:

$$\mathfrak{lg}(3) = sp\left\{ e_1 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, e_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}, e_3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \right\}$$

where $[e_1, e_2] = 0,\ [e_1, e_3] = e_1,\ [e_2, e_3] = e_1 + e_2$.

If we take $d \in Der(\mathfrak{lg}(3))$, then

$$d: \mathfrak{lg}(3) \to \mathfrak{lg}(3)$$
$$d([x, y]) = [d(x), y] + [x, d(y)]$$
$$d \in \mathfrak{lg}(3) \Longrightarrow d(e_1) = d_{11}e_1 + d_{12}e_2 + d_{13}e_3$$
$$d(e_2) = d_{21}e_1 + d_{22}e_2 + d_{23}e_3$$
$$d(e_3) = d_{31}e_1 + d_{32}e_2 + d_{33}e_3.$$

If we combain structure equations and derivation d, then we have

$$d = \begin{pmatrix} d_{11} & d_{12} & d_{13} \\ d_{21} & d_{22} & d_{23} \\ d_{31} & d_{32} & d_{33} \end{pmatrix}^T.$$

Hence,

$$d = \begin{pmatrix} d_{11} & d_{21} & d_{31} \\ 0 & d_{11} & d_{32} \\ 0 & 0 & 0 \end{pmatrix}.$$

Thus,

$$Der\big(\mathfrak{lg}(3)\big)=\left\{\begin{pmatrix}1&0&0\\0&1&0\\0&0&0\end{pmatrix},\begin{pmatrix}0&1&0\\0&0&0\\0&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},\begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix}\right\}.$$

2.1.1. Inner Derivation Algebra of $\mathfrak{lg}(3)$

In this section, we calculate inner derivations of the Lucky Guess Lie group of dimension three.

$$ad_{e_1}=\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix}$$

$$ad_{e_2}=\begin{pmatrix}0&0&1\\0&0&1\\0&0&0\end{pmatrix}$$

$$ad_{e_3}=\begin{pmatrix}-1&-1&0\\0&-1&0\\0&0&0\end{pmatrix}$$

$$In\big(\mathfrak{lg}(3)\big)=\left\{\begin{pmatrix}0&0&1\\0&0&0\\0&0&0\end{pmatrix},\begin{pmatrix}0&0&1\\0&0&1\\0&0&0\end{pmatrix},\begin{pmatrix}1&1&0\\0&1&0\\0&0&0\end{pmatrix}\right\}.$$

2.2. Lucky Guess Lie Group $LG(6)$ and its Lie Algebra $\mathfrak{lg}(6)$

In this section, we want to present Lucky Guess Lie Group of dimension six with its Lie algebra.

The Lucky Guess Lie Group of dimension six has the following form:

$$LG(6)\left\{\begin{pmatrix}1&0&z_1&0&x_1+y_1-z_1-1+e^{z_1}&0\\0&1&0&z_2&0&x_2+y_2-z_2-1+e^{z_2}\\0&0&1&0&-y_1-1+e^{z_1}&0\\0&0&0&1&0&-y_2-1+e^{z_2}\\0&0&0&0&e^{z_1}&0\\0&0&0&0&0&e^{z_2}\end{pmatrix}\middle|\, x_1,x_2,y_1,y_2,z_1,z_2\in\mathbb{R}\right\}$$

Genereal form of an element of $LG(6)$ is

$$\beta=\begin{pmatrix}1&0&z_1&0&x_1+y_1-z_1-1+e^{z_1}&0\\0&1&0&z_2&0&x_2+y_2-z_2-1+e^{z_2}\\0&0&1&0&-y_1-1+e^{z_1}&0\\0&0&0&1&0&-y_2-1+e^{z_2}\\0&0&0&0&e^{z_1}&0\\0&0&0&0&0&e^{z_2}\end{pmatrix}$$

and

$$\beta(0)=I,$$

and therefore, general form of an element of the Lucky Guess Lie algebra $\mathfrak{lg}(6)$ is

