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CHARACTERISTICS OF MAGNETOHYDRODYNAMIC MIXED CONVECTION IN A PARALLEL MOTION TWO-SIDED LID- DRIVEN DIFFERENTIALLY HEATED PARALLELOGRAMMIC CAVITY WITH VARIOUS SKEW ANGLES

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ABSTRACT

A numerical study is presented for mixed convection flow of air (Pr=0.71) within a parallel motion two sided lid-driven parallelogrammic cavity in the presence of magnetic field. The left and right lid-driven sidewalls of the parallelogrammic cavity are maintained at isothermal hot and cold temperatures respectively and slide from bottom to top in upward parallel direction with a uniform lid-driven velocity. A magnetic field of strength (B_{ox}) is subjected in the horizontal direction. The horizontal walls of the cavity are considered thermally insulated. The finite volume method has been used to solve the governing Navier–Stokes and energy conservation equations of the fluid medium in the parallelogrammic cavity in order to investigate the effect of magnetic field on the flow and heat transfer for various values of Richardson number, skew angle and Hartmann number. The values of the governing parameters are the Hartmann number ($0 \leq Ha \leq 75$), Richardson number ($0.01 \leq Ri \leq 100$) and skew angle ($-60° \leq \Phi \leq 60°$). The present numerical approach is found to be consistent and the results is presented in terms of streamlines and isotherm contours in addition with the average Nusselt number. It is found that as the Hartmann number increases the circulation of the rotating vortices is reduced and the conduction mode of heat transfer is dominant. Also, it is found that both Richardson number and direction of two sided lid-driven sidewalls affect the heat transfer and fluid flow in the parallelogrammic cavity.

NOMENCLATURE

B_{ox}	Magnetic induction in x-direction (Tesla)
g	Gravitational acceleration (m/s^2)
Gr	Grashof number
H	Height of the parallelogrammic cavity (m)
Ha	Hartmann number
k	Thermal conductivity of fluid (W/m.K)
n	Normal direction with respect to the left sidewall
$\overline{Nu}$	Averge Nusselt number
P	Dimensionless pressure
p	Pressure (N/m^2)
Pr	Prandtl number
Re	Reynolds number
Ri	Richardson number
T	Temperature (K)
T_c	Temperature of the cold surface (K)
T_h	Temperature of the hot surface (K)
U	Dimensionless velocity component in x-direction
u	Velocity component in x-direction (m/s)
V	Dimensionless velocity component in y-direction
v	Velocity component in y-direction (m/s)
V_P	Upward sliding velocity of two sided lid-driven sidewalls (m/s)
W	Width of the parallelogrammic cavity (m)
X	Dimensionless Coordinate in horizontal direction

x Cartesian coordinate in horizontal direction (m)
Y Dimensionless Coordinate in vertical direction
y Cartesian coordinate in vertical direction (m)

Greek Symbols

α Thermal diffusivity (m^2/s)
β Volumetric coefficient of thermal expansion (K^{-1})
θ Dimensionless temperature
Φ Sidewall inclination angle from vertical or skew angle (degree)
ν Kinematic viscosity of the fluid (m^2/s)
μ Dynamic viscosity of the fluid (kg.s/m)
ρ Density of the fluid (kg/m^3)
σ Fluid electrical conductivity (W/m. K)

