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Research Article

DEVELOPING OPTIMIZATION MODELS TO EVALUATE HEALTHCARE SYSTEMS

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ABSTRACT

This study investigated the use of public and private healthcare systems separately and interactively to find solutions to various problems the healthcare sector has faced including waiting time, cost, and quality of service. Patients, private and public healthcare providers, funders, and governments that make rules about healthcare were defined as components of healthcare systems in this study. Mathematical optimization models were developed by deriving scenarios to balance the components of healthcare systems. In addition to derivation of scenarios related to types of healthcare systems and decision variables, including copayment, treatment cost, reimbursement, and premium, numerical studies for the healthcare components were performed to determine which healthcare systems were useful. The minimum fee payable by patients for the easy access to healthcare schemes was calculated. While there was no remarkable change in the waiting time of the patients in the private healthcare system, the waiting time were reduced by 48.70% in the public healthcare system among the findings. The amount of reimbursement decreased significantly as a result of the interaction of healthcare systems. By reducing the amount of co-payment to the amount charged by the public healthcare system, it has been ensured that private healthcare providers treat more patients. Based on computational analyses, because of the agreements provided between the components, we have advocated that public and private healthcare systems must be unified to provide greater benefits for all the components of healthcare.
Keywords: Public healthcare system, private healthcare systems, healthcare components, mathematical optimization models, decision variables.

1. INTRODUCTION

Healthcare is one of the largest and fastest growing businesses besides manufacturing. Most of the countries, especially the U.S. do not have a global or local healthcare system but have a unique and complex system for healthcare service and delivery. This complexity causes many problems such as poor-quality service, duplicates, wrong treatment, high waiting time, medication error, etc. [1–4]. Healthcare structures have dynamic settings, multifaceted, and become large [5].

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Extraordinary costs, limited budget, and inadequate healthcare resources cause challenges in busy healthcare organizations [6, 7]. Healthcare systems have mainly two objectives, which are cost and healthcare services in order to provide profit [8]. Less cost and high patient satisfaction are desired by healthcare systems [9]. Generally, the levels of spending to receive service are measured by the satisfaction of patients [10, 11].

There are four main players that are patients, healthcare providers, funders, and the policy/rule maker governments in healthcare systems. Patients can, in some cases, lessen their out-of-pocket insurance and healthcare costs by limiting their utilization of provider services and/or by purchasing less insurance [12]. Patients try to have three objective functions which are having high quality care, minimizing healthcare expenses [13, 14], and easiness to access to healthcare service or delivery [15]. Providers such as physicians, nurses, physicians' assistants, registered nurses, administrators, etc. set their prices for patient services to maximize their profits; prices are determined by the procedure and diagnosis codes established by the insurance companies, which are pre-negotiated between providers and insurers [16–18]. Increasing patient satisfaction, providing high quality service, gaining experience, and maximizing profits were defined as health provider's goals in this study.

Private insurance companies and public funders strive to minimize costs by limiting payments to providers and/or by raising patient (or employer) premiums or implementing copayment systems [19]. The private insurance companies aim to increase the number of customers and want to prolong the insurance premiums along with public funders such as government wants to minimize patient's reimbursement and the patient's waiting time [20].

Patients wait for non-emergency treatment almost three months in some countries, even can extend into years [21]. Waiting time is one of the major problems in public healthcare system [22]. Many of studies show that the waiting time has the cost-impact of the funders and providers. Siciliani, Stanciole, and Jacobs (2009) researched the resistance of hospital costs with respect to waiting times. They found that hospital costs and waiting time has a U-shape correlation, because the waiting time on cost is nonlinear [22]. These kinds of studies consider in long term treatment; however, we consider daily waiting time in hospitals and it effects the waiting time cumulatively [23]. Under this circumstance, we try to develop a solution using interaction between healthcare components that gives patients an opportunity to select treatment among providers.

A Hoteling model with two districts to research the influence of patient mobility on health care quality, health care funding and welfare were studied by researches [24]. In addition, patient mobility that means patient becomes able to choose providers and elective treatments in the European Union (EU) has been studied in order to minimize the waiting time of patients. Cross-border healthcare guidelines using optimization models were developed by Andritsos and Tang in 2014 for Europe healthcare systems [25]. They have suggested that cross-border methodology yields low cost to patients and funders, and high health service utility. In addition, this provided flexibility in selecting hospitals which is helpful for patients [26–28]. They have examined cross-border healthcare policy (to choose the country patient receives a treatment) to increase patient mobility in order to minimize the waiting time in public hospitals and funder's cost [29]. Some of these studies care distance between providers or hospitals and some not. General overview, patients have free travelling to receive treatment among European Countries [30]. We have not added up to account the distance between the healthcare providers in this research.

Simulation technique is among the methods used to measure the quality of service in the healthcare system [31]. Generally, using simulation approach, along with optimization technique that depends on cost/benefit optimization is developed by researches in order to solve the healthcare systems problems [31]. Many of these studies have locally solved these kinds of problems such as improving clinic capacity [32], reducing waiting time in emergency department [33], re-scheduling [34], etc. to improve patient satisfaction [35]. Using optimization approach for healthcare system is inevitable, otherwise, it is difficult to solve problems analytically and statistically so as to try to minimize waste and maximize efficiency. The healthcare system has a complex structure, and mathematical optimization models help to cope with this system. In addition, having a complex structure of healthcare systems requires many factors to be taken into account in order to evaluate the quality of health services [36]. Multi-criteria decision-making (MCDM) method, as another approach, has been widely preferred by researchers to measure the quality of healthcare in the literature [37]. The importance of multiple factors affecting healthcare service quality in the decision-making process is specified by this method.

Many reasons have been stated for addressing the present study. The aim of this study was to develop models to increase the quality of the healthcare services both in public and private healthcare systems. There are three kinds of problems that belong to these organizations around the world. First, patients stay longer in public hospitals, so patient dissatisfaction and expenditure of public healthcare systems increase. The second problem is that the treatment cost is high, and patients need to pay a lot from their pocket to wait less belonging to private healthcare organizations. The third drawback is that healthcare expenditures provided by the government are very high. The objective of this study was to improve the optimization models to derivate strategies and analyze situations in which components of healthcare systems (government, patients, hospitals, funders) make decisions in order to find optimum and tangible solutions within these components in healthcare systems. It was aimed to make less healthcare expenditure on the behalf of the public, less waiting on behalf of the patient and more income on behalf of the private healthcare institutions by taking advantage of the healthcare components and taking advantage of the mathematical modeling technique.

This study consists of five main parts. A literature study was carried out by examining the healthcare systems in the first section. The objective functions and constraints were discussed to create optimization models in the second part of the study. The results of optimization models for healthcare systems were handled deeply in the third section. In the fourth section, the numerical results obtained in optimization models were interpreted according to healthcare systems. Finally, conclusion was included in the last section of the study.

