YILDIZ JOURNAL OF EDUCATIONAL RESEARCH

Journal website: www.eds.yildiz.edu.tr/yjer

2564-7431

Fuzzy Logic Modeling and Optimization of Academic Achievement of Students

Ahmet ŞAHİNER, Raziye AKBAY & Nurullah YILMAZ

Received: May, 5, 2018 ~ Accepted: June, 30, 2019
Online Published: June, 30, 2019

Suggested Citation: Şahiner, A., Akbay, R., & Yılmaz, N. (2019). Fuzzy logic modeling and optimization of academic achievement of students. *YILDIZ Journal of Educational Research*, *4*(1), 85-100. doi: 10.51280/yjer.2019.005

Abstract

The purpose of this study is to investigate the effects of students' attitudes towards the mathematics classes and mathematics teacher on the student's math's achievements. The participants of this study were 44 (9. 10. 11. and 12. grade) public high school students at an Anatolian city Isparta. An attitude inventory was used to measure the participants' attitudes towards the lesson and teacher. Students' academic success levels were determined using participants' grades at the end of first semester. Scores obtained from the attitude inventory were categorized by the experts. Finally, by using fuzzy logic modeling, the cases in which the minimum and maximum achievement level occurs are determined.

Keywords: Modelling, fuzzy logic, education, attitude

Öz

Çalışmanın amacı, öğrencilerin matematik dersine ve öğretmenine olan tutumun matematik başarısına olan etkisini incelemektir. Bu çalışma, Isparta ilinde bulunan 9., 10., 11. ve 12. sınıf öğrencisi olan 45 öğrenci üzerinde yapılmıştır. Öğrencilerin derse olan tutumunu ve öğretmene olan tutumunu ölçmek için tutum ölçeği geliştirilmiştir. Öğrencilerin başarılarını değerlendirmek için 1. Dönem sonu karne notları kullanılmıştır. Elde edilen veriler uzmanlar tarafından değerlendirilmiş ve derecelendirilmiştir. Son olarak, bulanık mantık modelleme kullanılarak maksimum ve minimum başarının hangi durumlarda oluştuğu belirlenmiştir.

Anahtar Kelimeler: Modelleme, bulanık mantık, eğitim, tutum

Introduction

The goal of learning is to benefit from knowledge. In the current Turkish general education system, learning is evaluated through acquirement of subject knowledge that is reflected by grades. In Turkey, mathematics is an obligatory class that Turkish students need to take during 12 year-old obligatory education. However, learning and teaching mathematics are thought to be difficult by the most of the Turkish students. From elementary education to high school, Turkish students are afraid of taking mathematics. (Dede and Argün, 2003; Filloy and Rojana, 1989; Wagner, 1981).

Mathematics is a subject that helps us understand the life and the world, and leads us to produce ideas about it. It also helps us develop critical thinking skills and therefore has an important place in each level of education. For that reason, students' success in math classes should not be ignored.

There are lots of factors affecting students' success in mathematics. Researchers indicated that these factors include but not limited to math talent, personal and family factors, parents' level of education, active participation in mathematics classes and the sufficiency of mathematics teachers teaching mathematics classes (Keskin & Sezgin, 2009; Savaş, Taş & Duru, 2010). Two other factors affecting students' mathematics success are emotions and attitudes toward mathematics. Emotions and attitudes are intertwined in a way that attitudes towards any person, object or situation can result in emotions toward it. Both attitudes and emotions such as liking or disliking are among important factors that effect mathematics success in a class is crucial to be successful in it. So, attitudes guide learning and there is a significant relationship between attitudes towards maths and math success (Yenilmez, 2010). In other words, one of the important reasons of math failure is the negative attitudes toward the mathematics.

Schools and teachers are foundations of teaching and learning. Particularly the behaviors of teacher strongly influence students learning. The lecturing style of the teachers and his/her pedagogic approach towards the students influence the attitudes of students formation towards a class or object. Teachers' behaviors are so important in developing positive attitudes towards classes. Therefore, teachers' behaviors have strong effect on students' success in a particular subject.

Data for the study was collected from high school students. For the study an attitude scale was applied to the students. Attitude scale measures attitudes towards a lesson and a teacher.