Subscripts

h Hot
c Cold

Abbreviations

MHD Magneto-hydrodynamics
SUR Successive Under Relaxation

1. INTRODUCTION

Magneto-hydrodynamics (MHD) is a branch of continuum mechanics, which deals with the dynamics of electrically conducting fluids in the presence of electromagnetic fields. Magneto-hydrodynamic equations are ordinary electromagnetic and hydrodynamic equations modified to take into account the interaction between the motion of the fluid and the electromagnetic field. The effect of the magnetic field on the convective heat transfer and the mixed convection flow of the fluid are of paramount importance in engineering. A combined free and forced convection flow of an electrically conducting fluid in a cavity with the presence of magnetic field is of special technical significance because of its frequent occurrence in many industrial applications such as liquid–metal cooling of nuclear reactors, electronic packages, electromagnetic casting and crystal growth. When an electrical current flows in a wire, the resistance of the wire causes a voltage drop along the wire and as a result electrical energy is lost. This lost in electrical energy is converted into thermal energy which called Joule heating [1-3].The magneto-hydrodynamics mixed convection with/without Joule heating is usually considered as a very up to date subject. Many researchers studied this subject for different geometries and boundary conditions. Chamkha [4] investigated the problem of unsteady, laminar, combined forced-free convection flow in a square cavity with the presence of internal heat generation or absorption and a magnetic field. Both the top and bottom horizontal walls of the cavity were insulated while the left and right vertical walls were kept at constant and different temperatures. The left vertical wall was moving in its own plane at a constant speed while all other walls were fixed. A uniform magnetic field was applied in the horizontal direction normal to the moving wall. A temperature-dependent heat source or sink was assumed to exist within the cavity. The governing equations and conditions were solved numerically using the alternating direct implicit (ADI) procedure. Two cases of thermal boundary conditions corresponding to aiding and opposing flows were considered. Comparisons with previously published work were performed and the results were found to be in excellent agreement. A parametric study was conducted and a set of representative graphical results was presented and discussed to illustrate the effect of the physical parameters on the solution. Hossain et al. [5] investigated the problem of combined buoyancy and thermo-capillary convection flow of an electrically conducting fluid filled in an enclosure in the presence of an external uniform magnetic field. They concluded that the flow rates were decreased when the horizontal direction of the external magnetic field was changed vertically. Abdelkhalek [6] presented numerical results for the effects of mass transfer on steady two-dimensional laminar MHD mixed convection owing to the stagnation flow against a heated vertical semi-infinite permeable surface. These results were obtained by solving the coupled non-linear partial differential equations describing the conservation of mass, momentum and energy by a perturbation technique. These results were presented to illustrate the influence of the Hartmann number, wall mass transfer coefficient, heat absorption coefficient, Prandtl number and the mixed convection or buoyancy parameter. Numerical results for the dimensionless velocity profiles, the temperature profiles, the local friction coefficient and the local Nusselt number were presented for various parameters. The effects of the different parameters on the velocity and temperature as well as the skin friction and wall heat transfer were presented graphically and discussed. Kandaswamy et al. [7] investigated numerically magneto-convection of an electrically conducting fluid in a square cavity with partially thermally active vertical walls. The active part of the left sidewall was at a higher temperature than the active part of the right sidewall. The top, bottom and the inactive parts of the sidewalls were thermally inactive. Nine different combinations of the relative positions of the active zones were considered. The governing equations were discretized by the control volume method. The results were obtained for Grashof numbers between 10^4 and 10^6, Hartmann numbers between 0 and 100 and Prandtl numbers between (0.054–2.05). The heat transfer characteristics were presented in the form of streamlines and isotherms. They concluded that the average Nusselt number decreased with an increase of Hartmann number and increased with an increase of Grashof number. Barletta and Celli [8] investigated combined forced and free MHD flow in a vertical channel with an adiabatic wall and an isothermal wall. The laminar, parallel and fully developed regime was considered. A uniform horizontal magnetic field was assumed to be applied to the fluid. The local balance equations were written in a dimensionless form and solved by taking into account the effects of Joule heating and viscous dissipation. The solutions were obtained both analytically by a power series method and numerically. The dimensionless governing parameters affecting the velocity and temperature profiles were the Hartmann number and the ratio between the Grashof number and the Reynolds number. Dual solutions were shown to exist for every value of the Hartmann number within a bounded range of the ratio between the Grashof number and the Reynolds number. They observed that outside this range, no parallel flow solutions of the problem exist. Rahman et al. [9] studied numerically magnetohydrodynamic (MHD) mixed convection in a lid-driven cavity along with Joule heating. The cavity consisted of adiabatic horizontal walls and differentially heated vertical walls, but it also contained a heat conducting horizontal circular cylinder located somewhere within the cavity. The governing equations were transformed into a non-dimensional form and resulting non-linear system of partial differential equations were then solved by using Galerkin weighted residual method of finite element formulation. The analysis was conducted by observing variations of the streamlines and isotherms for the size, locations and thermal conductivity of the cylinder at the Richardson number (Ri) ranging from 0.0 to 5.0, Prandtl number (Pr = 0.71) and Reynolds number (Re = 100) with constant physical properties. The results indicated that both the streamlines and isotherms strongly depended on the size and locations of the inner cylinder, but the thermal conductivity of the cylinder had a significant effect only on the isothermal lines. The variations of average Nusselt number on the hot wall and average fluid temperature in the cavity were also presented to show the overall heat transfer characteristics inside the cavity. Nithyadevi et al. [10] presented numerical results of magneto-convection in a square cavity subjected to sinusoidal temperature boundary conditions on the heating location of one of the sidewalls. Liquid metals (Pr = 0.054) was used as a coolant in nuclear reactors for thermodynamics systems. It was observed that the average Nusselt number increased with increase in Prandtl number and Grashof number and decreased with Hartmann number. The rate of heat transfer and vertical velocity oscillated for increasing amplitudes. In the steady state, the rate of heat transfer resonated to the periodic temperature at the hot region. For sufficiently large magnetic field (Ha = 100) the convective mode of heat transfer was converted into conductive mode. Rahman et al. [11] studied magnetohydrodynamic (MHD) mixed convection around a heat conducting horizontal circular cylinder placed at the center of a rectangular cavity along with Joule heating. Steady state heat transfer by laminar mixed convection had been studied numerically by solving the equations of mass, momentum and energy to determine the fluid flow and heat transfer characteristics in the cavity as a function of Richardson number, Hartmann number and the cavity aspect ratio. The results were presented in the form of average Nusselt number at the heated surface; average fluid temperature in the cavity and temperature at the cylinder center for the range of Richardson number, Hartmann number and aspect ratio. The streamlines and isotherms were also presented. It was found that the streamlines, isotherms, average Nusselt number, average fluid temperature and dimensionless temperature at the cylinder center strongly depended on the Richardson number, Hartmann number and the cavity aspect ratio. Rahman et al. [12] investigated numerically conjugate effect of Joule heating and magnetic force, acting normal to the left vertical wall of an obstructed lid-driven cavity saturated with an electrically conducting fluid. The cavity was heated from the right vertical wall isothermally. Temperature of the left vertical wall, which had constant flow speed, was lower than that of the right vertical wall. Horizontal walls of the cavity were considered adiabatic. The physical problem was represented mathematically by sets of governing equations and the developed mathematical model was solved by finite element formulation. Two cases of with and without obstacle for different values of Richardson number varying in the range 0.0 to 5.0 were considered. Results were presented in terms of streamlines, isotherms, average Nusselt number at the hot wall and average fluid temperature in the cavity for the magnetic parameter (Ha) and Joule heating parameter (J). The results showed that the obstacle had significant effects on the flow field at the pure mixed convection region and on the thermal field at the pure forced convection region. It was also found that the parameters (Ha) and (J) had notable effect on flow fields; temperature distributions and heat transfer in the cavity. Numerical values of average Nusselt number for different values of the aforementioned parameters had been presented in tabular form. Rahman et al. [13] performed a finite element analysis on the conjugated effect of Joule heating and magneto-hydrodynamic on double-diffusive mixed convection in a horizontal channel with an open cavity. Homogeneous flows were imposed throughout the channel. Consistent high temperatures and concentrations were imposed at the bottom wall of the cavity. The other sides of the cavity along with the channel walls were considered adiabatic. The effects of the various parameters (Richardson number, Hartmann number, Joule heating, buoyancy ratio and Lewis number) on the contours of streamline, temperature, concentration and density had been depicted. Moreover, the average Nusselt and Sherwood numbers as well as bulk temperature were presented for the aforementioned parameters. The results showed that the aforesaid parameters had noticeable effect on the flow pattern together with heat and mass transfer. Nasrin [14] investigated numerically the hydromagnetic natural convective flow and heat transfer characteristics in a square cavity with a solid circular heated obstacle located at the center. The left vertical surface of the cavity was uniformly heated of temperature (T_c) and other three surfaces were considered adiabatic. The obstacle was maintained with a constant heat temperature (T_h). The physical problem was represented mathematically by sets of governing equations and the developed mathematical model was solved by finite element simulation. The behavior of the fluid was studied in the ranges of Prandtl number (0.073–2.73), Hartmann number (0–50) and Joule heating parameter (1–7). It was found that the flow and temperature fields were strongly dependent on the above stated parameters for the ranges considered. The variation of the average Nusselt number (Nu) for various Prandtl number (Pr) was also presented. Sivasankaran et al. [15] investigated numerically mixed convection in a square cavity of sinusoidal boundary temperatures at the sidewalls in the presence of magnetic field. The horizontal walls of the cavity were considered adiabatic. The governing equations were solved by finite-volume method. The results were obtained for various combinations of amplitude ratio, phase deviation, Richardson number, and Hartmann number. It was found that the heat transfer rate increased with the phase deviation up to $\Phi = 90^o$ and then it decreased for further increase in the phase deviation. Also, the heat transfer rate increased with increasing the amplitude ratio. The flow behavior and heat transfer rate inside the cavity were strongly affected by the presence of the magnetic field. Nasrin and Parvin [16] presented a numerical simulation to analyze the mixed convection flow and heat transfer in a lid-driven cavity with sinusoidal wavy bottom surface in presence of transverse magnetic field. The enclosure was saturated with an electrically conducting fluid. The cavity vertical walls were insulated, while the wavy bottom surface was maintained at a uniform temperature higher than the top lid. In addition, the transport equations were solved by using the finite element formulation. The implications of Reynolds number (Re), Hartmann number (Ha) and number of undulations (λ) on the flow structure and heat transfer characteristics were investigated while, Prandtl number (Pr) and Rayleigh number (Ra) were considered fixed. The trend of the local heat transfer was found to follow a wavy pattern. The results illustrated that the average Nusselt number (Nu) at the heated surface increased with an increase of the number of waves as well as the Reynolds number, while decreased with increasing Hartmann number. Nasrin [17] made a numerical study of the flow characteristics on combined magneto-convection in a lid-driven differentially heated square enclosure. This problem was solved by using finite element method of the partial differential equations, which were the heat transfer and stream function in Cartesian coordinates. The tests were performed for different solid–fluid thermal conductivity ratio, cylinder location and Richardson number while the Prandtl number, Reynolds number, magnetic and Joule heating parameters were kept constant. One geometrical configuration was used with two undulations. The results showed that the heat conducting inner square cylinder affected the flow and the heat transfer rate in the enclosure. The trend of the local heat transfer was found to follow a wavy pattern. Results were presented in terms of streamlines, isotherms, average Nusselt number at the heated wavy wall, average temperature of the fluid in the enclosure and dimensionless temperature at the cylinder center for different combinations of the governing parameters. Rahman et al. [18] numerically studied the development of magnetic field effect on mixed convective flow in a horizontal channel with a bottom heated open enclosure. The enclosure considered had a rectangular horizontal lower surface and vertical side surfaces. The lower surface was at a uniform temperature (T_h) while other sides of the cavity along with the channel walls were adiabatic. The governing two-dimensional flow equations had been solved by using Galarkin weighted residual finite element technique. The investigations were conducted for different values of Rayleigh number (Ra), Reynolds number (Re) and Hartmann number (Ha). Various characteristics such as streamlines, isotherms and heat transfer rate in terms of the average Nusselt number (Nu), the Drag force (D) and average bulk temperature (θ_{av}) were presented. The results indicated that the mentioned parameters strongly affected the flow phenomenon and temperature field inside the cavity, while in the channel these effects were less significant. Oztop et al. [19] considered laminar mixed convection flow in the presence of magnetic field in a top sided lid-driven cavity heated by a corner heater. The corner heater was under isothermal boundary conditions with different length in bottom and right vertical walls. Finite volume technique was used to solve governing equations. The temperature of the lid was lower than that of heater. The study was performed for different Grashof and Hartmann numbers at Re = 100. They concluded that the heat transfer decreased with increasing of Hartmann number and the rate of reduction was higher for high values of the Grashof number. Nasrin [20] performed a numerical study using a finite element method to explore the mixed magneto-convective flow and heat transfer characteristics of fluid contained in a lid-driven cavity having a sinusoidal wavy vertical surface. A heat conducting square body was located at the centre of cavity. The cavity horizontal walls were perfectly insulated, while the corrugated right vertical surface was maintained at a uniform temperature higher than the left lid. The flow was assumed to be two-dimensional and Joule heating effect was considered. The flow pattern and the heat transfer characteristics inside the cavity were presented in the form of streamlines, isotherms, average temperature of the fluid and temperature of solid body centre for various values of Prandtl number, Richardson number and Hartmann number. The heat transfer rate was detected maximum for the highest Prandtl number and absence of magnetic field. From the other hand, the phenomenon of mixed convection in parallelogrammic cavities is not a new problem for research. A variety of numerical results had been published for last three decades due to its various engineering applications such as drying technologies, nuclear reactor systems, energy storage, lubrication technologies, food and metallurgical industries. Many research papers related with combined free and forced convection in one sided lid-driven or two-sided lid-driven enclosures had been studied in recent years by several authors under different conditions [21-28]. Based on the above literature review and according to our knowledge there is no research work has been published up to date considered the flow and thermal characteristics of two-dimensional mixed convection flow in a parallel motion two sided lid-driven parallelogrammic cavity by taking into account the effect of magnetic field. This significant point can be considered as an original contribution of the present research. Here, the flow and thermal characteristics of this complex geometry are analyzed by observing variations in streamlines and isotherms for different values of the Richardson number, Hartmann number and the inclination angle. We also have investigated the heat transfer characteristics by calculating the average Nusselt number on the hot surface of the cavity.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider steady, laminar, mixed convection flow in a two-dimensional parallelogrammic cavity of height (H) and width (W) filled with air. The physical model and coordinate system considered in this investigation are shown in Fig 1. Both the top and bottom walls of the cavity are insulated, while the left and the right sidewalls are maintained at a constant hot and cold temperature respectively. The magnetic field of strength (B_{ox}) is subjected in the horizontal direction, while the gravitational force acts in the vertical direction. Both the lid-driven left and the right sidewalls of the cavity are allowed to move in a parallel motion from bottom to top with an upward sliding velocity (V_p) in which the flow is driven by two facing walls. The flow inside the parallelogrammic cavity is assumed to be laminar, two-dimensional, incompressible, Newtonian and steady. Hartmann numbers are varied as $0 \le Ha \le 75$, the Richardson numbers ($Ri = Gr/Re^2$) are varied as $0.01 \le Ri \le 100$.This range of Richardson number is produced by considering the Grashof number (Gr) constant at $Gr = 10^4$, while the Reynolds number is varied as ($10 \le Re \le 1000$) by changing an upward sliding velocity (V_p). The cavity skew angle is varied as $-60° \le \Phi \le 60°$, respectively. The fluid properties are considered constant except for the density variation, which is modeled according to Boussinesq approximation while viscous dissipation effects are assumed to be negligible. In this work, the effects of magnetic field is considered whereas Joule heating is neglected. The governing non-dimensional mass, momentum and energy equations are as follows, respectively:-