2. METHODOLOGY

The problem formulation of this research was divided into two segments. The first part consists of the healthcare providers, funders and elective treatment for the patients as monopoly. Interactions between these healthcare components were studied in the second part of this section. Each healthcare component wants to take an advantage among others from different aspects, as in the examples of less waiting time for patients, minimization of the costs for public healthcare systems, request to increase the number of customers including patients and non-patients for funders, and increasing income for the private healthcare sectors. Lastly, we have considered funders profits using developed optimization models as monopoly and interactively in healthcare systems.

Contemplating two types of definitions among the healthcare components with index $i = s, p$ all notations were presented in Table 1. This index was situated as state/public and private healthcare systems. In the numerical procedures used in this study, the data registered to the healthcare system were considered only one time or one year or one season. For this reason, time factors were not measured in the optimization models established.

Literature research has showed that cost of queuing theory is inversely proportional to a certain level. Likewise, high waiting time was also known to cause high costs in hospital establishments. By considering the funder's reimbursement cost was related with the waiting time that was called disutility and formulated as $r(\alpha)w$, where r is reimbursement, α is the disutility coefficient associated with waiting w.[26] In this research, the relationship between patient satisfaction and waiting time was measured by the premium paid to obtain the healthcare services. This showed that; higher paying premiums led to shorter waiting times. The patients' satisfaction constraint was measured as equation below:

Table 1. Notations of the optimization models

Sets	**Definitions**
$i = \{s, p\}$	Sets among the state and private healthcare components
Parameters	**Definitions**
α_i	The coefficient associated with the waiting time i
β_i	Charge by private funders i
Decision Variables	**Definitions**
k_i	The unit cost for treating patient type i
c_i	Co-payment by the patient i who visit to the doctor each time
r_i	Reimbursement rate by insurance i
p_i	Payment to be insured by patient i
x_i	Proportion of population with insurance i
λ_i	Arrival rate of patient i
μ_i	Processing rate of patient i
Objective Functions	**Definitions**
w_i	Waiting time i
Rev_i	Expected Revenue of the healthcare systems type i
Abbreviations	**Definitions**
MSHSP	State Healthcare Systems in Monopoly Case for Provider
MPHSP	Private Healthcare Systems in Monopoly Case for Provider
MSHSF	State Healthcare Systems in Monopoly Case for Funders
MPHSF	Private Healthcare Systems in Monopoly Case for Funders
ISHSP	State Healthcare Systems in Interactive Case for Provider
IPHSP	Private Healthcare Systems in Interactive Case for Provider
ISHSF	State Healthcare Systems in Interactive Case for Funders
IPHSF	Private Healthcare Systems in Interactive Case for Funders

$$p_s \leq \alpha w_s \tag{1}$$

where w is indicated as the waiting time for treatment. Treatment processing time is exponentially distributed with mean $\frac{1}{\mu}$ and $M/M/1$ queuing model is applied to calculate that the expected waiting time of a patient was:

$$w = \frac{1}{\mu - \lambda} \tag{2}$$

According to the M/M/1 rule, the total waiting time of a patient in both the system and the queue was calculated as follows:

$$W_{total} = \frac{\lambda + \mu}{\mu * (\mu - \lambda)} \tag{3}$$

The queuing theory behaves the patient arrivals are Poisson distributed with rate λ whereas treatment times are exponentially distributed with rate μ. The ratio λ/μ is called utilization ρ. If this ratio is greater than 1, that says patients are arriving faster than they can be served, and so the waiting time will grow without bound. If the ratio is less than 1, the waiting time will reach some steady state averagely. In this study, the healthcare service time is not fixed, so the curve does trend down rapidly when μ exceeds 1 and λ falls in behind 1 (see Figure 1).

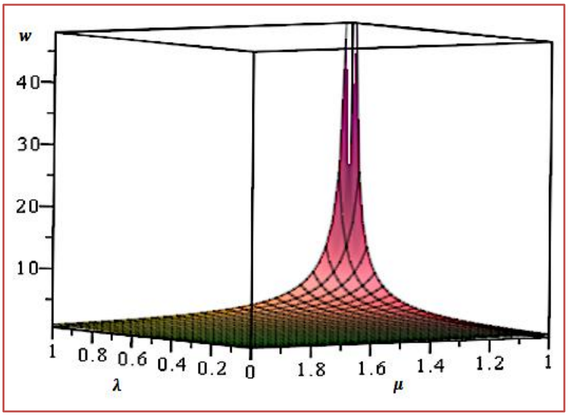

Figure 1. The relationship between λ and μ

2.1. Monopoly Healthcare Systems

The monopoly healthcare systems operate only patients and funders as if a patient has a public insurance, he/she can just get service from the public providers, and vice versa. If patients are eligible to have public insurance such as Medicaid, Medicare, SCHIP, Tricare, etc., the resources of the state or government cover all the patient's expenses. This situation causes high reimbursement cost for government and high waiting time in public hospitals [38]. The patient must pay more to have a private insurance to receive good service from healthcare providers.

2.1.1. Public Healthcare Provider

The patient who has public insurance receives service from public providers as exhibited in Figure 2. The patients state insurance does not cover the cost of health service from the private providers. Thereby the public hospitals must incur a high volume of patients, which leads to an increase in waiting time. All these negations result in unsatisfied patients and a high cost to the government.

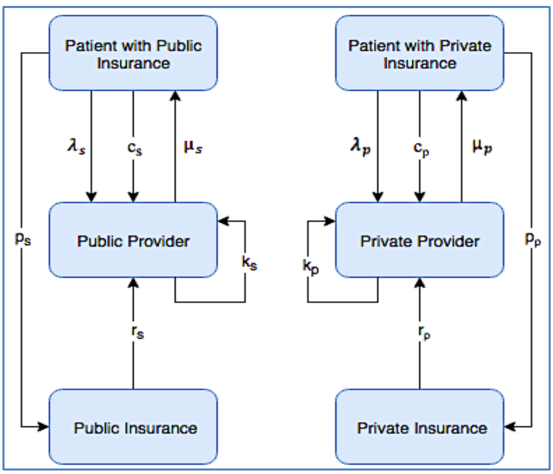

Figure 2. Monopoly healthcare systems

Using the denoted parameters, the public hospitals expected profit must be nonnegative and is computed since $\lambda_s(r_s + c_s) \geq \mu_s k_s = 0$. The constraint of the hospital's reimbursement and the unit cost for treating a patient were formulated as:

$$\lambda_s(r_s + c_s) \geq \mu_s k_s \tag{4}$$

where the proportion of private insured patients with multiply the insurances, premiums must be:

$$x_s p_s - \lambda_s r_s \geq 0 \tag{5}$$

By contemplating the patients waiting time who are treated in public providers could be invented as:

$$Waiting\ time_S = min \sum \frac{\lambda + \mu}{\mu(\mu - \lambda)} \tag{6}$$

Subject to 1, 4, and 5.