L.A. Zadeh developed fuzzy logic modeling (FLM) in 1965 (Zadeh, 1965; Zadeh, 1973; Zadeh, 1975a; Zadeh, 1975b; Zadeh, 1976). Classical logic is not sufficient to explain lots of problems in real life. With fuzzy logic in addition to wrong or right responses, a "partly true" response is emerged. There is no certainty in explaining indefiniteness by using linguistic factors. FLM is a quite successful method in indefinite situations. Fuzzy logic has been considered for many theoretical studies (Dubois &Prade, 1978; Grezegorzewski & Winiarska, 2014). It is also used for the solution of practical application. Initially, it was used in the control panel of steam machines (Mamdani, 1974; Mamdani & Assilian, 1975). Later on, it was used in many engineering studies (Sugeno & Takagi 1983; Takagi & Sugeno 1985; Kandel, 1986; Sakawa 1993). Currently, it is used in order to solve many real-life problems not only engineering and fundamental sciences but also social sciences (Zimmermann 1994; Saltan et al. 2007; Akbay et al. 2016).

Methodology

Data Collection

Data for this study was collected on a group of Anatolian High school students in an Anatolian city ISPARTA in the 2014 -2015 academic calendar educational year. 44 students (20 girls and 24 boys) joined the study. Inputs of this study are attitudes towards the teacher and attitudes towards the lesson. To evaluate the students' attitudes, an attitude scale was applied to the participants after obtaining approval from Isparta province department of education. Output of the study is the academic achievement. To measure academic achievement, mathematics grades of the students were used.

Fuzzy Logic Approach

According to the classical logic approach, an element either belongs to a set or not, that can be explained by using characteristic function $\boldsymbol{\chi_A}$ of a set $\boldsymbol{A}$ as the following:

$$_A=\begin{cases} 1, if \ \mathrm{x} \in \mathrm{A} \\ 0, \ if x \notin A. \end{cases}$$

On the other hand, according to fuzzy logic, a member may belong to multiple clusters. Therefore, sometimes we need fuzzy logic to describe some of the real-life problems better introducing partial membership.

As mentioned above, fuzzy logic modeling starts with the concept of a fuzzy set. A fuzzy set is a set without a crisp and clearly defined boundary. Assume X is an ordinary set, whose elements are denoted by x. Membership in an ordinary subset A of X can be expressed by the characteristic function, $\mu_A$from X to $\{0,1\}$ as follows:

Definition 1: If X is a collection of objects denoted by x, then a fuzzy set A in X is a set of ordered pairs

$$\mathrm{A}=\{(\mathrm{x}, \mu_{\mathrm{A}}(\mathrm{x})) : \mathrm{x} \in \mathrm{X}\}$$

where $\mu_A(\mathrm{x})$ is a membership function and it takes values between 0 and 1.

Definition 2: (Convex Fuzzy set) A fuzzy subset A of X is called convex if the following inequality holds:

$$\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2) \ \forall x_1, x_2 \in R, \lambda \in [0,1].$$

Definition 3 : Let A be a fuzzy subset of X; the support of A, denoted supp(A), is the crisp subset of X whose elements all have nonzero membership grades in A¸

$$\mathrm{supp}\ (\mathrm{A}) = \{\mathrm{x} \in \mathrm{X} | \mu_{\mathrm{A}}(\mathrm{x})>0\}.$$

Definition 4: h −cut of a set A is closed interval and membership degree provides the following feature:

$$\mathrm{A}_{\alpha}=\{\mathrm{x}: \mu_{\mathrm{A}}(\mathrm{x})>\mathrm{h}\}$$

If $\{x: \mu_A(x) \geq h\}$ is closed μ_A is upper semi-continuous.

Definition 5 : Let A and B are fuzzy subsets of a classical set X. We say that is a subset of B if A (t) ≤B (t), $\forall t \in X$.

Definition 6: If the degrees of at least member of fuzzy set are 1, the set is normal fuzzy set. There is an x for which $\mu_A(x) = 1$, if the set is not normal, the following statement $\frac{\mu_A(x)}{max\ \mu_A(x)}$ are used to normalize.