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \Pr\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \Pr\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + Ha^2 \Pr V + Ri \Pr \theta \tag{3}$$

$$U\frac{\partial \theta}{\partial X} + V\frac{\partial \theta}{\partial Y} = \left(\frac{\partial^2 \theta}{\partial X^2} + \frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

The previous dimensionless governing equations are converted into a non-dimensional forms by using the following dimensionless parameters:

$$\theta = \frac{T - T_C}{T_h - T_c}, \quad X = \frac{x}{H}, \quad Y = \frac{y}{H},$$
$$U = \frac{u}{V_p}, \quad V = \frac{v}{V_p}, \quad P = \frac{p}{\rho V_P^2} \tag{5}$$

The previous dimensionless numbers are defined as:-

$$\Pr = \frac{\upsilon}{\alpha} \qquad Gr = \frac{g\beta(T_h - T_c)H^3}{\nu^2} \qquad Ri = \frac{Gr}{Re^2}$$
$$Ha = B_{ox} H \sqrt{\frac{\sigma}{\rho \upsilon}} \qquad Re = \frac{V_p H}{\nu} \tag{6}$$

The Hartmann number (Ha) represents the effect of the electromagnetic field. While, the Richardson number (Ri) represents the relative strength of the natural convection and forced convection for the considered problem. If $Ri << 1$, then forced convection is dominant, while if $Ri >> 1$, then natural convection is dominant .For problems with Ri approximately 1.0 then the natural convection effects are comparable to the forced convection effects, i.e., the mixed convection is dominant. The rate of heat transfer is represented in terms of average Nusselt number at the hot left sidewall ($\overline{Nu_h}$) as follows:

$$\overline{Nu}_h = \int_0^H \left[\frac{\partial \theta}{\partial n}\right]_{X=0} dn \tag{7}$$

2.1 BOUNDARY CONDITIONS

On the solid walls no-slip boundary conditions are applied and the relevant non-dimensional boundary conditions of the present problem are expressed as follows:

1. The left sidewall of the parallelogrammic cavity is maintained at a uniform hot temperature (T_h), and sliding in upward direction at a uniform sliding velocity (V_p) so that :-

at $0 \le X \le \cos(\Phi)$, $\theta = 1$, $V = V_p \cos(\Phi)$ and $U = V_p \sin(\Phi)$ (8)

2. The right sidewall of the parallelogrammic cavity is maintained at a uniform cold temperature (T_c), and sliding in upward direction at a uniform sliding velocity (V_p) so that:-

at $1 \le X \le 1 + \cos(\Phi)$, $\theta = 0$, $V = V_p \cos(\Phi)$ and $U = V_p \sin(\Phi)$ (9)

3. The upper horizontal wall of the parallelogrammic cavity is considered adiabatic, so that:

$$\text{at} \quad Y = 1 \quad \frac{\partial \theta}{\partial Y} = 0 \quad , \quad U = V = 0 \qquad (10)$$

4. The lower horizontal wall of the parallelogrammic cavity is considered adiabatic, so that:

$$\text{at} \quad Y = 0 \quad \frac{\partial \theta}{\partial Y} = 0 \quad , \quad U = V = 0 \qquad (11)$$

3. SOLUTION METHOD AND VERIFICATION

The dimensionless governing equations associated with their boundary conditions are solved numerically using the finite volume method. The details of this method is well presented in [29].The hybrid-scheme, which is a combination of the central difference scheme and the upwind scheme, is used to discretize the convection terms. The momentum and energy balance equations are combinations of mixed elliptic-parabolic system of partial differential equations and the resulted set of algebraic equations are solved sequentially using the Successive Under Relaxation (SUR) method [30]. In order to couple the velocity field and pressure in the momentum equations, the well known SIMPLER-algorithm is utilized. To satisfy the required convergence an under-relaxation factor of 0.35 is used in the computational scheme. A staggered grid system, in which the velocity components are stored midway between the scalar storage locations also used. The computation process is terminated when the residuals for the continuity and momentum equations get below 10^{-6} and the residual for the energy equation gets below 10^{-9}. In order to capture the flow and thermal fields inside the parallelogrammic cavity accurately especially adjacent to the sharp corners, a grid clustering procedure close to solid boundaries is adopted. This is due to the strong expected temperature gradient adjacent the two sided lid-driven sidewalls. To check the sensitivity of the solution to the grid used, numerical experiments are performed with different sizes of non-uniform grids. Finally, a grid size of the (10000 node) is used in generating all solutions presented in this work as shown in Fig.2. In order to demonstrate the validity of the present numerical solution, the accuracy of the computational procedure is verified against the numerical results of Chamkha [4]. In his work, a laminar mixed convection problem was investigated in which the square cavity was filled with air (Pr = 0.71) with adiabatic horizontal walls, while the sidewalls were considered isothermal but kept at different temperatures. The left moving upwards wall was maintained at the hot temperature (T_H) while the non-moving vertical right wall was kept at the cold temperature T_C (aiding flow). A uniform magnetic field (B_{ox}) was applied in the horizontal direction normal to the moving sidewall. The comparison of the average Nusselt number distribution along the hot moving left sidewall is presented in Table 1. An excellent agreement can be observed between the results of the present work and the results of Chamkha [4] with an error occurs while extracting values. Therefore; the previous validation gives a fair interest in the present numerical scheme to treat with the present studied problem.