$$\dot{\beta} = \begin{pmatrix} 0 & 0 & \dot{z_1}(0) & 0 & \dot{x_1}(0)+\dot{y_1}(0)-\dot{z_1}+e^{z_1(0)}.\dot{z_1}(0) & 0 \\ 0 & 0 & 0 & \dot{z_2}(0) & 0 & \dot{x_2}(0)+\dot{y_2}(0)-\dot{z_2}(0)+e^{z_2(0)}.\dot{z_2}(0) \\ 0 & 0 & 0 & 0 & -\dot{y_1}(0)+e^{z_1(0)}.\dot{z_1}(0) & 0 \\ 0 & 0 & 0 & 0 & 0 & -\dot{y_2}(0)+e^{z_2(0)}.\dot{z_2}(0) \\ 0 & 0 & 0 & 0 & e^{z_1(0)}.\dot{z_1}(0) & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{z_2}.\dot{z_2}(0) \end{pmatrix}.$$

Here

$$\dot{\beta} = \dot{x_1}(0).\begin{pmatrix} 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} + \dot{x_2}(0).\begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

$$+\dot{y_1}(0).\begin{pmatrix} 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&-1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix} + \dot{y_2}(0).\begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&-1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}$$

$$+\dot{z_1}(0).\begin{pmatrix} 0&0&1&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0 \end{pmatrix} + \dot{z_2}(0).\begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&1 \end{pmatrix},$$

and the basis elements of the Lie algebra are

$$X_1 = \begin{pmatrix} 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}, X_2 = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix},$$

$$Y_1 = \begin{pmatrix} 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&-1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix}, Y_2 = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&-1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&0 \end{pmatrix},$$

$$Z_1 = \begin{pmatrix} 0&0&1&0&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0\\ 0&0&0&0&1&0\\ 0&0&0&0&0&0 \end{pmatrix}, Z_2 = \begin{pmatrix} 0&0&0&0&0&0\\ 0&0&0&1&0&0\\ 0&0&0&0&0&0\\ 0&0&0&0&0&1\\ 0&0&0&0&0&0\\ 0&0&0&0&0&1 \end{pmatrix}.$$

Thus, $\mathrm{Ig}(6) = span\{X_1, X_2, Y_1, Y_2, Z_1, Z_2\}$.

2.3. The Lie Brackets of Lucky Guess Lie Group $\boldsymbol{LG(6)}$

The Lucky Guess Lie algebra of dimension six has the same properties of dimension three case, and the list of the brackets is in the following;

1) $[X_1, X_2] = 0$ (1)
2) $[X_1, Y_1] = 0$ (2)
3)$[X_1, Y_2] = 0$ (3)
4) $[X_1, Z_1] = X_1$ (4)
5) $[X_1, Z_1] = 0$ (5)
6) $[X_2, Y_1] = 0$ (6)
7) $[X_2, Y_2] = 0$ (7)
8) $[X_2, Z_1] = 0$ (8)
9) $[X_2, Z_2] = X_2$ (9)
10) $[Y_1, Y_2] = 0$ (10)
11) $[Y_1, Z_1] = X_1 + Y_1$ (11)
12) $[Y_1, Z_2] = 0$ (12)
13) $[Y_2, Z_1] = 0$ (13)
14) $[Y_2, Z_2] = X_2 + Y_2$ (14)
15) $[Z_1, Z_2] = 0$ (15)

3. A GENERALIZATION OF LUCKY GUESS LIE GROUP $\boldsymbol{LG(3n)}$

In this section, we generalize the Lucky Guess Lie group of dimension three to dimension 3n and find its general form

$$LG(3n) = \left\{ \begin{pmatrix} I_n & Z_n & X_n + Y_n - I_n - Z_n + e^{Z_n} \\ 0_n & I_n & -Y_n - I_n + e^{Z_n} \\ 0_n & 0_n & e^{Z_n} \end{pmatrix} \right\},$$

$$Z_n = \begin{pmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n \end{pmatrix}, Y_n = \begin{pmatrix} y_1 & 0 & \dots & 0 \\ 0 & y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_n \end{pmatrix},$$

$$X_n = \begin{pmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n \end{pmatrix}, e^{Z_n} = \begin{pmatrix} e^{z_1} & 0 & \dots & 0 \\ 0 & e^{z_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{z_n} \end{pmatrix}.$$

