Journal of Thermal Engineering *http://eds.yildiz.edu.tr/journal-of-thermal-engineering/Articles*
Yildiz Technical University Press, Istanbul, Turkey *Manuscript Received October 08, 2014; Accepted October 15, 2014*
Vol. 1, No. 3, pp. 175-181, July, 2015.

This paper was recommended for publication in revised form by Regional Editor Somanchi V S S N V G Krishna Murthy

SEMIANALYTICAL METHODS FOR HEAT AND FLUID FLOW BETWEEN TWO PARALLEL PLATES

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Keywords: Semianalytical, Temperature distribution, Transient, Velocity distribution
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ABSTRACT

The present study attempts to investigate the effects the viscous dissipation on the unsteady temperature distribution in the conduction limit for both hydro-dynamically and thermally fully developed, laminar flow of Newtonian fluid between two asymmetrically heated infinitely long parallel plates. Utilizing the assumptions routinely employed in the literature, we devise here a semi-analytical formalism to investigate the temperature distribution for two different flow configurations, i.e., the poiseuille flow and the simple shear driven flow. In the analysis, we give focus to the viscous dissipative effect arises because of the two individual aspects in case of shear-driven flow: the shear heating produced by the movable upper plate along with fluid friction, while only due to the internal fluid friction in case of Poiseuille flow. Finally, we show the variation of velocity and the temperature distribution in the flow field for several non-dimensional parameters as emerge in the present study, and highlight their individual role in delineating the temperature distribution in the flow field, which essentially alters transient thermal transport characteristics of heat in different cases of flow dynamics.

INTRODUCTION

In the domain of the macroflows, there are so many practical applications where heat transfer normally occurs in the fluid flow system involving moving boundaries. Particularly, in many material processing applications such as extrusion, hot rolling, drawing, and continuous casting, materials continuously move in a channel. In such industrial applications, it is of great importance to encounter the heat transfer from the moving boundary to the surrounding fluid and vice-versa. However, the moving boundary deforms the fluid velocity profile, and shears the fluid layer near the boundary, results in local changes in velocity gradient. Thus the viscous dissipation effects may not be neglected in heat transfer analysis associated with moving boundaries [1]. The thermal energy generated due to the viscous dissipation is significant near the wall, which alters the heat transfer rates following the changes in the temperature profile. In order to obtain the actual heat transfer rate in the application of moving boundaries, it is important to take into account the effects of viscous dissipations using accurate velocity distribution. The first theoretical work studied by Brinkman [2] concerning the heat generation due to viscous dissipation has analyzed the effects of viscous heating for the flow of a single phase Newtonian fluid through a circular tube. The temperature distribution in the thermal entrance region has been examined considering the zero temperature of the wall and an insulated wall. The temperatures were found to be the highest, not surprisingly, within a small area near the wall region. The available literatures in the area of convective heat transfer have, however, considered the effects of viscous dissipation to be important in two cases: flow of very viscous fluids and flow in capillary tubes. Cheng and Wu [3] have carried out a numerical analysis to study the influence of viscous dissipation for the flow of Newtonian fluid in a parallel plate channel. The effects of viscous dissipation on laminar forced convection for the flow of Phan- Thien- Tanner fluid through a pipe and channel have been studied by Pinho and Oliveira [4]. Performing an analytical study, using a functional analysis method, Lahjomri et al. [5] have investigated the effects of viscous dissipation on the heat transfer of thermally-developing laminar Hartman flow through a parallel plate channel with the aid of a magnetic field. In a study of thermal development of forced convection in a parallel plate channel filled by porous medium, Nield et al. [6] have investigated the effects of viscous dissipation with the thermal boundary condition of uniform wall temperature including axial conduction effects. Following a Numerical study, Duwairi et al. [7] have investigated the heat transfer effects of a viscous fluid squeezed and extruded between two parallel plates. The study considered constant wall temperature and revealed the interactive effects of squeezing and extrusion parameters on the velocity, temperature profile and on the heat transfer characteristics.