TABLE 1 COMPARISON OF THE AVERAGE NUSSELT NUMBER AT THE HEATED LEFT MOVING UPWARDS SIDEWALL FOR AIDING FLOW CONDITIONS (Gr=100, Pr=0.71, Φ= 0° and Re=1000) WITH THOSE OF PREVIOUS STUDY.

Ha	Average Nusselt number		Error %
	Chamkha [4]	Present work	
0	2.2692	2.2588	-0.45
10	2.1050	2.1132	+0.38
20	1.6472	1.6587	+0.69
50	0.9164	0.9211	+0.51

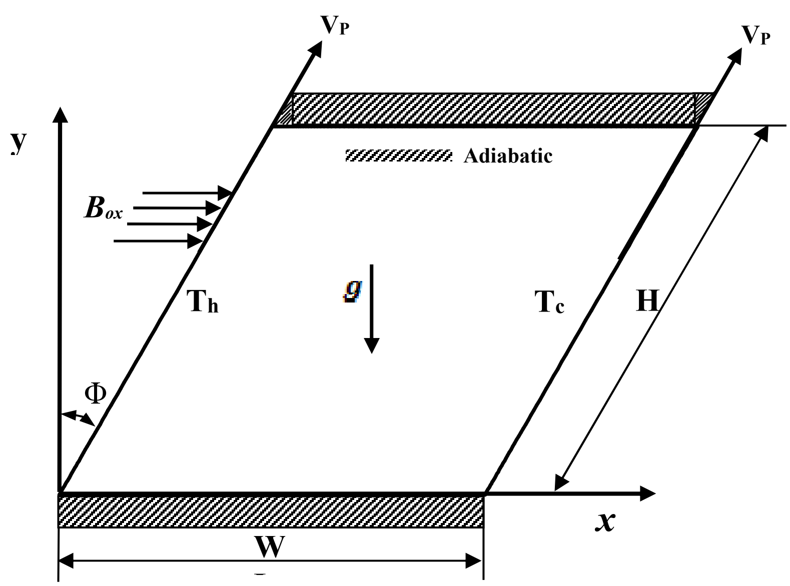

FIGURE 1 SCHEMATIC DIAGRAM OF THE PHYSICAL DOMAIN.

4. RESULTS AND DISCUSSION

The flow and thermal fields' characteristics in a parallelogrammic cavity which is lid-driven at two sided left and right walls while the bottom and top walls are considered adiabatic are explained in this section. In the present numerical analysis, the following parametric domains of the dimensionless groups are considered : the Hartmann number is varied as $0 \leq Ha \leq 75$, the working fluid is air with Prandtl number (Pr) = 0.71**,** the Richardson number (Ri) is varied at $0.01 \leq Ri \leq 100$ while the cavity skew angle, Φ, is varied as $-60° \leq \Phi \leq 60°$. Figs (3-8) illustrate the computed streamlines and isotherms for various values of Hartmann number, Ha = 25, 50 and 75 at $0.01 \leq Ri \leq 100$ and $-60° \leq \Phi \leq 60°$ .It can be observed from the streamlines distributions, that the Hartmann number has a strong role in the fluid circulation. When the effect of magnetic field is slight or when the Hartmann number is low (Ha = 25), the strength of circulation is strong and the flow is characterized by various major and minor rotating vortices of relatively high velocity which can be observed to fill all the parallelogrammic cavity. Some of these vortices are rotate in clockwise direction and the others are rotate in counterclockwise direction. These vortices are generated due to the movement of the two sided lid-driven sidewalls. But, when the Hartmann number increases or when the effect of magnetic field is strong, the strength of circulation decreases gradually and the convection role diminishes because the magnetic field slow down the movement of the fluid in the cavity. Therefore; it can be concluded that the application of magnetic field in the normal direction to the two-sided lid-driven walls causes to reduce the strength of fluid circulation and as a result the vortices begin to enlarge in upward and downward directions. It is clear that the flow within parallelogrammic cavity takes place by a mixture between the natural convection represented by the buoyancy forces due to the temperature difference between two sided lid-driven sidewalls and forced convection due to shear force coming from the sliding of two sided lid-driven walls in upward direction. With respect to isotherms, when the Hartmann number is low (Ha = 25), the isothermal lines are non-uniform in shape and clustered as expected, adjacent the hot left sidewall indicating a sharp temperature gradient which can be observed in this wall. The isothermal lines have a shape similar to that observed when the convection effect is dominant. As Hartmann number increases (Ha = 50 and 75), isothermal lines inside the parallelogrammic cavity become more linear and parallel and approach gradually towards the conduction pattern of isothermal lines. Therefore; the effect of the increase in magnetic field on the isothermal lines is that they are become more straighten out since the magnetic field resists the flow and the convection effect is significantly reduced inside the parallelogrammic cavity. Also, it can be observed in Figs (3-8) that when the Richardson number is very small (i.e., Ri = 0.01 and 0.1) then the forced convection effect is strong and the flow is generated mainly due to shear force which is produced as a result of the sliding of the two sided lid-driven walls at uniform velocity. For this reason, it can be noticed that the flow consists from two primary rotating vortices of elliptical shape .When there is no effect to the sidewalls inclination angle (i.e., Φ = 0), the effects of Hartmann number and Richardson number are control the flow field inside the parallelogrammic cavity. From the other hand, when the inclination angles increase from $\Phi = 30^o$ to $\Phi = 60^o$, the size of vortices near the hot left sidewall changes and begins to enlarge towards the cold right sidewall and as the inclination angles increase, the size of these vortices increases. While, the size of vortices adjacent to the cold right sidewall changes and begins to enlarge towards the hot left sidewall when the inclination angles decreases from Φ = -30° to Φ= -60°.With respect to isotherms, it is evident that the conduction heat transfer regime has become the dominant mode of energy transport in the cavity since, slight heat is noticed to be carried away from the lid-driven hot left sidewall into the cavity. When the Richardson number equals one (i.e., Ri = 1), then mixed convection is dominant. In this case, the effect of enclosure inclination angle remains the same where the core of vortices near the hot left sidewall begins to move towards the cold right sidewall and its moving increases as the inclination angle increases from $\Phi = 30^o$ to $\Phi = 60^o$ or decreases from Φ = -30° to Φ = -60°.This is because the buoyancy force effect balances the effect of the two-sided lid-driven walls. Moreover, the isotherms are in general symmetrical and parallel to the sidewalls of the cavity, which gives a clear indication of conduction and mixed convection dominated heat transfer in the cavity, while the effect of sidewall skew angle is slight at this case. When the Richardson number is high (i.e., Ri = 10 and 100), then natural convection is dominant. In this case, the effect of the sidewall skew angle has a significant influence on the flow and thermal fields. This is because when the Richardson number is high, the buoyancy force effect becomes greater than the effect of the two-sided lid-driven sidewalls. Therefore; the size of rotating vortices becomes larger and clearly visible than the corresponding vortices when the Richardson number equals one (i.e., Ri = 1). Also, when the inclination angles increase from $\Phi = 30^o$