Expanded- See Appendix A1.

2.1.2. Private Healthcare Provider

The patient who has private insurance obtains service from private providers. Although public health services cost less, the patients who have private insurance are not eligible to receive health service from public providers. Hence the public providers operate in a high volume of capacity as displayed in Figure 2.

The private provider has the patient arrival rate λ and processing rate μ. The hospitals expected profit is $\lambda_p(r_p + c_p) - \mu_p k_p$. The constraint of the hospital's reimbursement and the unit cost for treating a patient was formulated as:

$$\lambda_p(r_p + c_p) \geq \mu_p k_p \tag{7}$$

The number of customers x_i who are insured by funders do not count the total number of patients in all area where necessitate an insurance. Where the proportion of private insured patients with multiply the insurances' premiums must be:

$$x_p p_p \geq \lambda_p r_p \tag{8}$$

Otherwise, the private hospitals profits become negative. Therefore, we assume that the waiting time in private healthcare sector is close to zero. The patient's satisfaction constraint was measured as:

$$p_p \geq \alpha w_p \tag{9}$$

where w is symbolized as the waiting time for treatment. By deliberating the private providers or hospitals profit could be formulated as:

$$MaxRev_p = \sum \lambda_p(r_p + c_p) - \mu_p k_p \tag{10}$$

Subject to 7, 8, and 9.

Expanded- See Appendix A2.

2.1.3. Public Healthcare Funders

Patients insured by the governments have a very long treatment time, namely length of stay in hospitals. This waiting time is reflected in the cost of the state/government budget. People who work in the public sector and those who have low income enjoy the benefit of this type of policy. But even states with a social healthcare system structure are receiving cheap premiums for their citizens who have this insurance and citizens have the obligation to have this insurance in some states. For this reason, public insurance is required to be registered at least $50 [38], (or 88,29 ₺ for Turkey [39]). However, as the amount of premium of healthcare insurance increases, the scope of the insurance coverage expands. The aim of public healthcare insurance companies is to have the maximum income, but the cost of patients treated in the hospitals should not exceed the income. Otherwise, healthcare insurance institutions, the state/government, will be exposed to high costs. The objective function was constructed as follows:

$$Max\ Rev_S = \sum_n^N [x_s p_s - \lambda_s r_s] \quad (11)$$

Expanded- See Appendix A3.

The annual income of state insurance should not be less than the cost of the patient being treated within that year. The constraint was formulated as below:

$$r_s \lambda_s + \lambda_s c_s + \mu_s k_s \leq x_s p_s \quad (12)$$

2.1.4. Private Healthcare Funders

Private health insurance companies want to save more customers. However, the patients who are the customers agree with the hospitals where they will be treated. Because private health insurance companies want to pay at least the reimbursement they will pay to the hospitals. For private health insurance companies, the objective function was expressed as follows:

$$MaxRev_p = \sum x_p p_p - \lambda_p r_p \quad (13)$$

There were two constraints on private health insurance. First, the amount of reimbursement to be paid to the hospital must be greater than or equal to the amount of the co-payment and the treatment fee of a patient with private health insurance. The first constraint was formulated as:

$$\lambda_p r_p \geq \lambda_p c_p + \mu_p k_p \quad (14)$$

The second constraint is that the premium to be paid by the clients with private health insurance should be equal to or greater than the total reimbursement to be paid to the hospital. The second constraint was defined as:

$$\lambda_p r_p \leq x_p p_p \quad (15)$$

2.2. Interactive Healthcare Systems

2.2.1. Healthcare Providers

The arrival rate λ and discounted reimbursement rate r play the main roles in the problem formulations. If the patient who has a public insurance, he or she cannot access to receive a treatment in private hospitals or although having a public insurance, the patient must pay for his/her treatment's cost. This constraint cases less patient's mobility to choose hospitals. We advocate the patient who has either private or public insurance obtains service from the private providers with the different or same reimbursement, he/she can access treatment in public or private providers (see Figure 3).

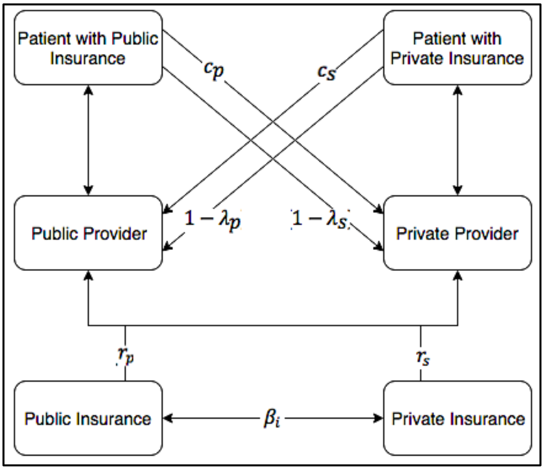

Figure 3. Interaction between components of healthcare systems

The patient who has either private or public insurance obtains service from the private providers with the same reimbursement and the unit cost of hospital capacity. Patients can be shared between providers, so the arrival and the processing rate constraint which is related to the capacity of private hospitals was:

$$\lambda_p + (1-\lambda_s) \leq \mu_p + \mu_s \tag{16}$$

Patients can be shared between providers, so the arrival and the processing rate constraint in public hospitals was:

$$\lambda_s + (1-\lambda_p) \leq \mu_s + \mu_p \tag{17}$$

Otherwise, the waiting time must be unwanted situations and it increases exponentially. Considering the state provider profit should be nonnegative and shown as below:

$$\lambda_s r_s + \lambda_s c_s + (1-\lambda_p) r_s + (1-\lambda_p) c_s - \mu_s k_s \geq 0 \tag{18}$$

In this case, where the proportion of private insured patients with multiply the insurances premiums must be zero, because customers who have private insurances have higher payment than public insured customers. Therefore, the state provider's profit was revised as:

$$\lambda_s (r_s + c_s) + (1-\lambda_p)(r_s + c_s) - (\mu_s + \mu_p) k_s \geq 0 \tag{19}$$

The private providers profit was formulated as:

$$Expected\ Rev_p = \max \sum \lambda_p (r_p + c_p) + (1-\lambda_s)(r_s + c_s) - \mu_p k_p - \mu_s k_s \tag{20}$$

The public hospital's waiting time was reformulated as:

$$Waiting\ time_s = \min \left[\frac{\lambda_s}{\mu_s(\mu_s + \lambda_s)} + \frac{1-\lambda_p}{(1-\mu_p)\left((1-\mu_p) + (1-\lambda_p)\right)} \right] \tag{21}$$

Expanded- See Appendix A4.
Expanded- See Appendix A5.