In a study, the effect of viscous heating on the stability of Taylor-Couette flow has been investigated experimentally by White and Muller [8]. The analysis of laminar forced convection in a pipe for Newtonian fluid of constant properties has been performed by Aydin [9,10] by taking the effect of viscous dissipation into account. In Part-1, both hydro-dynamically and thermally fully developed convection has been studied, while Part-2 of the study has considered the hydro-dynamically developed but thermally developing case. In both cases, two different types of thermal boundary condition have been considered, namely, constant heat flux (CHF) and constant wall temperature (CWT). The variations of dimensionless radial temperature and Nusselt number have been obtained for different values of Brinkman number under both wall heating and cooling. The analytical work done by Aydin and Avci [11] has dealt with the convective heat transfer problem for the plane Poiseuille flow with an emphasis on the viscous dissipation effect. The energy equation has been solved for thermally developed and developing cases separately with the boundary condition of CWT and CHF, respectively. In both cases, the flow has been considered to be hydrodynamically developed. It has been found from the study that with the increasing intensity of viscous dissipation (increase in Brinkman number), the heat transfer decreases up to a critical value, and that is attributed to the internal heat generation due to the viscous dissipation effect. In another work, Aydin and Avci [12] have studied the laminar forced convective heat transfer problem in a Couette- Poiseuille flow with an emphasis on the viscous dissipation effect. In a recent study, Francisa and Tso [13] have extended the work of Aydin and Avci [11] and have investigated the viscous dissipation effects on fixed parallel plates with constant heat flux boundary condition. Various analytical expressions of Nusselt number as a function Brinkman number has been obtained by several researchers as apparent from the reported investigation. The survey shows the effects of viscous dissipation on laminar heat transfer on a Poiseuille flow in stationary parallel plates for Newtonian as well as non-Newtonian fluids. The steady state laminar heat transfer to a plane Poiseuille-Couette flow of a Newtonian fluid with simultaneous pressure gradient and axial movement of one of the plates has been investigated by Hudson and Bankoff [14] and Sestak and Rieger [15].

All the research mentioned above have dealt with the effect of viscous dissipation on convective heat transfer in a Poiseuille flow and combined Couette-Poiseuille flow for a hydro-dynamically fully developed flow between two parallel plates, considering both the thermally fully developed and developing cases. To the Authors knowledge, no work, so far, has been reported till date wherein the limiting temperature profile for an unsteady simple shear driven and Poiseuille flow has been delineated giving the intricate interactions of different dimensionless parameters including the effect of viscous dissipation in a comprehensive way.

The objective of this paper is to explore the temperature distribution in the conduction limit for an unsteady fully developed simple shear driven and Plane Poiseuille flow of a Newtonian fluid between two asymmetrically heated parallel plates, which in essence predicts the thermal transport characteristics of heat in the conduction limit. For this, a detailed study is carried out to investigate the effect of viscous dissipation on the limiting temperature profile between two parallel plates subjected to the unequal constant temperature thermal boundary condition. In the analysis part, we have used two definitions of Eckert number based on the temperature difference between two plates, as well as detailed the intermediate steps to facilitate the checking by interested audience in the research community.

NOMENCLATURE

C_p	specific heat at constant pressure, J/kg K
E	Eckert number, $V_C^2 / C_p (T_L - T_0)$
i	number of nodal points
k	thermal conductivity of fluid, W/m K
L	length as shown in Figs. 1 and 2, m
Pr	Prandtl number, ν / α
t	time, s
T_0	temperature of the lower plate, K
T_L	temperature of the upper plate, K
U	dimensionless velocity, V_x / V_C
V_C	reference velocity, m
V_x	velocity in the x-direction, m
x	Coordinate in the x-direction, m
X	dimensionless Coordinate in the x-direction, x/L
y	Coordinate in the y-direction, m
Y	dimensionless Coordinate in the y-direction, Y/L
Greek Letters	
α	thermal diffusivity, $k / \rho C_p$, m^2/s
β	viscosity parameter, $\mu / \rho C_p$, K s/m^2
ν	Kinematic viscosity, μ / ρ , m^2/s
θ	dimensionless temperature, $(T - T_0)/(T_L - T_0)$
τ	Fourier number, $\alpha t / L^2$

PROBLEM FORMULATION AND ANALYSIS

Physical Considerations. For the analysis, we consider a channel between two parallel plates of infinite length, height L and width b as shown in Fig. 1. Fluid is flowing in the axial (x) direction, while the flow is influenced by the movement of the upper plate. The flow is considered to be both hydro-dynamically and thermally fully developed. The no-slip boundary conditions are assumed to be valid at both the plates for both hydro-dynamically and thermally. In addition to the consideration of fully developed flow, few more assumptions considered for the study are: newtonian fluid; incompressible fluid flow; there is no heat generation and thermo-physical properties are constant; axial conduction is neglected in the fluid and through the wall.