to $\Phi = 60^{o}$, the size of vortices near the hot left sidewall increases. While, the size of vortices adjacent the cold right sidewall increases when the inclination angles decreases from $\Phi = -30°$ to $\Phi = -60°$.With respect to isotherms, when the Richardson number is high, they are changed and begin to crowd near the hot left sidewall since the effect of natural convection is high at this case, while the forced convection and the shear force effect is weak and this causes to increase the temperature gradients inside the parallelogrammic cavity. Also, a thermal boundary layer adjacent to the two-sided lid-driven sidewalls can be detected due to the high circulation strength inside the parallelogrammic cavity. The effects of Hartmann number on the average Nusselt number ($\overline{Nu_h}$) along the hot left sidewall for various values of Richardson number and inclination angles are presented in Figs (9-13). It is evident from these figures that the average Nusselt number ($\overline{Nu_h}$) decreases with increasing the Hartmann number for all values of Richardson number and skew angles. The reason of this behavior because when the Hartmann number increases, the magnetic field effect becomes strong enough to reduce the fluid circulation and makes a clear drop in the temperature gradient adjacent the hot left sidewall and as a result reduces the average Nusselt number. Also, it is found that the average Nusselt number decreases rapidly when the Richardson number is low due to the reduction in the convection role and the sudden drop in the temperature gradient. As the Richardson number increases, this reduction decreases and the behavior of the average Nusselt number becomes more uniform and steady. This behavior can be observed for all studied range of sidewall skew angles. Figs (14-16) demonstrate the variation of the average Nusselt number along the hot left sidewall for various values of Hartmann number and inclination angles at Ri = 0.01, 1.0 and 100 respectively. It is observed that for the case with no magnetic field (i.e., Ha = 0), the average Nusselt number increases very rapidly with increasing the inclination angles until $\Phi = \mathbf{0}°$ and then decreases with increasing the inclination angles. Moreover, it is found that when the Richardson number is low (i.e., forced convection is dominant), the average Nusselt number has a remarkable value on the hot left sidewall even when the Hartmann number increases. But, when the Richardson number is high (i.e., natural convection is dominant), the average Nusselt number decreases rapidly with increasing the Hartmann number. Therefore; the magnetic field has a clear effect on the natural convection region rather than the forced convection region.

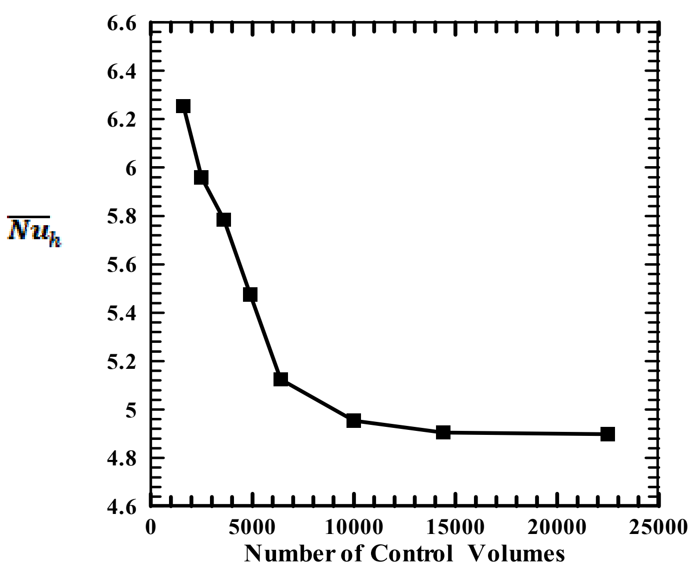

FIGURE 2 CONVERGENCE OF AVERAGE NUSSELT NUMBER ALONG THE HEATED LEFT SIDEWALL WITH GRID REFINEMENT AT Re=1000, Pr = 0.71, Ri= 0.01, Ha=75 and Φ = 60°.