Definition 7: [Grezegorzewski, Winiarska, 2014] A fuzzy set A is called a fuzzy number if the following properties hold:

- is normal,
- is fuzzy convex, i.e.

  $\mu_A(\lambda x_1 + (1-\lambda)x_2) \geq \mu_A(x_1) \wedge \mu_A(x_2)$ $\forall x_1, x_2 \in$R, $\lambda \in [0,1]$
- μ_A isuppersemi-continuous,
- supp(A)isbounded,where supp$(A) = cl\ (\{x \in X: \mu_A(x) > 0\})$ and cl is the closure operator.

Definition 8: The core of a fuzzy number A is the set of all points that surely belong to A, i.e.,

Core (A) $=\{x \in R: \mu_A(x) = 1\}$,

Now, we define some operations in fuzzy sets;

Definition 9: Intersection: the membershipfunction of the intersection of two fuzzy sets A and B is defined as:

$\mu_{A \cap B}(x)$=min $(\mu_A(x), \mu_B(x))$, $\forall x \in X$

Definition 10: Union: the membership function of the union is defined as:

$\mu_{A \cup B}(x)$=max $(\mu_A(x), \mu_B(x))$, $\forall x \in X$

Definition 11: Complement: the membership function of the complement is defined as:

$\mu_{\bar{A}}(x)=1-\mu_{A}(x),\ x \in X$

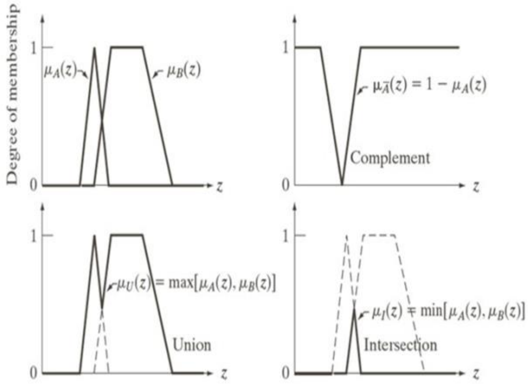

Figure 1: Operations on fuzzy sets.

When research studies regarding the application fuzzy logic were examined, the results indicates that two types of Fuzzy Inference Systems (FIS) namely Mamdani and Takagi-Sugeno are common in the literature. It has been argued that, each of these FIS outmaneuvers each other from time to time but there is no conclusive evidence showing superiority of one of them. Although ,both of these systems have membership for input, the output for them is different. In comparison to mamdani system that has membership for output. In Sugeno systems, the output membership function is polynomial that consists of coeffient and input. Although , it is easy create Mamdani system; for Sugeno system calculation is easy. In Mamdani system, defuzzification is made to results. In comparision to Mamdani system that has a set of fuzzy sets as output , Sugeno system consists of a fixed or linear output.

Mamdani systems consist of 3 parts. These parts are:

1. Fuzzification – convert the classical data or crisp data into fuzzy data or Membership Functions (MFs)

2. Fuzzy Inference Process – combine membership functions with the control rules to derive the fuzzy output

3. Defuzzification – use different methods to calculate each associated output and put them into a table: the lookup table. Picks up the output from the lookup table based on the current input during an application [Bai, Yang].

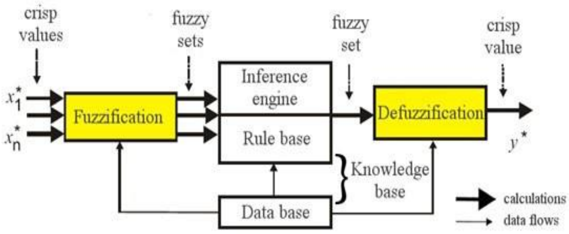

Figure 2: Fuzzy system.

In the fuzzification stage, the crisp values become fuzzy values and the membership functions are defined for the inputs and outputs. Among different kinds of membership functions such as triangular, trapezoidal, Z-shaped, Gaussian, sigmoidal, S-shaped whose graphs are shown in Fig. 3 we use triangular and trapezoidal membership functions defined as the following:

Triangular fuzzy number (TFM): The membership function of a TFM $\mu_A(x)$ has the following form:

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x < a_2, \\ \frac{a_3-x}{a_3-a_2}, & a_2 < x < a_3, \\ 0, & x > a_3. \end{cases}$$

where $a_1, a_2, a_3 \in R$ and $(x, \mu_A(x)) \in A$.

