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ANALYTICAL TEMPERATURE DISTRIBUTION ON A TURBINE BLADE SUBJECT TO COMBINED CONVECTION AND RADIATION ENVIRONMENT

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ABSTRACT

The overall operating cost and operation of a turbine is greatly influenced by the durability of the hot section components operating at very high temperatures. Modern day turbine blades become a critical component for the designers as it receives heat and as a result produces great thermal stresses due to variation of high temperature. Thus the turbine blade metal temperature distribution and temperature gradients are the most important parameters to determine the blade life. In this paper the analysis is done by developing an analytical method to find the temperature distribution in lumped system of combined convection-radiation effect on a turbine blade.

INTRODUCTION

The inlet temperature of the turbine engines has been steadily increasing with the development of new engine. The thermal efficiency of a turbine largely depends on the high operating temperatures in the engines as it produces more work. This results in extremely high temperature gases exiting the combustor and entering the other stages. Turbine blades experience severe thermal stress and fatigue as a result of exposure to these high-temperature gases. In particular, the tips of gas turbine rotor blades are subjected to large thermal loads, resulting in damage to the blade tips.

Turbine blade metal temperature distribution and temperature gradients are the most important parameters determining the blade life. The blade failure mechanisms are low cycle fatigue, high cycle fatigue and thermal fatigue, environmental attack and creep. Reyhani et al. [1] used the conjugate heat transfer method for finding temperature distribution and blade life and concluded that the minimum life occurs at the same point as the same point as maximum temperatures and indicated that the most dominant factor for blade creep life is temperature.

Most scientific problems such as heat transfer are inherently of nonlinearity. Except a limited number of these problems, most of them do not have analytical solution. Therefore, these nonlinear equations should be solved using other methods. Some of them are solved using numerical techniques and some are solved using the analytical method of perturbation.

Rajabi and Ganji [2] used both homotopy and perturbation techniques to solve the temperature distribution in lumped system of combined convection-radiation. The homotopy perturbation (HPM) is compared with the perturbation method (PM) and both methods have got nearly the same results. Ganji and Rafei [3] used the HPM for solving the nonlinear Hirota-Satsuma coupled Kdv partial differential equations. The obtained solutions are compared with Adomian Decomposition Method (ADM) and concluded that HPM is to overcome the difficulties arising in calculation of adomian polynomials. Ganji and Rajabi [4] used both homotopy and perturbation methods to solve heat transfer problems with high nonlinearity order.

A new closed-form analysis was established by Kundu and Lee [5] for the temperature profile with the help of Differential Transformation Method (DTM) for calculating the maximum heat transfer of an annular stepped fins internal heat generation and radiation effects. An integral DTM was introduced to determine the actual heat-transfer rate when heat was generated inside annular stepped fins under nonlinear radiation surface conditions.

Kundu and Lee [6] studied the effect of wet surface, the variable conductivity, and the heat transfer coefficient of different profiles on the temperature and fin efficiencies. The new expression based on the transformed method was formulated appropriately to determine the heat transfer rate as nonlinear terms associated with it. Kundu and Barman [7] has made an analysis on design analysis of annular fins under dehumidifying conditions with a polynomial relationship between humidity ratio and saturation temperature and proposed the DTM to determine the temperature field in wet fins of rectangular and triangular geometries. The fin performance of triangular fins subject to simultaneous heat and mass transfer has been studied by Kundu et al. [8] and they adopted DTM for solving the nonlinear governing differential equation of fully wet fins.

From the above literature survey, it can be highlighted that DTM is a power full method to solve a highly nonlinear differential equation analytically. For the implementation of DTM to solve any nonlinear equation, linearization is not required. Hence, in this paper, the differential transformation method is applied to solve the nonlinear problem arising in the analysis of determination of temperature distribution on turbine blade. Another closed form solution is established by linearization of the radiation term.