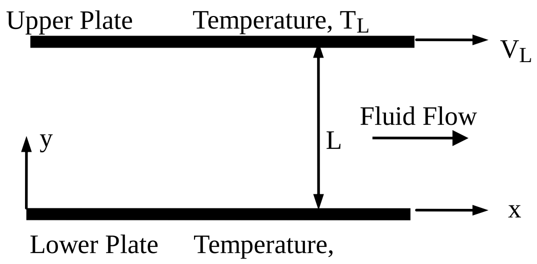

Fig. 1. Schematic diagram describing the case of shear driven flow problem.

Analysis of the problem. The relevant equations specific to the problem considered in the present study are the incompressible continuity, momentum and energy equations. The governing equations are as follows:

Continuity Equation:

$$\partial V_x / \partial x = 0 \tag{1}$$

Momentum Equation:

$$\partial V_x / \partial t = \nu \, \partial^2 V_x / \partial y^2 \tag{2}$$

Energy Equation:

The prime focus of the present study is to delineate the temperature distribution in the conduction limit including the effect of viscous dissipation in the analysis. This, essentially allow us to discard the advective term in the energy equation, while the viscous dissipation term is taken into account. The governing transport equation of thermal energy for the problem considered in the present study looks like the following form as given by

$$\partial T / \partial t = \alpha \, \partial^2 T / \partial y^2 + \beta \left(\partial V_x / \partial y \right)^2 \tag{3}$$

where $\alpha = k / \rho c_p$, $\beta = \mu / \rho c_p$ and the last term on the right hand side of the above equation is the viscous dissipation term. To solve the governing transport equations as mentioned above, it is essential to define the necessary initial and boundary conditions very commensurate to the problem considered here. However, we define the initial and boundary conditions for the problem considered in the present study as given below.

Initial conditions:

$$\left. \begin{array}{l} V_x = 0 \\ T = T_o \end{array} \right\} \quad \text{for} \quad t = 0 \tag{4a}$$

Boundary conditions:

$$\left. \begin{array}{ll} y = L \rightarrow & \begin{cases} V_x = \begin{cases} V_L; (\, for\ shear\ driven\ flow) \\ 0\ ; (\, for\ poiseuille\ flow) \end{cases} \\ T = T_L \end{cases} \\ y = 0 \rightarrow & \begin{cases} V_x = 0 \\ T = T_0 \end{cases} \end{array} \right\} \quad \forall\, t > 0 \tag{4b}$$

Unsteady Shear-Driven Flow: Upper plate is moving and lower plate fixed. We consider a case where the upper plate is moving at constant speed V_L , while the lower plate is fixed. Furthermore, we assume that the upper plate is at constant temperature T_L and the lower plate temperature is T_0 . However, in order to express Eqs. (2) and (3) in a non-dimensional framework, it is essential to define the non-dimensional parameters suitably. From the physical considerations discussed above, following non-dimensional parameters are chosen:

$$U = \frac{V_x}{V_C}, Y = \frac{y}{L}, \ \theta = \frac{T - T_0}{T_L - T_0} \ \text{and} \ \tau = \frac{\alpha t}{L^2}. \tag{5}$$

It is to be noted here that the term V_C in Eq. (5) is the reference velocity used to normalise the flow velocity. For a shear driven flow, we consider the velocity of the upper plate as the reference velocity. However, with the aid of the above non-dimensionless parameters, Eq. (2)-(3) may be normalised to yield the following:

$$\frac{\partial U}{\partial \tau} = \Pr \frac{\partial^2 U}{\partial Y^2} \tag{6}$$

$$\frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial Y^2} + Pr.E \left(\frac{\partial U}{\partial Y} \right)^2 \tag{7}$$

where $\Pr = \nu / \alpha$ is the Prandtl Number and $E = V_C^2 / C_p (T_L - T_0)$ is the Eckert Number based on the temperature difference between two plates.