3.1. Generators of the Generalized of Lucky Guess Lie Algebra $\boldsymbol{\mathfrak{lg}(3n)}$

Let us denote by X_i^{2n+i} the $3n \times 3n$ matrices having 1 at ith row and $(2n+i)$th column. Then,

$$X_1^{2n+1} = \begin{pmatrix} 0 & 0 & \cdots & \boxed{1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \cdots & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{pmatrix},$$

$$X_2^{2n+2} = \begin{pmatrix} 0 & 0 & \cdots & 0 & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \boxed{1} & \cdots & 0 \\ \vdots & \vdots & \cdots & \ddots & \cdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{pmatrix},$$

$$\vdots$$

$$X_n^{3n} = \begin{pmatrix} 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \ddots & \cdots & \cdots & \boxed{1} \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \ddots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & \cdots & 0 \end{pmatrix}.$$

Let us denote by Y_i^{2n+i} the $3n \times 3n$ matrices having first 1 at ith row and $(2n+i)$th column. 0_{n-1} is an $(n-1) \times 1$ column matrix with zero entries. Then,

$$Y_1^{2n+1} = \begin{pmatrix} 0 & 0 & \cdots & \boxed{1} & 0 & \cdots & 0 \\ 0 & 0 & 0 & 0_{n-1} & 0 & 0 & 0 \\ \vdots & \cdots & \ddots & \boxed{-1} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & 0 & \cdots & 0 \end{pmatrix},$$

$$Y_2^{2n+2} = \begin{pmatrix} 0 & 0 & \cdots & \cdots & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \boxed{1} & \cdots & \vdots \\ \vdots & \cdots & \ddots & \cdots & 0_{n-1} & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \boxed{-1} & \cdots & 0 \\ \vdots & \cdots & \cdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & \cdots & 0 & \cdots & 0 \end{pmatrix},$$

$$\vdots$$

$$Y_n^{3n} = \begin{pmatrix} 0 & 0 & \cdots & \cdots & \cdots & 0 \\ 0 & 0 & \cdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \ddots & \cdots & \cdots & \boxed{1} \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0_{n-1} \\ \vdots & \cdots & \cdots & \ddots & \cdots & \boxed{-1} \\ \vdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{pmatrix}.$$

Let us denote by Z_i^{n+i} the $3n \times 3n$ matrices having first 1 at ith row and $(n+i)$th column. $0_{(n-1)\times 1}$ is an $(n-1) \times 1$ column matrix with zero entries and $0_{1\times(n-1)}$ is a $1 \times (n-1)$ row matrix with zero entries. Then,

$$Z_1^{n+1} = \begin{pmatrix} 0 & \cdots & \boxed{1} & 0_{1\times(n-1)} & 0 & \cdots & \cdots & 0 \\ \vdots & \cdots & 0 & \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \vdots & \vdots & 0 & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & 0 & \boxed{1} & 0 & \cdots & \vdots \\ \vdots & \ddots & \cdots & \vdots & 0_{(n-1)\times 1} & \cdots & \cdots & 0 \\ 0 & \cdots & \cdots & 0 & \boxed{1} & 0 & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots & 0 & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \ddots & \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & 0 \end{pmatrix},$$

$$Z_2^{n+2} = \begin{pmatrix} 0 & \cdots & \cdots & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & 0 & \boxed{1} & 0 & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \vdots & 0 & \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \cdots & 0 & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & \boxed{1} & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \cdots & \cdots & 0_{(n-1)\times 1} & \vdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \boxed{1} & 0 & \cdots & 0 \\ \vdots & \cdots & \cdots & \ddots & \cdots & 0 & \cdots & \cdots & \vdots \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & 0 \end{pmatrix},$$

$$\vdots$$

$$Z_n^{2n} = \begin{pmatrix} 0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \cdots & \vdots \\ \vdots & \cdots & \vdots & \vdots & \cdots & 0 & \cdots & \cdots & \vdots \\ 0 & \cdots & \cdots & \cdots & 0 & \boxed{1} & 0 & \cdots & 0 \\ \vdots & \ddots & \cdots & \cdots & \cdots & 0 & \vdots & \cdots & 0 \\ 0 & \cdots & \cdots & \cdots & 0 & \vdots & 0 & \cdots & 0 \\ \vdots & \cdots & \cdots & \ddots & \cdots & \vdots & \cdots & \cdots & \boxed{1} \\ \vdots & \cdots & \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & 0_{(n-1)\times 1} \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & \cdots & \cdots & \boxed{1} \end{pmatrix}$$