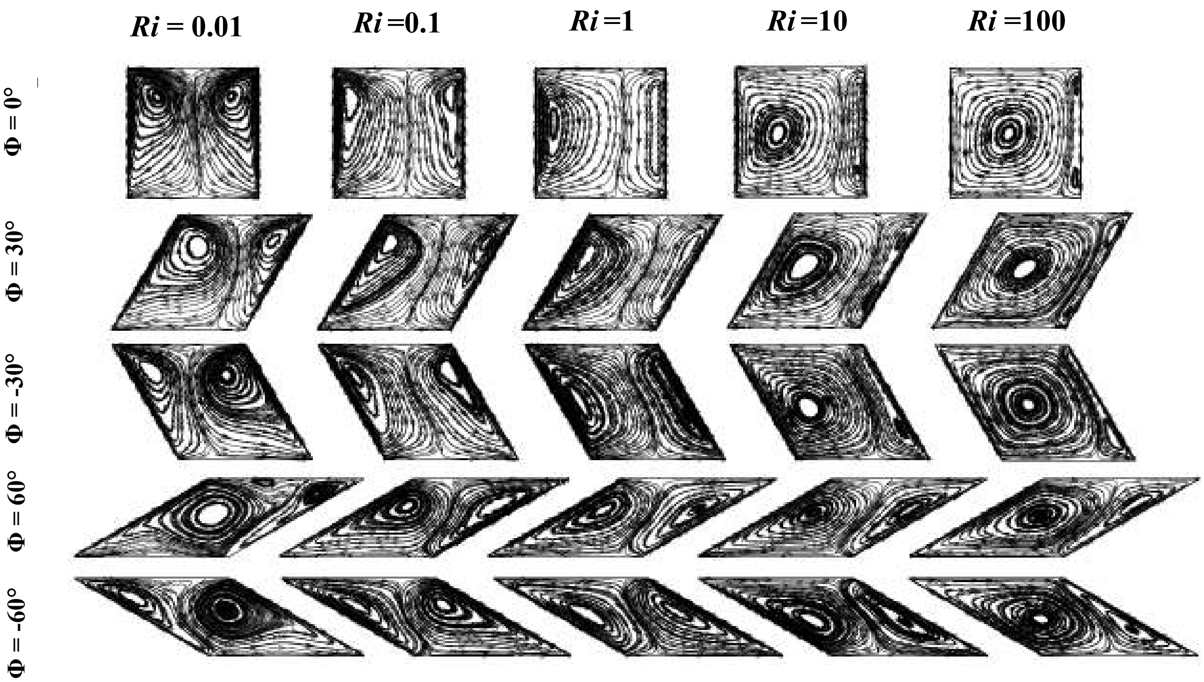

FIGURE 3 VARIATION OF STREAMLINES FOR DIFFERENT Ri and Φ WITH Ha=25

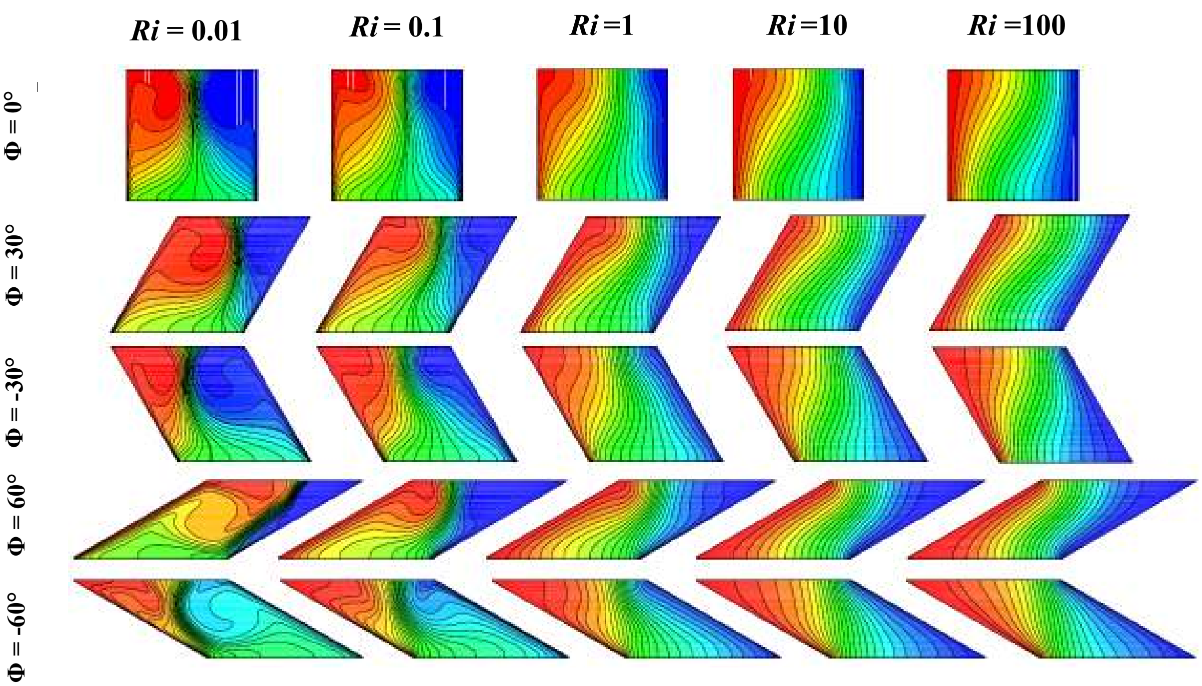

FIGURE 4 VARIATION OF ISOTHERMS FOR DIFFERENT Ri and Φ WITH Ha=25

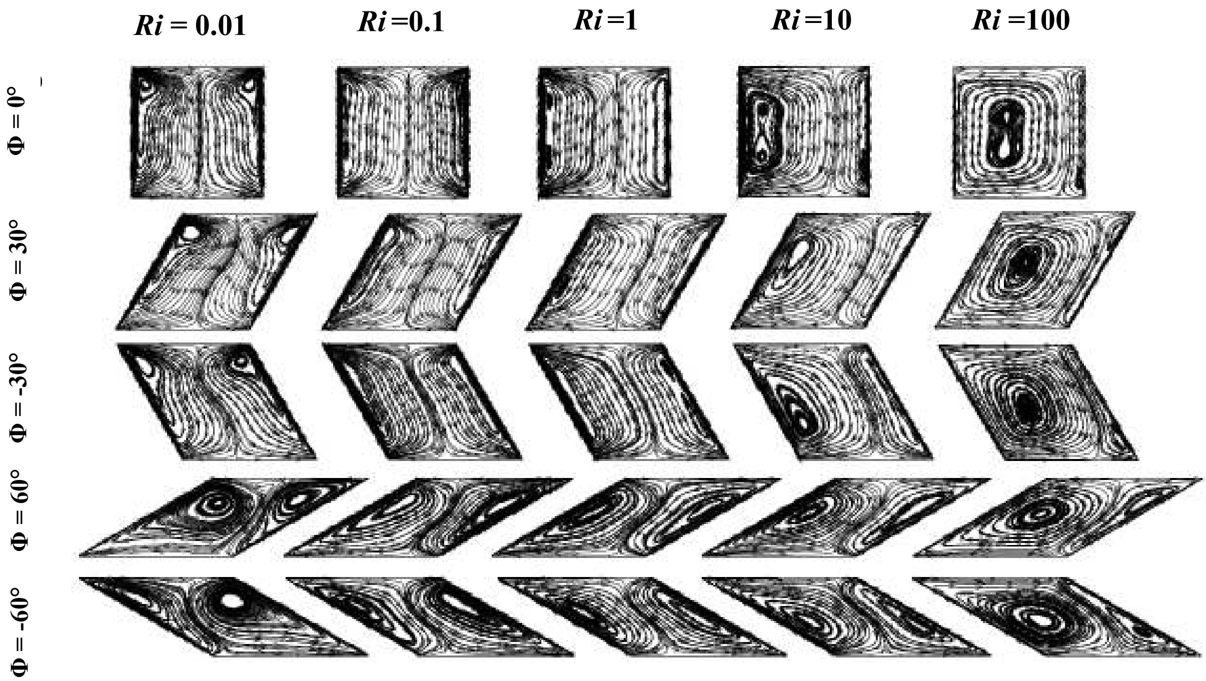

FIGURE 5 VARIATION OF STREAMLINES FOR DIFFERENT Ri and Φ WITH Ha=50

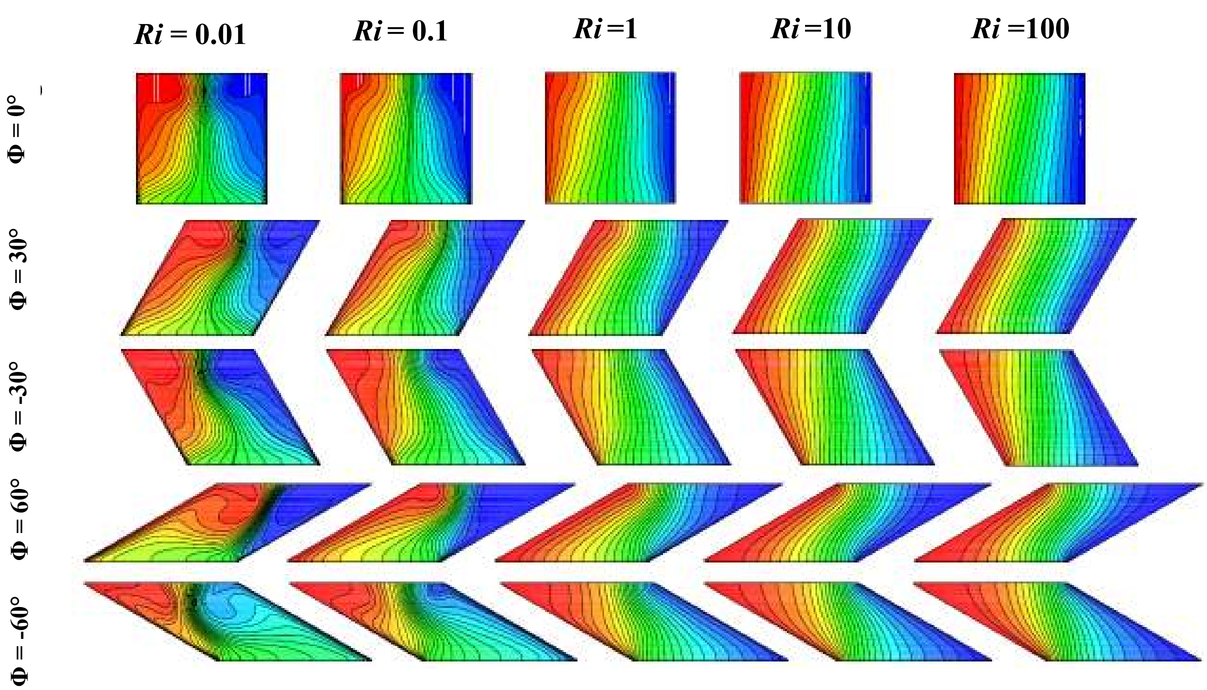

FIGURE 6 VARIATION OF ISOTHERMS FOR DIFFERENT Ri and Φ WITH Ha=50

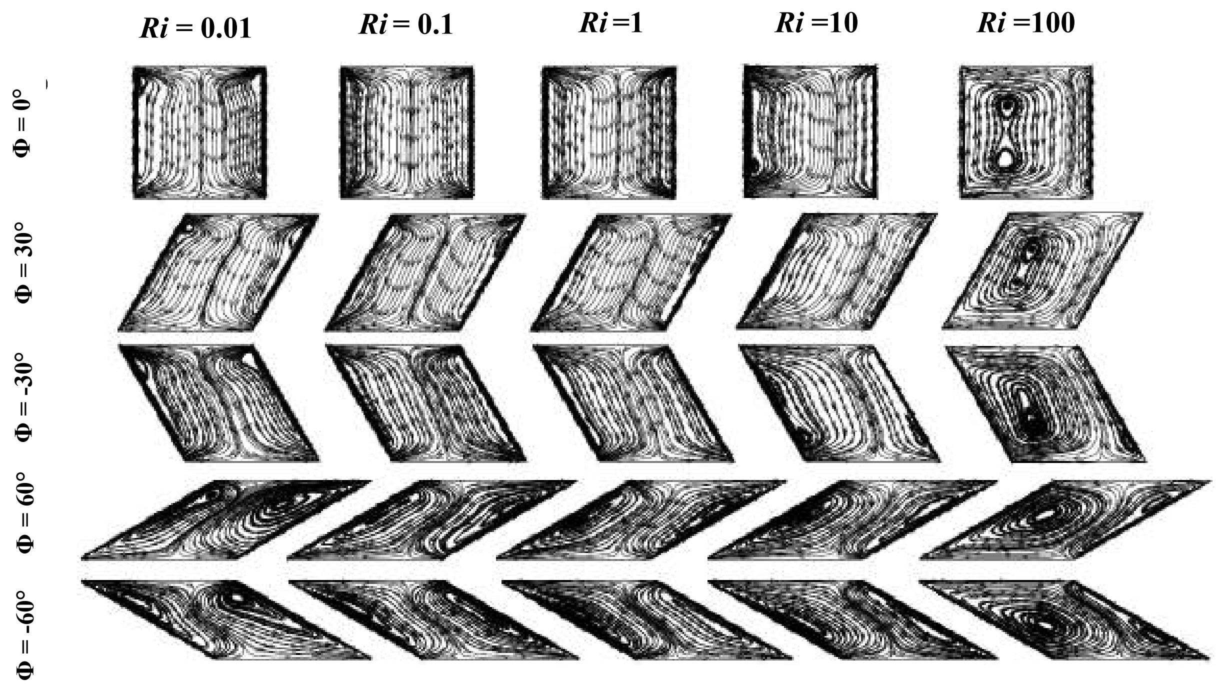

FIGURE 7 VARIATION OF STREAMLINES FOR DIFFERENT Ri and Φ WITH Ha=75