Trapezoidal fuzzy number (TRFM): The membership function of a TRFM $\mu_A(x)$ has the following form:

$$\mu_A(x) = \begin{cases} 0, & x < a_1, \\ \frac{x-a_1}{a_2-a_1}, & a_1 < x < a_2, \\ 1\ , & a_2 < x < a_3\ , \\ \frac{a_4-x}{a_4-a_3}, & a_3 < x < a_4, \\ 0, & x > a_4. \end{cases}$$

where $a_1, a_2, a_3, a_4 \in R$ and $(x, \mu_A(x)) \in A$.

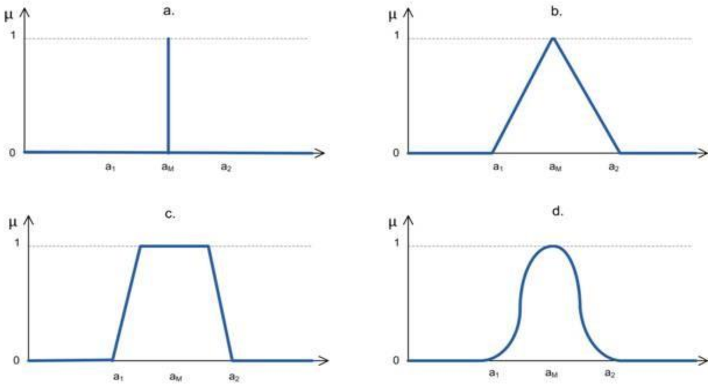

Figure 3: The membership functions.

For the inference engine, the linguistic rules ‘’if …then… ‘’ which are commands of the system behavior are set up. These rules determine the relations between inputs and outputs. Accordingly, it takes minimum values for inputs fuzzy sets.

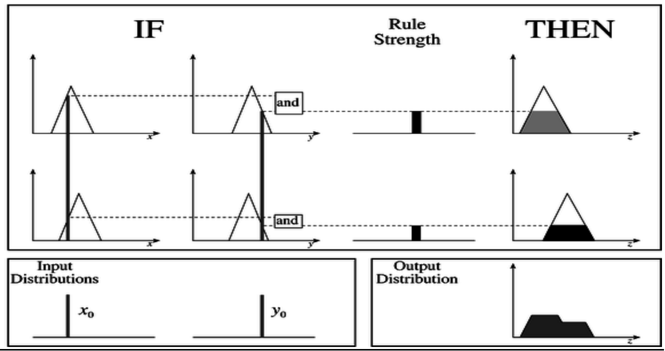

Figure 4: Inference engine.

Later the value obtained by the union of the fuzzy output set is found. At the last stage in defuzzification, fuzzy values are transformed into crisp values. At this stage we have fuzzy sets for output but we need crisp numbers instead. So we will use defuzzification. There are different types of defuzzification methods such as: centroid, bisector, middle of maximum (MOM), smallest of maximum (SOM), and largest of maximum (LOM). We prefer to use centroid technique in this study. Centroid of gravity method returns the output by calculating the centroid of area formed by the aggregated fuzzy sets of the consequents.

Results (Application of Fuzzy Logic)

The parameters Attitude Towards Teacher (ATT) and Attitude Towards Lesson (ATL) effect the Academic Achievement (AA). As a result of application of attitude scales to the students and correspond the mathematics grades of students, the data collection is obtained. The obtained data collection is presented in Table 1.

Table 1:*The obtained data collection*

No	ATL	ATT	AA
1	30	78	50
2	29	88	86
3	62	83	71
4	54	59	95
5	46	98	78
6	25	79	52
7	28	82	57
8	38	96	60
9	46	82	82
10	22	72	58
11	44	90	75
12	64	72	65
13	33	86	54
14	39	90	27
15	33	87	53
16	51	71	31
17	41	89	34
18	91	93	93
19	76	93	87
20	62	89	85
21	82	86	66
22	41	75	30
23	61	91	75
24	54	75	44
25	32	84	25
26	55	85	51
27	41	60	31
28	83	94	58
29	33	79	30
30	54	92	65
31	67	93	28
32	44	78	40
33	61	83	26
34	32	71	50
35	22	71	38
36	87	99	68
37	50	94	43
38	54	87	45
39	56	57	20
40	64	86	40
41	58	90	53
42	45	75	43
43	23	57	42
44	51	55	30

To set up a model, we build a fuzzy logic model whose input variables are ATT and ATL and the output variable is AA (academic achievement). The general structure of the model is shown in Fig. 5.