3.2. Lie Brackets of Generators of Generalized of Lucky Guess Lie Algebra $\mathfrak{lg}(3n)$

The Lucky Guess Lie algebra of dimension $3n$ has same properties of dimension three case, and the list of the brackets is in the following: $(1 \le i, j \le n)$,

1) $\left[X_i^{2n+i}, X_j^{2n+j}\right] = 0$ (16)

2) $\left[X_i^{2n+i}, Y_j^{2n+j}\right] = 0$ (17)

3) $\left[X_i^{2n+i}, Z_j^{n+j}\right] \delta_{ij}. X_i^{2n+i}$ (18)

4) $\left[Y_i^{2n+i}, Y_j^{2n+j}\right] = 0$ (19)

5) $\left[Y_i^{2n+i}, Z_j^{n+j}\right] = \delta_{ij}. (X_i^{2n+i} + Y_i^{2n+i})$ (20)

6) $\left[Z_i^{n+i}, Z_j^{n+j}\right] = 0$ (21)

4. ALGEBRAIC AND TOPOLOGICAL PROPERTIES OF THE GENERALIZED LUCKY GUESS LIE GROUP $LG(3n)$

Lemma 4.1: The Lucky Guess Group $LG(3n)$ is closed Lie group in $GL(3n, \mathbb{R})$.
Proof: To prove, we use the same technique in [4]. Let $(A_r)_{r>0}$ be any sequence of elements in $LG(3n)$ where each A_r is of the form

$$A_r = \begin{pmatrix} I_n & {Z_n}^r & {X_n}^r + {Y_n}^r - I_n - {Z_n}^r + e^{{Z_n}^r} \\ 0_n & I_n & -{Y_n}^r - I_n + e^{{Z_n}^r} \\ 0_n & 0_n & e^{{Z_n}^r} \end{pmatrix}, r > 0, \text{ where}$$

$$Z_n{}^r = \begin{pmatrix} z_1{}^r & 0 & \dots & 0 \\ 0 & z_2{}^r & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n{}^r \end{pmatrix}, Y_n{}^r = \begin{pmatrix} y_1{}^r & 0 & \dots & 0 \\ 0 & y_2{}^r & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_n{}^r \end{pmatrix},$$

$$X_n{}^r = \begin{pmatrix} x_1{}^r & 0 & \dots & 0 \\ 0 & x_2{}^r & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n{}^r \end{pmatrix},\ e^{Z_n{}^r} = \begin{pmatrix} e^{z_1{}^r} & 0 & \dots & 0 \\ 0 & e^{z_2{}^r} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{z_n{}^r} \end{pmatrix}.$$ Since $\mathbb{R}$ is complete, as $r \to \infty$

we have x_i, y_i, z_i such that $x_i{}^r \to x_i$, $y_i{}^r \to y_i$ and $z_i{}^r \to z_i$ for each i. Therefore, $A_r \to A$ as $r \to \infty$,

$$A = \begin{pmatrix} I_n & Z_n & X_n + Y_n - I_n - Z_n + e^{Z_n} \\ 0_n & I_n & -Y_n - I_n + e^{Z_n} \\ 0_n & 0_n & e^{Z_n} \end{pmatrix}, Z_n = \begin{pmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n \end{pmatrix},$$

$$Y_n = \begin{pmatrix} y_1 & 0 & \dots & 0 \\ 0 & y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_n \end{pmatrix},\ X_n = \begin{pmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n \end{pmatrix}, e^{Z_n} = \begin{pmatrix} e^{z_1} & 0 & \dots & 0 \\ 0 & e^{z_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{z_n} \end{pmatrix}.$$