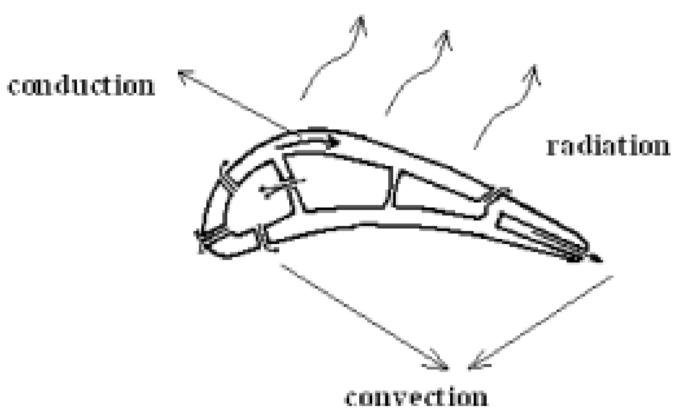

FIGURE 1 KINDS OF HEAT TRANSFER FROM GAS TURBINE BLADES.

DEVELOPMENT OF MATHEMATICAL MODEL

We consider the lumped system with a body of surface area A, volume V, density D, thermal conductivity k, specific heat C_p, initial temperature T_0, and surrounding temperature T_a. The transient response of the solid (blade surface) can be determined by equating an energy balance with combined convection and radiation heat transfer.

$$Ah(T_a - T) + A\sigma\varepsilon(T_a^4 - T^4) = \rho C_p V\left(\frac{\partial T}{\partial t}\right) \tag{1}$$

To normalized the above equation, the following dimensionless parameters are defined as

$$\tau = \alpha t A^2/V^2 \ ; \ L = V/A \ ; \ \theta = T/T_0 \ ; \ \theta_a = T_a/T_0 \tag{2}$$

Equation (1) reduces to dimensionless form by using Eq. (2):

$$\frac{\partial\theta}{\partial\tau} + Bi\left[(\theta - \theta_a) + R_p\left(\theta^4 - \theta_a^4\right)\right] = 0 \tag{3}$$

where

$$Bi = (hL)/k \ ; \ R_p = \sigma\varepsilon T_a^3/h \tag{4}$$

The initial condition taken for the solution of Eq. (3) is in dimensionless form as

$$\text{at } \tau = 0, \quad \theta = 1 \tag{5}$$

Equation (3) is a highly non linear. A method based on the DTM is considered to develop an analytical solution. A brief description of DTM is given in the following section.

Differential transformation Method

This classical Taylor series method is one of the earliest analytical techniques to many problems, especially ordinary differential equations. However, since it requires a lot of symbolic calculation for the derivatives of functions, it takes a lot of computational time for higher order derivatives. Hence an updated version of Taylor series, called the Differential Transformation Method (DTM) is introduced here.

The differential equation for the initial- value can be described as

$$\frac{dy}{dt} = f(t, y), \ a \le t \le b \tag{6}$$

With initial condition,

$$y(a) = \alpha \tag{7}$$

If y (t) is analytic in the time domain t then let

$$\phi(t,k)=\frac{d^k y(t)}{dt^k} \quad \forall t \in T \tag{8}$$

At $t=t_i$, $\varphi(t,k)=\varphi(t_i,k)$, where k belongs to the set of non-negative integer, denoted as the *k* domain. Therefore, Eq. (8) can be rewritten as

$$Y_i(k)=\phi(t_i,k)=\left[\frac{d^k y(t)}{dt^k}\right]_{t=t_i} \quad \forall t \in T \tag{9}$$

where $Y(k)$ is called the spectrum of $y(t)$ at $t=t_i$ in the *k* domain. If $y(t)$ is analytic then $y(t)$ can be represented as

$$y(t)=\sum\frac{(t-t_i)^k}{k!} \tag{10}$$

Equation (10) is known as the inverse transformation of $Y(k)$
If $Y(k)$ is defined as

$$Y(k)=M(k)\left[\frac{d^k q(t)y(t)}{dt^k}\right]_{t-t_0} \tag{11}$$

where $k=0,1,2,.....\infty$. Using the differential transform, a differential equation in the domain of interest can be transformed to an algebraic equation in the *t* domain and $y(t)$ can be obtained by finite-term Taylors series plus a remainder, as

$$x(t)=\frac{1}{q(t)}\sum_{k=0}^{n}\left(\frac{(t-t_0)^k}{H}\right)X(k)+R_{n+1}(t) \tag{12}$$

where, $M(k)=H^k/k!$.