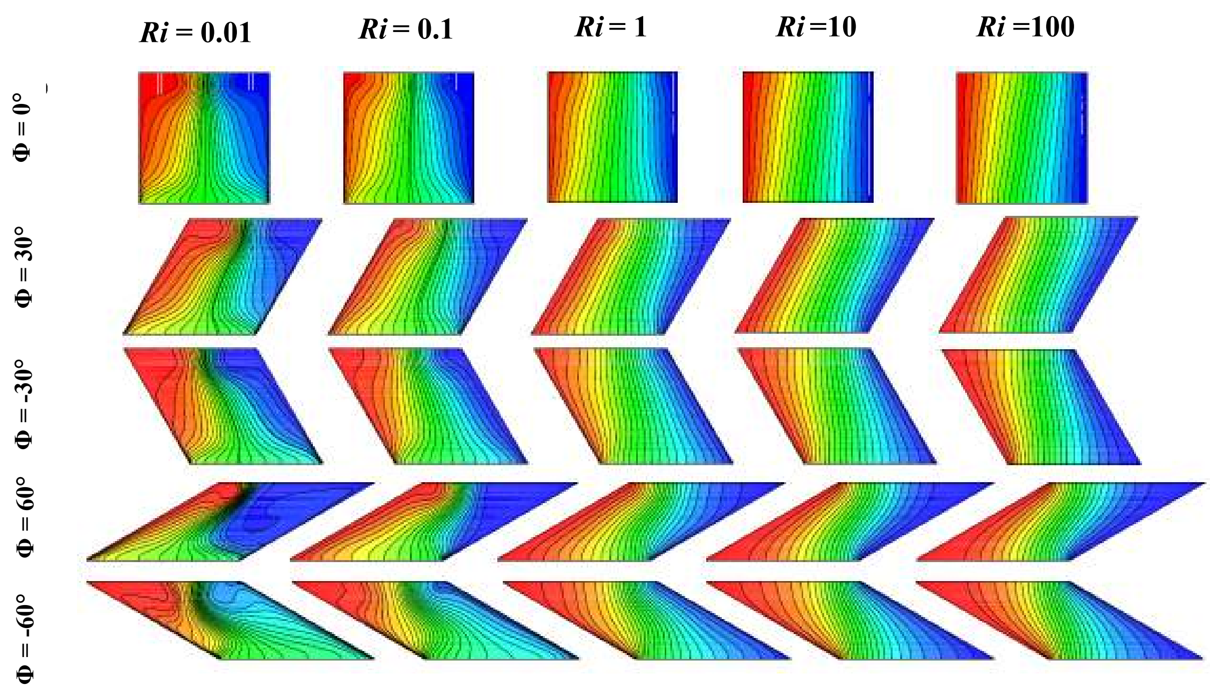

FIGURE 8 VARIATION OF ISOTHERMS FOR DIFFERENT Ri and Φ WITH Ha=75

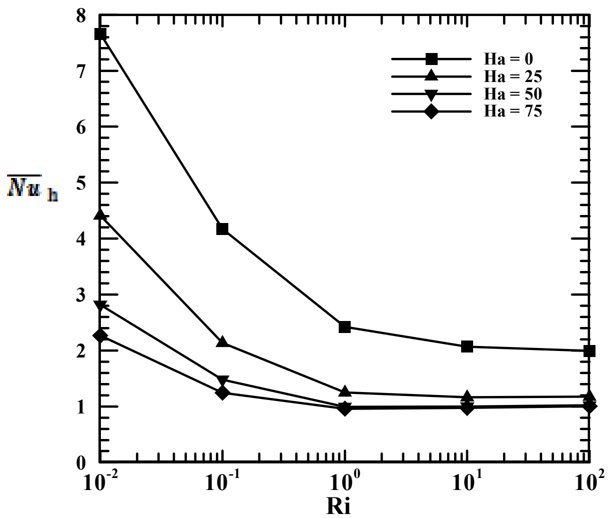

FIGURE 9 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF RICHARDSON NUMBER AT Φ= 0°.

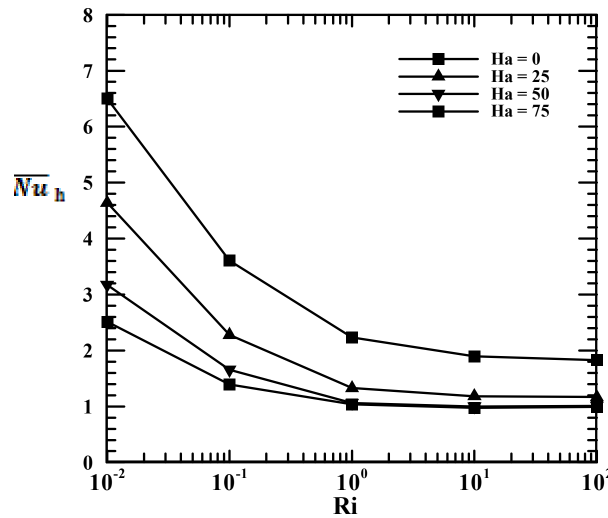

FIGURE 11 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF RICHARDSON NUMBER AT Φ= -30°.

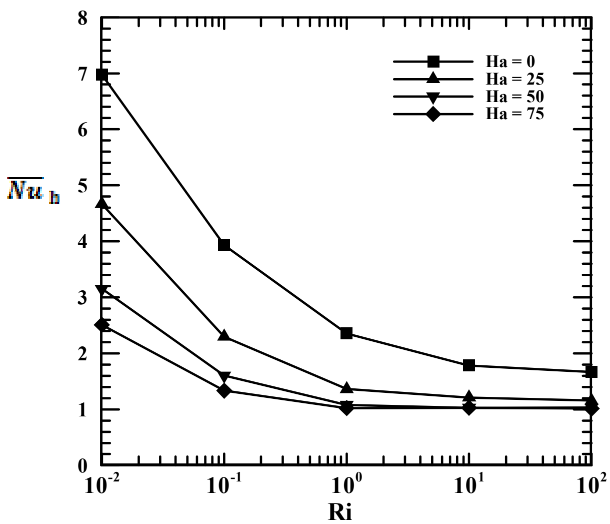

FIGURE 10 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF RICHARDSON NUMBER AT Φ= 30°.