2.2.2. Healthcare Funders

Private insurance companies want to increase the number of customers and expected revenues as well. This is indicated as the x_i proportion of population with insurance. There are two reasons why patients with private health insurance do not choose state hospitals. These are the high premium payments for patients with private health insurance and the high waiting time in state hospitals, which is the reason for dissatisfaction.

While providing the interaction between health systems, there are four main constraints in terms of insurance institutions. The first constraint is the fact that a patient with a state health insurance is treated in a private hospital and the resulting cost is the interaction between the insurance companies.

$$x_s p_s + (1 - \lambda_s)\beta = (1 - \lambda_s) r_p + r_s \lambda_s + c_s \lambda_s + k_s \mu_s \qquad (22)$$

Expanded- See Appendix A6.

When a patient with a state insurance is sent for treatment to a private hospital, it means that the state health insurance company pays for treatment. The fact that the state health insurance company is a nonprofit organization should not harm the institution as well as not profit. This constraint equation was expressed as:

$$\lambda_p r_p + (1 - \lambda_s)\beta + (1 - \lambda_s) c_p + \lambda_p c_p + \left(\mu_p + 1 - \mu_s\right) k_p \leq x_p p_p \qquad (23)$$

The relationship between the reimbursement that private health insurance companies will pay to private hospitals and the amount they will pay to state insurance β should be balanced. This balance will ensure that private health insurance institutions will not suffer any loss.

$$\beta = r_p \qquad (24)$$

Private insured customers and the number of treated patients was formulated as:

$$x_p p_p - \mu_p r_p \geq 0 \qquad (25)$$

The number of customers, the arrival, and the processing rate constraint in private and public hospitals was defined in equation 17. In this case, the relationship between hospitalization rates of customers with both private and state healthcare systems is shown in the following equation.

$$\lambda_s + \lambda_p \leq x_s, x_p \qquad (26)$$

Considering high waiting time in public health system and low capacity of private health system are mainly triggered to build a new model for both health systems. In the developed optimization model, the objective function must be maximized so that the revenues of private health insurance companies are not negatively affected. The objective function equation was expressed as:

$$Expected\ \ Rev_p = max \sum \left[x_p p_p - (1 - \lambda_s)\beta - \lambda_p r_p \right] \qquad (27)$$

Subject to 22, 23, 24, 25, and 26.

3. COMPUTATIONAL RESULTS

Some numerical examples presented for each optimization model developed were discussed in this section of the study. The results of the numerical example show the accuracy and validity of optimization models.

3.1. Monopoly Healthcare Systems

3.1.1. Public Healthcare Components

Created optimization models for monopoly of public healthcare system was considered the waiting time that was aimed to minimize. If the decision variables were p_s, λ_s, and μ_s in the state healthcare system, the relationship between the waiting time of a patient and was given in

Table 2. If the value of p_s is between \$100.00 and \$499.00, then the r value is at least \$501.00. Otherwise the model was unsolvable.

Table 2. The range of values for decision variables $\boldsymbol{p_s}$, $\boldsymbol{\lambda_s}$, and $\boldsymbol{\mu_s}$

Iterations No	**Decision Variables**			**Objective Function**	**NLP Status**
	$\boldsymbol{p_s}$	$\boldsymbol{\lambda_s}$	$\boldsymbol{\mu_s}$	**Waiting Time**	
-	\$100.00	-	-	-	Infeasible
26	\$150.00	0.15	0.85	0.8235	Feasible
26	\$200.00	0.20	0.80	0.8000	Feasible
26	\$250.00	0.25	0.75	0.7500	Feasible
26	\$300.00	0.70	0.30	0.5716	Feasible
27	\$350.00	0.35	0.65	0.4610	Feasible
27	\$400.00	0.40	0.60	0.3330	Feasible
27	\$450.00	0.45	0.55	0.1818	Feasible
27	\$499.00	0.49	0.51	0.0392	Feasible
27	\$500.00	0.50	0.50	-	Infeasible

Note: Lindo/Lingo Optimization Software Output

Lindo/Lingo optimization software was used for the solution of the developed optimization models. The optimization software has made many iterations for the public healthcare system optimization model in monopoly healthcare system. While no results could be achieved until the twenty-sixth iteration, the optimization model yielded results in the twenty-seventh iteration. Patient waiting time is shown in Figure 4 that there is an inverse connection with the patient treatment process (processing rate). It was observed that the waiting time no longer changes when the patient's arrival rate reaches a certain point with the processing rate.

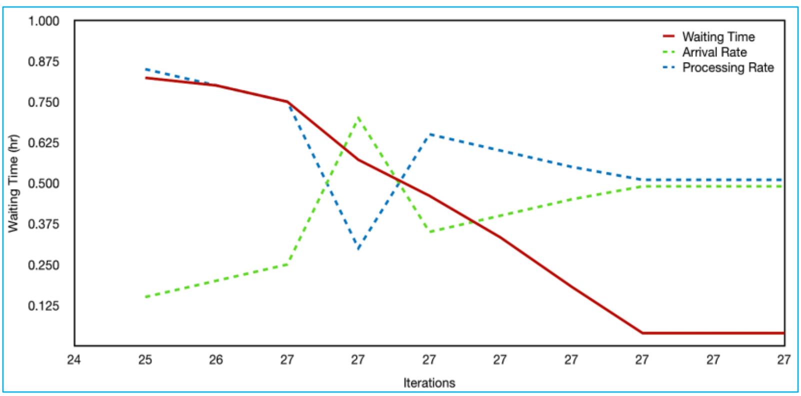

Figure 4. Waiting time interval according to $\boldsymbol{\lambda_s}$, and $\boldsymbol{\mu_s}$

The waiting time is based on reimbursement and the cost of treatment a patient dependently. In a state hospital, the values below which a patient's treatment cost is minimized without a limit (unconstrained) in the decision variables are indicated in Table 3. However, if the minimum cost is achieved, a patient's waiting time in the hospital is at least 19.00 minutes. However, this waiting time will increase even more with the increase in the proportion of patients and the prolongation of the treatment period. There is no limit to the decision variables for these models with the limiting number of major iterations has been reached.