In order to obtain the temperature distribution in the flow field, we need to solve the above set of equations [Eqs. (5-6)] get the temperature profile. However, to do that we utilize the boundary conditions as given in Eqs. 7(a)–(b). In a non-dimensional form, the above set of boundary conditions may be expressed as given below:

$$\text{Initial conditions:} \quad \left. \begin{array}{l} U = 0 \\ \theta = 0 \end{array} \right\} \quad \text{for} \quad \tau = 0 \tag{8a}$$

Boundary conditions:

$$\left. \begin{array}{ll} Y = 1; & \begin{cases} U = 1 \\ \theta = 1 \end{cases} \\ Y = 0; & \begin{cases} U = 0 \\ \theta = 0 \end{cases} \end{array} \right\} \quad \forall\, \tau > 0 \tag{8b}$$

Solving Eq. (6) with the boundary conditions given in Eq. (8a,b), the expression of the velocity distribution obtained is presented below [16–23]:

$$U = Y + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^n}{n}.e^{-\left(n^2\pi^2 Pr\tau\right)}.\sin(n\pi Y) \tag{9}$$

In order to obtain deeper insight about the thermal transport characteristics of heat, next, we appeal to obtain the temperature distribution in the flow field θ. The temperature distribution in the conduction limiting considering the effect of viscous dissipation into account can be obtained on solving Eq. (7). In order to do so, we use the boundary conditions as given in Eqs. (8a) and (b) and plug in the closed form of expression of the velocity distribution [Eq. (9)] into the energy equation. After plugging the velocity distribution into the energy equation and having a bit manipulations of simple algebra, equation (9) takes the form as given by

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial Y^2} + Pr\cdot E\left[1 + 2\sum_{n=1}^{\infty}(-1)^n.e^{-\left(n^2\pi^2 P_r.\tau\right)}\cos(n\pi Y)\right]^2 \tag{10}$$

Following Taylor series central difference and implicit schemes, the above equation can be expressed as

$$\theta_i^{m+1} = \frac{1}{\left(1+\dfrac{2\Delta\tau}{\Delta Y^2}\right)}\left[\theta_i^m + \frac{\Delta\tau}{\Delta Y^2}\left(\theta_{i+1}^m + \theta_{i-1}^m\right) + \mathrm{Pr}E\Delta\tau\left\{1 + 2\sum_{n=1}^{\infty}(-1)^n e^{-\left(n^2\pi^2\,\mathrm{Pr}\,\tau_m\right)}\cos\left(n\pi Y_i\right)\right\}^2\right] \tag{11}$$

It is important to mention here that the term ΔY in above equation can be expressed as

$$\Delta Y = 1/(i-1),\ Y_i = (i-1)\Delta Y \tag{12}$$

Where i is the number of nodal points and From Eq. (11), and using the boundary conditions given in Eq.(8a,b) we can delineate the variation of limiting temperature distribution in the flow field towards the understanding of the thermal transport characteristics of heat in an unsteady viscous dissipative shear driven flow between two asymmetrically heated parallel plates. We discuss the parametric variation of the temperature distribution in the results and discussions section of the present paper. In the next sub-sections, we make an attempt to obtain the temperature distribution in a plane poiseuille flow.

Unsteady Poiseuille Flow. In this section, we demonstrate our application considering the case of a plane poiseuille flow. We remain stick to the other conditions unlike the previous case except the movement of the upper plate. We assume that the fluid is bounded by two parallel plates fixed at y=-Land Y=L as shown in Fig. 2. We also consider that the fluid is initially at rest and an axially applied pressure gradient set the fluid in motion suddenly along the *x*-direction. We use the following dimensionless parameters to cast the governing equations into the corresponding non-dimensional form.

$$U = \frac{V_x}{V_m},\ Y = \frac{y}{L},\ \theta = \frac{T - T_0}{T_L - T_0}\ \text{and}\ \tau = \frac{\alpha t}{L^2}. \tag{13}$$

In above equation, V_m is the mean velocity of the flow. Using above scales, we arrive at the following set of the non-dimensional governing equations as written below:

$$\frac{1}{Pr}\frac{\partial U}{\partial\tau} = \frac{\partial^2 U}{\partial Y^2} - \frac{1}{2} \tag{14}$$

$$\frac{\partial\theta}{\partial\tau} = \frac{\partial^2\theta}{\partial Y^2} + E\cdot Pr\cdot\left(\frac{\partial U}{\partial Y}\right)^2 \tag{15}$$

where $Pr = \frac{\nu}{\alpha}$ is the Prandtl Number and $E = \frac{V_m^2}{C_P(T_L - T_0)}$ is the Eckert Number based on the temperature difference between two plates. Equations (14)–(15) are subject to the following boundary conditions