Using the above properties of DTM, Eq. (3) can be written as a differential transform function as

$$\begin{aligned}&(i+1)\Phi(i+1)+Bi\,\Phi(i)-Bi\,\theta_a\delta(i)\\&+R_p\sum_{j=0}^{i}\sum_{l=0}^{j}\sum_{m=0}^{k}\Phi(m)\Phi(k-m)\Phi(j-l)\Phi(i-j)\\&-R_p\theta_a^4\,\delta(i)=0\end{aligned} \tag{13}$$

For $i=0$, Eq. (13) reduces to

$$\Phi(1)=Bi-Bi\,\Phi(0)+R_p\theta_a^4-R_p\Phi^4(0) \tag{14}$$

For $i\geq 1$, Eq. (13) can be written as

$$\Phi(i+1)=-\frac{1}{(i+1)}\left[\begin{aligned}&Bi\,\Phi(i)\\&+R_p\sum_{j=0}^{i}\sum_{l=0}^{j}\sum_{m=0}^{k}\Phi(m)\Phi(k-m)\Phi(j-l)\Phi(i-j)\end{aligned}\right] \tag{15}$$

Applying DTM to Eq. (5)

$$\Phi(0)=1 \tag{16}$$

After knowing all the differential functions from Eq. (14) – (16), temperature of the turbine blade can be evaluated readily from the following expression:

$$\theta=\sum_{i=0}^{\infty}\tau^i\Phi(i) \tag{17}$$

Approximate solution is also possible if the radiation term in Eq. (3) is linearized. Now a linearization of the radiation term is to made as

$$\theta^4 \approx 4\theta\theta_a^4-3\theta_a^4 \tag{18}$$

Equation (3) can be expressed by using Eq. (18) as

$$\frac{d\theta}{d\tau}+(\theta-\theta_a)\left(Bi+4\theta_a^4R_p\right)=0 \tag{19}$$

Equation (19) is solved with the initial condition (5) as

$$\theta=\theta_a+(1-\theta_a)e^{-\left(B_i+4\theta_a^3R_p\right)\tau} \tag{20}$$

RESULTS AND DISCUSSION

The governing equating with the nonlinear term results in a complicated analysis. Hence the DTM is used for temperature distribution and to solve non linear equations of unsteady conduction in a turbine blade.

Figure 2 shows the variation of dimensionless temperature as a function of dimensionless time with different surrounding temperature. From Fig. 2, it can be found that the value of dimensionless temperature (θ) for different value of (θa) is decreasing with the increasing value of (τ) and after some time θ becomes constant. Here the value of other parameters like Biot number (Bi) and radiation parameter (R_p) are kept constant as 0.01 and 0.1, respectively. For a high surrounding temperature, less time is provided to reach under steady condition.