Lemma 4.2: The group $LG(3n)$ is a connected, simply connected and non-compact Lie group.
Proof: Firstly, we prove that $LG(3n)$ is non-compact with the help of Frobenius norm. To prove, we use the same techinque in [4]. The Frobenius norm of an arbitrary element of $LG(3n)$ is given by

$$\left\| \begin{pmatrix} I_n & Z_n & X_n + Y_n - I_n - Z_n + e^{Z_n} \\ 0_n & I_n & -Y_n - I_n + e^{Z_n} \\ 0_n & 0_n & e^{Z_n} \end{pmatrix} \right\|_F$$

$$= \sqrt{trace\left[\begin{pmatrix} I_n & Z_n & X_n + Y_n - I_n - Z_n + e^{Z_n} \\ 0_n & I_n & -Y_n - I_n + e^{Z_n} \\ 0_n & 0_n & e^{Z_n} \end{pmatrix} . \begin{pmatrix} I_n & Z_n & X_n + Y_n - I_n - Z_n + e^{Z_n} \\ 0_n & I_n & -Y_n - I_n + e^{Z_n} \\ 0_n & 0_n & e^{Z_n} \end{pmatrix}^T \right]}$$

$$= \sqrt{n + \textstyle\sum_{i=1}^{n} \left(z_i^2 + 1\right) + (x_i + y_i - z_i - 1 + e^{z_i})^2 + (-y_i - 1 + e^{z_i})^2 + e^{2z_i}}.$$

Thus, $LG(3n)$ is not clearly bounded for all $x_i, y_i, z_i \in \mathbb{R}$. Hence $LG(3n)$ is not compact. Secondly, we verify $LG(3n)$ is connected and simply-connected:

Let g be a mapping from $\mathbb{R}^{3n}$ to $LG(3n)$ such that

$$g: \mathbb{R}^{3n} \to LG(3n)$$

$$g(x_1, \dots, x_n, y_1, \dots, y_n, z_1, \dots, z_n) = \begin{pmatrix} I_n & Z_n & X_n + Y_n - I_n - Z_n + e^{Z_n} \\ 0_n & I_n & -Y_n - I_n + e^{Z_n} \\ 0_n & 0_n & e^{Z_n} \end{pmatrix},$$

where $Z_n = \begin{pmatrix} z_1 & 0 & \dots & 0 \\ 0 & z_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & z_n \end{pmatrix}$, $Y_n = \begin{pmatrix} y_1 & 0 & \dots & 0 \\ 0 & y_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & y_n \end{pmatrix}$, $X_n = \begin{pmatrix} x_1 & 0 & \dots & 0 \\ 0 & x_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & x_n \end{pmatrix}$, $e^{Z_n} = \begin{pmatrix} e^{z_1} & 0 & \dots & 0 \\ 0 & e^{z_2} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & e^{z_n} \end{pmatrix}$.

Since g is a homoemorphism and $\mathbb{R}^{3n}$ is a connected and simply-connected space, $LG(3n)$ is also connected and simply-connected.

Lemma 4.3: The group $LG(3n)$ is solvable and non-nilpotent Lie group.
Proof: $\mathrm{lg}(3n) = span\{X_1, \dots, X_n, Y_1, \dots, Y_n, Z_1, \dots, Z_n\}$,
$\mathrm{lg}(3n)^{(0)} = span\{X_1, \dots, X_n, Y_1, \dots, Y_n, Z_1, \dots, Z_n\}$,
$\mathrm{lg}(3n)^{(1)} = span\{X_1, \dots, X_n, Y_1, \dots, Y_n\}$,
$\mathrm{lg}(3n)^{(2)} = \{0\}$,
$\vdots$
$\mathrm{lg}(3n)^{(k)} = \{[X,Y] \mid X, Y \in \mathrm{lg}(3n)^{(k-1)}\} = 0$. The derived series vanishes for some $k \in$ □, $\mathrm{lg}(3n)$ is solvable.
$\mathrm{lg}(3n)_{(0)} = span\{X_1, \dots, X_n, Y_1, \dots, Y_n, Z_1, \dots, Z_n\}$,
$\mathrm{lg}(3n)_{(1)} = span\{X_1, \dots, X_n, Y_1, \dots, Y_n\}$,
$\mathrm{lg}(3n)_{(2)} = span\{X_1, \dots, X_n, Y_1, \dots, Y_n\}$,
$\vdots$
$\mathrm{lg}(3n)_{(k)} = \{[X,Y] \mid X \in \mathrm{lg}(3n)_{(k-1)}, Y \in \mathrm{lg}(3n)\} = span\{X_1, \dots, X_n, Y_1, \dots, Y_n\}$. The lower central series does not vanish for some $k \in \mathbb{N}$, $\mathrm{lg}(3n)$ is not nilpotent.

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