Boundary conditions:

$$Y = \pm 1;\quad \begin{cases} U = 0 \\ \theta = 1 \end{cases}\quad \forall\,\tau > 0 \tag{16b}$$

$$\text{Initial conditions: }\left.\begin{matrix} U = 0 \\ \theta = 0 \end{matrix}\right\}\ \text{for}\quad \tau = 0 \tag{16a}$$

The solution of Eq. (14) with the above set of boundary conditions yields the velocity distribution in the flow field as

$$U = 1 - Y^2 - \frac{32}{\pi^3}\sum_{n=1}^{\infty}\frac{(-1)^n}{(2n+1)^3}\exp\left\{-\frac{(2n+1)^2\pi^2\,\mathrm{Pr}\tau}{4}\right\}\times\cos\left\{\frac{(2n+1)\pi Y}{2}\right\} \tag{17}$$

On the other hand, using the above velocity distribution and following the Taylor series central difference scheme we attempt to solve the thermal energy equation as given in Eq. (15). Finally, we arrive at the following expression, which in essence depicts the limiting temperature distribution in the flow field as given by

$$\theta_i^{m+1} = \theta_i^m + \frac{\Delta\tau}{\Delta Y^2}\left(\theta_{i+1}^m + \theta_{i-1}^m\right) + \mathrm{Pr}E\,\Delta\tau\left[-2Y_i + \frac{16}{\pi^2}\times\right.$$
$$\left.\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)^2}\exp\left\{\frac{(2n+1)^2\pi^2\,\mathrm{Pr}\tau_m}{4}\right\}\cos\left\{\frac{(2n+1)Y_i}{2}\right\} \tag{18}$$

The solution of Eq. (18) with the thermal boundary conditions given in Eq. (16) capitulate the limiting temperature distribution in the flow field, which we will see in the next section.

RESULTS AND DISCUSSIONS

In an effort to bring out the effect of viscous dissipation on the temperature profile, we present here some particular cases to investigate the thermal characteristic of heat in the limiting condition. Using the semi-analytical technique described above, we obtain the expressions of the temperature profile for both the cases of flow configurations. Moreover, those expressions turn out to be most important to analyze the objective of the present study. In this section, we present several plots and briefly discuss them.

Variation of velocity and limiting temperature in an unsteady Shear-Driven flow. Figure 3 illustrates the constant speed. A closer look at Fig. 3 reveals that the moment at which the upper plate starts to move in the axial direction, the fluid layer adjacent to the upper plate also starts moving with the movable upper plate, while majority of the fluid in the flow field remain stationary. However, with increasing time, the entire flow field responses with the movement of the upper plate as one can find from the non-zero velocity of the fluid layer located even very close to the stationary lower plate, which is intuitive and quite expected. Interestingly, with the further increase in time, to be more specific at a particular value of $\tau = 2$, the velocity profile becomes linear irrespective of the value of Prandtl number Pr, which is suggestive of reaching a steady state velocity profile in the flow field. Therefore, it can be inferred from the above figure that the solution becomes steady when magnitude of the dimensionless time is greater or equal to 2 for all types of Newtonian fluid.

The variation of non-dimensional temperature distribution in the conduction limit specific to the case of shear flow condition and including the effect of viscous heating has been delineated in Fig 3. To highlight the individual effect of different parameters, materialized during non-dimensionalisation of the thermal energy equation, on the limiting temperature profile, we display several plots of the temperature considering different values of Pr, E and τ as one can clearly see on closely looking at Fig. 4. It is important to mention here that, unlike the case of velocity distribution, the limiting temperature profile also becomes steady and fully developed at $\tau = 2$. It is seen from the above figure that for the value of $E = 1$ and at the steady state condition ($\tau = 2$), the dimensionless fluid temperature increases almost in a linear fashion from zero at the lower plate to satisfy the boundary conditions imposed at the upper plate. The limiting temperature profile in a steady state condition and for a given value of Pr, however, increases with increasing value of Eckert number E. Moreover, the temperature profile at a relatively higher E no longer remain linear; rather the profile initially shows an increasing trend then attains a maximum value very close to the upper plate and, finally, meet the upper plate temperature following a gradual increasing trend as one can observe from Fig. 4. The increasing trend of the dimensionless temperature with the increasing value of E for a given Pr as seen up to the location of $Y = 0.8$ dimensionless velocity distribution in the flow field specific to the case when the upper plate is moving at in the flow field is essentially because of the enhanced effect of viscous dissipation. Conversely, the movement of the upper plate carries away the adjacent fluid layer along with it to satisfy the no slip boundary condition and, hence, a drop in the fluid temperature nearer at the upper plate is observed as reflected in the above figure.