Table 3. Outputs of Providers in Monopoly Healthcare Systems

Variables	Public	Private
c_i	[\$48.52$_2$...\$53.52$_1$]	[\$50.00$_1$, \$50.00$_2$]
k_i	[\$64.48$_2$...\$69.97$_1$]	[\$50.00$_2$...\$150.00$_1$]
p_i	[\$72.22$_1$... \$95.85$_2$]	[\$98.97$_1$… \$-$_2$]
r_i	[\$163.03$_2$...\$422.24$_1$]	[\$1000.00$_1$, \$1000.00$_2$]
x_i	[57.42$_1$...8.08$_2$]	[-$_1$...-$_2$]
λ_i	[0.4003$_1$...0.6598$_2$]	[0.1000$_2$...0.0149$_1$]
μ_i	[0.2039$_1$...0.4925$_2$]	[0.0588$_1$...0.1000$_2$]
Rev_i	[\$169.06$_1$...\$246.89$_2$]	[\$42,250.00$_2$...\$64,842.56$_1$]
w_i	[17.7072min_2...19.6650min_1]	[4.6451min_1...8.0000 min_2]

NOTE: [1]-Results of the constrained optimization model,
[2]-Results of the unconstrained optimization model.

Table 3 shows that there are three major decision-making variables that concern the patient. If the average waiting time of the patients is to be reduced to 18.68 minutes, the average co-payment amount to be paid by the patients or the health insurance clients is \$51.02 and the amount to pay for the insurance premium is \$84.04 averagely. Only the co-payment charge for these amounts is encouraging some patients to take advantage of the private health system.

3.1.2. Private Healthcare Components

The model is run for private monopoly healthcare system that gives delivery to only private insurer patients. In the private health sectors expected revenue of \$42,250.00 when a certain limit is placed on the decision variables to capture the maximum income ratio (see Table 3). This amount is caught by treating very few patients and without waiting for the patient to wait.

Bearing the constraints in mind, not only the values of the decision variables change, but also the income of the private health sector changes and its expected revenue of \$64,842.60 Patient waiting times have been observed to decrease with increasing patient treatment cost. This trend is expected. The system recognizes the number of customers with health insurance unlimitedly while there are no constraints related to the objective function. A certain proportion of the number of customers and the proportion of patients required for treatment also show the quality of the health care provided.

Scenarios were created based on the decision variables for monopoly private health system. The values of the other decision variables are fixed to determine the limit of a selected decision variable. Hereby, it is examined how decision variables affect the objective function value (see Table 4).

Table 4. Outputs of private healthcare providers according to monopoly case

	Decision Variables	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Maximum	Minimum	Average
Optimum	k_p	\$150.00	\$ 50.00	\$ 150.00	\$ 150.00	\$ 150.00	\$ 50.00	\$ 125.00
	c_p	\$50.00	\$ 50.00	\$150.00	\$ 50.00	\$ 150.00	\$ 50.00	\$ 75.00
	r_p	>\$1.00 <1,000.00	\$ 1,000.00	\$1,000.00	\$ 1,000.00	\$ 1,000.00	\$ 1,000.00	\$ 1,000.00
	p_p	\$100.00	\$ 100.00	\$100 10.00	\$ 170.00	\$ 170.00	\$ 100.00	\$ 117.5.00
	λ_p	0.1000	0.1000	0.0925	0.0588	0.1000	0.0588	0.0851
	μ_p	0.1000	0.1000	0.1000	0.0588	0.1000	0.0588	0.0836
	x_p	100	100	100	100	100	100	100
	w_p	8.0000 min	8.0000 min	7.071 min	4.7059 min	4.7059 min	8.0000 min	6.944 min
	Profit	\$ 32,249.90	\$42,250.00	\$ 26,750.00	\$99,450.00	\$ 99,450.00	\$ 26,750.00	\$ 50,174.90
Improvements	k_p	16.66_l %	60.00_p %	16.66_l %	16.66_l %	\$ 150.00	\$ 50.00	\$ 125.00
	c_p	33.33_p %	33.33_p %	100_l %	33.33_p %	\$ 150.00	\$ 50.00	\$ 75.00
	r_p	Stable	Stable	Stable	Stable	\$ 1,000.00	\$ 1,000.00	\$ 1,000.00
	p_p	14.89_p %	14.89_p %	14.89_p %	44.68_l %	\$ 170.00	\$ 100.00	\$ 117.50
	λ_p	14.89_l %	14.89_l %	$14.8l_p$ %	44.68_p %	0.1000	0.0588	0.0851
	μ_p	16.32_l %	16.32_l %	9.62_l %	42.26_p %	0.1000	0.0588	0.0836
	x_p	Stable	Stable	Stable	Stable	100	100	100
	w_p	15.207_l %	15.207_l %	1.828_l %	32.230_p %	4.7059	8.0000 min	6.944 min
	Profit	35.725_l %	15.794_l %	46.686_l %	98.206_p %	\$ 99,450.00	\$ 26,750.00	\$ 50,174.9.00

Note 1: Improvements are calculated according to the average values. *Note 2:* *p* = profit, *l* = loss

Scenario 1: r_p is unknown, $r_p \geq 0$ and decision variables that are $c_p, k_p, p_p, x_p, \lambda_p,$ and μ_p are constants. According to the Table 5, the patient remains steady when the number of patients spent for treatment reaches a certain number. In this case, although the number of patients does not increase, the waiting time of the patient increases with the increase of the treatment duration.

Scenario 2: k_p is unknown, $k_p \geq 0$ and decision variables that are $c_p, r_p, p_p, x_p, \lambda_p,$ and μ_p constants. It is seen that by decreasing the k_p value according to the first scenario and increasing the objective function value without changing the waiting time. It is observed that the cost of treatment has not changed with the increase of the time required for the treatment. This situation is undesirable for healthcare providers.

Scenario 3: c_p unknown, $c_p \geq 0$ and decision variables that are $r_p, k_p, p_p, x_p, \lambda_p,$ and μ_p constants. In Scenario 3, the release of the co-payment amount, which is the most important cost factor in the healthcare system, leads to a decrease in the waiting time of the patients according to Scenario 1 and Scenario 2. It has been found that 13.14% of benefits are provided for patients and healthcare facilities.

Scenario 4: p_p unknown is unknown, $p_p \geq 0$ and decision variables that are $r_p, k_p, x_p, \lambda_p,$ and μ_p constants. The 4th scenario gives the highest value of the model. Despite the increase in insurance premium, the waiting time has decreased. Compared to the other scenarios, scenario 4 appears to benefit 69.9% of scenario 1 and 2, and 13.14% of scenario 3 in terms of both patient and healthcare providers. The negative side of this scenario is an average increase of \$70.00 in the amount of insurance premium.

3.1.3. Public Healthcare Funders:

The diversity shown by state insurance premiums can be contracted with some private healthcare providers and some patients can be treated in contracted hospitals. However, the cost of annual patient treatment should not exceed the annual income of citizens with state insurance (see Table 5).