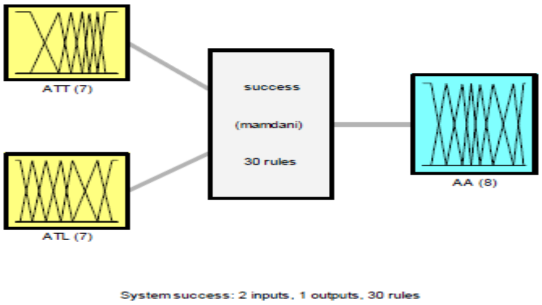

Figure 5: System success: 2 inputs, 1 outputs, 30 rules

Fuzzy logic shows experience and preference through its membership functions. Scores that can be obtained from ATT are classified into seven categories. The scores between 50 and 71 are categorized as 1, the scores between 57 and 77 are categorized as 2, the scores between 71 and 82.5 are categorized as 3, the scores between 77 and 86 are categorized as 4, the scores between 82.5 and 90 are categorized as 5, the scores between 86 and 94 are categorized as 6 and the scores between 90 and 100 are categorized as 7. The results are shown in Fig. 6:

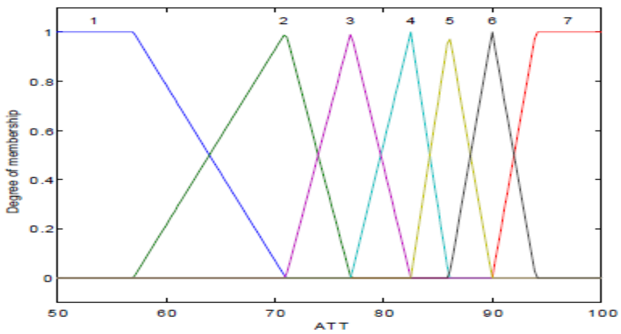

Figure 6: ATT (Attitude Towards the Teacher)

ATL are also classified into seven categories. The scores between 20 and 32 are categorized as 1, the scores between 23 and 42 are categorized as 2, the scores between 32 and 53 are categorized as 3, the scores between 42 and 61.5 are categorized as 4, the scores between 53 and 80 are categorized as 5, the scores between 61.5 and 91 are categorized as 6 and the scores between 80 and 95 are categorized as 7. These categories were mathematically modeled by fuzzy sets, as shown in Fig. 7:

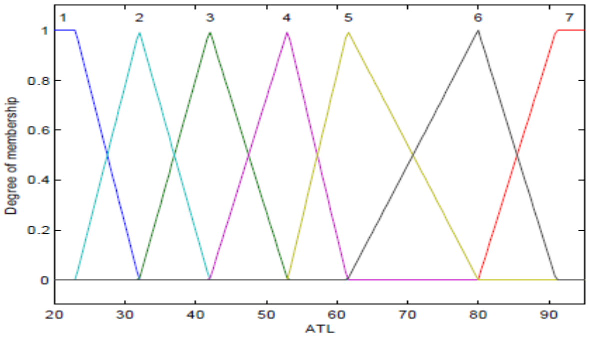

Figure 7: ATL (Attitude Towards the Lesson)

The output of academic achievements are classified into eight categories. The scores between 25 and 41 are categorized as 1, the scores between 30.5 and 52 are categorized as 2, the scores between 41 and 58 are categorized as 3, the scores between 52 and 64 are categorized as 4, the scores between 58 and 73 are categorized as 5, the scores between 64 and 87 are categorized as 6, the scores between 73 and 94 are categorized as 7 and the scores between 87 and 95 are categorized as 8,. These categories are mathematically modeled by fuzzy sets, as shown in Fig. 8:

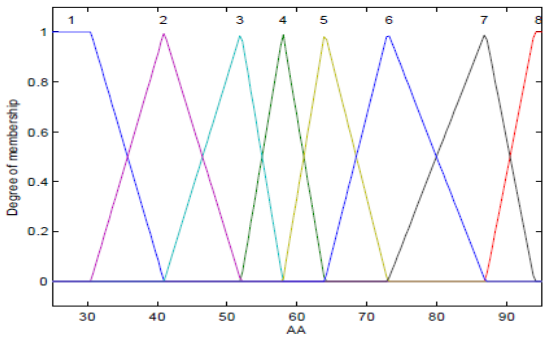

Figure 8: AA (Academic Achievement)

The rule structure of the model is designed based on how the experts interpret the characteristics of the variables of the system. It is possible to write down a lot of if-then statements. Fuzzy rules and some of the rules used in the model are as the following:

If ATT is 2 and ATL is 2 then AA is 3

If ATT is 2 and ATL is 1 then AA is 2

If ATT is 3 and ATL is 3 then AA is 2

If ATT is 1 and ATL is 1 then AA is 2

If ATT is 1 and ATL is 4 then AA is 1

If ATT is 3 and ATL is 3 then AA is 1

If ATT is 6 and ATL is 4 then AA is 5

After simulating the Mamdani method, based on the constructed model, the surfaces for academic achievement is plotted in Fig. 9.

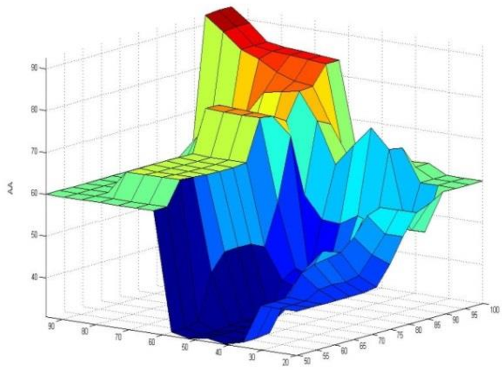

Figure 9: Surface view of the system

The correlation coeffient R^2 obtained using regression analysis shows that the experimental results and fuzzy logic results for the academic achievement are close to each other at the rate of 94%. It is shown in Fig. 10.

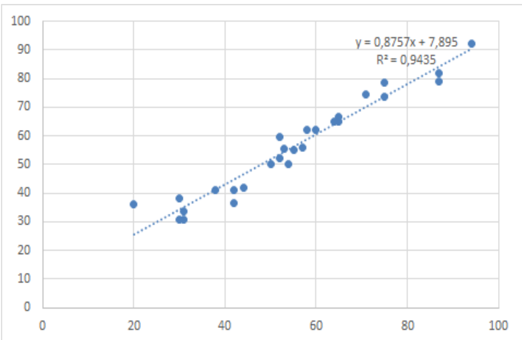

Figure 10: The correlation coefficient

Discussion

In this study fuzzy logic has been used and the effect of students' attitude toward both the teacher and the mathematics has been analyzed by means of attitude scales. The data is composed by measuring the level of the attitude towards the teachers and the lesson, by considering the averages of grades at the end of the first term as academic success. The attitudes towards the lesson and the teacher have been assumed as an input and the success as for output factor. By using fuzzy logic between the input and the output factors, a model has been generated. The aim of this model is to state the relation the input and the output factors and to have an idea about the success on mathematical education. We deduce that just one positive attitude towards the lesson or the teacher can not been seen enough for success when examined the obtained grade. We also conclude that if there is not a high positive attitude to teacher and the mathematics, an advanced level success is not observed or vice versa. There should be positive attitude for both the teacher and the lesson for achievement. When there is negative attitude toward them, there will be low success. After forming, the output data has been obtained by being entered input factors. When analyzed the real values and fuzzy logic model values, it is observed that there is a close relation between real values and fuzzy logic model values. 94% correlation coefficient is found when compared real value and system value by being taken into consideration fuzzy logic model. The high level academic success can be acquired with interest for both the teacher and the lesson.

Conclusion and Recommendation

For further study; since there are many factors affecting the learning by changing input factors, any other studies can be conducted. Some different modeling techniques can be practiced for this kinds of problems and the comparison of these modeling techniques can be searched. Furthermore, proposed method can be effective in terms of precision and credibility of results for large data sets.