It is interesting to observe from the figure under focus is that, a point exists in the flow field where two different steady state ($\tau = 2$) temperature profile obtained for the same value of $Pr = 2$ but at different E, intersects each other. Moreover, for the combination of $Pr = 2; \tau = 2$ an anomaly of increasing trend of temperature even at lower value of E ($E = 1$) as seen up to the location of $Y = 0.8$ in the flow field is somewhat interesting albeit the natural trend of decreasing temperature for the same value of E is observed afterwards till the upper plate. This anomalous behaviour of the dimensionless temperature as observed up to location of $Y = 0.8$ is attributable to the interactive effects of the Eckert number E and Prandtl number Pr on the viscous dissipation effect. However, the movement of the upper plate results in the decrease in the fluid temperature, which one can clearly see from the fact that the dimensionless temperature closer to the upper plate decreases. Notably, the decreasing rate of temperature for lower value of E becomes faster in comparison to the same obtained at higher value of E.

Variation of velocity and limiting temperature in an unsteady Poiseuille flow. In this section we show the variation of dimensionless velocity and temperature distribution for the case of unsteady poiseuille flow of Newtonian fluid between two asymmetrically heated parallel plates. In Fig. 5, we depict the variation of dimensionless velocity for different values of Prandtl number. It is worth mentioning that the velocity profile corroborates the fully developed profile at $\tau = 2$. As Pr increases, the axial velocity at a given location in the flow field increases and the velocity profile essentially overlay with the fully developed profile leaving relatively higher magnitude of boundary layer thickness, which is as expected.

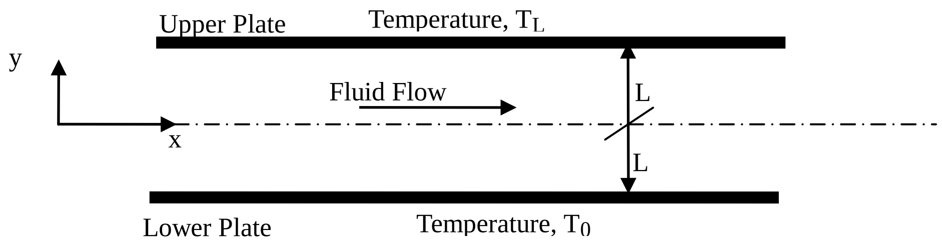

Fig. 2 Schematic diagram describing the case of plane Poiseuille Flow problem

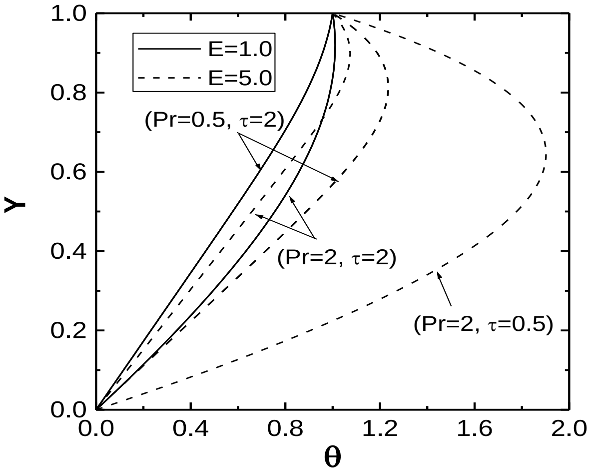

Fig. 3 Variation of dimensionless velocity for different values of Pr and τ in a shear-driven flow condition