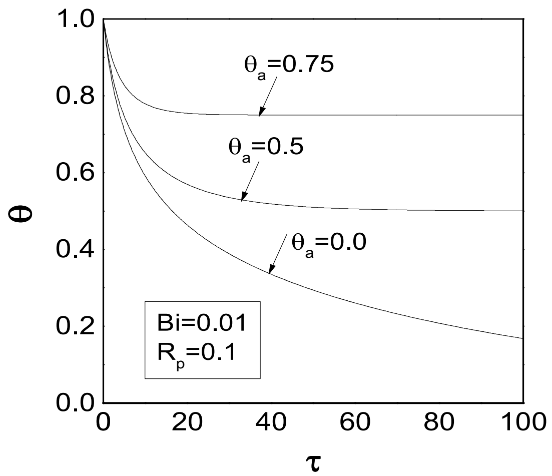

FIGURE 2 DIMENSIONLESS TEMPERATURE (θ) VS DIMENSIONLESS TIME (τ) FOR DIFFERENT θ_a

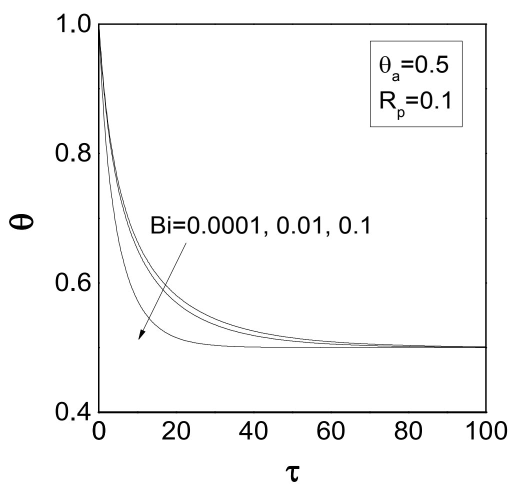

FIGURE 3 DIMENSIONLESS TEMPERATURE (θ) VS DIMENSIONLESS TIME (τ) FOR DIFFERENT Bi

Figure 3 depicts the temperature on the turbine surface with time for different Biot number values. From Fig. 3, it is clear that the value of dimensionless temperature (θ) for different values of Biot number (B_i) is decreasing sharply with increasing value of (τ) and after some time (θ) becomes constant. The value of other parameters like (θa) and (R_p) are considered as 0.5 and 0.1 respectively. From this figure, it can be highlighted that the temperature does not change significantly with variation of Bi. It may be important from the design point of view.

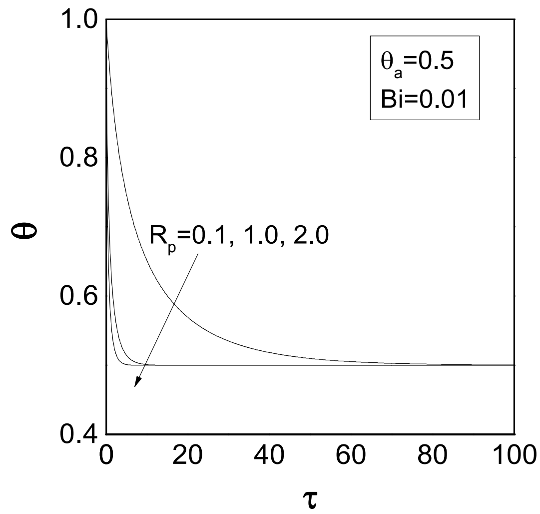

FIGURE 4 EFFECT OF R_p ON DIMENSIONLESS TEMPERATURE (Θ) UNDER TRANSIENT CONDITION

To know the radiation effect on transient temperature response for a turbine blade, Fig. 4 is illustrated. From Fig. 4, it is found that the value of dimensionless temperature (θ) for different values of (R_p) is decreasing sharply with the increasing value of (τ) and after some time (θ) becomes constant. The values of other parameters like (θa) and (Bi) are considered as 0.5 and 0.01 respectively. The effect of radiation becomes dominated with lower values of its parameter.

CONCLUSIONS

The study investigates the effect of temperature distribution of a gas turbine blade with some design parameters. The mathematical model is for a 2-dimensional profile of a gas turbine engine blade. The differential Transformation method is used for calculating the temperature distribution on a turbine blade. The proposed approximate analytical model can estimate the temperature distribution on a turbine blade under both convective and radiative environments.

The major findings can be enumerated as follows:

(1) Dimensionless temperature (θ) for a constant θ_a decreases with the increasing value of dimensionless time (τ). A higher θ_a requires more time to attend steady condition.

(2) The effect of Bi on temperature response under lumped system of analysis is insignifiant.

(3) The radiation parameter Rp reduces time scale to have maintaining unsteady condition.

NOMENCLATURE

A	area (m^2)
Bi	Biot number, hL/k
C_p	Specific heat (KJ/KgK)
DTM	differential transform method
h	Coefficient of convection (W/m^{2}K)
HPM	homotopy perturbation method
k	Thermal conductivity (W/mK)
L	characteristics length, V/L (m)
PM	perturbation method
R_p	dimensionless radiation parameter, $R_p = \sigma\varepsilon T_a^3/h$
t	time (sec)
T	temperature (^{0}C)
T_0	initial temperature (^{0}C)
T_a	Surrounding temperature (^{0}C)
V	Volume (m^3)
x	Coordinate (m)
X	dimensionless Coordinate, x/L

Greek symbols

α	thermal diffusivity (m^2/sec)
ε	emissivity
ρ	density (Kg/m^3)
σ	Stefan-Boltzman constant (-)
θ	dimensionless temperature, T/T_0
θ_a	dimensionless temperature, T_a/T_0
Φ	differential transform function of θ

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