Table 5. Outputs of Funders in Monopoly Healthcare Systems

Variables	Public	Private
Rev_i	$ 1235.20	$ 442,948.59
c_i	$ 53.52	$ 220.00
k_i	$ 70.00	$ 170.00
p_i	$ 73.06	$ 900.00
r_i	$ 352.92	$ 300.00
x_i	65.260	0.2000
λ_i	0.1000	0.9900
μ_i	0.2500	100.1
w_i	15.0001 min	15.95min

In view of this optimization model, private health insurance requires less reimbursement to increase annual income. At this point, private health care providers will not accept this agreement. Private health insurance companies are required to have an agreement with state hospitals. This will have a negative impact both on patients with private health insurance and on state hospitals. Private health insurance companies must push some limits. For example, it is necessary to increase the amount of reimbursement to be paid to hospitals by increasing insurance premiums. Another limitation is that the amount of reimbursement to be paid to the hospital should not be less than the cost of the treatment of the patient. Otherwise, the incomes of private hospitals will be reduced.

3.2. Interactive Healthcare Systems

3.2.1. Healthcare Providers

How the objective functions of the private health sector and the state health sector changed because of the interaction between the private health system and the state health system. The most important goal of the private health system is to increase the annual income to the maximum level without affecting patient waiting time. Feasible results were given in Table 6.

3.2.2. Healthcare Funders

The issue to be considered in this section is that a patient with state health insurance can be treated in a private hospital. To be able to do this, some constraints must be recognizable. For example, there should be a special rate between private health insurance companies and state-owned health companies. That is, private health insurance companies have to pay for the state health insurance institution as if the patient had private health insurance that goes to the private hospital. The state health insurance should be provided not to loss by considering the reimbursement paid to the private hospital. Taking these constraints into account, the change in annual revenues has been shown in the monopoly system of private health insurance companies in Table 6.

Table 6. Networks between Healthcare Providers and Funders

Decision Variables	Private Providers	Public Providers	Private Funders	Public Funders
Rev_i	$ 13,200.00	-	$ 34,730.00	-
k_i	$ 300.00	-	$122.90	$ 64.00
c_i	$ 200.00	-	$53.02	$ 48.00
p_i	-	-	$350.00	$ 83.93
r_i	$20,00.00	-	-	-
λ_i	0.1000	0.2000	0.0475	0.0475
μ_i	0.1000	0.2000	0.1589	0.1344
x_i	-	-	100	57.64
w_i	6.0000 min	9.5833 min	9.0500min	8.0600min

4. DISCUSSION OF COMPUTATIONAL OUTPUTS

We have discussed which healthcare system gives better results with this study. Firstly, certain features or constraints of two systems were determined and compared. We have focused on four elements which were patients, providers, funders, and governments of the healthcare systems in terms of their benefits. It has been tried to determine how these elements are affected by the established optimization models. When the components of healthcare were taken into consideration, the treatment quality and the way the fees were affected by the patients were separately examined.

The results of the optimization models developed for healthcare systems; financial affairs have been discussed in the previous section to provide the best decision in terms of the healthcare components. Decision variables in both objective functions and constraint equations for healthcare system components played an important role. Some specific constraints of these models have been identified and added to get healthy results. Having more than one component causes more than one objective function in the healthcare systems. It could be noted that while one component achieves the desired result for the objective function, the change in the results of the objective functions of the other components was inevitable. From this point of view, it was observed that the results obtained were close to optimum values. So, the results were feasible and within the feasible (solution) region.

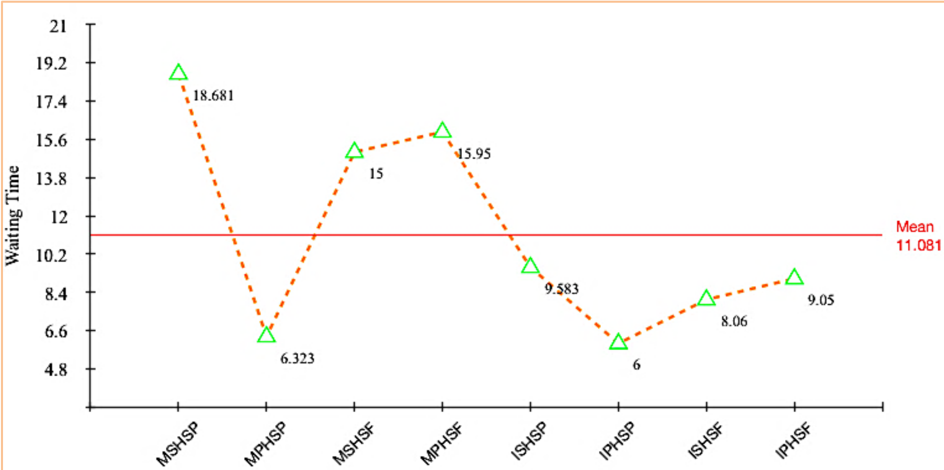

Figure 5. The waiting time of patients according to considered healthcare systems

As a result, it was seen that the values of the decision variables belonging to the healthcare system are closer to each other. It was determined that the waiting times of patients who want to be treated were close to each other in both private and public healthcare institutions. In the public healthcare system for providers or hospitals, the waiting time for patients has decreased from 18.681 minutes to 9.5833 minutes (see Figure 5). However, it was determined that the amount to be paid to the public healthcare insurance has increased from $50.00 (for USA healthcare systems) to $83.903 (see Figure 6). These pricing schemes show the minimum payable amounts for patients.

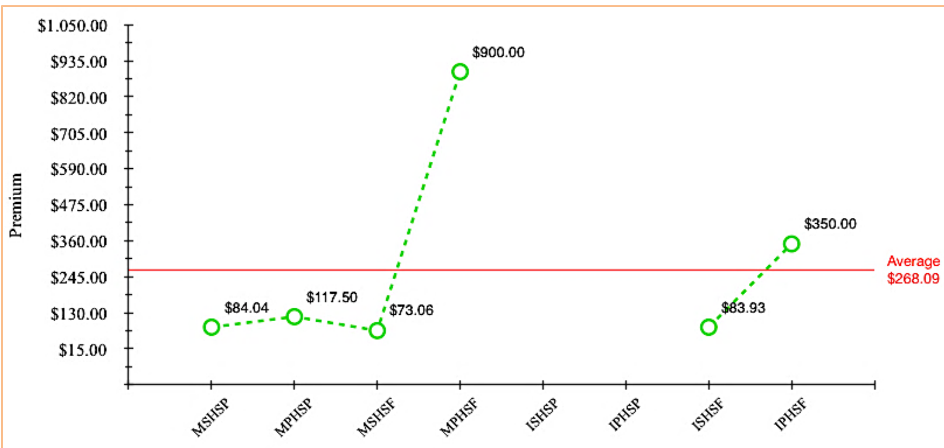

Figure 6. The levels of premium as decision variable among healthcare systems

Unfortunately, in some citizens of states are being treated in a limited manner when they cannot pay high premiums. As a matter of fact, this policy of the states' health insurance seems to be wrong. Because the patients who pay a small premium and the patients who pay too much premium are in the same public hospitals, the waiting times are the same and higher. The density of the public hospitals is reduced by the referral to the private healthcare institutions of the patients who have more premiums. As a result, patient waiting time in public hospitals has been reduced and private healthcare sectors' annual income will increase.