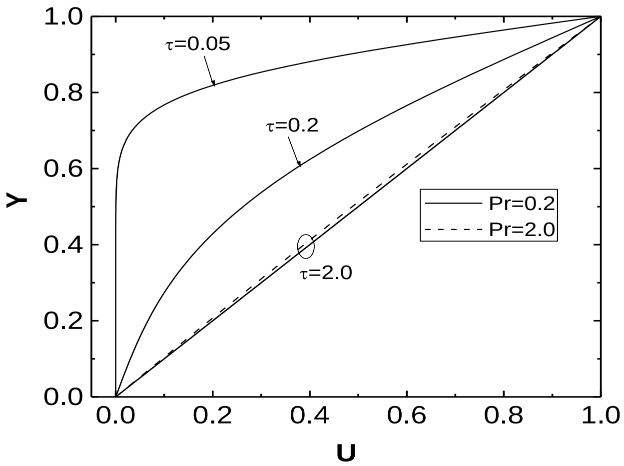

Fig. 4 Variation of dimensionless temperature for different values of Pr, E and τ in a shear-driven flow condition

Figure 6 displays the variation of dimensionless temperature profile for different values of Prandtl number Pr and Eckert number E. Notably, for a given Pr, the relatively higher fluid temperature as observed with increasing value of Eckert number E is essentially attributable to the viscous dissipation effect owing to fluid frictional heating. On the other hand, the effect of decreasing value of Pr is getting reflected more clearly on the variation of dimensionless temperature profile as manifested in terms of an increasing value of the thermal boundary layer thickness. Actually, a precise look at Figs. 5 and 6 clearly gives us a direct comparison between the magnitude of thermal and hydrodynamic boundary layer thickness, which, in turn, explores the undeviating consequences of the effect of Prandtl number on the limiting temperature profile as delineated above.

CONCLUSIONS

In the present work, we investigate the influences of the viscous dissipation on the limiting temperature profile for an unsteady shear driven and Poiseuille flow of Newtonian fluid between two asymmetrically heated parallel plates. We have presented a semi-analytical formalism in an exhaustive way to solve the governing transport equation of energy for the case of unequal but constant temperature thermal boundary conditions. We have demonstrated the interplay between different non-dimensional parameters in dictating the limiting temperature profile. The influential role of viscous dissipation is found to be of great importance in the variation of temperature distribution in the flow field; hence an emphasis on viscous dissipation is given to include the effect of shear stress induced by the axial movement of the upper plate in addition to the effect of viscous dissipation due to the internal fluid heating in case of shear driven flow and the fluid frictional heating in case of plane Poiseuille flow.

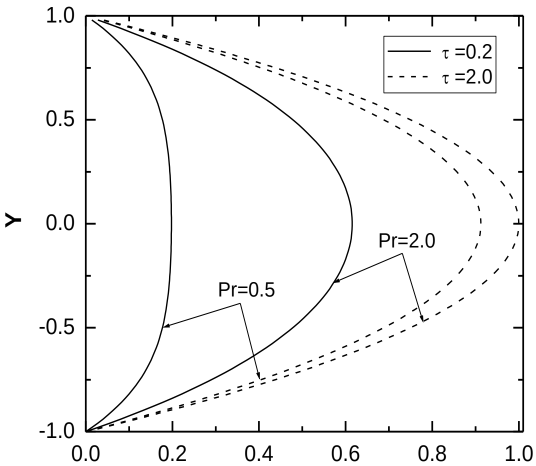

Fig. 5 Variation of dimensionless velocity for different values of Pr and τ in a Poiseuille flow condition

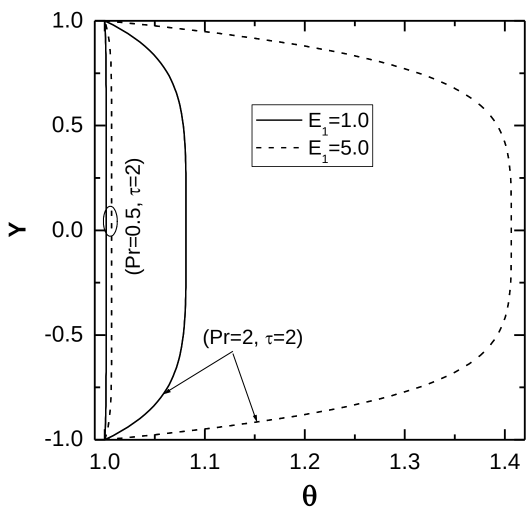

Fig. 6 Variation of dimensionless temperature for different values of Pr, E and τ in a Poiseuille flow condition

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