Acknowledgments

This study has been supported by the Scientific Research Projects unit of Suleyman Demirel University (4508-YL1-15).

References

Akbay, R. Şahiner, A. & Yılmaz, N. (2016). Determining of The Achievement of Students by Using Classical and Modern Optimization Techniques *International Journal of Physics and Chemistry Education 8(1),* 3–13.

Chai Y., Jia L. & Zhang Z., (2009). Mamdani Model based Adaptive Neural Fuzzy Inference System and its Application, *International Journal of Computational Intelligence 5(1),* 22-29.

Dede Y., Argün Z., (2003). Why do students have difficulty with algebra?, *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi, 24*, 180-185

Duatepe A., Çilesiz S., (1999). Matematik tutum ölçeğinin geliştirilmesi, *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*,16-17, 45- 52.

Dubois D., Prade H., (1978). Operations on fuzzy numbers, *International Journal of Systems Science* , 6, 613-626.

Dubois D., Prade H., (1980). *Fuzzy Sets and Systems: Theory and Applications*, New York, Academic Press.

Filloy, E.ve Rojana, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics*, *9*(2), 19-25.

Grezegorzewski, P.,Winiarska, K.P. (2014). Naturel trapezoidal approximations of fuzzy number, *Fuzzy Sets and Systems*, *250*, 90-109.

Kandel, A., (1986). *Fuzzy mathematical techniques with applications*, Massachusetts, Addison-Wesley.

Karagöz, S., Zülfikar, H., Kalaycı, T. (2014). Öğrenme sürecine ilişkin değerlendirmeler ve fuzzy karar verme tekniği ile sürece dair bir uygulama, *İstanbul Üniversitesi Sosyal bilimler Dergisi*, 2014(1), 56-71.

Mamdani, E. H., 1974, Application of fuzzy algorithms for control of simple dynamic plant, *Proceedings of IEEE, 121(12)*, 1585–1588.

Mamdani, E. H. &Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller, *International Journal of Man-Machine Studies* 7(1), 1-13.

Sakawa M., (1993). Fuzzy Sets and Interactive Multi objective optimization, New York, Plenum Press.

Saltan M., Saltan S. & Şahiner A., (2007). Fuzzy logic modelling of deflection behavior against dynamic loading in flexible pavements, *Construction and Building Materials, 21*, 1406-1414.

Savaş E., Tas S. & Duru A., (2010). Matematik'te öğrenci başarısını etkileyen faktörler, *İnönü Üniversitesi Eğitim Fakültesi Dergisi*, *11(1)*,113-132.

Sugeno M., Takagi T., (1983). Multi-dimensional Fuzzy Reasoning, *Fuzzy Sets and Systems 9(2),*313-325.

Takagi, T. , Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control, *IEEE Trans. Syst., Man and Cybernetics 15* , 116–132.

Yenilmez, K. (2010). Ortaöğretim öğrencilerinin matematik dersine yönelik umutsuzluk düzeyleri, *Hacettepe Üniversitesi Eğitim Fakültesi Dergisi*, 38, 307-317.

Wagner, S. (1981). Conservation of equation and function under transformations of variable. *Journal For Research in Mathematics Education*, 12(2), 107-118.

Zadeh L. A. (1965). Fuzzy sets, *Information and Control, 8*, 338-353.

Zadeh L. A. (1973). Outline of a new approach to the analysis of complex systems and decision processes, *IEEE Transactions on Systems, Man and Cybernetics 3*, 28–44.

Zadeh L. A. (1975a). The concept of a linguistic variable and its application to approximate reasoning: part 1, *Information Sciences 8(1)*, 199–249.

Zadeh L. A., (1975b), The concept of a linguistic variable and its application to approximate reasoning: part 2, *Information Sciences 8(1),* 301–357.

Zadeh L. A., (1976), The concept of a linguistic variable and its application to approximate reasoning: part 3, *Information Sciences 9(2),* 43–80.

Zimmermann, H. J. (1994). *Fuzzy Set Theory-and Its Applications*. Boston, Kluwer Academic.