The scenarios of the monopoly of the private healthcare system showed that the average amount that a person must pay for private health insurance was $117.50. In the interactive case, those who should have private health insurance have to pay at least $350.00. Both government and private healthcare institutions are able to receive any kind of health care with high healthcare insurance premiums. It was seen that the amount of co-payments that a patient has to pay to the hospital for treatment in any hospitals was high in the private healthcare system and this amount was reduced to a minimum level in the interactive healthcare system (see Figure 7).

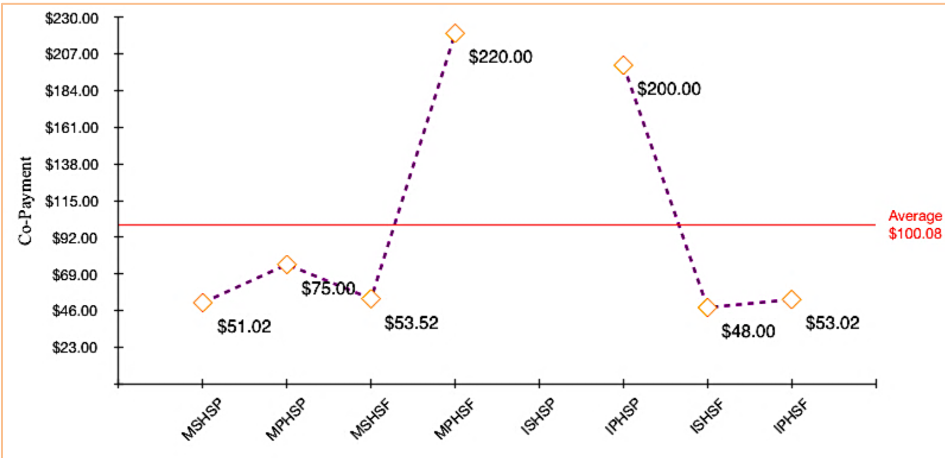

Figure 7. The levels of co-payment as decision variable among healthcare systems

The greatest change in the transition of the healthcare system from the monopoly to the interactive system was seen in the amount of reimbursement. In particular, it has been found that the amount of reimbursement, which was a high cost for government, decreased. The reimbursement rate in the private healthcare system was dropped from $2,000.00 to $275.00 (see Figure 8).

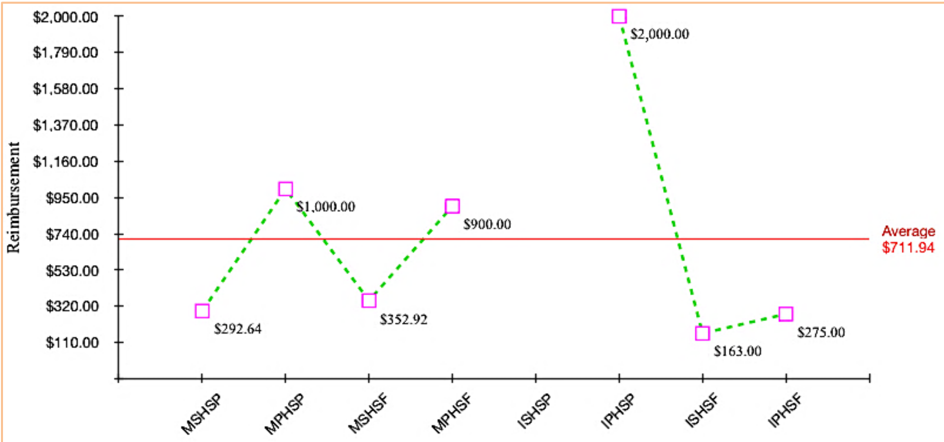

Figure 8. The levels of reimbursement as decision variable among healthcare systems

The average cost of treatment for a patient was calculated as $ 131.30. This value was computed the average of the results obtained for all types of healthcare systems. The highest treatment cost was calculated as $ 300.00, while this value was dropped to $ 64.00 (see Figure 9).

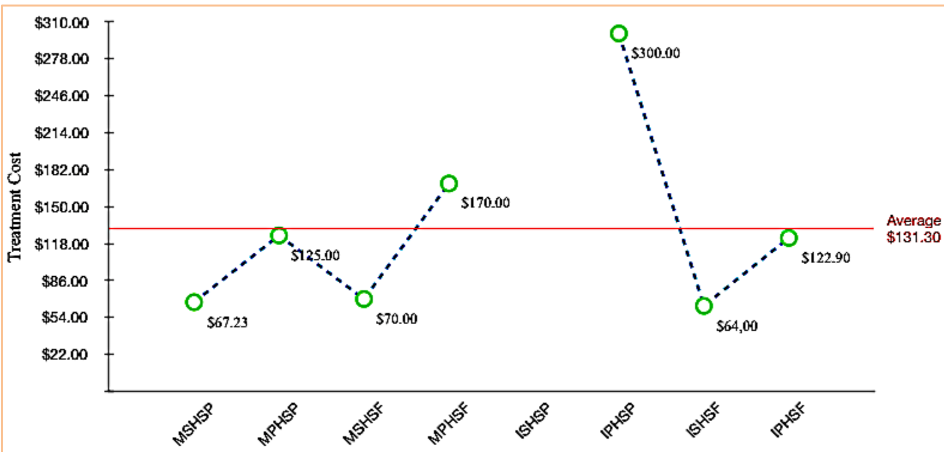

Figure 9. The levels of treatment cost as decision variable among healthcare systems

5. CONCLUSION

In industry or health environment, the main goal is to develop the potential quality for products, increase patient or customer satisfaction and system or resources of health facilities performance. Many companies lose profit due to wrong applications of methods. It causes defection of products or services. In this research, health systems have been extensively examined and specific methods developed to solve important problems. Having many components in healthcare systems makes complicated problems; therefore, optimization technique is inevitable to overcome these problems in healthcare area. Because optimization models are developed specific to real problems and poured into mathematical environment. Development of optimization models become imperative as the problems in the field of health are unique.