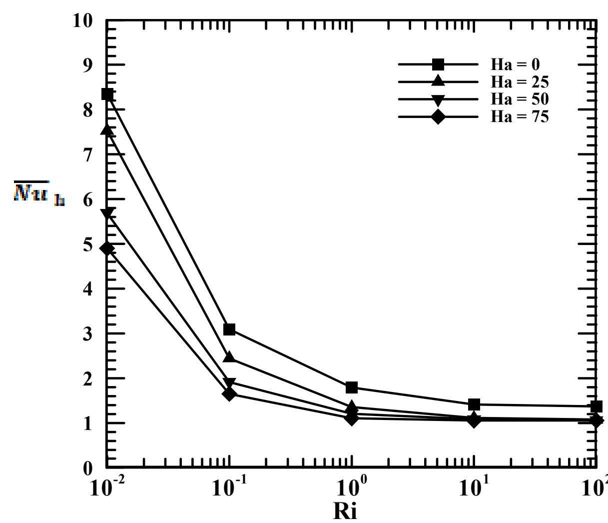

FIGURE 12 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF RICHARDSON NUMBER AT Φ= 60°.

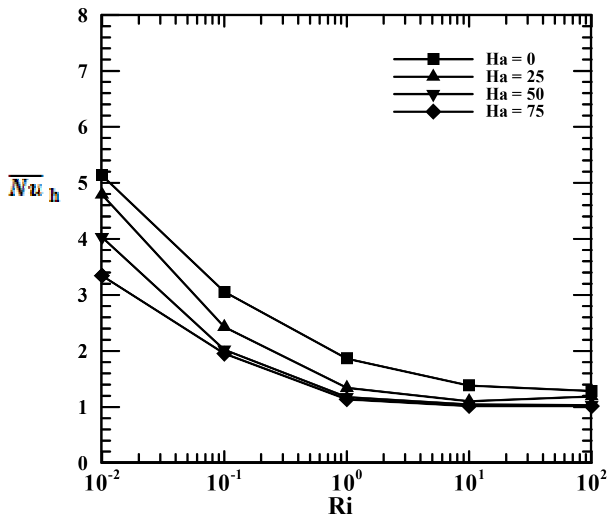

FIGURE 13 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF RICHARDSON NUMBER AT Φ= -60°.

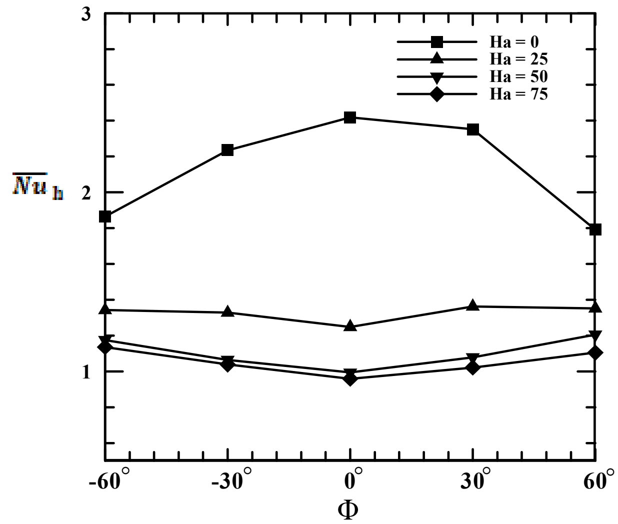

FIGURE 15 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF SKEW ANGLE AT Ri= 1.

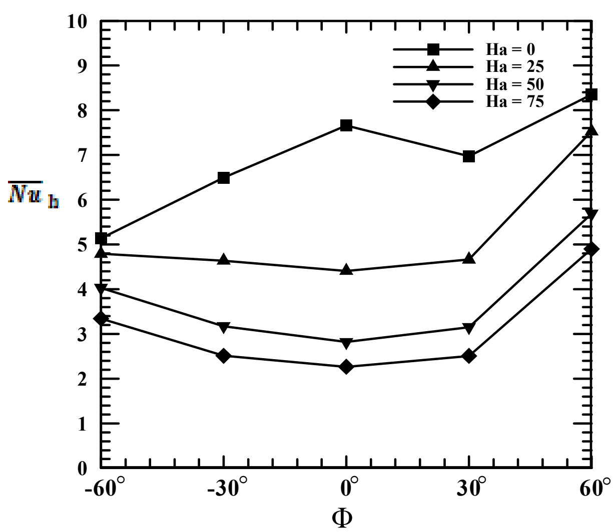

FIGURE 14 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF SKEW ANGLE AT Ri= 0.01.

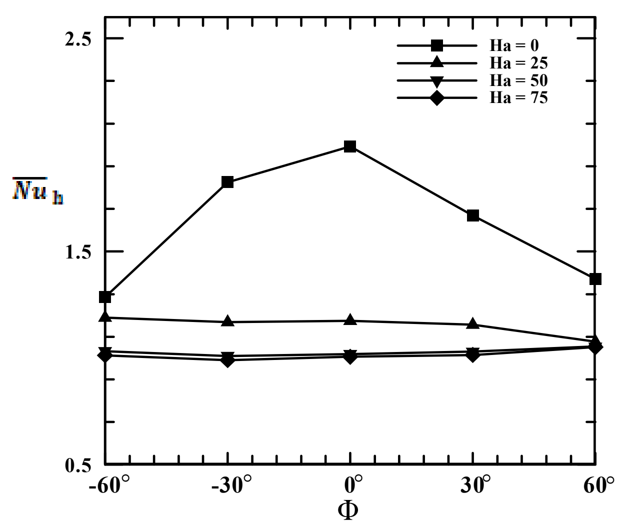

FIGURE 16 AVERAGE NUSSELT NUMBER ALONG THE LEFT HEATED WALL FOR DIFFERENT (Ha) AS A FUNCTION OF SKEW ANGLE AT Ri= 100.

5. CONCLUSIONS

The following conclusions can be detected from the present work results:

1- When the Hartmann number is low, the effect of convection and the strength of circulation is high and different major and minor rotating vortices which observed to fill all the parallelogrammic cavity. An opposite behavior can be observed when the Hartmann number is high.

2- The isotherm lines are substantially linear in shape and the conduction mode of heat transfer is dominant when the Hartmann number is high. As the Hartmann number decreases the isotherm lines begin to confuse and become non-uniform in shape. This reflects the fact that the heat transfer is mostly occurred by convection when the Hartmann number is low.

3- The flow field is characterized by a two primary rotating vortices and the forced convection is dominant when the Richardson number is very small. The vortices size adjacent the hot left sidewall begins to enlarge towards the cold right sidewall when the inclination angles increase from $\Phi = 30^o$ to $\Phi = 60^o$, while it enlarges towards the hot left sidewall when the inclination angles decrease from $\Phi = -30^o$ to $\Phi = -60^o$.

4-When the Richardson number is very small, conductive distortion of isothermal lines starts to appear adjacent the hot left sidewall of the parallelogrammic cavity.

5-The mixed convection is dominant when the Richardson number equals one (i.e., Ri = 1), and the core of vortices near the hot left sidewall begins to move towards the cold right sidewall and its moving increases as the inclination angle increases from $\Phi = 30^o$ to $\Phi = 60^o$ or decreases from $\Phi = -30°$ to $\Phi = -60°$. From the other side, the isotherms are symmetrical and parallel to the sidewalls of the cavity, while the conduction and mixed convection are dominant in the cavity.

6-The natural convection is dominant when the Richardson number is high (i.e., Ri = 10 and 100), where the rotating vortices grow further and occupy most of the cavity region and the effect of the inclination angle becomes significant. For isotherms, the convective mode of isothermal lines can be observed due to the strong influence of the convective current.

7-The average Nusselt number ($\overline{Nu_h}$) decreases significantly with increasing the Hartmann number for all values of Richardson number and inclination angles.

8- For the case with no magnetic field (i.e., Ha = 0), the average Nusselt number increases very rapidly with increasing the inclination angles until $\Phi = \mathbf{0}°$ and then decreases with increasing the inclination angles.

9- For all the sidewall skew angles, the average Nusselt number decreases rapidly when the Richardson number is low. But, when the Richardson number increases, the behavior of the average Nusselt number becomes more uniform and steady.

10- When the Richardson number is low, the average Nusselt number has a high value on the hot left sidewall even when the Hartmann number increases. But, when the Richardson number is high, the average Nusselt number decreases rapidly with increasing the Hartmann number.

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