The main aim for this study was to increase the quality of the services for patients both in public and private healthcare sectors. The feasible solutions have been obtained by creating optimization models with the scenarios provided to create a balance between the state and the private healthcare system. The minimum fee payable by patients for access to healthcare schemes was determined. While there was no remarkable change in the waiting time of the patients in the private healthcare system, the waiting time were reduced by 48.70% in the public healthcare system. The amount of reimbursement decreased significantly as a result of the interaction of healthcare systems. By reducing the amount of co-payment to the amount charged by the public healthcare system, it has been ensured that private healthcare providers treat more patients. The public healthcare systems expected profit was provided to be nonnegative. Our findings of this study led to the parties (patients, provider, funders, and governments) that make up the healthcare system have achieved positive results, especially for patients being in the center of healthcare systems. With these results, it is shown that no sides of healthcare components have lost.

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Appendix of Mathematical Equations

A1 *Mathematical expansion of objective function equation and constraints for public health sector in monopoly case*

- *Expended 0:*

$$Waiting\ time_S = \min\left[\frac{\lambda+\mu}{\mu(\mu-\lambda)}\right] \quad \text{E [1]}$$

Expended 1:

$$Waiting\ time_S = \left[\frac{\mu_S}{\mu_S(\mu_S+\lambda_S)}\right] + \left[\frac{\lambda_S}{\mu_S(\mu_S+\lambda_S)}\right] \quad \text{E [2]}$$

A2 *Mathematical expansion of objective function equation for private health sector in monopoly case*

- When the mathematical expansion of the objective function equations is made, it turns out that there is no linear equation (see *expanded*). In this case, optimization model for private provider in monopoly case is a non-linear problem.
  - Expanded 0:

$$Max\ Rev_p = \sum \lambda_p(r_p + c_p) - \mu_p k_p \quad \text{E [3]}$$

- Expanded 1:

$$Max\ Rev_p = \left(\frac{1}{2}\right) r_p {\lambda_p}^2 - \left(\frac{1}{2}\right) r_p \lambda_p + \left(\frac{1}{2}\right)\lambda^2 - \left(\frac{1}{2}\right) c_p \lambda_p - \mu_p k_p \lambda_p \quad \text{E [4]}$$

- The most important factor in increasing annual incomes in private health care is the number of patients treated in one year. For this reason, private health care will want to increase the number of patients. This is called maximization of the objective function without increasing patient waiting time. In simple terms, the rate of patient arrival is calculated as follows:

$$\lambda p = \left[\frac{[2k_p\mu_p + c_p + r_p]}{r_p} + c p\right] \quad \text{E [5]}$$

- The treatment time required for the patient to be treated:

$$\mu p = \frac{1}{2\left[\frac{c_p\lambda_p + r_p\lambda_p - c_p - r_p}{k_p}\right]} \quad \text{E [6]}$$

A3 *Mathematical expansion of the objective function of the Public Health Funders in monopoly case*

$$Max\ Rev_S = \left(\frac{1}{2}\right) p_s \left(x_s + \frac{-\left(\frac{1}{2}\right)p_s - r_s\lambda_s}{p_s}\right)^2 - \frac{\left(\frac{1}{2}\right)\left(-\left(\frac{1}{2}\right)p_s - r_s\lambda_s\right)^2}{p_s} \quad \text{E [7]}$$

A4 *Mathematical expansion of the objective function of the Private healthcare system in interactive case*

- Expanded 0:

$$Exp\ Rev_p = \max \sum [\lambda_p (r_p + c_p) + (1 - \lambda_S)(r_S + c_S) - \mu_p k_p - \mu_S k_S] \quad \text{E [8]}$$

- Expanded 1:

$$Max\ Rev_p = \lambda_p \lambda_S c_p + \lambda_p \lambda_S r_P + \frac{3}{2\lambda_S r_p} + \frac{3}{2\lambda_S c_p} - k_p\,\mu_p\,\lambda_S - k_p\,\mu_S\,\lambda_S - \frac{1}{2r_p}\lambda_S{}^2 - \frac{1}{2c_p}\lambda_S{}^2 \quad \text{E [9]}$$

A5 *Mathematical expansion of the objective function of the Public Healthcare System in interactive case*

- Expanded 0:

$$Waitingtime_s = \sum \left[\frac{\mu_s + \lambda_s}{\mu_s(\mu_s - \lambda_s)} + \frac{1 - \mu_p + (1 - \lambda_p)}{(1 - \mu_p)(1 - \mu_p - 1 + \lambda_p)}\right] \quad \text{E [10]}$$

- Expanded 1:

$$Waiting\ Time_s = \frac{\mu_s}{\mu_s(\mu_s - \lambda_s)} + \frac{\lambda_s}{\mu_s(\mu_s - \lambda_s)} + \frac{2}{(1 - \mu_p)(-\mu_p + \lambda_p)} - \frac{\mu_p}{(1 - \mu_p)(-\mu_p + \lambda_p)} - \frac{\lambda_p}{(1 - \mu_p)(-\mu_p + \lambda_p)} \quad \text{E [11]}$$

- Expanded 2:

$$Waiting\ Time_s = \mu_p^2 \mu_s + \mu_p^2 \lambda_s - \mu_p \mu_s \lambda_p - \mu_p \lambda_p \lambda_s - \mu_p \mu_s(\mu_s - \lambda_s) - \lambda_p \mu_s(\mu_s - \lambda_s) - \mu_p \mu_s - \mu_p \lambda_s + \mu_s \lambda_p + \lambda_p \lambda_s + 2\mu_s(\mu_s - \lambda_s))/(\mu_s(\mu_s - \lambda_s)\mu_p^2 - \mu_s(\mu_s - \lambda_s)\mu_p \lambda_p - \mu_p \mu_s(\mu_s - \lambda_s) + \lambda_p \mu_s(\mu_s - \lambda_s) \quad \text{E [12]}$$

A6 *Mathematical expansion of the β*

The amount of β appears to be related to the number of patients with state health insurance that are treated in private hospitals.

$$\begin{cases} [\beta = \beta]: 1 - \lambda_s \ and\ [\beta = \beta]: 0 \le -c_s \lambda_s - k_s \mu_s + x_s p_s + r_p \lambda_s - r_s \lambda_s - r_p \\ \left[\beta \le \frac{-[c_s \lambda_s + k_s \mu_s - p_s x_s - r_p \lambda_s + r_s \lambda_s + r_p]}{(-1 + \lambda_s)}\right]: 0 < \lambda_s \\ [\frac{-[c_s \lambda_s + k_s \mu_s - p_s x_s - r_p \lambda_s + r_s \lambda_s + r_p]}{(-1 + \lambda_s)}] \ge \beta]: 0 > \lambda_s \\ [\,]: otherwise \end{cases} \quad \text{E [13]}$$